

# An analytical solution for the head distribution in a tidal leaky confined aquifer extending an infinite distance under the sea

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## Abstract

A mathematical model is developed to investigate the effects of tidal fluctuations and leakage on the groundwater head of leaky confined aquifer extending an infinite distance under the sea. The leakages of the offshore and inland aquitards are two dominant factors controlling the groundwater fluctuation. The tidal influence distance from the coast decreases significantly with the dimensionless leakage of the inland aquitard ( $u_i$ ). The fluctuation of groundwater level in the inland part of the leaky confined aquifer increases significantly with the dimensionless leakage of the offshore aquitard ( $u_o$ ). The influence of the tidal propagation parameter of an unconfined aquifer on the head fluctuation of the leaky confined aquifer is comparatively conspicuous when  $u_i$  is large and  $u_o$  is small. In other words, ignoring water table fluctuation of the unconfined aquifer will give large errors in predicting the fluctuation, time lag, and tidal influence distance of the leaky confined aquifer for large  $u_i$  and small  $u_o$ . On the contrary, the influence of the tidal propagation parameter of a leaky confined aquifer on the head fluctuation of the leaky confined aquifer is large for large  $u_o$  and small  $u_i$ .

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**Keywords:** Tidal aquifer; Leaky confined aquifer; Leakage; Analytical solution; Coupled confined–unconfined aquifer

## 1. Introduction

The influence of the dynamic interaction between groundwater and seawater is an interesting topic for hydrologists. Since the 1950s, problems and researches on groundwater dynamics in response to tidal fluctuation in a coastal aquifer system had attracted much attention. These studies include coastal aquifer parameter estimation, beach dewatering, marine environment, marine retaining structures, and seawater intrusion e.g., [1–3,5–7,17,21,22]. Some researchers focused on the field problems of a single aquifer system [10,14,18,20,23]. The problems were solved in analogy to heat conduction in a semi-infinite solid subject to periodic temperature variations normal to the infi-

nite dimension [8,9] where inland amplitudes of the waves decrease significantly with distance. On the other hand, a coastal aquifer system may be considered as an unconfined aquifer, a leaky confined aquifer, or an aquitard between them [4,11–13,15].

Jiao and Tang [11,13] used an analytical solution to investigate the influence of leakage on the tidal response in a coastal leaky confined aquifer system. They showed that both tidal amplitude of groundwater head in the aquifer and the distance over which the seawater disturbs the aquifer could be significantly reduced because of leakage. Li and Jiao [16] presented an analytical solution for tidal-induced groundwater fluctuation in a coastal leaky confined aquifer system extending under the sea to investigate the influences of tidal efficiency, roof length, and leakage of the semi-permeable layer on tide-induced groundwater fluctuations. They assumed that the leakage of the offshore

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aquitard is the same as that of the inland aquitard and the water table fluctuation in the unconfined aquifer is negligible. They also showed that there exists a finite threshold value ( $L_u$ ) of roof length ( $L_f$ ). When  $L_f \geq L_u$ , the tidal propagation in the inland aquifer will behave as if the roof length were infinite.

Recently, Li et al. [15] utilized perturbation approach to derive an approximate solution for examining dynamic effects of the overlying aquifer. Volker and Zhang [24] used the finite element program 2DFEMFAT to assess the errors induced by neglecting water level changes in the unconfined aquifer of a leaky aquifer system being subject to tidal sea boundary condition. Jeng et al. [12] presented an analytical solution for the tidal response in a fully coupled leaky confined aquifer system with considering the effects of the water table fluctuations in the unconfined aquifer. They concluded that the dynamic effects were important under a relatively large leakage and phreatic aquifer transmissivity. Ignoring those effects could lead to errors in estimating aquifer properties based on tidal signals.

This paper focuses on groundwater dynamics in response to tidal fluctuation in a coastal leaky confined aquifer extending an infinite distance under the sea. The leakage of the offshore aquitard, which may be formed by sedimentary depositional process of the offshore current [19], is assumed different from that of the inland aquitard. Thus, the leakage effects of both inland and offshore aquitards on the head distribution of the tidal leaky confined aquifer are considered and discussed. The objective of this paper is to develop a mathematical model for describing the hydraulic head distribution in a tidal leaky confined aquifer that extends an infinite distance under the sea and has hydraulic connection with an upper unconfined aquifer. Based on the derived solution of this model, the joint effects of various parameters, such as the leakages of the inland and offshore aquitards, on the behavior of the groundwater level fluctuations in the inland part of the leaky confined aquifer can be clearly explored.

## 2. Mathematical model

### 2.1. Governing equation and boundary conditions

Consider a coastal aquifer system consisting of an unconfined and underlying leaky confined aquifer hydraulically connected via a leaky aquitard as shown in Fig. 1. The effects of tidal fluctuations on both the unconfined and the leaky confined aquifers are considered. The unconfined aquifer terminates at the coast, while the aquitard and the leaky confined aquifer extend infinitely under the sea. The bottom of the leaky confined aquifer is impermeable. In addition, the leakages of the offshore and inland aquitards are assumed to have different values.

The origin of the  $x$ -axis is at the intersection of the mean sea surface and the beach face. The  $x$ -axis is horizontal, positive landward, and perpendicular to the coastal line. Consider that the aquifer material is homogeneous and isotropic and the thickness of the unconfined aquifer is very large when compared to the magnitude of the tidal fluctuations, therefore allowing linearity of the governing flow equations. The flow velocity in the leaky confined aquifer is essentially horizontal, and there is a vertical leakage through the aquitard. The reference hydraulic head in the whole system is assumed uniform and equals  $h_{MSL}$ . In addition, the aquitard storage is assumed negligible and leakage is linearly proportional to the difference in head between the unconfined aquifer and leaky confined aquifer [2,15,16]. Under these assumptions, the governing equations of the head fluctuation for the inland unconfined and the leaky confined aquifer ( $x > 0$ ) are respectively [2,15,16]

$$S_1 \frac{\partial h_1}{\partial t} = T_1 \frac{\partial^2 h_1}{\partial x^2} + L_i(h_2 - h_1) \tag{1}$$

and

$$S_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + L_i(h_1 - h_2) \tag{2}$$

where  $h_1$  and  $h_2$  are the hydraulic heads in the unconfined and leaky confined aquifer, respectively;  $S_1$  and  $S_2$ , as well

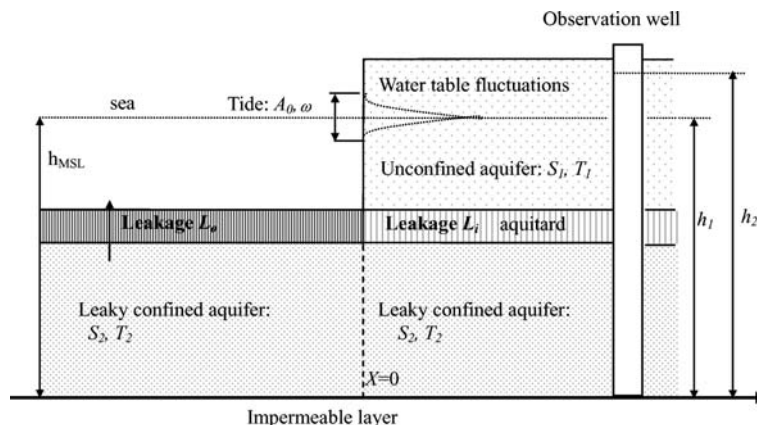


Fig. 1. Schematic diagram of a leaky confined aquifer extending an infinite distance under the sea.

as  $T_1$  and  $T_2$  are the storativities and transmissivities of these two aquifers, respectively;  $L_i$  is the leakage of the inland aquitard. The governing equation of the head fluctuation for offshore aquifer ( $x < 0$ ) is

$$S_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + S_2 T_e \frac{dh_s}{dt} + L_o(h_s - h_2) \quad (3)$$

where  $h_s$  is hydraulic head of the sea tide,  $L_o$  is the leakage of the offshore aquitard, and  $T_e$  is tidal efficiency, reflecting the fluctuation of groundwater level caused by compression of both the aquifer skeleton and groundwater due to the tidal loading above the offshore aquitard [16]. The leakage is defined as the ratio of the hydraulic conductivity of the aquitard to the thickness of the aquitard. The hydraulic conductivity and/or thickness of the inland aquitard may differ from those of the offshore aquitard due to different depositional sediment facies [19]. The tidal boundary may be written as

$$h_1(0, t) = h_s(t) = h_{\text{MSL}} + A_0 \cos(\omega t) \quad (4)$$

where  $h_1(0, t)$  is the hydraulic head at  $x = 0$ ,  $A_0$  is the amplitude of the tidal fluctuation, and  $\omega$  is the tidal speed. Also  $\omega = 2\pi/t_0$  where  $t_0$  is the tidal period. The continuity conditions of the hydraulic head and flux at  $x = 0$  respectively require

$$\lim_{x \downarrow 0} h_2(x, t) = \lim_{x \uparrow 0} h_2(x, t) \quad (5)$$

and

$$\lim_{x \downarrow 0} \frac{\partial h_2(x, t)}{\partial x} = \lim_{x \uparrow 0} \frac{\partial h_2(x, t)}{\partial x} \quad (6)$$

The boundary conditions for  $x = \pm\infty$  may be expressed as

$$\lim_{x \rightarrow \infty} \frac{\partial h_1}{\partial x} = 0 \quad (7)$$

$$\lim_{x \rightarrow \infty} \frac{\partial h_2}{\partial x} = 0 \quad (8)$$

$$\lim_{x \rightarrow -\infty} \frac{\partial h_2}{\partial x} = 0 \quad (9)$$

which state that the slopes of the hydraulic head approach zero at the remote boundary.

### 2.2. Closed-form solutions

Detailed derivations of the solutions for the governing Eqs. (1)–(3) with the appropriate boundary conditions are given in Appendix 1 and the results are

$$h_1(x, t) = h_{\text{MSL}} + \text{Re}[A_0(\alpha_1 e^{-\lambda_1 x} + \alpha_2 e^{-\lambda_2 x})e^{-i\omega t}], \quad x > 0 \quad (10)$$

$$h_2(x, t) = h_{\text{MSL}} + \text{Re}[A_0(\alpha_1 \beta_1 e^{-\lambda_1 x} + \alpha_2 \beta_2 e^{-\lambda_2 x})e^{-i\omega t}], \quad x > 0 \quad (11)$$

$$h_2(x, t) = h_{\text{MSL}} + \text{Re}[A_0(\alpha_3 e^{\lambda_3 x} + \beta_3)e^{-i\omega t}], \quad x < 0 \quad (12)$$

where  $\text{Re}$  denotes the real part of the complex expression,  $i = \sqrt{-1}$ , and parameters  $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2,$  and  $\lambda_3$  are respectively defined as

$$\alpha_1 = \frac{\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_3 \lambda_3}{\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_1 \lambda_1 - \beta_1 \lambda_3} \quad (13)$$

$$\alpha_2 = \frac{\beta_3 \lambda_3 + \beta_1 \lambda_1 - \beta_1 \lambda_3}{\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_1 \lambda_1 \beta_1 \lambda_3} \quad (14)$$

$$\alpha_3 = \frac{\beta_1 \beta_2 \lambda_2 + \beta_1 \beta_3 \lambda_1 - \beta_1 \beta_2 \lambda_1 - \beta_2 \beta_3 \lambda_2}{\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_1 \lambda_1 - \beta_1 \lambda_3} \quad (15)$$

$$\beta_1 = \frac{L_i - T_1 B_1 - i\omega S_1}{L_i} \quad (16)$$

$$\beta_2 = \frac{L_i - T_1 B_2 - i\omega S_1}{L_i} \quad (17)$$

$$\beta_3 = \frac{L_o - i\omega S_2 T_e}{L_o - i\omega S_2} \quad (18)$$

$$\lambda_1 = -\sqrt{B_1} \quad (19)$$

$$\lambda_2 = -\sqrt{B_2} \quad (20)$$

$$\lambda_3 = \left( \frac{L_o - i\omega S_2}{T_2} \right)^{0.5} \quad (21)$$

The parameters  $B_1$  in (16) and (19) and  $B_2$  in (17) and (20) are respectively defined as

$$B_1 = -a - \sqrt{a^2 - b} \quad (22)$$

$$B_2 = -a + \sqrt{a^2 - b} \quad (23)$$

where  $a$  and  $b$  are respectively defined as

$$a = \frac{L_i}{2T_1 T_2} \left( T_1 + T_2 - \frac{i\omega T_1 S_2}{L_i} - \frac{i\omega T_2 S_1}{L_i} \right) \quad (24)$$

$$b = -\frac{L_i}{T_1 T_2} \left( \frac{\omega^2 S_1 S_2}{L_i} + i\omega S_1 + i\omega S_2 \right) \quad (25)$$

### 3. Results and discussion

Eqs. (10) and (11) are the solutions for the groundwater heads in inland parts of the unconfined and confined aquifers, respectively. Eq. (12) is the solution for the groundwater heads in offshore part of the confined aquifer. For convenience of discussion, some parameters used in Li and Jiao [16] are adopted herein. The tidal propagation parameter of the unconfined aquifer ( $a_1$ ) is defined as

$$a_1 = \sqrt{\omega S_1 / 2T_1} = \sqrt{\pi S_1 / T_1 t_0} \quad (26a)$$

Similarly, the tidal propagation parameter of the leaky confined aquifer ( $a_2$ ) is

$$a_2 = \sqrt{\omega S_2 / 2T_2} = \sqrt{\pi S_2 / T_2 t_0} \quad (26b)$$

In addition, the dimensionless inland leakage ( $u_i$ ) is expressed as

$$u_i = \frac{L_i}{\omega S_2} \quad (26c)$$

Likewise, the dimensionless offshore leakage ( $u_o$ ) is

$$u_o = \frac{L_o}{\omega S_2} \quad (26d)$$

In the following sections, the effects of the dimensionless leakages of the offshore and inland aquitards on the fluctuation of groundwater level are investigated. The tidal influence distance from the coast and tidal amplitude in response to various values of the dimensionless leakages of inland and offshore aquitards are examined. The influences of the tidal propagation parameters are also discussed. In addition, the effects of the water table fluctuations are also observed. The amplitude of the tidal fluctuation,  $A_0$ , is assumed constant and the normalized groundwater amplitude, expressed as  $|H_2|/A_0$  or simply  $HA$ , is defined as the ratio of the groundwater fluctuation amplitude of the inland confined leaky aquifer to the tide amplitude.

3.1. Effect of the offshore leakage on the leaky confined aquifer and related tidal influence distance

Fig. 2 shows the normalized groundwater amplitude versus the dimensionless landward distance from coastline when  $u_o$  varies from 0 to 5 with parameters  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$ ,  $a_1 = 10a_2$ ,  $T_e = 0.5$ , and  $u_i = 0$ . This figure demonstrates that the present analytical solutions denoted by solid line closely conform to that of Van der Kamp [23], as represented by the dot symbol, when  $u_o = u_i = 0$ . In addition, the figure also shows that the  $HA$  increases significantly with  $u_o$  and decreases with the dimensionless landward distance from coastline.

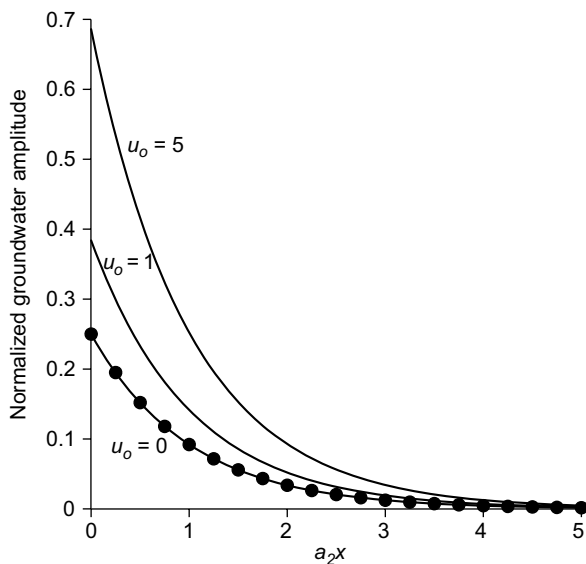


Fig. 2. Normalized groundwater amplitude versus the dimensionless landward distance from coastline ( $a_2x$ ) when the dimensionless offshore leakage ( $u_o$ ) varies from 0 to 5 with parameters  $a_1 = 1.23 \times 10^{-2} \text{ m}^{-1}$ ,  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$ ,  $T_e = 0.5$ , and  $u_i = 0$ . Solid line denotes the present solution, and the symbol ● stands for [23].

3.2. Effect of the inland leakage on the leaky confined aquifer and water table fluctuation

Fig. 3 exhibits the normalized groundwater amplitude versus the dimensionless landward distance from coastline when  $u_i$  varies from 1 to 30 with parameters  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$ ,  $a_1 = 10a_2$ ,  $T_e = 0$  for (a)  $u_o = u_i$  and (b)  $u_o = 1$ . The solid line denotes the present solution and the triangle symbol denotes the one without considering the fluctuation of groundwater level in the unconfined aquifer. The triangle symbol of Fig. 3a stands the solution

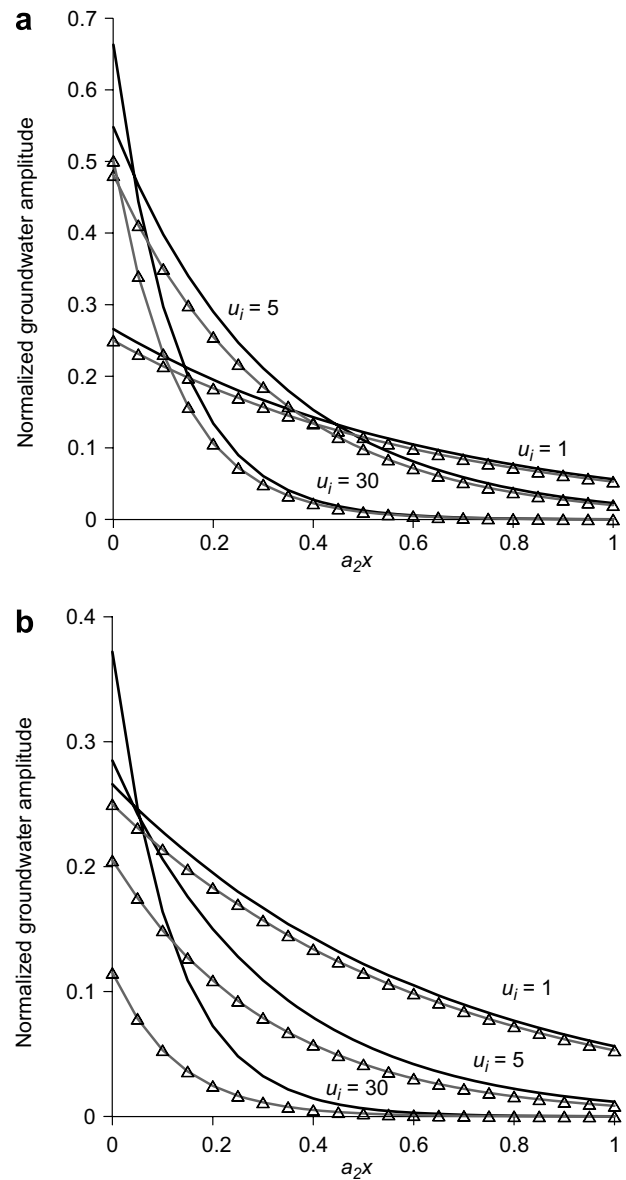


Fig. 3. Normalized groundwater amplitude versus the dimensionless landward distance from coastline ( $a_2x$ ) when the dimensionless inland leakage ( $u_i$ ) varies from 1 to 30 with parameters  $a_1 = 1.23 \times 10^{-2} \text{ m}^{-1}$ ,  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$ ,  $T_e = 0$  when (a)  $u_o = u_i$  and (b)  $u_o = 1$ . The solid line denotes the present solution and the triangle symbol denotes the one without considering the fluctuation of groundwater level in the unconfined aquifer.

of Li and Jiao [16] when the roof length extends to infinity. It is interesting to note that under the condition of  $u_o = u_i$ , the present solution can be shown equal to that of Li and Jiao [16] if the roof length extends to infinity. Obviously, the values of the normalized groundwater amplitude for the solution with considering the effect of the water table fluctuation are larger than that of neglecting such an effect. Fig. 3a and b indicate that the effect of the water table fluctuations is large when  $u_o$  is small and  $u_i$  is large. Accordingly, the assumption of a constant water table may lead to a significant error in the prediction of groundwater amplitude for small  $u_o$  and large  $u_i$ . However, the error decreases with the dimensionless landward distance from

coastline. In addition, Fig. 3a and b also show that the tidal influence distance decreases significantly with  $u_i$ .

3.3. The influences of the tidal propagation parameters on the temporal head fluctuation of the leaky confined aquifer

The influences of the tidal propagation parameters on the temporal head fluctuation of the leaky confined aquifer are examined herein. Fig. 4 displays the temporal normalized groundwater fluctuation in the leaky confined aquifer when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  with parameters  $T_e = 0$ ,  $x = 162 \text{ m}$ , and  $a_1 = 1.23 \times 10^{-2} \text{ m}^{-1}$  for (a)  $u_o = 30$ ,  $u_i = 1$ ;  $u_o = 1$ ,  $u_i = 30$ , (b)  $u_o = u_i = 1$ ;

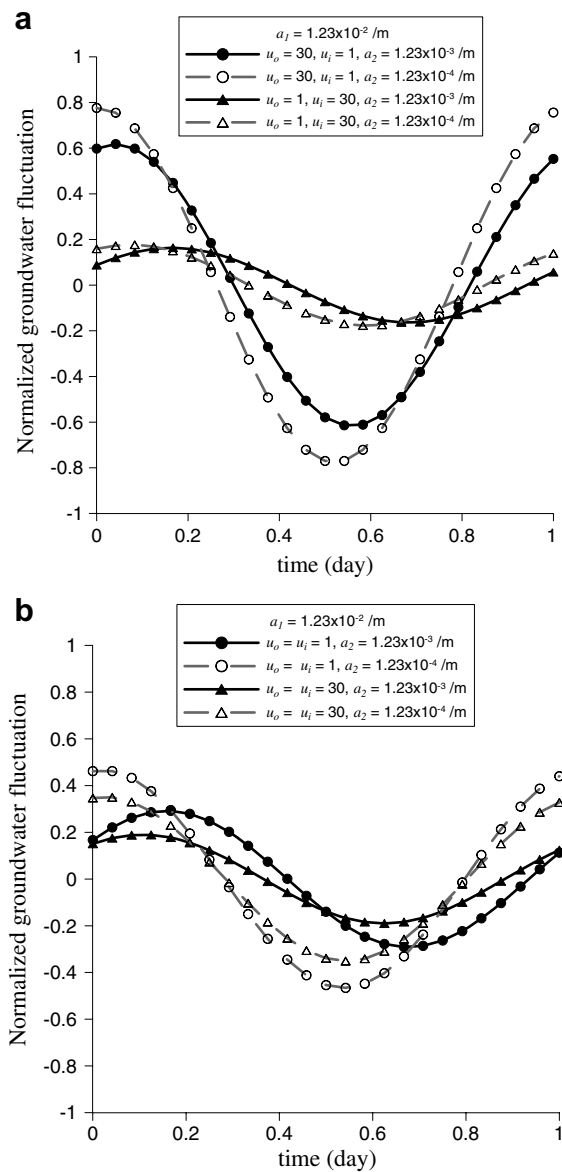


Fig. 4. Temporal normalized groundwater fluctuation in the leaky confined aquifer when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  with parameters  $T_e = 0$ ,  $x = 162 \text{ m}$ , and  $a_1 = 1.23 \times 10^{-2} \text{ m}^{-1}$  for (a)  $u_o = 30$ ,  $u_i = 1$ ;  $u_o = 1$ ,  $u_i = 30$ , (b)  $u_o = u_i = 1$ ;  $u_o = u_i = 30$ .

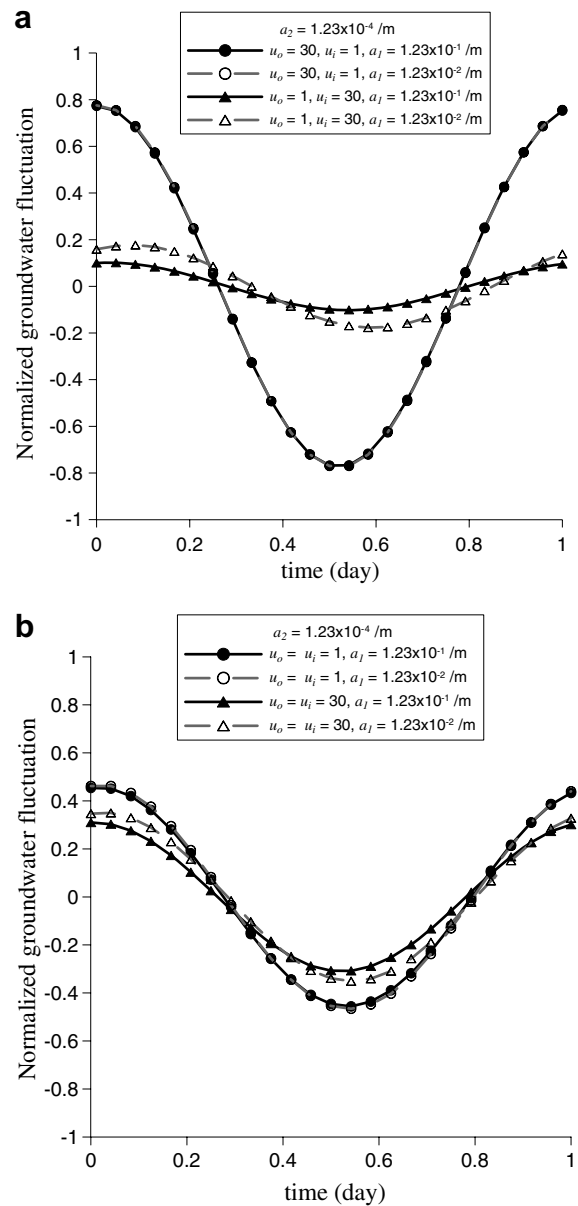


Fig. 5. Temporal normalized groundwater fluctuation in the leaky confined aquifer when  $a_1$  varies from  $1.23 \times 10^{-1}$  to  $1.23 \times 10^{-2} \text{ m}^{-1}$  with parameters  $T_e = 0$ ,  $x = 162 \text{ m}$ , and  $a_2 = 1.23 \times 10^{-4} \text{ m}^{-1}$  for (a)  $u_o = 30$ ,  $u_i = 1$ ;  $u_o = 1$ ,  $u_i = 30$ , (b)  $u_o = u_i = 1$ ;  $u_o = u_i = 30$ .



$u_o = u_i = 30$ . Fig. 4a exhibits that  $HA$  changes from 0.614 to 0.770 and time lag changes from 0.542 day to 0.500 day when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  for parameters  $u_o = 30$ ,  $u_i = 1$ . On the other hand, that  $HA$  changes from 0.163 to 0.177 and time lag changes from 0.667 day to 0.583 day when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  for parameters  $u_o = 1$  and  $u_i = 30$ . Obviously, the  $HA$  increases with decreasing  $a_2$  and the time lag decreases with  $a_2$  when  $u_o = 30$ ,  $u_i = 1$  and  $u_o = 1$ ,  $u_i = 30$ . Furthermore, the change in  $HA$  is large and the change in time lag is small when relatively  $u_o$  is large and  $u_i$  is small. Fig. 4b shows that the change in  $HA$  is 0.175 and the change in time lag is 0.125 day when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  for  $u_o = u_i = 1$ . On the other hand, the change in  $HA$  is 0.161 and the change in time lag is 0.083 day when  $a_2$  varies from  $1.23 \times 10^{-3}$  to  $1.23 \times 10^{-4} \text{ m}^{-1}$  for  $u_o = u_i = 30$ . Accordingly, both  $HA$  and time lag decrease when  $u_o$  and  $u_i$  are equally increased. For  $u_o = u_i = 1$  or  $u_o = u_i = 30$ , the  $HA$  with  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$  is smaller than that with  $a_2 = 1.23 \times 10^{-4} \text{ m}^{-1}$ ; however, the time lag with  $a_2 = 1.23 \times 10^{-3} \text{ m}^{-1}$  is larger than that with  $a_2 = 1.23 \times 10^{-4} \text{ m}^{-1}$ . Fig. 4a and b indicate that the influence of  $a_2$  on the groundwater fluctuation in the leaky confined aquifer is large when relatively  $u_o$  is large and  $u_i$  is small.

Fig. 5 shows that temporal normalized groundwater fluctuation in the leaky confined aquifer when  $a_2 = 1.23 \times 10^{-4} \text{ m}^{-1}$  and  $a_1$  varies from  $1.23 \times 10^{-1}$  to  $1.23 \times 10^{-2} \text{ m}^{-1}$  with parameters  $T_e = 0$  and  $x = 162 \text{ m}$  for (a)  $u_o = 30$ ,  $u_i = 1$  or  $u_o = 1$ ,  $u_i = 30$ , (b)  $u_o = u_i = 1$  or  $u_o = u_i = 30$ . This figure reveals that the influence of  $a_1$  on the groundwater fluctuation is insignificant when  $u_i = 1$ . The changes in  $HA$  and time lag are larger for  $u_o = 1$  than those for  $u_o = 30$  when  $a_1$  varies from  $1.23 \times 10^{-1}$  to  $1.23 \times 10^{-2} \text{ m}^{-1}$  and  $u_i = 30$  as demonstrated in Fig. 5. Those results indicate that the influence of  $a_1$  on the groundwater fluctuation is relatively large for small  $u_o$  and large  $u_i$ .

#### 4. Conclusions

A mathematical model is developed to investigate the influence of leakages and tidal efficiency on tidal responses in a coupled coastal aquifer system consisting of an unconfined aquifer, aquitard, and leaky confined aquifer. The unconfined aquifer terminates at the coast while the aquitard and leaky confined aquifer extend an infinite distance under the sea. Detailed investigations are carried out to explore the effects of various parameters on the behavior of the groundwater level fluctuation of the inland leaky confined aquifer. The offshore and inland leakages are the two dominant factors controlling the groundwater fluctuations if offshore leakage differs from inland leakage. The tidal influence distance decreases significantly with the leakage of the inland aquitard. The groundwater fluctuation in the inland part of the leaky confined aquifer increases significantly with the leakage of the offshore aquitard. The values of the normalized groundwater amplitude

with considering the effects of the water table fluctuations are larger than those of neglecting such effects. In addition, the effects of the water table fluctuations are very significant when dimensionless offshore leakage is smaller and dimensionless inland leakage is larger.

The influence of the tidal propagation parameter of the unconfined aquifer on the head fluctuation of the leaky confined aquifer is comparatively conspicuous when the dimensionless inland leakage is large and the dimensionless offshore leakage is small. On the contrary, the influence of the tidal propagation parameter of the leaky confined aquifer on the head fluctuation of the leaky confined aquifer is large when the dimensionless offshore leakage is large and the dimensionless inland leakage is small.

This paper has derived the analytical solution for the present model to explore the effects of tidal fluctuations and leakage on the groundwater head of leaky confined aquifer extending an infinite distance under the sea. However, the elastic storage of the leaky aquitard as well as the non-linear Boussinesq equation for representing the upper water table aquifer is not considered. Future extension to comprise these two subjects in the present model may lead to a more complete representation for the real-world tidal aquifer system.

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#### Appendix 1. Derivation of the solutions to (1)–(9)

Let  $H_1(x, t)$  and  $H_2(x, t)$  be complex functions of the real variables  $x$  and  $t$  that satisfy Eqs. (1)–(9). Assume that  $h_1(x, t)$  and  $h_2(x, t)$  are the solutions to Eqs. (1)–(9) and follow that

$$h_1(x, t) = h_{\text{MSL}} + \text{Re}[H_1(x, t)] \quad (\text{A.1})$$

$$h_2(x, t) = h_{\text{MSL}} + \text{Re}[H_2(x, t)] \quad (\text{A.2})$$

where  $\text{Re}$  denotes the real part of the complex expression and  $i = \sqrt{-1}$ .

Now suppose

$$H_1(x, t) = A_0 X_1(x) e^{-i\omega t} \quad (\text{A.3})$$

$$H_2(x, t) = A_0 X_2(x) e^{-i\omega t} \quad (\text{A.4})$$

where  $X_1(x)$  and  $X_2(x)$  are unknown functions of  $x$ . Substituting Eqs. (11) and (12) into those eight equations, which  $H_1(x, t)$  and  $H_2(x, t)$  satisfy, and dividing the results by  $Ae^{-i\omega t}$  yields the result for inland aquifer ( $x > 0$ ) as

$$X_1'' + \frac{i\omega S_1 - L_i}{T_1} X_1(x) + \frac{L_i}{T_1} X_2(x) = 0 \quad (\text{A.5})$$

$$X_2'' + \frac{i\omega S_2 - L_i}{T_2} X_2(x) + \frac{L_i}{T_2} X_1(x) = 0 \quad (\text{A.6})$$

and the results for offshore aquifer ( $x < 0$ ) as

$$X_2''(x) + \frac{i\omega S_2 - L_o}{T_2} X_2(x) = \frac{i\omega T_e S_2 - L_o}{T_2} \quad (\text{A.7})$$

The tidal boundary of Eq. (4) may be rewritten as

$$X_1(0) = 1 \quad (\text{A.8})$$

The continuity conditions of Eqs. (5) and (6) may be replaced as

$$\lim_{x \downarrow 0} X_2(x) = \lim_{x \uparrow 0} X_2(x) \quad (\text{A.9})$$

$$\lim_{x \downarrow 0} X_2'(x) = \lim_{x \uparrow 0} X_2'(x) \quad (\text{A.10})$$

Moreover, the boundary conditions of Eqs. (7)–(9) may be rewritten as

$$\lim_{x \rightarrow \infty} X_1' = 0 \quad (\text{A.11})$$

$$\lim_{x \rightarrow \infty} X_2' = 0 \quad (\text{A.12})$$

$$\lim_{x \rightarrow -\infty} X_2' = 0 \quad (\text{A.13})$$

Accordingly, the general solutions to Eqs. (A.5)–(A.7) for inland aquifer ( $x > 0$ ) are

$$X_1(x) = \alpha_1 e^{-\lambda_1 x} + \alpha_2 e^{-\lambda_2 x} \quad (\text{A.14})$$

$$X_2(x) = \alpha_1 \beta_1 e^{-\lambda_1 x} + \alpha_2 \beta_2 e^{-\lambda_2 x} \quad (\text{A.15})$$

and for offshore aquifer ( $x < 0$ ) is

$$X_2(x) = \alpha_3 e^{\lambda_3 x} + \beta_3 \quad (\text{A.16})$$

where parameters  $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2,$  and  $\lambda_3$  are defined in Eqs. (13)–(25).

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