Study of VSC Reliable Designs With Application to Spacecraft Attitude Stabilization

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***Abstract*—This brief investigates variable structure reliable control (VSRC) issues of a set of second-order nonlinear systems and their application to spacecraft attitude stabilization. Both passive and active reliable designs are presented. To achieve the active task, an observer to identify faults as they occur in the spacecraft actuators is also presented. These VSRC laws do not require the solution of a Hamilton–Jacobi (HJ) equation, which is essential in the optimal approaches such as linear quadratic Riccati (LQR) and H^∞ reliable designs. As a matter of fact, this approach can relax the computational burden for solving the HJ equation. Simulation results regarding spacecraft attitude stabilization with comparisons among the VSRCs and the LQR reliable designs are also given. It is shown from these simulations that the active VSRC is the most flexible, robust and effective method because it does not need to prespecify susceptible actuators and because it allows more space for the control parameter adjustment.**

***Index Terms*—Hamilton–Jacobi (HJ) equation, nonlinear systems, reliable control, spacecraft attitude stabilization, variable structure control.**

I. Introduction

DUE TO the growing demands for system reliability in a highly automated industrial system and in aerospace missions, where repair and maintenance often cannot be achieved immediately, the study of reliable control has become of paramount importance and has attracted considerable attention (see, e.g., [2]–[4], [7], [9], [12]–[16], [20], [22], and [24]–[29]). The objective of reliable control is to design an appropriate controller in such a way that the closed-loop system can tolerate the abnormal operations of some specific control components and retain the overall system stability with acceptable system performance. In practical applications, the reliable control concept has been commonly used in the design of many control systems, especially for spacecrafts that require high level of security and reliability. Engineers have incorporated the reliable control concept, along with redundancy in design, so that when faults occur, the spacecraft is able to continue operating safely without prompt supports from the ground. In fact, when some of the control components happen to experience abnormal operation, the spacecraft is guided to a back-up system or a fault mode so that the mission can be continued, and that the cost is reduced as much as possible.

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From the approach viewpoint, reliable control can be classified as active [2]–[4], [7], [14], [20], [22], [28], [29] or passive [9], [13], [16], [24]–[27]. In an active reliable control system, faults are detected and identified by a fault detection and diagnosis (FDD) mechanism. Then the controllers are reconfigured in real time in accordance with the online detection results. In contrast, the passive approach exploits the system inherent redundancy to design a fixed controller so that the closed-loop system can achieve an acceptable performance not only during normal operations but also when various components fail without FDD and controller reconfiguration, which is important when the available reaction time is short after the occurrence of a fault. Although the performance of active reliable control is generally superior to that of a passive one under various fault conditions because of the controller reconfiguration feature, the active approach needs a reliable FDD to detect and diagnose possible faults. The confirmation time in the FDD after the fault occurrence is very important, and the performance of active schemes depend on this. In this brief, we consider both passive and active reliable control issues.

Among the existing reliable control studies, several approaches have been proposed. For instance, Boskovic and Mehra [4] investigated an active reliable control issue for actuator failures through a multiple models technique. Liang *et al.* [14] dealt with active reliable output tracking control issues. Moerder *et al.* [20] employed a self-repairing flight control system concept to reconfigure control strategy. Zhang and Jiang [28], [29] explored the active fault-tolerant control against partial actuator failures. On the other hand, the passive approaches include the linear matrix inequality (LMI)-based approach [16], the algebraic Riccati equation (ARE)-based approach [25], the coprime factorization approach [26], and the Hamilton–Jacobi (HJ)-based approach [13], [27]. Among the previously mentioned passive studies, only the HJ-based approach deals with the reliable issues for nonlinear systems, which is the case that this brief concerns. Although the HJ equation has been used in solving many control problems (see, e.g., [1], [8], [10], and [13]), it is known that the HJ equation is, in general, difficult to solve. A power series method [8] may alleviate the difficulty through computer calculation, while the obtained solution is only an approximate one, and the computation load grows fast when the system is complicated. Owing to these potential drawbacks of the HJ-based approach, this brief investigates the reliable control issues from the variable structure control (VSC) viewpoint.

It is known that the VSC schemes have the advantages of fast response and small sensitivity to system parameter uncertainties and disturbances (see, e.g., [6] and [17] and the references therein). Thus, it has been widely applied to control a variety of systems [6], [10], [14], [17], [22], [23]. In this brief, we employ the VSC technique to design reliable controllers. The resulted systems are shown to be able to tolerate the outage of actuators, and these reliable laws are easily implemented considering that the controllers do not require a solution of an HJ equation. Thus, the VSC approach can also alleviate the computational burden for solving the HJ equation.

The rest of this brief is organized as follows. Section II describes both passive and active VSC reliable designs. The application of these reliable designs to a spacecraft is explored in Section III. Finally, Section IV gives the conclusions.

II. Design of Reliable Controllers

Consider a set of n second-order nonlinear differential equations as given by

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \qquad \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + \mathbf{d}. \tag{1}$$

Here, $\mathbf{x}_1 := (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $\mathbf{x}_2 := (x_{n+1}, \ldots, x_{2n})^T \in \mathbb{R}^n$, $\mathbf{x} := \left(\mathbf{x}_1^T, \text{ and } \mathbf{x}_2^T\right)^T \in \mathbb{R}^{2n}$ denote system states, $\mathbf{u} := (u_1, \ldots, u_{n+m})^T \in \mathbb{R}^{n+m}$ are control inputs, $\mathbf{d} := (d_1, \ldots, d_n)^T \in \mathbb{R}^n$ denote possible model uncertainties and external disturbances, $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$ and $G(\mathbf{x}) \in \mathbb{R}^{n\times(n+m)}$ are smooth functions, and $(\cdot)^T$ denotes the transpose of a vector or a matrix. In addition, for the interest of this brief, we assume that $\mathbf{f}(\mathbf{0}) = \mathbf{0}$. Note that, in the description of (1) we have assumed that the system has control input redundancy.

The main goal of this brief is to synthesize a control law under which the stabilization task can be achieved even when the system experiences actuator outage with the number of healthy actuators being no less than n. In this section, both passive and active designs will be presented.

A. Passive Reliable Controllers

In passive reliable designs, the actuators are predivided into two groups $\mathcal{F}$ and $\mathcal{H}$, within which we assume that during the operation, all of the actuators in $\mathcal{H}$ must be healthy while those in $\mathcal{F}$ are allowed to fail. System (1) then becomes

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \qquad \dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}} + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + \mathbf{d}. \tag{2}$$

Since the nonsingularity assumption of $G_{\mathcal{H}}(\mathbf{x})$ is necessary for the existence of equivalent control in VSC design when all the actuators in $\mathcal{F}$ fail to operate [6], we assume that the preselected healthy actuators satisfying $\mathbf{u}_{\mathcal{H}} \in \mathbb{R}^n$, and $G_{\mathcal{H}}(\mathbf{x}) \in \mathbb{R}^{n\times n}$ is a nonsingular matrix. This assumption implies that the preselected susceptible actuators have assumed to be as many as possible.

To begin with, we first consider the design of $\mathbf{u}_{\mathcal{H}}$. Suppose that some or all of the actuators in $\mathcal{F}$ work abnormally, (2) becomes $\dot{\mathbf{x}}_1 = \mathbf{x}_2$ and $\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + G_{\mathcal{F}}(\mathbf{x})(\mathbf{u}_{\mathcal{F}}^* - \mathbf{u}_{\mathcal{F}}) + \mathbf{d}$, where $\mathbf{u}_{\mathcal{F}}^*$ and $\mathbf{u}_{\mathcal{F}}$ denote the actual and the designed control values for those actuators in $\mathcal{F}$, respectively. The idea of this approach is to treat the faulty term $G_{\mathcal{F}}(\mathbf{x})(\mathbf{u}_{\mathcal{F}}^* - \mathbf{u}_{\mathcal{F}})$ as an additional disturbance and organize a control law to compensate for the fault. Details are given as follows. Choose a sliding surface as

$$\mathbf{s} = \mathbf{x}_2 + M\mathbf{x}_1 = \mathbf{0} \tag{3}$$

where $M \in \mathbb{R}^{n\times n}$ is a positive-definite matrix. It is noted that if the system states keep staying on the sliding surface, the reduced model will have the form $\dot{\mathbf{x}}_1 + M\mathbf{x}_1 = \mathbf{0}$, which implies that $\mathbf{x}_1 \to \mathbf{0}$ and $\mathbf{x}_2 = -M\mathbf{x}_1 \to \mathbf{0}$ exponentially. That is, the stabilization performance will be fulfilled and the main goal of this brief is then achieved. From (3), we have

$$\dot{\mathbf{s}} = \mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}}^* + \mathbf{d} + M\mathbf{x}_2. \tag{4}$$

In order to compensate for the effect of disturbance and fault, we impose the next assumption.

Assumption 1: There exists nonnegative scalar functions $\rho_i(\mathbf{x}, t)$ such that, for $i = 1, \ldots, n$, $|(G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}}^*)_i| + |d_i| \le \rho_i(\mathbf{x}, t)$, where $(\cdot)_i$ denotes the ith entry of a vector.

Following the VSC design procedure [17], the VSC law is designed to be

$$\mathbf{u}_{\mathcal{H}} = -G_{\mathcal{H}}^{-1}(\mathbf{x})\{\mathbf{f}(\mathbf{x}) + M\mathbf{x}_2 + \Lambda_{\mathcal{H}} \cdot \operatorname{sgn}(\mathbf{s})\}. \tag{5}$$

Here, $\Lambda_{\mathcal{H}} = \operatorname{diag}(\rho_1(\mathbf{x}, t) + \eta_1, \ldots, \rho_n(\mathbf{x}, t) + \eta_n)$ with $\eta_i > 0$ for all $i = 1, \ldots, n$, $\operatorname{sgn}(\cdot)$ denotes the sign function and $\operatorname{sgn}(\mathbf{s}) := (\operatorname{sgn}(s_1), \ldots, \operatorname{sgn}(s_n))^T$. Under the control $\mathbf{u}_{\mathcal{H}}$, it follows from (4) and Assumption 1 that $\mathbf{s}^T\dot{\mathbf{s}} \le -\sum_{i=1}^n \eta_i \cdot |s_i|$. This inequality implies that the system states will reach the sliding surface in a finite time [17]. In fact, the larger the constants $\eta_1, \ldots, \eta_n$ we selected, the shorter the first time the system states reach the sliding surface [17].

In addition to the design of $\mathbf{u}_{\mathcal{H}}$ as discussed before, we now investigate the design of $\mathbf{u}_{\mathcal{F}}$ to promote the overall system performance when some or all of the actuators in $\mathcal{F}$ are healthy. The governing equations are now given by (2). From (2), (3), and (5), we have $\mathbf{s}^T\dot{\mathbf{s}} \le \mathbf{s}^T G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}} - \sum_{i=1}^n \eta_i \cdot |s_i|$. Clearly, one of the choices of $\mathbf{u}_{\mathcal{F}}$ to make system states approach the sliding surface faster than in the case of $\mathbf{u}_{\mathcal{F}} = \mathbf{0}$ is

$$\mathbf{u}_{\mathcal{F}} = -\Lambda_{\mathcal{F}} \cdot \operatorname{sgn}\left(G_{\mathcal{F}}^T(\mathbf{x})\mathbf{s}\right) \tag{6}$$

where $\Lambda_{\mathcal{F}} = \operatorname{diag}(\eta_{n+1}, \ldots, \eta_{n+m})$ and $\eta_{n+i} \ge 0$ for all $i = 1, \ldots, m$. These derivations show that the magnitude of control gains η_{n+i}, $i = 1, \ldots, m$, of actuators in $\mathbf{u}_{\mathcal{F}}$ that guarantee stabilization performance may vary from 0 to the allowable maximum control input magnitude. That is, it allows the situation of actuators in $\mathbf{u}_{\mathcal{F}}$ to be total failure, partial failure, attenuation or amplification in any order and any combination. These lead to the following result.

Theorem 1: Suppose that Assumption 1 holds. Then the origin of (2) is locally asymptotically stable (LAS) under the control laws given by (5) and (6) even when some or all of the actuators in $\mathcal{F}$ experience abnormal operation.

B. Active Reliable Controllers

The passive reliable design discussed before does not need the information of FDD, but does need to prespecify those actuators that are allowed to fail. It should be noted that it is, in general, very difficult to define the healthy and the susceptible actuators before faults occur. Although passive reliable control might achieve stabilization performance, it is a conservative method in that its controllers are designed based on the faulty system without any change in control law even when faults occur. Due to the lack of FDD information, the passive reliable design often overestimates the magnitude of faults. This overestimation might result in undesirable performances, including wasting of control energy and causing the designed control to exceed the allowable maximum control input magnitude. To improve the performance of passive reliable design, in the following, we consider the active control issue.

Before the occurrence of faults, the engineers may take any kind of control to fulfill their desired system performances. When fault happens, the control system is likely to be unstable and may yield an undesirable transient (see, e.g., [11]). After the fault is detected and diagnosed, the control law is guided to switch to an active reliable control and the system states are expected to converge, as described in the following. We assume without loss of any generality that faults happen at control channels $u_{k+1},\ldots,u_{n+m}$, $k\geq n$, and that the actual output values of these faulty control channels are successfully detected and diagnosed as $u_j^*=\hat{u}_j+\Delta u_j$ for $j=k+1,\ldots,n+m$. Here, $\hat{u}_j$ and Δu_j denote the estimated control value and estimated error, respectively. With the same definition of the sliding surface as given by (3), we have $\dot{\mathbf{s}}=M\mathbf{x}_2+\mathbf{f}(\mathbf{x})+G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}}+G_{\mathcal{F}}(\mathbf{x})(\hat{\mathbf{u}}_{\mathcal{F}}+\Delta\mathbf{u}_{\mathcal{F}})+\mathbf{d}$. Here, $\mathbf{u}_{\mathcal{H}}=(u_1,\ldots,u_k)^T$, $\mathbf{u}_{\mathcal{F}}=(u_{k+1},\ldots,u_{n+m})^T$, $\hat{\mathbf{u}}_{\mathcal{F}}=(\hat{u}_{k+1},\ldots,\hat{u}_{n+m})^T$, $\Delta\mathbf{u}_{\mathcal{F}}=(\Delta u_{k+1},\ldots,\Delta u_{n+m})^T$, $G_{\mathcal{H}}(\mathbf{x})=[\mathbf{g}_1(\mathbf{x}),\ldots,\mathbf{g}_k(\mathbf{x})]$ and $G_{\mathcal{F}}(\mathbf{x})=[\mathbf{g}_{k+1}(\mathbf{x}),\ldots,\mathbf{g}_{n+m}(\mathbf{x})]$, and $\mathbf{g}_j(\mathbf{x})$ denote the jth column of $G(\mathbf{x})$. This time we treat $G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}}+\mathbf{d}$ as a disturbance and impose the following assumption.

Assumption 2: There exists nonnegative scalar functions $\sigma_i(\mathbf{x},t)$ such that, for $i=1,\ldots,n$, $|(G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}})_i|+|d_i|\leq\sigma_i(\mathbf{x},t)$.

Clearly, the least upper bound $\sigma_i(\mathbf{x},t)$ for the uncertainties and disturbances in Assumption 2 is, in general, much less than the one $\rho_i(\mathbf{x},t)$ given in Assumption 1 if the estimated errors $|\Delta u_j|$ are small for $j=k+1,\ldots,n+m$. Indeed, the more accurate the fault diagnosis is, the smaller the estimated upper bound will be. Using the same design procedures as those for a passive approach, the VSC laws for those of healthy actuators are designed to be

$$\mathbf{u}_{\mathcal{H}}=-G_{\mathcal{H}}^T(\mathbf{x})\left(G_{\mathcal{H}}(\mathbf{x})G_{\mathcal{H}}^T(\mathbf{x})\right)^{-1}\times\{M\mathbf{x}_2+\mathbf{f}(\mathbf{x})+G_{\mathcal{F}}(\mathbf{x})\hat{\mathbf{u}}_{\mathcal{F}}+\Lambda_{\mathcal{H}}\cdot\mathrm{sgn}(\mathbf{s})\}\quad(7)$$

where $\Lambda_{\mathcal{H}}=\mathrm{diag}(\sigma_1(\mathbf{x},t)+\eta_1,\ldots,\sigma_n(\mathbf{x},t)+\eta_n)$ and $\eta_i>0$ for $i=1,\ldots,n$. Note that, $\mathbf{u}_{\mathcal{H}}$ as given by (7), contains an extra term $G_{\mathcal{F}}(\mathbf{x})\hat{\mathbf{u}}_{\mathcal{F}}$ involving the information of diagnosis. As a result, $\mathbf{s}^T\dot{\mathbf{s}}=\mathbf{s}^T\{G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}}+\mathbf{d}-\Lambda_{\mathcal{H}}\cdot\mathrm{sgn}(\mathbf{s})\}\leq-\sum_{i=1}^n\eta_i|s_i|$ by the use of Assumption 2. These lead to the following result.

Theorem 2: Suppose that (1) experiences actuator faults at the control channels $u_{k+1},\ldots,u_{n+m}$, $k\geq n$, with estimated values $\hat{u}_j$ and errors Δu_j. If, in addition, the faults and disturbances satisfy Assumption 2, then the origin of system (1) is LAS under the control laws given by (7).

III. Application To Spacecraft Attitude Stabilization

A spacecraft attitude model in a circular orbit (see, e.g., [5], [19], and [21]) can be described in the form of (1) with $n=3$ [5], [15]. The six state variables denote the three Euler's angles (ϕ,θ,ψ) and their derivatives. For simplicity, we assume in this study, that thruster is the only applied control force and there is an actuator redundancy to perform the reliable task. By letting $\mathbf{x}=(\phi,\theta,\psi,\dot{\phi},\dot{\theta},\dot{\psi})^T$ and $\mathbf{f}(\mathbf{x})=(f_1(\mathbf{x}),f_2(\mathbf{x}),f_3(\mathbf{x}))^T$, the overall system dynamics is described as follows [15]:

$$\begin{aligned}f_1(\mathbf{x})=&\,\omega_0x_6cx_3cx_2-\omega_0x_5sx_3sx_2+\frac{I_y-I_z}{I_x}\\&\times\Big[x_5x_6+\omega_0x_5cx_1sx_3sx_2+\omega_0x_5cx_3sx_1\\&\quad+\omega_0x_6cx_3cx_1+\frac{1}{2}\omega_0^2s(2x_3)c^2x_1sx_2\\&\quad+\frac{1}{2}w_0^2c^2x_3s(2x_1)-\omega_0x_6sx_3sx_2sx_1\\&\quad-\frac{1}{2}\omega_0^2s^2x_2s^2x_3s(2x_1)-\frac{1}{2}\omega_0^2s(2x_3)sx_2s^2x_1\\&\quad-\frac{3}{2}\omega_0^2c^2x_2s(2x_1)\Big]\\f_2(\mathbf{x})=&\,\omega_0x_6sx_3cx_1+\omega_0x_4cx_3sx_1+\omega_0x_6cx_3sx_2sx_1\\&+\omega_0x_5sx_3cx_2sx_1+\omega_0x_4sx_3sx_2cx_1+\frac{I_z-I_x}{I_y}\\&\times\Big[x_4x_6+\omega_0x_4cx_1sx_3sx_2+\omega_0x_4cx_3sx_1\\&\quad-\omega_0x_6sx_3cx_2-\frac{1}{2}\omega_0^2s(2x_2)s^2x_3cx_1\\&\quad-\frac{1}{2}w_0^2cx_2sx_1s(2x_3)+\frac{3}{2}\omega_0^2s(2x_2)cx_1\Big]\\f_3(\mathbf{x})=&\,\omega_0x_4sx_1sx_3sx_2-\omega_0x_6cx_1cx_3sx_2-\omega_0x_5cx_1sx_3cx_2\\&+\omega_0x_6sx_3sx_1-\omega_0x_4cx_3cx_1+\frac{I_x-I_y}{I_z}\\&\times\Big[x_4x_5+\omega_0x_4cx_3cx_1-\omega_0x_4sx_3sx_2sx_1\\&\quad-\omega_0x_5sx_3cx_2-\frac{1}{2}\omega_0^2s(2x_3)cx_2cx_1\\&\quad+\frac{1}{2}w_0^2s^2x_3sx_1s(2x_2)-\frac{3}{2}\omega_0^2s(2x_2)sx_1\Big]\end{aligned}$$

$$G(\mathbf{x})=\begin{pmatrix}0.67&0.67&0.67&0.67\\0.69&-0.69&-0.69&0.69\\0.28&0.28&-0.28&-0.28\end{pmatrix}.$$

Here, I_x, I_y, and I_z are the inertia with respect to the three body coordinate axes, ω_0 denotes the constant orbital rate, and c and s denote the cos and sin functions, respectively.

In the following, we present an observer to detect the occurrence of thruster faults and to diagnose their location and magnitude for the purpose of active reliable mission. A simulation example is then given to demonstrate the performance of the proposed schemes.

A. FDD Observer Design

Since the three Euler rates can be expressed in terms of angular velocity vector, which is available through accelerometer and gyroscope [18], in this section, we assume that all of the state variables are available for measurement and that $G(\mathbf{x})$ in

(1) is a constant matrix. The main idea of this design is to decouple the control input so that the fault associated with each channel can be distinguished and diagnosed. Details are given as follows. Consider the coordinate transformation $\mathbf{z}_1 = \mathbf{x}_1$ and $\mathbf{z}_2 = P\mathbf{x}_2$, where $P := (\mathbf{g}_1\ \mathbf{g}_2\ \mathbf{g}_3)^{-1}$ and $\mathbf{g}_i$ denotes the ith column of $G(\mathbf{x})$. Equation (1) with the new state variables becomes

$$\dot{\mathbf{z}}_1 = P^{-1}\mathbf{z}_2, \qquad \dot{\mathbf{z}}_2 = \mathbf{f}^{\text{new}}(\mathbf{z}) + G^{\text{new}}(\mathbf{z})\mathbf{u} + P\mathbf{d} \tag{8}$$

where

$$\begin{aligned} \mathbf{f}^{\text{new}}(\mathbf{z}) &= P\mathbf{f}(\mathbf{z}_1, P^{-1}\mathbf{z}_2) \\ G^{\text{new}}(\mathbf{z}) &= PG(\mathbf{z}_1, P^{-1}\mathbf{z}_2) \\ &= \begin{pmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & l_3 \end{pmatrix}. \end{aligned} \tag{9}$$

Note that all constants l_1, l_2, and l_3 are nonzero since every three columns taken out from $G(\mathbf{x})$ are linearly independent. Moreover, from (8) and (9), z_{i+3} is only affected by actuators u_i and u_4 for $i = 1, 2, 3$. It is known that the tracking performance is hard to achieve with only one or two actuators; therefore, in the following, we only consider the cases of a single actuator fault.

With the aid of the transformed system (8), an observer is proposed to be

$$\dot{\xi}_i = f_i^{\text{new}}(\mathbf{z}) + u_i + l_i u_4 + k_i \cdot (z_{i+3} - \xi_i), \qquad i = 1, 2, 3 \tag{10}$$

and $k_i > 0$. Define the residual signals to be

$$r_i = z_{i+3} - \xi_i, \qquad i = 1, 2, 3. \tag{11}$$

We will claim that any single actuator fault can be detected and diagnosed. To see this, suppose that only the first actuator experiences a fault with actual value u_1^*. Define

$$\Delta u_1 = u_1^* - u_1 \tag{12}$$

the error between actual and designed control values. It follows from (8) and (9) that

$$\begin{aligned} \dot{z}_{i+3} &= f_i^{\text{new}}(\mathbf{z}) + u_i + l_i u_4 + \delta_i \cdot \Delta u_i + (P\mathbf{d})_i \\ i &= 1, 2, 3 \end{aligned} \tag{13}$$

where $\delta_1 = 1$ and $\delta_2 = \delta_3 = 0$. From (10), (11), and (13), we have

$$\dot{r}_i = -k_i r_i + \delta_i \cdot \Delta u_i + (P\mathbf{d})_i, \qquad i = 1, 2, 3. \tag{14}$$

Clearly, the constant k_i determines the convergence rate of r_i, and r_i depends on both Δu_i and $(P\mathbf{d})_i$. Suppose the disturbance is small enough to be neglected, then after a short time transient

$$r_i \to \delta_i \cdot \frac{\Delta u_i}{k_i}. \tag{15}$$

Thus, the fault at the first actuator only affects r_1, and the actual value u_1^* can be diagnosed from (12) and (15) as $u_1 + k_1 r_1$ at an exponential rate of k_1. Similarly, it is easy to show that the ith actuator fault, for $i = 1, 2, 3$, only affects r_i, and that the fourth actuator fault affects all of r_1, r_2 and r_3. Moreover, the actual value of u_i^* can be easily diagnosed at an exponential rate. Details are omitted.

B. Simulation Results

The parameters of this example are chosen as follows. The spacecraft parameters: $I_x = I_z = 2000\ \text{N} \cdot \text{m} \cdot \text{s}^2$, $I_y = 400\ \text{N} \cdot \text{m} \cdot s^2$, $\omega_0 = 1.0312 \times 10^{-3}$ rad/s, and $|u_i| \le 1$ for all i. The VSRC parameters: $M = 2I_3$, $\eta_i = 0.4$ for all η_i in $\Lambda_{\mathcal{H}}$ and $\Lambda_{\mathcal{F}}$. $\rho_i(\mathbf{x}, t) := \|(G_{\mathcal{F}}^T(\mathbf{x}))_i\|_1 + \|d_i\|_\infty$ and $\sigma_i(\mathbf{x}, t) := \|d_i\|_\infty$, where $(\cdot)_i$ and $\|\cdot\|_1$ denote the ith column of a matrix and the 1-norm of a vector, respectively, and $\|d_i\|_\infty := \sup_t |d_i(t)|$. The sign function is replaced by the saturation function with boundary layer width 0.05 to alleviate the chattering produced by the sign function. The FDD parameters: $k_i = 10$ for i =1, 2, 3, and the alarm is fired if $\max_{1 \le i \le 3} |r_i| \ge 0.01$. The initial states and the desired attitude are $\mathbf{x}(0) = (-0.7, -0.07, 1.5, 0.3, 1.3, -0.2)^T$ and $\mathbf{x}_d(t) \equiv \mathbf{0}$, respectively. To compare the results of VSRC with those produced by LQR reliable design [13], we adopt the numerical scheme of [8] to approximate the LQR reliable controllers up to order 3 with quadratic performance being chosen as $\int (\mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}) dt$, $Q = I_6$, and $R = I_4$. Before alarm, both the active reliable designs of VSC and LQR adopt their traditional nonreliable designs as if all the actuators are available. For instance, the nonreliable VSC-type controller has the form of (7) with all the actuators being healthy. Whenever there is an alarm, the associated active reliable controllers are then activated according to the FDD information. With the threshold setting and from (15), the alarm is fired if $|\Delta u_i|$ is greater than 10% of the actuators' constraint and the system is free from disturbances. In general, the selection of threshold is a tradeoff between the probability of false alarm and the probability of missed detection [28]. The threshold can be set lower to promote the sensitivity of FDD; however, a lower threshold setting might result in a false alarm due to disturbances and/or measurement noises. Since (14) represents a low pass filter $1/(j\omega + k_i)$ which can greatly attenuate high frequency noises, therefore, the suggested threshold should be selected considering the values of k_i, and the estimated magnitude and frequency spectra of possible disturbances.

Numerical simulations are summarized in Tables I–III. Among these, the passive controllers of LQR and VSRC are designed by considering u_2 as the susceptible actuator. These three tables display the simulation results when the system experiences u_1 outage, u_2 outage, and u_2 outage with an external disturbance of magnitude $|d_i| \le 0.2$ for each i, respectively. Both the outages of u_1 and u_2 are assumed to happen at $t = 1$. To investigate the performance of active VSRC with different control parameters, we also consider the case (denoted by Active VSRC2) of $\eta_1 = 1$, $\eta_2 = 1$ and $\eta_3 = 0.7$. A typical scenario is shown in Figs. 1 and 2 for u_1 experiencing outage. It is shown from Fig. 1 that both the active designs and the passive LQR design are able to achieve the attitude stabilization task, while that of the passive VSRC fails since the outage actuator u_1 does not coincide with the susceptible actuator u_2. Although the passive LQR also achieves the attitude stabilization, the system states are observed convergent to the

TABLE I
PERFORMANCES OF VSRC AND LQR RELIABLE DESIGNS (REGARDING u_2 AS SUSCEPTIBLE ACTUATOR) WHEN u_1 FAILS

Performance index	Controller				
	Passive LQR	Active LQR	Passive VSRC	Active VSRC	Active VSRC2
Time when $\max_i \lvert x_i \rvert < 0.01$	15.028	9.168	X	9.609	5.123
$\int \mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}$	10.054	9.130	X	14.095	10.376
$\int \mathbf{u}^T R \mathbf{u}$	2.416	2.494	X	0.489	2.164
$\lVert \mathbf{u} \rVert_\infty$	1	1	X	0.561	1

TABLE II
PERFORMANCES OF VSRC AND LQR RELIABLE DESIGNS (REGARDING u_2 AS SUSCEPTIBLE ACTUATOR) WHEN u_2 FAILS

Performance index	Controller				
	Passive LQR	Active LQR	Passive VSRC	Active VSRC	Active VSRC2
Time when $\max_i \lvert x_i \rvert < 0.01$	8.564	8.501	5.611	9.873	5.365
$\int \mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}$	8.455	8.461	10.127	14.530	9.655
$\int \mathbf{u}^T R \mathbf{u}$	1.806	1.886	1.891	0.495	1.427
$\lVert \mathbf{u} \rVert_\infty$	1	1	1	0.561	1

TABLE III
PERFORMANCES OF VSRC AND LQR RELIABLE DESIGNS WITH DISTURBANCE (REGARDING u_2 AS SUSCEPTIBLE ACTUATOR) WHEN u_2 FAILS

Performance index	Controller				
	Passive LQR	Active LQR	Passive VSRC	Active VSRC	Active VSRC2
Time when $\max_i \lvert x_i \rvert < 0.01$	X	X	4.850	7.240	4.810
$\int \mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}$	X	X	9.922	11.333	9.573
$\int \mathbf{u}^T R \mathbf{u}$	X	X	2.268	1	2.033
$\lVert \mathbf{u} \rVert_\infty$	X	X	1	0.857	1

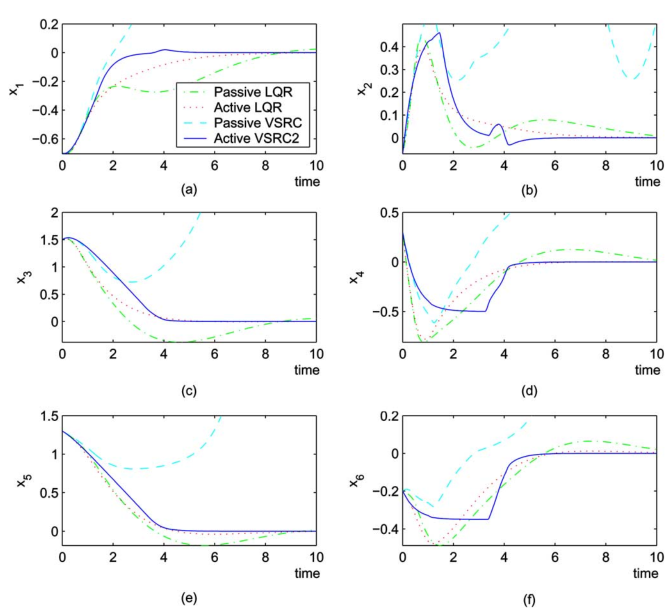

Fig. 1. Time histories of the six system states.

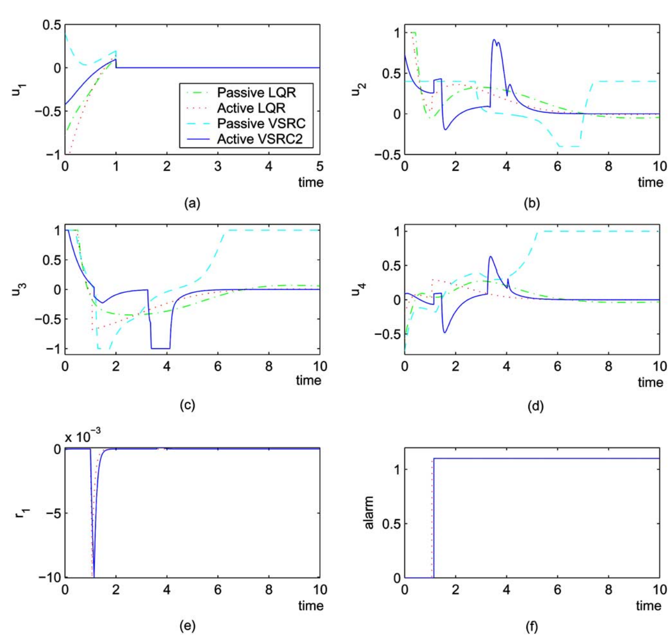

Fig. 2. (a)–(d) Time histories of controls. (e) Residual signals of the FDD. (f) Alarm signals.

desired attitude much slower than those of active designs. From Fig. 2(e) the magnitude of the first residual signals exceed the threshold for both active designs at around $t_{\rm LQR} = 1.071$ and $t_{\rm VSRC2} = 1.143$, respectively, which can also be seen from the alarm signals given in Fig. 2(f). The alarm $t_{\rm LQR}$ for LQR design is noted a little earlier than $t_{\rm VSRC2}$ because the initial magnitude of faults (at $t = 1$) by LQR and VSC designs have the relation $|\Delta u_1|_{\mathrm{LQR}} = 0.146 > |\Delta u_1|_{\mathrm{VSRC2}} = 0.097$, which can be seen from Fig. 2(a), and the effect of Δu_1 on r_1, as given by (14) and (15). After the fault has been detected and diagnosed, the associated active controller is activated and the magnitude of the residual is soon decreased, as shown in Fig. 2(e). The six states for active designs are observed, as expected, from Fig. 1 to converge to zero. The associated controls are shown in Fig. 2(a)–(d). It is noted from these figures that there are several peaks for the control curves of the Active VSRC2. Physically, these peaks correspond to apply the maximum acceleration to the satellite, followed by applying brake to force the satellite keeping at the desired attitude. The curves of x_1 and x_2 in Fig. 1(a)–(b) are also observed to have a small peak near $t = 4$. These two peaks are the effect of the saturation of u_3, which can be shown from Fig. 2(c). Table I summarizes these performances. It is shown from this table that most of the performances of active LQR are better than those of passive LQR, especially for the convergence time (i.e., the time when $\max_i |x_i| < 0.01$) and the quadratic performance $\int \mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u}$. On the other hand, the performances of Active VSRC2 except for the quadratic performance, which is known optimal when adopting the LQR design, are better than those of active LQR.

Table II considers the case when the outage actuator coincides with the susceptible actuator. It implies that the passive and the active LQR designs use the same controller after alarm. Table II shows that the active LQR scheme achieves a smaller convergence time, but consumes a little more energy than the passive one. Similarly, the passive and the active VSRCs also use the same type of controller after alarm, however, under the same η_i, the convergence time of the passive VSRC is much shorter than that of active VSRC because the passive VSRC uses a larger control effort, appeared in $\rho_i(\mathbf{x}, t)$, due to overestimation of the fault. This can also be shown from the energy consumption $\int \mathbf{u}^T R\mathbf{u}$, in which the value for passive VSRC is much larger than that of active VSRC. It is worth noting that the active VSRC requires the smallest $\|\mathbf{u}\|_\infty$ to perform the stabilization task. When tuning up the value of η_i, the performances of active VSRC (i.e., those of Active VSRC2) are shown to be significantly improved. Again, the performances of Active VSRC2 are better than those of LQRs except for the quadratic performance. Finally, Table III presents the results when the system is corrupted with disturbance. The scenarios for VSRCs are similar to those of Table II with smaller convergence time and quadratic performance but larger energy consumption, because $\rho_i(\mathbf{x}, t)$ and $\sigma_i(\mathbf{x}, t)$ for VSRCs in Table III are higher than those for Table II due to disturbances. The alarms for active VSRCs in this case are fired at around $t_{\mathrm{VSRC}} = 1.14$ and $t_{\mathrm{VSRC2}} = 1.04$, respectively. On the other hand, the associated states of LQR reliable designs are found to oscillate near the origin (i.e., $\max_i |x_i| \not< 0.01$). From this example, it can be concluded that the proposed active VSRC is more robust than those of LQRs.

IV. Conclusion

VSC-type stabilization laws have been proposed in this brief to study the reliable control issues of a set of second-order nonlinear systems. Unlike the reliable laws of optimal approaches, these VSRCs do not depend on a solution of a HJ equation. By the use of the VSC technique, the proposed VSRCs have been shown to be able to achieve the stabilization task. An illustrative example was also given to demonstrate the use of the main results and compare system performances with those by LQR reliable designs. It is shown from the example that the active VSRC is more flexible and effective than the other designs except for the quadratic performance, which is optimal when adopting the LQR scheme, because it does not need to prespecify susceptible actuators and because it allows more space for control parameter adjustment. In addition, the active VSRC is shown more robust than the LQR reliable schemes, especially, for system with uncertainties and/or disturbances.

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