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Electron transport of a driven three-level system in an asymmetric double quantum dot irradiated by an external field

Y. Y. Liao and D. S. Chuu*
Department of Electrophysics, National Chiao-Tung University, Hsinchu 300, Taiwan

Y. N. Chen
Department of Physics and National Center for Theoretical Sciences, National Cheng-Kung University, Tainan 701, Taiwan
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Electron tunneling through a three-level system in an asymmetric double quantum dot irradiated by an external field is investigated. For a resonant external field, two symmetric peaks occur in the current spectrum. If the field frequency is detuned, unequal contributions from two channels lead to two asymmetric peaks with population inversion, which can be observed with increasing Rabi frequency. On the other hand, as the ground states in two dots are equal, a suppression of current occurs around the resonant frequency. In contrast, an enhanced behavior is found for the case of unequal ground states.

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I. INTRODUCTION

Due to the zero dimensionality, quantized energy levels, and stable states, transport properties of the electrons in quantum dots have been studied extensively.[1,2] With the advance of nanotechnologies, quantum dots can be laterally fabricated from a two-dimensional electron gas in a heterostructure. Together with the gate technology, the leads and single (multi-) quantum dot are able to form a quantum device. Since the features of the dots are controllable, the study of external influences on quantum dots becomes an important topic.[3,4]

Recently, quantum dot systems in the presence of time-varying external fields manifest some interesting effects ranging from photon-assisted tunneling[5] to electron pumping.[6] In a recent experiment, the transport spectroscopy has been measured in coupled double quantum dots under microwave fields.[5] The photon-assisted resonances are found due to a modulated gate voltage. The phenomenon involves the emission or absorption of a microwave photon. In addition to electron tunneling, other systems based on time-dependent influences also give rise to some promising physics and applications.[7–11]

On the theoretical side, many transport studies under driving fields were focused on two-level systems in single or double quantum dot.[12–17] The transport property is basically related to the energy difference between two levels. Although many investigations are focused on the artificial two-level systems, to the best of our knowledge, however, electron tunneling through a three-level system still receives little attention.[18] In the present work, we thus propose to study the electron tunneling through a three-level system in a double quantum dot. By applying an external field on the device, two peaks are found in the current spectrum. Besides, the positions of two peaks are varied with different strengths of the external field. A crossover from the three-level to the two-level system is further pointed out, depending on the frequency of the external field. By analyzing the contributions from the two channels in the right dot, a population inversion can be observed in the system. Furthermore, we also study the effect of field frequency on the transport in the case of various energy differences between the ground states in two dots. It is found that the current shows a suppressed or an enhanced behavior.

II. MODEL

With the advances of nanotechnologies, the double-dot (charge qubit) system in the Coulomb blockade regime can now be fabricated and measured as demonstrated in Refs. 19 and 20. The charging energies of the two quantum dots, left and right, are ~4 and ~1 meV, respectively, and the average spacing between single-particle states is ~0.5 and ~0.25 meV.[19] According to the experimental parameters, we thus consider a three-level system defined in a double quantum dot system, as shown in Fig. 1. The large charging energies enable one to reasonably assume that there is maximally one extra electron in this double-dot system.[19–22] Furthermore, since the single-particle spacing for the left and right dots is different (~0.25 meV), it is plausible to neglect the excited state in the left dot; i.e., when a continuous field (with energy close to 0.25 meV) irradiates on the system, it only gives rise to the contribution of the first excited state of

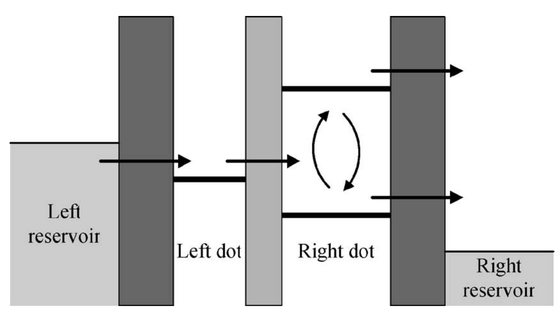

FIG. 1. Schematic view of a three-level system which consists of the ground state in the left dot and the ground state and first excited state in the right dot in a double quantum dot device. An external field irradiates on the device and leads to the transition between two states in the right dot.

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the right dot. We further estimate the transition between the ground state of the left dot and the excited state of the right dot. It is found that this contribution is much smaller than the contribution between the ground and the excited states of the right dot. In order to simplify the system, we reasonably concentrate on the transition between two states in the right dot. For the tunneling between two dots, the electron is only allowed to tunnel between two ground states. In this case, the effective Hilbert space of the electronic system can be defined by four states: empty (no electron exists in both dots), left (one electron in the ground state of left dot), right (one electron in the ground state of right dot), and excited states (one electron in the excited state of right dot), corresponding to $|0\rangle=|N_L,N_R,N_E\rangle$, $|L\rangle=|N_L+1,N_R,N_E\rangle$, $|R\rangle=|N_L,N_R+1,N_E\rangle$, and $|E\rangle=|N_L,N_R,N_E+1\rangle$, respectively. The total Hamiltonian of the system is

$$H=H_{res}+H_{dot}+H_V+H_T+H_{ep}. \tag{1}$$

The first term describes the electron reservoir contributions,

$$H_{res}=\sum_{k\in L}\varepsilon_k^L c_k^\dagger c_k+\sum_{k\in R}\varepsilon_k^R d_k^\dagger d_k, \tag{2}$$

where c_k ($c_k^\dagger$) is the annihilation (creation) operator in the left lead (L) with wave vector k and d_k ($d_k^\dagger$) is the annihilation (creation) operator for the right lead (R). The term H_{dot} describes the contributions of three states in the double-dot system,

$$H_{dot}=\varepsilon_L\hat{n}_L+\varepsilon_R\hat{n}_R+\varepsilon_E\hat{n}_E, \tag{3}$$

where the energy levels ε_L, ε_R, and ε_E represent the ground state in the left dot, the ground state, and the first excited state in the right dot, respectively. The operators of the three states are given by $\hat{n}_L=|L\rangle\langle L|$, $\hat{n}_R=|R\rangle\langle R|$, and $\hat{n}_E=|E\rangle\langle E|$. The dot-lead coupling can be written as

$$H_V=\sum_k V_k^L c_k^\dagger\hat{s}_L+\sum_k V_k^R d_k^\dagger\hat{s}_R+\sum_k V_k^E d_k^\dagger\hat{s}_E+\text{H.c.}, \tag{4}$$

with the operators $\hat{s}_L=|0\rangle\langle L|$, $\hat{s}_R=|0\rangle\langle R|$, and $\hat{s}_E=|0\rangle\langle E|$, and the tunneling matrix elements V_k^α for α (=L, R, and E). The term H_T describes the tunneling between the ground states in two dots,

$$H_T=T_c(\hat{P}+\hat{P}^\dagger), \tag{5}$$

where the operator $\hat{P}$ ($\hat{P}^\dagger$) is defined by $|L\rangle\langle R|$ ($|R\rangle\langle L|$) and the tunnel matrix element T_c determines the strength of the tunneling process. In the dipole and rotating-wave approximations, the last term H_{ep} which describes the interaction between electron and external field in the right dot can be expressed as

$$H_{ep}=-\frac{\gamma}{2}(\hat{Q}e^{-i\omega t}+\hat{Q}^\dagger e^{i\omega t}), \tag{6}$$

where γ is the Rabi frequency, ω is the field frequency, and the operator $\hat{Q}$ ($\hat{Q}^\dagger$) denotes $|E\rangle\langle R|$ ($|R\rangle\langle E|$). The Rabi frequency relates to the field strength and the electric dipole moment for the transition $|R\rangle\leftrightarrow|E\rangle$.[23,24]

An analytical expression for the stationary current can be solved from the master equation.[17] One can obtain an equation of motion for the time-dependent expectation values of the operators $\hat{n}_L$, $\hat{n}_R$, $\hat{n}_E$, $\hat{P}$, and $\hat{Q}$. After the Laplace transformation (e.g., $n_L(z)=\int_0^\infty dt\, e^{-zt}\langle\hat{n}_L\rangle_t$), the corresponding equations can be written as

$$n_L(z)=-i\frac{T_c}{z+\Gamma_L}[P(z)-P^\dagger(z)]+\frac{\Gamma_L}{z+\Gamma_L}[1/z-n_R(z)-n_E(z)],$$

$$n_R(z)=i\frac{T_c}{z+\Gamma_R}[P(z)-P^\dagger(z)]+i\frac{\gamma}{2(z+\Gamma_R)}\times[Q(z+i\omega)-Q^\dagger(z-i\omega)],$$

$$n_E(z)=-i\frac{\gamma}{2(z+\Gamma_R)}[Q(z+i\omega)-Q^\dagger(z-i\omega)],$$

$$P(z)=-i\frac{2T_c[2(z-i\omega+i\Delta L)+\Gamma_R]}{A}[n_L(z)-n_R(z)]+\frac{2\gamma T_c}{A}Q^\dagger(z-i\omega),$$

$$Q(z)=-i\frac{\gamma[2(z-i\Delta L)+\Gamma_R]}{2B}[n_E(z-i\omega)-n_R(z-i\omega)]+\frac{\gamma T_c}{B}P^\dagger(z-i\omega), \tag{7}$$

where

$$A=[2(z-i\omega+i\Delta L)+\Gamma_R][2(z-i\Delta\varepsilon)+\Gamma_R]+\gamma^2,$$

$$B=[2(z-i\Delta L)+\Gamma_R][z-i\Delta R+\Gamma_R]+2T_c^2,$$

with the parameters $\Delta\varepsilon=\varepsilon_L-\varepsilon_R$, $\Delta L=\varepsilon_E-\varepsilon_L$, and $\Delta R=\varepsilon_E-\varepsilon_R$, respectively. The tunneling rates between the reservoirs and dots are assumed to be energy independent: $\Gamma_\alpha=2\pi\Sigma_k|V_k^\alpha|^2\delta(\varepsilon_\alpha-\varepsilon_k^{L/R})$ with α (=L, R, and E). Note that, according to the calculations, some particular relations between the states are attributed to the influence of the external field. For example, it is found that a collective effect on the right dot can be generated in the presence of the driving field. This gives rise to a contribution of the transition between the ground state of the left dot and the excited state of the right dot. These contributions are further incorporated into Eq. (7). We can solve Equation (7) algebraically and subsequently obtain the stationary current (in units of e) from the tunneling between two dots,

$$I=iT_c(P-P^\dagger)_{t\to\infty}. \tag{8}$$

To simplify the parameters of the system, the tunneling rates are assumed to be identical ($\Gamma_L=\Gamma_R=\Gamma_E$). According to Refs. 19 and 20, the tunneling rate (Γ), tunneling coupling (T_c), and energy spacing ($\varepsilon_E-\varepsilon_R$) in this work are set equal to the values of 1 μeV, 1 μeV, and 0.25 meV, respectively.

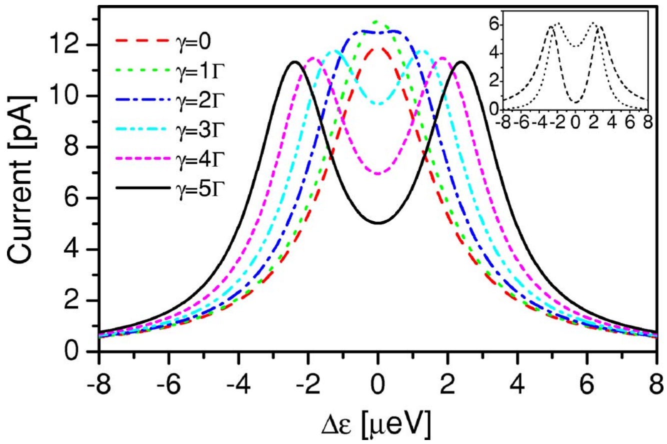

FIG. 2. (Color online) Current as a function of energy difference $\Delta\varepsilon$ between two ground states for different Rabi frequencies. The inset shows the currents I_R (dashed curve) and I_E (dotted curve) for $\gamma=5\Gamma$.

III. RESULTS AND DISCUSSIONS

We first consider that the field frequency is in resonance ($\Delta\omega=\omega-\Delta R=0$) and apply the external voltages to vary the energy difference between the ground states.[19–22] The current can be written as

$$I=\frac{4T_c^2\Gamma}{12T_c^2+4(\Delta\varepsilon)^2+\Gamma^2+f(\gamma,\Delta\varepsilon)}, \tag{9}$$

where $f(\gamma,\Delta\varepsilon)$ is directly dependent on the Rabi frequency. For a two-level system ($\gamma=0$), the current is

$$I=\frac{4T_c^2\Gamma}{12T_c^2+4(\Delta\varepsilon)^2+\Gamma^2} \tag{10}$$

and shows a maximum response at $\Delta\varepsilon=0$ (see Fig. 2).[15] However, as a resonant field is applied to the device, the current shows an interesting behavior. With the increase of the field strengths, two symmetric peaks occur in the current spectrum. Compared to the case of $\gamma=0$, the maximum current does not locate at the point ($\Delta\varepsilon=0$). The positions of the peaks are mainly dependent on the related parameters such as the tunneling rates, tunnel coupling, and Rabi frequency. For the case of strong field, the Rabi frequency becomes dominant. Accordingly, it is found that the interval between the two peaks is roughly equal to the value of the Rabi frequency.

We also analyze the components of the current in this device. For the right dot, the ground and first excited states can contribute to the transport, as shown in Fig. 1. In the stationary case, Eq. (8) is equivalent to the contributions of two states in the right dot. The current can be rewritten as

$$I=I_R+I_E, \tag{11}$$

$$I_R=\Gamma n_R, \tag{12}$$

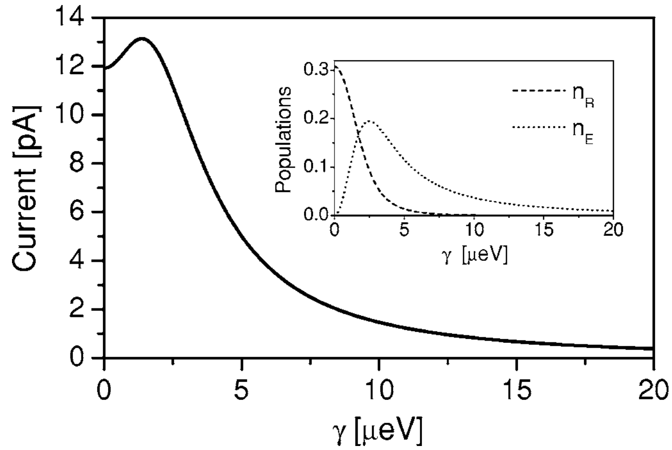

FIG. 3. Dependence of the current on Rabi frequency, corresponding to the populations n_R and n_E (inset). The conditions are fixed to be $\Delta\varepsilon=0$ and $\Delta\omega=0$.

$$I_E=\Gamma n_E, \tag{13}$$

where n_R and n_E are the populations in the right dot. From the inset in Fig. 2, we find that electron tunneling through two channels behaves similarly and contributes equally to the current.

In order to study the influence of the external field on the transport, in Fig. 3 we illustrate the curve of the Rabi-frequency-dependent current. For simplicity, the conditions are chosen to be $\Delta\varepsilon=0$ and $\Delta\omega=0$. The current can be written as

$$I=\frac{4T_c^2\Gamma}{12T_c^2+\Gamma^2+f(\gamma)}, \tag{14}$$

$$f(\gamma)=\gamma^2\frac{\gamma^2-6T_c^2+\Gamma^2}{\gamma^2+2T_c^2+\Gamma^2}. \tag{15}$$

If the Rabi frequency γ is zero, the current is $4T_c^2\Gamma/(12T_c^2+\Gamma^2)$. As the frequency γ increases, a crossover from enhanced behavior to suppressed behavior in the transport spectrum is found due to the competition among the Rabi frequency γ, tunneling coupling T_c, and tunneling rate Γ [see Eq. (15)]. In particular, it is found that a *population inversion* is observed in the inset, approximately corresponding to the tendency of the current. To analyze the properties of the device in detail, we simplify the parameters by letting $T_c=\Gamma$ and the populations of two states in the right dot can be subsequently taken as

$$n_R=\frac{12\Gamma^4}{\gamma^4+8\gamma^2\Gamma^2+39\Gamma^4}, \tag{16}$$

$$n_E=\frac{4\gamma^2\Gamma^2}{\gamma^4+8\gamma^2\Gamma^2+39\Gamma^4}. \tag{17}$$

In the limit of $\gamma\rightarrow 0$, the population in the ground state mainly contributes to the current. However, as one increases γ, there exists a crossing point when *population inversion* occurs, as can be seen from the inset of Fig. 3. This is be-

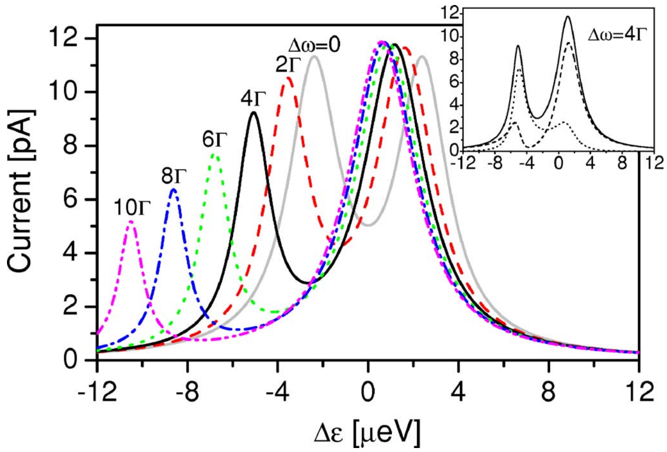

FIG. 4. (Color online) Current as a function of energy difference $\Delta\varepsilon$ for different nonresonant fields ($\Delta\omega$) and fixed Rabi frequency ($\gamma=5\Gamma$). The inset shows that the total current I (solid curve) is composed of two channels in the right dot: the electron tunneling out through the ground level I_R (dashed curve) and first excited level I_E (dotted curve) for $\Delta\omega=4\Gamma$.

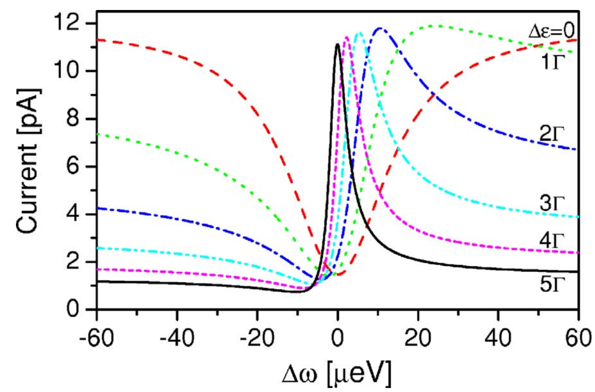

FIG. 5. (Color online) Current as a function of frequency difference $\Delta\omega$ ($=\omega-\Delta R$) for different energy differences $\Delta\varepsilon$ between two ground states with fixed $\gamma=10\Gamma$.

cause the population of the excited states first increases and then decreases with a rate of γ^{-2} [Eq. (17)], which is slower than γ^{-4} for the population in ground state [Eq. (16)].

Instead of studying the strength of the field, let us now turn our attention to the detuned field frequency. In Fig. 4, the current shows two asymmetric peaks. Though this may be similar to the result of Ref. 25, our system with the reservoirs shows some different consequences. If the detuning is further increased, the main (larger) peak is close to the value $\Delta\varepsilon=0$, while another one is far away from the main peak and deeply suppressed. This reflects a crossover from a three-level to a two-level system. In particular, as can be seen in the inset, the current (solid curve) consists of two components I_R (dashed curve) and I_E (dotted curve), and both components contribute different degrees to the peaks of current. The main (secondary) peak mainly results from the contribution of current I_R (I_E), respectively, i.e., the populations behave inversely. The important feature is a large asymmetry in the magnitudes of the populations n_R and n_E contributed to the current peaks, e.g., 4:1 for the main one. To describe the appearance, one can expect that the external field establishes a particular relationship among the states. The distributions of populations are sensitive to the related parameters in the double-dot system. Under the condition $\Delta\omega\neq 0$, compared to the symmetric current (Fig. 2), the electron transferred among these states shows an unbalanced behavior such that two channels in the right dot unequally contribute to the current peaks.

Figure 5 shows the dependence of the current on the detuning for various energy differences between two ground states. At $\Delta\varepsilon=0$, a symmetric and antiresonant behavior appears in the transport spectrum (red dashed curve). The current is greatly suppressed and the maximum response is located at resonant frequency ($\Delta\omega=0$). By altering the field frequencies, the ratio of the maximum to minimum current is about 8:1. This indicates that the operation of the external field can drastically influence the transport. When the energy difference is further increased, it is found that an asymmetric and enhanced current occurs. In addition, the profile is squeezed and the location of maximum current is no longer fixed. This can be understood from the splitting and shifting of energy states modified by strong driving field in the right dot. For example, when $\Delta\varepsilon=5\Gamma$ (black solid curve), the maximum current is approximately located at the resonant frequency ($\Delta\omega=0$), and the energy shift from the external field (Rabi splitting) is roughly equal to the value of 5Γ. Therefore, one can utilize a suitable field to effectively control the transport, such as a drastic drop (enhancement) in current for the case of $\Delta\varepsilon=0$ (5Γ). The highly sensitive response in the current spectrum might be very useful for the purpose of being a switch.

A few remarks about the comparison of our model with the related studies should be addressed here. For population inversion, the degree of the inversion in our work is manipulated by the strength of external field, while in Ref. 26 the inversion is achieved for the case of strong interdot Coulomb repulsion. As for the switching phenomenon, Chan *et al.*[27] investigated a double-dot working as a switch through strong capacitive coupling provided by a floating interdot capacitor. Furthermore, Ono *et al.*[28] showed that the Pauli spin blockade can be used to block current in a double dot. Different from the switching systems in Refs. 27 and 28, our results suggest that the three-level system driven by an external field can advantageously play the role of switching in electron transport with a flexible characteristic via the control of the external field.

In order to realize our model, we suggest the experimental system studied by Fujisawa *et al.*[19] An important requirement is the asymmetric double-dot structure. Although the work of Fujisawa *et al.* concentrated on two-level system, it is easily generalized to a three-level case with an adequate external field,[29,30] i.e., an additional channel is opened to contribute to the transport. Based on the successful studies and advanced nanofabrication technologies, we believe our model can be experimentally verified under proper arrangements.

IV. CONCLUSION

The transport of a three-level system under the influence of the external field is studied. When a resonant field irradiates on the double-dot device, the current shows two symmetric peaks. For the case of strong field, the Rabi frequency dominates the interval between two peaks. In contrast, two asymmetric peaks display in a nonresonant field. When the detuning is increased, we find a crossover from the three-level to the two-level system. The population inversion can be observed by varying the frequency or strength of the external field. Moreover, we also study the frequency-dependent current by modulating the energy difference between the ground states. It is clearly shown that a suppressed (enhanced) behavior occurs due to the interplay among the states and external field.

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*Electronic address: dschuu@mail.nctu.edu.tw

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