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Elastic constants identification of symmetric angle-ply laminates via a two-level optimization approach

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Abstract

A two-level optimization method for elastic constants identification of symmetric angle-ply laminates is presented. Measured axial and lateral strains of two symmetric angle-ply laminates with different fiber angles are used in the proposed method to identify four elastic constants of the composite laminates. In the first-level optimization process, the theoretically and experimentally predicted axial and lateral strains of a $[(45°/-45°)_2]_s$ laminate are used to construct the error function which is a measure of the differences between the experimental and theoretical predictions of the axial and lateral strains. The identification of the material constants is then formulated as a constrained minimization problem in which the best estimates of the shear modulus and Poisson's ratio of the laminate are determined by making the error function a global minimum. The problem of this level of optimization is then solved using a multi-start global minimization algorithm. In the second-level optimization process, the shear modulus and Poisson's ratio determined in the previous level of optimization are kept constant while the Young's moduli of the second angle-ply laminate with fiber angles other than 45° are identified using the same minimization technique that has been used in the previous level. The accuracy of the proposed method are studied by means of a number of numerical examples on the material constants identification of symmetric angle-ply laminates made of different composite materials. Finally, static tensile tests of $[(45°/-45°)_2]_s$ and $[(30°/-30°)_2]_s$ laminates made of Gr/ep composite material are performed to measure the strains of the laminates. The experimental data are then used to identify the elastic constants of the laminates. The excellent results obtained in the experimental investigation have demonstrated the feasibility and applications of the proposed method.

Keywords: A. Angle-ply laminate; B. Elastic constants; Strain analysis; C. Identification; Constrained minimization problem

1. Introduction

The use of correct material elastic constants in analyzing a structure is very important if a realistic prediction of the mechanical behavior of the structure is desired. In general, the determination of the material elastic constants can be accomplished by performing material testing of a number of specimens. For instance, according to the ASTM [1] specifications, the four elastic constants (E_1, E_2, G_{12}, and ν_{12}) of composite materials can be determined by testing in laboratory three types of standard laminated composite specimens with different fiber angles, namely, 0°, 90°, and ±45°. As well known, the preparation and testing of the standard specimens are quite tedious and time consuming processes. To facilitate the material constants verification process during the fabrication of a composite structure, it is always desirable to have a simple yet effective method that can determine the correct material elastic constants in an efficient way. In recent years, the identification of material constants of structural components has drawn close attention and many different identification techniques have thus been proposed. For instance, a number of nondestructive evaluation techniques have been proposed for the determination of material properties of laminated composite parts [2]. A number of researchers used 12–16 experimental eigenfrequencies to identify elastic constants of laminated composites [3–6]. Shin and Pande [7] developed

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a two-step method to identify the elastic constants of orthotropic materials. In their method, the monitored data of a structure are used in the first step to recursively train a neural network based constitutive model embedded in a finite element code and in the second step, the required material parameters are computed from the trained neural network based constitutive model. Marin et al. [8] used the boundary element method and the strain/displacement measurements on the boundary to identify the elastic constants of isotropic materials. Wang and Kam [9] proposed a constrained minimization method to identify five material constants of shear deformable laminated composite plates using measured strains and/or displacements. Grédiac et al. [10] proposed a method constructed on the basis of the principal of virtual work to identify the material stiffness coefficients of an orthotropic laminate using measured deformational data of the laminate.

In this paper, the determination of elastic constants of composite materials is treated as an inverse problem, which is solved using a two-level optimization method. In the proposed two-level optimization method, axial and lateral strains of two symmetric angle-ply laminates with different fiber angles are measured in the static tensile tests of the laminates. An error function is established at each level of optimization to measure the sum of the differences between the experimental and theoretical predications of the axial and lateral strains of the two symmetric angle-ply laminates. A multi-start global minimization technique is then used to identify material constants G_{12} and ν_{12} by minimizing the error function of the first-level of optimization and material constants E_1 and E_2 by minimizing the error function of the second-level of optimization. The accuracy and feasibility of the proposed method are demonstrated by means of several numerical examples on the elastic constants identification of composite laminates made of different materials. Experimental investigation is performed to illustrate the applications of the proposed method.

2. Composite laminate analysis

Consider the symmetric angle-ply laminate of size $a \times b \times h$ subjected to in-plane axial stress resultant N_x in x-direction as shown in Fig. 1. The relations between the stress resultant and strains expressed in the matrix form are [11]

$$\begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

where ε_x, ε_y and γ_{xy} are axial, lateral, and shear strains, respectively.

The laminate in-plane stiffness coefficients are

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(m)} \, \mathrm{d}z \quad (i,j = x,y,s) \tag{2}$$

where h is laminate thickness ; $\overline{Q}_{ij}^{(m)}$ $(i,j = x,y,s)$ are the transformed lamina stiffness coefficients of the mth layer with fiber angle θ_m. For an orthotropic lamina, the lamina stiffness coefficients are expressed as

$$\underline{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{3}$$

with

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \\ Q_{12} &= Q_{21} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \end{aligned} \tag{4}$$

where E_1, E_2 are Young's moduli in fiber and transverse directions, respectively; ν_{ij} is Poisson's ratio for transverse strain in the j-direction when stressed in the i-direction; G_{12} is shear modulus in the 1–2 plane. The relations between $\overline{Q}_{ij}$ and Q_{ij} can be found in the literature [11].

The inversion of Eq. (1) gives

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & 0 \\ a_{xy} & a_{yy} & 0 \\ 0 & 0 & a_{ss} \end{bmatrix} \begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

where a_{ij} $(i,j = x,y,s)$ are in-plane compliance coefficients which are functions of the elastic constants (E_1, E_2, G_{12}, ν_{12}) and the layer fiber angles of the laminate. Hence, from the above equation, the axial and lateral strains can be obtained, respectively, as

$$\varepsilon_x = a_{xx} N_x \tag{6}$$

and

$$\varepsilon_y = a_{xy} N_x \tag{7}$$

It is noted that the strains (ε_x, ε_y) can be determined directly from the above equations provided that the elastic constants of the composite laminate are available. In many cases, however, strains are usually available while elastic constants are unknown. The purpose of this paper is thus to determine the elastic constants using the measured strains of the laminate. In tackling the problem of elastic constants identification, it is noted that the four elastic con-

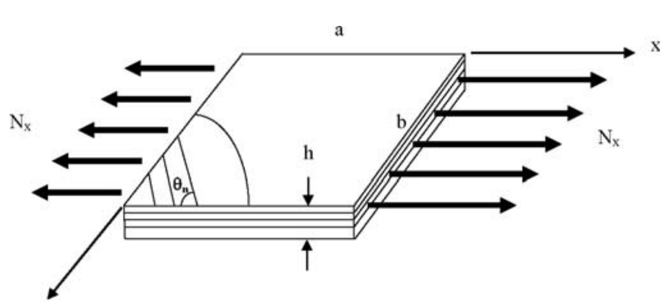

Fig. 1. Geometry and loading condition of composite laminate.

stants of an angle-ply laminate cannot be determined directly if merely two strain components of the laminate are used in solving Eqs. (6) and (7) simultaneously. To overcome this difficulty, herein two different symmetric angle-ply laminates with four measured strains are used in a two-level optimization method to solve the inverse problem.

3. Identification of material constants

The problem of material constants identification of symmetric angle-ply laminates is formulated as a minimization problem. Two pairs of axial and lateral strains measured from two symmetric angle-ply laminates subjected to tensile testing are used to identify four elastic constants of the laminates in the minimization problem. The minimization problem of elastic constants identification is expressed as

$$\begin{aligned} &\text{minimize} \quad e(\underline{x}) = \sum_{i=1}^{2}[(\varepsilon_{xi}^{*} - \varepsilon_{xi})^2 + (\varepsilon_{yi}^{*} - \varepsilon_{yi})^2] \\ &\text{subject to} \quad \underline{x}^{\mathrm{L}} \leqslant \underline{x} \leqslant \underline{x}^{\mathrm{U}} \end{aligned} \tag{8}$$

where $e(\underline{x})$ is the error function measuring the differences between the predicted and measured strains; $\underline{x} = [E_1, E_2, G_{12}, \nu_{12}]$, the estimates of the elastic constants; ε_{xi}^{*}, ε_{yi}^{*} are respectively the measured axial and lateral strains of the ith laminate; ε_{xi}, ε_{yi} are respectively the predicted axial and lateral strains of the ith laminate; $\underline{x}^{\mathrm{L}}$, $\underline{x}^{\mathrm{U}}$ are respectively the lower and upper bounds of the elastic constants. The above minimization problem is hereafter termed as the one level optimization problem. It is noted that the direct solution of the one level optimization problem using any of the conventional optimization techniques may encounter great difficulty in producing acceptable results. A detailed sensitivity study has shown that the solution of the one level optimization problem is very sensitive to the variations of the strains and may produce erroneous results even when the variations of the measured strains are relatively small. For instance, if the strains are independent random variables and the coefficients of variation of the elastic constants are approximated from those of the strains using the first order second moment method [12], the calculated coefficients of variation for G_{12} and ν_{12} can exceed 50% and 270%, respectively, when the coefficients of variation of the measured strains are merely 5% for a Gr/ep composite laminate. Since the existence of noise in measurement data is inevitable, the above formulation of the material constants identification problem then becomes inappropriate if the determination of accurate elastic constants is desired. Furthermore, it worths noting that the large differences among the values of the four elastic constants make the gradients of the error function with respect to E_1, E_2 and G_{12} much larger than that respect to ν_{12}. Therefore, in searching for the solution, the search direction will be so significantly dominated by the gradients of the error function with respect to E_1, E_2 and G_{12} that the solution may have great difficulty to converge. Herein, a two-level optimization method is proposed to solve the minimization problem of equation (8). In the proposed method, the first level optimization problem is expressed as

$$\begin{aligned} &\text{minimize} \quad e(\underline{x}_1) = [(\varepsilon_{x1}^{*} - \varepsilon_{x1})^2 + (\varepsilon_{y1}^{*} - \varepsilon_{y1})^2] \cdot \xi \\ &\text{subject to} \quad x_{1i}^{\mathrm{L}} \leqslant x_{1i} \leqslant x_{1i}^{\mathrm{U}}; \quad i = 1\text{–}4 \end{aligned} \tag{9}$$

where $e(\underline{x}_1)$ is the first level error function measuring the differences between the predicted and measured strains of the first symmetric angle-ply laminate; $\underline{x}_1 = [E_1^{(1)}, E_2^{(1)}, G_{12}^{(1)}, \nu_{12}^{(1)}]$ the estimates of the elastic constants at the first level with $x_{11} = E_1^{(1)}$, $x_{12} = E_2^{(1)}$, $x_{13} = G_{12}^{(1)}$, and $x_{14} = \nu_{12}^{(1)}$; ε_{x1}^{*} and ε_{y1}^{*} are the measured axial and lateral strains of the first angle-ply laminate, respectively; x_{1i}^{L}, x_{1i}^{U} are the lower and upper bounds of the elastic constants. ε_{x1}, ε_{y1} are respectively the predicted strains determined in the strain analysis of the first angle-ply laminate using the trial values of the elastic constants; ξ is an amplification factor which is used to increase the sensitivity and avoid the occurrence of numerical under flow of the error function. A detailed numerical study has shown that for the Gr/ep and Gl/ep materials under consideration, if the magnitudes of the strains are in the range from 10^{-3} to 10^{-5}, the value of ξ is best chosen in the range from 10^5 to 10^7.

The above constrained minimization problem of Eq. (9) is first converted into an unconstrained minimization problem by creating the following general augmented Lagrangian [13]

$$\bar{\Psi}_1(\tilde{\underline{x}}_1, \underline{\mu}, \underline{\eta}, r_p) = e_1(\tilde{\underline{x}}_1) + \sum_{j=1}^{4}\left[\mu_j z_j + r_p z_j^2 + \eta_j \phi_j + r_p \phi_j^2\right] \tag{10}$$

with

$$\begin{aligned} &z_j = \max\left[g_j(\tilde{x}_{1j}), \frac{-\mu_j}{2r_p}\right] \\ &g_j(\tilde{x}_{1j}) = \tilde{x}_{1j} - \tilde{x}_{1j}^{\mathrm{U}} \leqslant 0 \\ &\phi_j = \max\left[H_j(\tilde{x}_{1j}), \frac{-\eta_j}{2r_p}\right] \\ &H_j(\tilde{x}_{1j}) = \tilde{x}_{1j}^{\mathrm{L}} - \tilde{x}_{1j} \leqslant 0; \quad j = 1\text{–}4 \end{aligned} \tag{11}$$

where μ_j, η_j, r_p are multipliers; max [*,*] takes on the maximum value of the numbers in the bracket. The modified design variables $\tilde{\underline{x}}_1$ are defined as

$$\tilde{\underline{x}}_1 = \left[\frac{E_1^{(1)}}{\alpha_1}, \frac{E_2^{(1)}}{\alpha_2}, \frac{G_{12}^{(1)}}{\alpha_3}, \frac{\nu_{12}^{(1)}}{\alpha_4}\right] \tag{12}$$

It is noted that the normalization factors α_i are used to prevent E_1, E_2, and G_{12} from dominating the search direction of the solution and make the modified design variables have appropriate contributions to the search direction. A sensitivity study has shown that the solution of the above minimization problem can have excellent convergence rate if the normalization factors are chosen in such a way that they make the modified design variables less than 10. It is noted that the modified design variables $\tilde{\underline{x}}_1$ are only used in the minimization algorithm while the original design variables $\underline{x}_1$ are used in the strain analysis of the composite laminate. The update formulas for the multipliers μ_j, η_j and r_p are

$$\begin{aligned} \mu_j^{n+1} &= \mu_j^n + 2r_p^n z_j^n \\ \eta_j^{n+1} &= \eta_j^n + 2r_p^n \phi_j^n; \quad j = 1-4 \\ r_p^{n+1} &= \begin{cases} \gamma_0 r_p^n & \text{if} \quad r_p^{n+1} < r_p^{\max} \\ r_p^{\max} & \text{if} \quad r_p^{n+1} \geqslant r_p^{\max} \end{cases} \end{aligned} \tag{13}$$

where the superscript n denotes iteration number; γ_0 is a constant; $r_p^{\max}$ is the maximum value of r_p. Following the guideline given in the literature [13], the parameters μ_j^0, η_j^0, r_p^0, γ_0 and $r_p^{\max}$ are chosen as

$$\begin{aligned} &\mu_j^0 = 1.0, \quad \eta_j^0 = 1.0, \quad j = 1-4 \\ &\gamma_0 = 2.5, \quad r_p^{\max} = 100, \quad r_p^0 = 0.4 \end{aligned} \tag{14}$$

The constrained minimization problem of Eq. (9) has thus become the solution of the following unconstrained optimization problem.

$$\text{Minimize} \quad \underline{\bar{\Psi}}\left(\tilde{\underline{x}}_1, \underline{\mu}, \underline{\eta}, r_p\right) \tag{15}$$

The solution of the above unconstrained optimization problem is straightforward by using the previously proposed unconstrained multi-start stochastic global optimization algorithm. In the adopted optimization algorithm, the objective function of Eq. (10) is treated as the potential energy of a traveling particle and the search trajectories for locating the global minimum are derived from the equation of motion of the particle in a conservative force field [14]. The design variables, i.e., elastic constants that make the potential energy of the particle, i.e., objective function, the global minimum constitute the solution of the problem. In the minimization process, the side constrains in Eq. (9) are observed and a series of starting points for the design variables of Eq. (12) are selected at random from the region of interest. The lowest local minimum along the search trajectory initiated from each starting point is determined and recorded. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved. The multi-start optimization procedure is terminated once the condition that a target probability, typically 0.995, has been exceeded is satisfied. The estimates of elastic constants G_{12} and v_{12} determined at this level of minimization are treated as the true values and thereafter kept constant in the second level minimization problem which is expressed as

$$\begin{aligned} &\text{minimize} \quad e(\underline{x}_2) = [(\varepsilon_{x2}^* - \varepsilon_{x2})^2 + (\varepsilon_{y2}^* - \varepsilon_{y2})^2] \cdot \xi \\ &\text{subject to} \quad x_{2i}^{\mathrm{L}} \leqslant x_{2i} \leqslant x_{2i}^{\mathrm{U}}; \quad i = 1-2 \end{aligned} \tag{16}$$

where $\underline{x}_2 = [E_1^{(2)}, E_2^{(2)}]$, the estimates of elastic constants E_1 and E_2 at the second level with $x_{21} = E_1^{(2)}$ and $x_{22} = E_2^{(2)}$; ε_{x2}^* and ε_{y2}^* are the measured axial and lateral strains of the second angle-ply laminate, respectively; ε_{x2}, ε_{y2} are respectively the predicted axial and lateral strains determined in the strain analysis of the second laminate using the trial values of E_1 and E_2. Again the above second level minimization problem is solved using the same optimization technique as described in the first level minimization problem. For comparison purpose, an error analysis of the present two-level optimization method has also been performed using the same data of the Gr/ep composite laminate studied in the error analysis of the one level minimization problem of Eq. (8). It has been shown that the calculated coefficients of variation for G_{12} and v_{12} are less than 3.66% and 0.14% when those of the strains are also 5%. It is obvious that the errors produced by the present two-level problem are much less than those produced by the one-level problem of Eq. (8). Finally, it worths pointing out that the use of 0° and 90° specimens in the present two-level identification method can only determine E_1, E_2 and v_{12} but not G_{12}.

4. Experimental investigation

Several symmetric angle-ply laminates, with layups of $[(30°/-30°)_2]_s$ and $[(45°/-45°)_2]_s$ were fabricated and used in the experimental study of the material constants identification of the composite laminates. The laminates made of graphite/epoxy (Gr/ep) prepreg tapes supplied by Toray Co., Japan were composed of same number of Gr/ep laminae. The material constants of the Gr/ep lamina were first determined using three types of the standard specimens in accordance with the ASTM standards of D3039 and D3518 [1] and their average values and coefficients of variation are given as follows:

$$\begin{aligned} &E_1 = 146.5\ \text{GPa}\ (0.7\%) \\ &E_2 = 9.22\ \text{GPa}\ (1.2\%) \\ &G_{12} = 6.84\ \text{GPa}\ (3.2\%) \\ &v_{12} = 0.3\ (0.19\%) \end{aligned} \tag{17}$$

In the above equation, the values in the parentheses denote the coefficients of variation (COV). The average values in the above equation are treated as the actual values of the composite material.

For the present experimental investigation, the dimensions of the symmetric laminates are shown in Fig. 2. The laminates comprised 8 laminae in which the thickness of each lamina was 0.125 mm. It worths noting that the actual strains of the angle-ply laminates can be determined by substituting the material constants of Eq. (17) into Eq. (5). The actual strains of the angle-ply laminates subjected to $F = 0.5$ kN are tabulated in Table 1.

The angle-ply laminates were then subjected to tensile tests in which two strain gages were used to measure the axial and lateral strains at the mid-span of each of the lam-

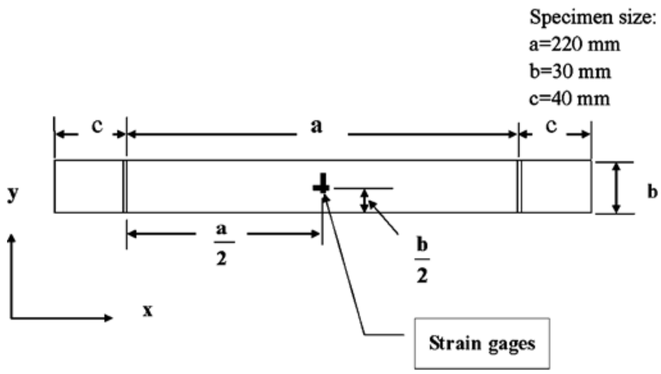

Fig. 2. Dimensions of symmetric angle-ply laminate subject to tensile test.

Table 1
Actual strains of the Gr/ep $[(\theta°/-\theta°)_2]_s$ laminates subjected to $F = 0.5$ kN

Fiber angle θ	Strain	
	$\varepsilon_x(10^{-4})$	$\varepsilon_y(10^{-4})$
$[(15°/-15°)_2]_s$	1.388	−1.352
$[(30°/-30°)_2]_s$	2.930	−3.695
$[(45°/-45°)_2]_s$	7.119	−5.064
$[(60°/-60°)_2]_s$	12.75	−3.695

inates. The strain gages produced by KYOWA, Japan had 3 mm gage length and 2.07 ± 1.0% gage factor. The tensile tests of the laminates were performed using the MTS (Type 810) testing machine with test speed in the range of 0.005–0.01 mm/s. In the tensile testing of the symmetric angle-ply laminates with different layups, three specimens of same layup were tested and the load-strain relations of the specimens were constructed to produce the strain statistics for the identification of material constants. For instance, Figs. 3 and 4 show the typical load-strain curves for the $[(45°/-45°)_2]_s$ and $[(30°/-30°)_2]_s$ laminates, respectively. Table 2 lists the measured strain pairs (axial and lateral strains) and their COV and average values of the angle−ply laminates. It is noted that the COV of the measured strains are less than or equal to 1.8%. The differences between the actual and average values of the measured strains are less than or equal to 2.7%. The experimental strains will then be used in the present method to identify the material constants of the composite laminates.

5. Results and discussion

The aforementioned two-level optimization method will be applied to the material characterization of symmetric angle-ply laminates. Chosen to be reasonably large for commonly used fiber-reinforced composite materials, the upper and lower bounds of the material constants are expressed as

$$\begin{aligned} &0 < E_1 < 1000 \text{ GPa} \\ &0 < E_2 < 50 \text{ GPa} \\ &0 < G_{12} < 20 \text{ GPa} \\ &-1.0 < \nu_{12} < 0.5 \end{aligned} \tag{18}$$

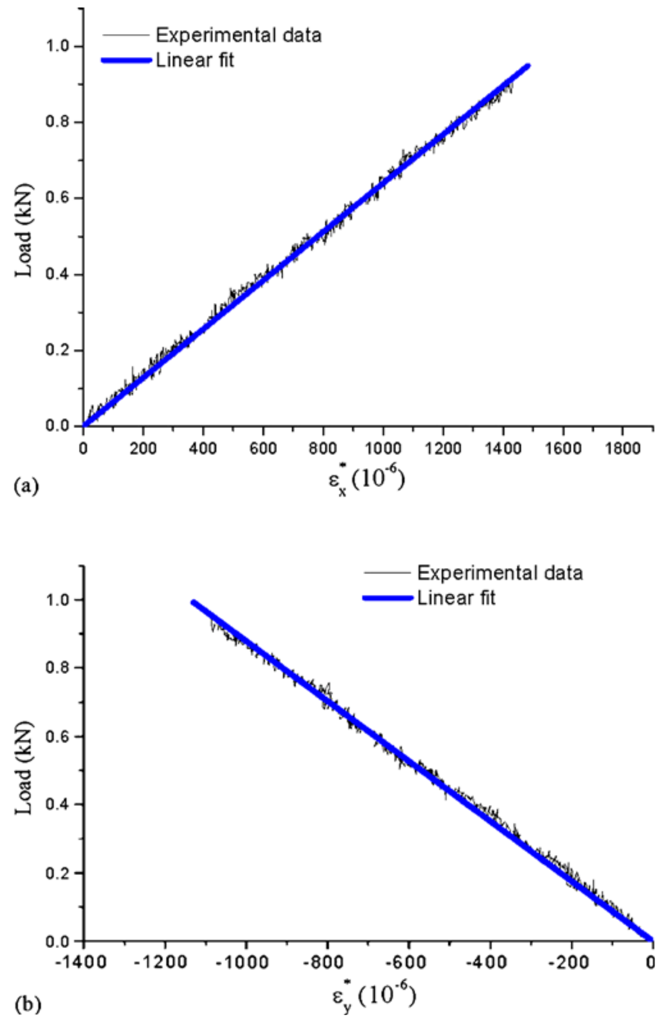

Fig. 3. Load–strain relations of the $[(45°/-45°)_2]_s$ laminate: (a) axial direction and (b) lateral direction.

The modified design variables of Eq. (12) are obtained via the use of the following normalization factors:

$$\alpha_1 = 1000, \quad \alpha_2 = 100, \quad \alpha_3 = 10, \quad \alpha_4 = 1 \tag{19}$$

The value of the amplification factor ξ in Eq. (9) and (16) is set to be 10^6. As mentioned before, the use of the above values for the normalization and amplification factors can help increase the convergence rate of the solution. A number of numerical examples are first given to illustrate the accuracy and feasibility of the proposed method in identifying elastic constants of different composite materials. In the numerical study of the material characterization of Gr/ep composite laminates, the actual strains in Table 1 are treated as the ‘‘measured’’ strains for identifying the material constants given in Eq. (17). The $[(45°/-45°)_2]_s$ laminate is used as an example to show how the material constants are identified in the first-level optimization problem. In this case, five starting points are randomly generated in getting the global minimum with probability exceeding 0.995. The numbers of iterations required for identifying the lowest local minima for the starting points and the material constants identified at the global mini-

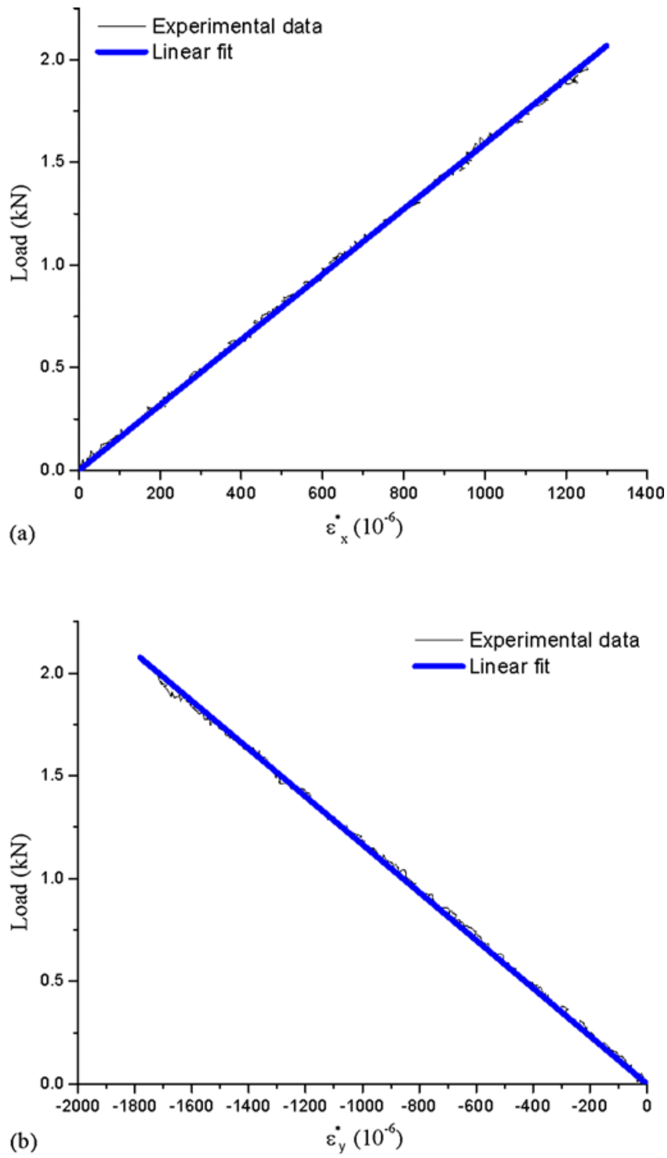

Fig. 4. Load–strain relations of the [(30°/−30°)$_2$]$_s$ laminate: (a) axial direction and (b) lateral direction.

mum are tabulated in Table 3. It is noted that the exact values of G_{12} and ν_{12} can be obtained in the solution of the first-level optimization problem for the [(45°/−45°)$_2$]$_s$ laminate. Furthermore, it worths noting that all the starting points produce the same global minimum. This implies that the identified elastic constants G_{12} and ν_{12} of the [(45°/−45°)$_2$]$_s$ laminate are independent of the starting points chosen in the feasible region and the use of one starting point is enough to determine G_{12} and ν_{12}. As for the symmetric angle-ply laminates with other fiber angles, the identified material constants obtained at the first level optimization are listed in Table 4. In view of the results in Tables 3 and 4, it is noted that except the [(45°/−45°)$_2$]$_s$ laminate, none of the adopted symmetric angle-ply laminates is able to identify the exact values of G_{12} and ν_{12}. Therefore, the [(45°/−45°)$_2$]$_s$ laminate is chosen at the first level optimization to determine G_{12} and ν_{12}. When proceeding to the second level optimization problem, the values of G_{12} and ν_{12} identified in the previous level of optimization using the [(45°/−45°)$_2$]$_s$ laminate are kept constant during the minimization process and the other material constants, namely, E_1 and E_2 are identified using the measured strains of the second symmetric angle-ply laminate with fiber angle different from 45°. In solving the second-level optimization problem, the multi-start global minimization procedure is again used to randomly generate a number of starting points for the chosen symmetric angle-ply laminate, search for the lowest local minima from the starting points, and identify the global minimum with probability greater than or equal to 0.995. As an example, for the case where the "measured" strains of the [(30°/−30°)$_2$]$_s$ laminate are used to solve the second-level optimization problem, the starting points, the lowest local minima for the starting points, the numbers of iterations required for obtaining the lowest local minima, and the global minimum so obtained are listed in Table 5. It is noted that the small numbers of starting points and iterations required to obtain the global minimum have demonstrated the efficiency of the present method. Furthermore, it is also noted that all the starting points produce the same global minimum. This implies that the use of one starting point chosen in the feasible region is enough to identify elastic constants G_{12} and ν_{12}. For comparison, the material constants identified using different symmetric angle-ply laminates at the second level optimization are given in Table 6. It is noted that all the adopted symmetric angle-ply laminates can produce excellent estimates of E_1 and E_2 with errors less than 0.05%. Next, consider the material characterization of Gl/ep composite laminates. The data required for the study are listed in Table 7. The identified material constants at the first-level optimization using different symmetric angle-ply laminates are listed in Table 8. It is noted that among the adopted angle-ply laminates, the [(15°/−15°)$_2$]$_s$ and [(60°/−60°)$_2$]$_s$ laminates are unable to produce the global minima and only the [(45°/−45°)$_2$]$_s$ laminate can produce excellent estimates of G_{12} and ν_{12}. At the second-level optimization, the best estimates of E_1 and E_2 obtained using different symmetric angle-ply laminates are listed in Table 9. It is noted that excellent results can also be obtained for E_1 and E_2 irrespective of the fiber angles of the laminates. Therefore, it is obvious that the present two-level optimization method is capable to produce excellent results in the identification of the material constants for different composite materials provided that the [(45°/−45°)$_2$]$_s$ laminate and another symmetric angle-ply laminate with fiber angle different from 45° are used at the first- and second-level optimizations, respectively. For illustration purpose, the strains of the [(45°/−45°)$_2$]$_s$ and [(30°/−30°)$_2$]$_s$ laminates together with the optimization algorithm DBCONF of IMSL [15] are also used to solve the one level minimization problem of Eq. (8). It has been shown that the optimization algorithm DBCONF is unable to make the solution converge and thus no results are obtained. A sensitivity analysis on the variations of the identified elastic constants

Table 2
Statistics of measured strains of Gr/ep $[(\theta°/-\theta°)_2]_s$ laminates subjected to $F = 0.5$ kN

Fiber angle θ	$\varepsilon_x^*(10^{-4})$				$\varepsilon_y^*(10^{-4})$			
	Specimen no.	Measured	Average	COV	Specimen no.	Measured	Average	COV
$[(45°/-45°)_2]_s$	1	7.081 (−0.5%)[a]	7.081	0.2%	1	−5.224 (+3.2%)	−5.202	1.8%
	2	7.098 (−0.3%)	(−0.5%)		2	−5.101 (+0.7%)	(+2.7%)	
	3	7.063 (−0.8%)			3	−5.282 (+4.3%)		
$[(30°/-30°)_2]_s$	1	3.061 (+2.9%)	2.958	1.8%	1	−3.772 (+2.1%)	−3.773	1.1%
	2	2.911 (−0.6%)	(+1%)		2	−3.737 (+1.1%)	(+1%)	
	3	2.947 (+0.6%)			3	−3.689 (−0.2%)		

[a] Value in parentheses denotes the percentage difference between actual and measured strains.

Table 3
Identified material constants of the Gr/ep $[(45°/-45°)_2]_s$ laminate for the first-level optimization problem

Starting point no.	Stage	Material constant				No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	
1	Initial	736.71	45.75	13.08	0.36	10
	Final	130.22	18.80	6.84	0.30	
2	Initial	445.59	3.31	17.87	0.26	8
	Final	130.22	18.80	6.84	0.30	
3	Initial	529.27	25.58	14.04	0.43	7
	Final	130.22	18.80	6.84	0.30	
4	Initial	429.96	17.10	14.77	0.28	9
	Final	130.22	18.80	6.84	0.30	
5	Initial	633.82	32.28	19.87	0.37	10
	Final	130.22	18.80	6.84	0.30	
Global minimum		130.22 (11.1%)[a]	18.80 (103.9%)	6.84 (0%)	0.30 (0%)	Probability 0.997

[a] Value in parentheses denotes the percentage difference between identified and actual data.

Table 4
Identified material constants using the Gr/ep $[(\theta°/-\theta°)_2]_s$ laminates in the first-level optimization problem

Fiber angle θ	Identified material constant			
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
15°	150.79 (2.9%)[a]	14.30 (55.1%)	1.95 (71.5%)	0.32 (6.7%)
30°	184.31 (25.8%)	31.14 (237.7%)	3.11 (54.5%)	0.30 (0%)
60°	No global minimum			

[a] Value in parentheses denotes the percentage difference between identified and actual data.

Table 5
Identified material constants of the Gr/ep $[(30°/-30°)_2]_s$ laminate for the second-level optimization problem

Starting point no.	Stage	Material constants		No. of iterations
		E_1 (GPa)	E_2 (GPa)	
1	Initial	743.76	14.95	5
	Final	146.52	9.22	
2	Initial	684.98	21.97	7
	Final	146.52	9.22	
3	Initial	936.11	6.47	7
	Final	146.52	9.22	
4	Initial	743.97	43.59	8
	Final	146.52	9.22	
5	Initial	896.04	36.76	10
	Final	146.52	9.22	
Global minimum		146.52 (0.01%)[a]	9.22 (0%)	Probability 0.997

[a] Value in parentheses denotes the percentage difference between identified and actual data.

obtained at the second level of optimization has also been performed via the use of the first order second moment method. The results of the sensitivity analysis have shown that amongst the laminates under consideration, the variations of E_1 and E_2 identified from the laminates with fiber angles of 15° and 30° are around 10% less than those of the 60° laminate. Therefore, it is recommended that the 45degr laminate be used in the first level optimization and either the 15° or 30° laminate in the second level of optimization for identifying the elastic constants of the Gr/ep and Gl/ep materials.

To demonstrate its applications, the present method is now used to identify the material constants of the symmetric angle-ply laminates, which have been tested using the measured strain pairs (measured axial and lateral strains)

Table 6
Material constants E_1 and E_2 identified at second level optimization using the Gr/ep $[(\theta°/\theta°)_2]_s$ laminates

Fiber angle θ	Identified material constant	
	E_1 (GPa)	E_2 (GPa)
15°	146.45 (0.03%)[a]	9.22 (0%)
30°	146.52 (0.01%)	9.22 (0%)
60°	146.57 (0.05%)	9.22 (0%)

[a] Value in parentheses denotes percentage difference between identified and actual data.

Table 7
Data for material characterization of Gl/ep angle-ply laminates subject to $F = 0.1$ kN

Layup	Actual strains	
	$\varepsilon_x(10^{-5})$	$\varepsilon_y(10^{-}5)$
$[(15°/-15°)_2]_s$	9.874	−4.322
$[(30°/-30°)_2]_s$	15.26	−9.876
$[(45°/-45°)_2]_s$	26.55	−13.71
$[(60°/-60°)_2]_s$	36.44	−9.876

Table 8
Identified material constants of Gl/ep $[(\theta°/-\theta°)_2]_s$ optimization laminates obtained at the first-level

Fiber angle θ	Identified material constant			
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}
15°	No global minimum			
30°	42.41 (9.9%)[a]	27.93 (237.7%)	3.42 (17.4%)	0.26 (0%)
45°	30.87 (20.0%)	12.99 (57.1%)	4.14 (0%)	0.26 (0%)
60°	No global minimum			

[a] Value in parentheses denotes percentage difference between identified and actual data.

Table 9
Estimates of E_1 and E_2 for the Gl/ep $[(\theta°/-\theta°)_2]_s$ laminates obtained at the second-level optimization

Fiber angle θ	Estimate of material constant	
	E_1 (GPa)	E_2 (GPa)
15°	38.60 (0%)[a]	8.27 (0%)
30°	38.62 (0.05%)	8.28 (0.12%)
60°	38.60 (0%)	8.27 (0%)

[a] Value in parentheses denotes percentage difference between identified and actual data.

Table 10
Identified material constant using measured strains of the Gr/ep$[(45°/-45°)_2]_s$ laminates at the first-level optimization

Specimen no.	Measured strain		Identified material constant			
	ε_x^+ (10^{-4})	ε_y^+ (10^{-4})	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}
1	7.081	−5.224	147.01 (0.3%)[a]	19.09 (107.0%)	6.77 (1.0%)	0.297 (1%)
2	7.098	−5.101	134.79 (8.0%)	18.88 (104.8%)	6.83 (0.1%)	0.296 (1.3%)
3	7.063	−5.282	154.45 (5.4%)	19.22 (108.5%)	6.75 (1.3%)	0.298 (0.67%)
Average strain	7.081	−5.202	144.97 (1.0%)	19.05 (106.6%)	6.78 (0.9%)	0.297 (1%)

[a] Value in parentheses denotes percentage difference between identified and actual data.

Table 11
Identified E_1 and E_2 using measured strains of the Gr/ep $[(30°/-30°)_2]_s$ laminates at the first-level optimization

Specimen no.	Measured strain		Identified elastic constant	
	$\varepsilon_x^+(10^{-4})$	$\varepsilon_y^+(10^{-4})$	E_1 (GPa)	E_2 (GPa)
1	3.016	−3.772	140.42 (4.15%)[a]	8.45 (8.35%)
2	2.911	−3.737	149.77 (2.23%)	9.18 (0.43%)
3	2.947	−3.689	145.36 (0.78%)	9.56 (3.69%)
Average strain	2.958	−3.733	145.10 (0.96%)	9.06 (1.73%)

[a] Value in parentheses denotes percentage difference between identified and actual data.

of the specimens in Table 2. Each of the measured strain pairs as well as the average strain pair (average axial and lateral strains) of the $[(45°/-45°)_2]_s$ laminates in Table 2 are used at the first-level optimization to determine the estimates of G_{12} and v_{12} of which the values are listed in Table 10. It is noted that the use of any of the measured strain pairs or the average strain pair of the $[(45°/-45°)_2]_s$ laminates can produce excellent estimates of G_{12} and v_{12} with errors less than or equal to 1.3%. At the second-level optimization, the values of G_{12} and v_{12} are set as 6.78 GPa and 0.297, respectively and the measured strains of the $[(30°/-30°)_2]_s$ laminate are used to identify E_1 and E_2. The identified estimates of E_1 and E_2 using different sets of measured strain pairs of the $[(30°/-30°)_2]_s$ laminates given in Table 2 obtained at this level are listed in Table 11. Again it is noted that all the adopted measured strain pairs as well as the average strain pair can produce excellent estimates of E_1 and E_2 with errors less than 8.35%. It worths mentioning that the use of any of the other sets of the estimates of G_{12} and v_{12} in Table 10 in the identification process at the second-level optimization is also able to procedure excellent estimates of E_1 and E_2. Furthermore, in view of the results with errors less than or equal to 1.73% obtained for the average strains in Tables 10 and 11, it seems the use of the average strain pairs in the present method can produce results with better and consistent accuracy. Furthermore, the coefficients of variation of the identified elastic constants calculated using the sample data in Tables 10 and 11 are around 0.6%, 0.3%, 4.8%, and 8.8% for G_{12}, v_{12}, E_1, and E_2, respectively. The small coefficients of variation obtained for the identified elastic constants have thus further demonstrated the accuracy and repeatability of the present identification technique. For comparison purpose, the one level optimization problem of Eq. (8) has also been solved using the present multi-start global minimization technique. The solution of the one level minimization problem produces the error percentages of 0.85%, 11.3%, 1.2%, and 66.7% for E_1, E_2, G_{12}, and ν_{12}, respectively. The relatively high errors produced in the estimations of E_2 and ν_{12} have further demonstrated the fact that the elastic constants identified from the one level minimization problem of Eq. (8) are very sensitive to measurement noise.

6. Conclusions

A method for the identification of four material constants of fiber reinforced composite materials using four measured strains obtained from two symmetric angle-ply laminates with different fiber angles subjected to static tensile testing has been presented. The proposed method has been established on the basis of a two-level optimization method together with a multi-start global minimization technique. Measured axial and lateral strains of a $[(45°/-45°)_2]_s$ laminate and a symmetric angle-ply laminate with different fiber angle have been used, respectively, in the first-level optimization problem to identify G_{12} and ν_{12} and in the second-level optimization problem to identify E_1 and E_2. A number of numerical examples have been given to demonstrate the capability of the present method in identifying the exact values of the material constants of different laminated composite materials. It has also been shown in a sensitivity study that the use of a $[(45°/-45°)_2]_s$ laminate in the first level of optimization together with one of the $[(15°/-15°)_2]_s$ and $[(30°/-30°)_2]_s$ laminates in the second level of optimization can produce satisfactory results for identifying the elastic constants of Gr/ep and Gl/ep materials. Static tensile tests of several symmetric angle-ply laminates have been performed to measure the axial and lateral strains of the laminates of which the experimental data have been used to study the feasibility and accuracy of the present method. The study has shown that the present method can produce good estimates of the elastic constants for the laminates in an effective and efficient way. The errors in the identification of material constants G_{12} and ν_{12} are less than or equal to 1.3% while those for E_1 and E_2 are less than or equal 8.35%. The use of the average measured strains in the identification may produce results with better and consistent accuracy. The present method is simple and effective and thus has the potential to become a standard testing procedure for the determination of elastic constants.

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