Performance Modeling of Integrated Mobile Prepaid Services

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***Abstract*—Prepaid personal communication service users have outnumbered postpaid users. This paper studies the charging issues of an integrated Global System for Mobile Communications (GSM) and General Packet Radio Service (GPRS) prepaid service, where a single prepaid account provides a user both voice and data services. The call setup and charging procedures for GSM and GPRS are presented using the Customized Applications for Mobile network Enhanced Logic network architecture. To reduce the probability of terminating both ongoing voice and data calls, we suggest that no new call be admitted when the user credit is below a threshold. An analytic model has been developed to evaluate the performance of the approach. Computer simulations have also been used to verify the results. The numeric results indicate that the force-termination probability can be significantly reduced by choosing an appropriate threshold of the user credit.**

***Index Terms*—Customized Applications for Mobile network Enhanced Logic (CAMEL), General Packet Radio Service (GPRS), prepaid services.**

I. Introduction

DUE TO advances in wireless communication technologies, the personal communication service (PCS) market has grown exponentially over the past ten years. Prepaid service is a telecommunication service that requires subscribers to pay before they make calls. This service was offered in Europe and Asia in 1982 and became popular in the U.S. in 1992 [2]. In 1997, there were about 60 million GSM subscribers around the world, and 8% of them subscribed to prepaid service. At the end of the third quarter of in 2001, 50% of the subscribers worldwide were prepaid users [8]. In the U.S., the prepaid calling market grew by 56% to about $2 billion in 1998 and was expected to maintain a high growth rate to 2005 [14]. Asian countries such as the Philippines, Australia, Hong Kong, Singapore, and Taiwan, R.O.C., have already shown successful examples for prepaid services.

Four billing technologies have been used in mobile prepaid service, namely 1) the hot-billing approach, 2) the service-node approach, 3) the intelligent network (IN) approach, and 4) the handset-based approach. We have studied the performance of the hot-billing approach in [4] and the service-node approach in [5]. However, these studies focused only on voice services. As the average revenue per user (ARPU) decreases, operators must offer more value-added services to stimulate user demand. By providing data services and multimedia services, the ARPU is expected to increase [6].

General Packet Radio Service (GPRS) reuses the Global System for Mobile Communications (GSM) infrastructure and provides "always-on" connection, flexible radio resource allocation, and fast data transmission to customers [15]. Although GPRS is just being launched, it is estimated that it will attract 110 million GPRS users across western Europe by 2006, which may generate € 23 billion in mobile data revenue [7]. By providing integrated prepaid service, which includes voice service and data service in the GSM and GPRS network, the operator can offset the decline of the voice revenue.

In GSM, billing records are generated by the mobile switching center (MSC) when a voice call terminates. The information in the record includes type of service, date/time of usage, user identification, and location information [9]. In GPRS, the charging information of data service is collected by the serving GPRS support node (SGSN) and the gateway GPRS support node (GGSN). The charging information collected by the SGSN includes the charging ID, access point name (APN) ID, location of the mobile station (MS), and amount of data transmitted through the radio interface with quality-of-service (QoS) profiles [1], [11]. The GGSN collects the information of external data network usage including the source address and the destination address of the packets and the amount of transmitted packets to the external network.

Customized Applications for Mobile network Enhanced Logic (CAMEL) integrates the IN techniques into the mobile telecommunication network, enabling the operator to provide service to its subscribers inside and outside the home network. In phase 3, CAMEL specified the capability to control GPRS sessions and Packet Data Protocol (PDP) contexts [12]. Very few studies have been done on the analysis of the prepaid services with concurrent sessions. We have included two related papers recently published. Cai presented secure authorization mechanisms for prepaid services [3]. Lin investigated the credit allocation problem of concurrent service sessions [10]. In this paper, we studied how to provide GSM and GPRS prepaid services under the CAMEL architecture. We design CAMEL message flows for GPRS prepaid service based on the attach/detach state model and PDP state control mode. In addition, we study the performance of the GSM/GPRS network where the charges (i.e., voice charge and data charge) of a subscriber are debited in one account. The prepaid credit may

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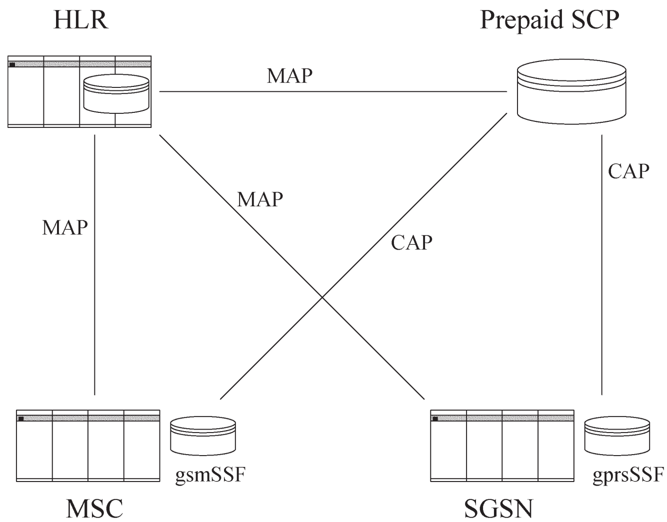

Fig. 1. CAMEL architecture for integrated GSM/GPRS prepaid services.

deplete when the subscriber is in either a voice conversation or a data transport session, or both. It is preferable for the billing system to avoid terminating both ongoing voice call and data sessions simultaneously when the prepaid credit runs out. We proposed a simple algorithm to reduce this probability and an analytic model to evaluate the improvement.

This paper is organized as follows: Section II describes GSM and GPRS prepaid service based on the CAMEL architecture. Section III presents the analytic models. The numeric results are discussed in Section IV. The conclusions are given in Section V.

II. Prepaid Services in CAMEL

Fig. 1 depicts the CAMEL network architecture for the integrated GSM and GPRS prepaid service. In this architecture, we add three components in the GSM/GPRS network for prepaid service, namely 1) prepaid service control point (P-SCP), 2) GSM service switching function (gsmSSF), and 3) GPRS service switching function (gprsSSF). The SGSN (MSC) interfaces with the P-SCP via the gprsSSF (gsmSSF) for CAMEL session control. The P-SCP integrates the billing of GSM and GPRS services in one account. When a subscriber originates a voice call, the gsmSSF issues a CAMEL application part (CAP) message "InitialDP (trigger detection point request)" to the P-SCP. The P-SCP checks the account status and starts a countdown timer if the call is allowed. When a subscriber originates a data session (activate a PDP context), the gprsSSF issues a CAP message "InitialDPGPRS (trigger detection point request)" to the P-SCP. The P-SCP checks the account status and gives predefined units of quota to the gprsSSF for the charging of the data session.

Fig. 2 illustrates the voice call origination procedure, which comprises the following steps.

Step 1) The prepaid customer initiates a call by dialing the called party's telephone number.

Step 2) The MSC/gsmSSF encounters an IN call setup trigger. The call setup process is suspended, and a CAP message "InitialDP" is sent to the P-SCP. The message includes the MS Integrated Services Digital Network, location information of the MS, and the called party telephone number. The P-SCP determines whether or not the customer can make the call by querying its database. Based on threshold processing parameters defined in the prepaid billing system, the P-SCP may deny or accept the call. (In this example, we assume that the call is accepted.)

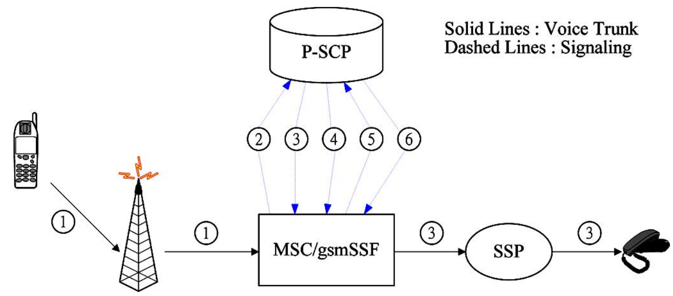

Fig. 2. IN prepaid call origination procedure.

Step 3) The P-SCP asks the MSC to resume the call setup procedure with a CAP message "ApplyCharging," and the call is eventually connected. The P-SCP starts a countdown timer. The rate of credit decrement (from the current balance) is derived from the carrier-defined threshold parameters, rate plan, destination, and time/date dependence.

Step 4) The call terminates when either the balance depletes or the call completes. If the countdown timer ends before the customer terminates the call, the P-SCP instructs the MSC with a message "ReleaseCall" to terminate the call. For normal call completion, this step does not exist.

Step 5) Once the call is completed, the MSC/gsmSSF encounters an IN call-release trigger, which sends a disconnect message "EventReportBCSM" to the P-SCP, indicating the completion time of the call.

Step 6) The P-SCP rates the completed call and updates the customer's prepaid balance accordingly. Then, it sends the current balance and cost of the call to the MSC. The MSC releases the call.

According to the popular view on GPRS, data volume is the natural measure for GPRS resource usage and should be the crucial parameter to drive the charge. This proposed charging principle reflects the GPRS vision that the user may stay always online and may send/receive data as the need arises. Fig. 3 illustrates the data call procedure and charging by data volume when the PDP context is activated, which comprises the following steps.

Step 1) A MS performs GPRS PDP context activation.

Step 2) The SGSN/gprsSSF sends an "InitialDPGPRS" message to the P-SCP, containing a service indicator as key to GPRS prepaid service. The P-SCP invokes the operator-defined GPRS prepaid data service. Tariff and authorization features are applied to the request for service. The P-SCP sends the result with predefined units of quota in an "ApplyCharging GPRS" message to the gprsSSF.

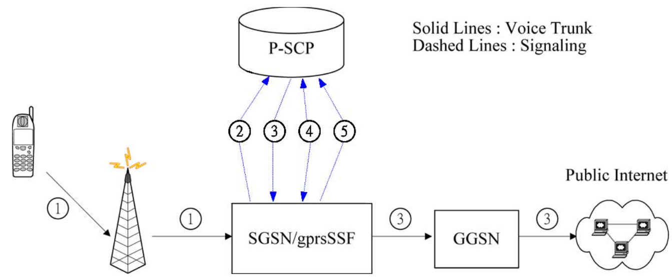

Fig. 3. Basics of prepaid GPRS call procedure.

Step 3) The SGSN receives the response that either authorizes or rejects the call. If authorized, the SGSN sends a "Create PDP Context Request" message to the GGSN, which contains the MS identifiers, location, QoS, and APN. The session of data access is continued.

Step 4) The SGSN also starts to decrement the quota of accessible data volume. When the quota is depleted, the SGSN suspends the session (data queued in buffer) and sends an "ApplyChargingReportGPRS" message to the P-SCP asking additional quota. The P-SCP starts a new check to the user account and sends the result with predefined units of quota in an "ApplyChargingGPRS" message to the gprsSSF again. The SGSN receives the response and lets the session to continue. The checking-quota process is repeated during the session.

Step 5) On call termination, the SGSN sends an "Apply-ChargingReportGPRS" message to the P-SCP, indicating the charge of the service. The SGSN sends a "PDP Context Disconnection" message to the MS.

Step 6) The account status notification feature can be enabled at the P-SCP. The P-SCP sends a status notification message to the short message service center, which is forwarded to the prepaid subscriber.

III. Analytic Model

In this section, we present an analytic model for GSM and GPRS prepaid services. For a GSM/GPRS network providing prepaid services, there are two types of calls, namely 1) voice calls and 2) data calls. The voice calls and data calls that originate from an MS are assumed to be Poisson with rates λ_1 and λ_2, respectively. The call holding time of voice calls and data calls is assumed to be exponentially distributed with mean $1/\mu_1$ and $1/\mu_2$, respectively.

Let B be the prepaid credit. We assume that the prepaid credit of voice service and data service shares the same account and is decremented by the P-SCP in a real-time fashion. When the credit of the subscriber is depleted, the P-SCP terminates ongoing calls. Let $P_{f,v}$, $P_{f,d}$, and $P_{f,vd}$ be the probabilities that a voice call, data call, and both types of calls are forced to terminate when the credit depletes, respectively. Note that

$$P_{f,v} + P_{f,d} + P_{f,vd} = 1. \tag{1}$$

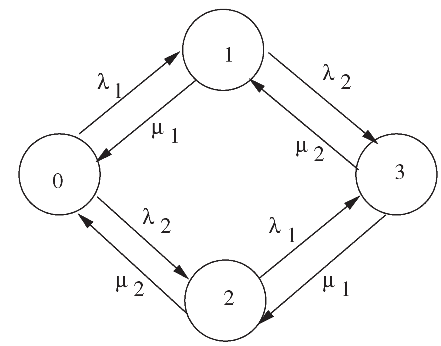

Fig. 4. State transition diagram of prepaid GSM/GPRS services.

Our previous study on prepaid voice calls shows that the decrement of a subscriber's credit by the successive calls of the subscriber can be modeled as a renewal process since the charge of each call is independent and has an identical distribution. Our study also indicated that the time when the prepaid credit depletes can be treated as a random observer of a renewal process if the initial credit is much larger than the charge of one call [4]. Hence, $P_{f,v}$, $P_{f,d}$, and $P_{f,vd}$ can be derived by using the theory of Markov chain. In this process, state 0 represents that the subscriber has no ongoing call. State 1 (state 2) represents that the subscriber is making a voice call (data call). State 3 represents that the subscriber is making a voice call and a data call simultaneously. Fig. 4 illustrates the state transition diagram of the prepaid GSM/GPRS services.

Let π_i be the steady-state probability that the system is at state i. Let r_1 be the charge rate of voice calls and r_2 be that of data calls. If the initial credit is sufficiently large, the probability that the credit depletes at state i is proportional to the fraction of credit depleted at state i, i.e., $P_{f,v} : P_{f,d} : P_{f,vd} = \pi_1 r_1 : \pi_2 r_2 : \pi_3(r_1 + r_2)$. From Fig. 4, we have the balance equations as follows:

$$\begin{aligned}
(\lambda_1 + \lambda_2)\pi_0 &= \mu_1\pi_1 + \mu_2\pi_2 \\
(\lambda_2 + \mu_1)\pi_1 &= \lambda_1\pi_0 + \mu_2\pi_3 \\
(\lambda_1 + \mu_2)\pi_2 &= \lambda_2\pi_0 + \mu_1\pi_3 \\
(\mu_1 + \mu_2)\pi_3 &= \lambda_2\pi_1 + \lambda_1\pi_2.
\end{aligned} \tag{2}$$

Note that

$$\sum_{i=0}^{3} \pi_i = 1. \tag{3}$$

From (2) and (3), π_0, π_1, π_2, and π_3 can be expressed as follows:

$$\begin{aligned}
\pi_0 &= \frac{\mu_1\mu_2}{\lambda_1\mu_2 + \lambda_2\mu_1 + \lambda_1\lambda_2 + \mu_1\mu_2} \\
\pi_1 &= \frac{\lambda_1\mu_2}{\lambda_1\mu_2 + \lambda_2\mu_1 + \lambda_1\lambda_2 + \mu_1\mu_2} \\
\pi_2 &= \frac{\lambda_2\mu_1}{\lambda_1\mu_2 + \lambda_2\mu_1 + \lambda_1\lambda_2 + \mu_1\mu_2} \\
\pi_3 &= \frac{\lambda_1\lambda_2}{\lambda_1\mu_2 + \lambda_2\mu_1 + \lambda_1\lambda_2 + \mu_1\mu_2}.
\end{aligned} \tag{4}$$

From (1)–(4), we have

$$P_{f,v} = \frac{r_1\lambda_1\mu_2}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}$$
$$P_{f,d} = \frac{r_2\lambda_2\mu_1}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}$$
$$P_{f,vd} = \frac{(r_1+r_2)\lambda_1\lambda_2}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}. \quad (5)$$

For prepaid GSM/GPRS services, a subscriber would be annoyed if both ongoing voice call and data call are forced to terminate simultaneously when the credit is depleted. To enhance the user's satisfaction, we propose an algorithm, namely, no-more-call (NMC), to reduce the force-termination probability. It is described as follows.

Algorithm NMC: When the user credit is less than b, no more new calls, except ongoing calls, can be served.

Note that b can be sufficiently small such that the remaining credit accommodates only ongoing calls.

A. Analytic Model for NMC

In this section, we present an analytic model to derive the force-termination probability of the NMC algorithm. The credit depletes in one of the following cases.

- Case I) The subscriber is only in a voice conversation when $B = b$.
  - Case Ia) The voice call depletes all remaining credit and is forced to terminate.
  - Case Ib) The voice call is completed, and there is remaining credit.
- Case II) The subscriber is only using the data service when $B = b$.
  - Case IIa) The data call depletes the remaining credit and is forced to terminate.
  - Case IIb) The data call is completed, and there is remaining credit.
- Case III) The subscriber has both an ongoing voice call and an ongoing data call when $B = b$.
  - Case IIIa) Both calls deplete the remaining credit and are forced to terminate.
  - Case IIIb) The data call terminates before the credit is depleted. The voice call depletes the remaining credit and is forced to terminate.
  - Case IIIc) The voice call terminates before the credit is depleted. The data call depletes the remaining credit and is forced to terminate.
  - Case IIId) Both calls are completed, and there is remaining credit.

Let $P^*_{f,v}$ be the probability that a voice call is forced to terminate in NMC. If the subscriber's initial credit is sufficiently larger than b, Prob(Case I) $= P_{f,v}$, Prob(Case II) $= P_{f,d}$, and Prob(Case III) $= P_{f,vd}$. Let x and y be the amount of credit depleted by the voice call and data call after the remaining credit equals to b, but the calls are allowed to complete normally. Then, $P^*_{f,v}$ can be expressed as

$$\begin{aligned}
P^*_{f,v} &= \text{Prob(Case I)} \times \text{Prob(Case Ia|Case I)} \\
&\quad + \text{Prob(Case III)} \times \text{Prob(Case IIIb|Case III)} \\
&= P_{f,v} \int_{x=b/r_1}^{\infty} \mu_1 e^{-\mu_1 x} dx \\
&\quad + P_{f,vd} \int_{y=0}^{\frac{b}{r_1+r_2}} \int_{x=\frac{b-r_2 y}{r_1}}^{\infty} \mu_1 e^{-\mu_1 x} \mu_2 e^{-\mu_2 y} dx dy \\
&= \frac{r_1\lambda_1\mu_2 e^{-\left(\frac{b}{r_1}\right)\mu_1}}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2} \\
&\quad + \left[\frac{(r_1+r_2)\lambda_1\lambda_2 e^{-\left(\frac{b}{r_1}\right)\mu_1}}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}\right] \\
&\quad \times \left[\frac{\mu_2}{\mu_2 - \left(\frac{r_2}{r_1}\right)\mu_1}\right] \left\{1 - e^{-\left(\frac{b}{r_1+r_2}\right)\left[\mu_2 - \left(\frac{r_2}{r_1}\right)\mu_1\right]}\right\}.
\end{aligned} \quad (6)$$

Let $P^*_{f,d}$ be the probability that only a data call is forced to terminate in NMC and $P^*_{f,vd}$ be the probability that both the voice and data calls are forced to terminate in NMC. Using approaches similar to those previously mentioned, $P^*_{f,d}$ and $P^*_{f,vd}$ can be expressed as

$$\begin{aligned}
P^*_{f,d} &= \text{Prob(Case II)} \times \text{Prob(Case IIa|Case II)} \\
&\quad + \text{Prob(Case III)} \times \text{Prob(Case IIIc|Case III)} \\
&= P_{f,d} \int_{y=b/r_2}^{\infty} \mu_2 e^{-\mu_2 y} dy \\
&\quad + P_{f,vd} \int_{x=0}^{\frac{b}{r_1+r_2}} \int_{y=\frac{b-r_1 x}{r_2}}^{\infty} \mu_1 e^{-\mu_1 x} \mu_2 e^{-\mu_2 y} dy dx \\
&= \frac{r_2\lambda_2\mu_1 e^{-\left(\frac{b}{r_2}\right)\mu_2}}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2} \\
&\quad + \left[\frac{(r_1+r_2)\lambda_1\lambda_2 e^{-(\frac{b}{r_2})\mu_2}}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}\right] \\
&\quad \times \left[\frac{\mu_1}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2}\right] \left\{1 - e^{-\left(\frac{b}{r_1+r_2}\right)\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]}\right\}
\end{aligned} \quad (7)$$

$$\begin{aligned}
P^*_{f,vd} &= P_{f,vd} \times \text{Prob(Case III)(Case IIIa|Case III)} \\
&= P_{f,vd} \int_{x=\frac{b}{r_1+r_2}}^{\infty} \int_{y=\frac{b}{r_1+r_2}}^{\infty} \mu_1 e^{-\mu_1 x} \mu_2 e^{-\mu_2 y} dy dx \\
&= \left[\frac{(r_1+r_2)\lambda_1\lambda_2}{r_1\lambda_1\mu_2 + r_2\lambda_2\mu_1 + (r_1+r_2)\lambda_1\lambda_2}\right] e^{-(\mu_1+\mu_2)\left(\frac{b}{r_1+r_2}\right)}.
\end{aligned} \quad (8)$$

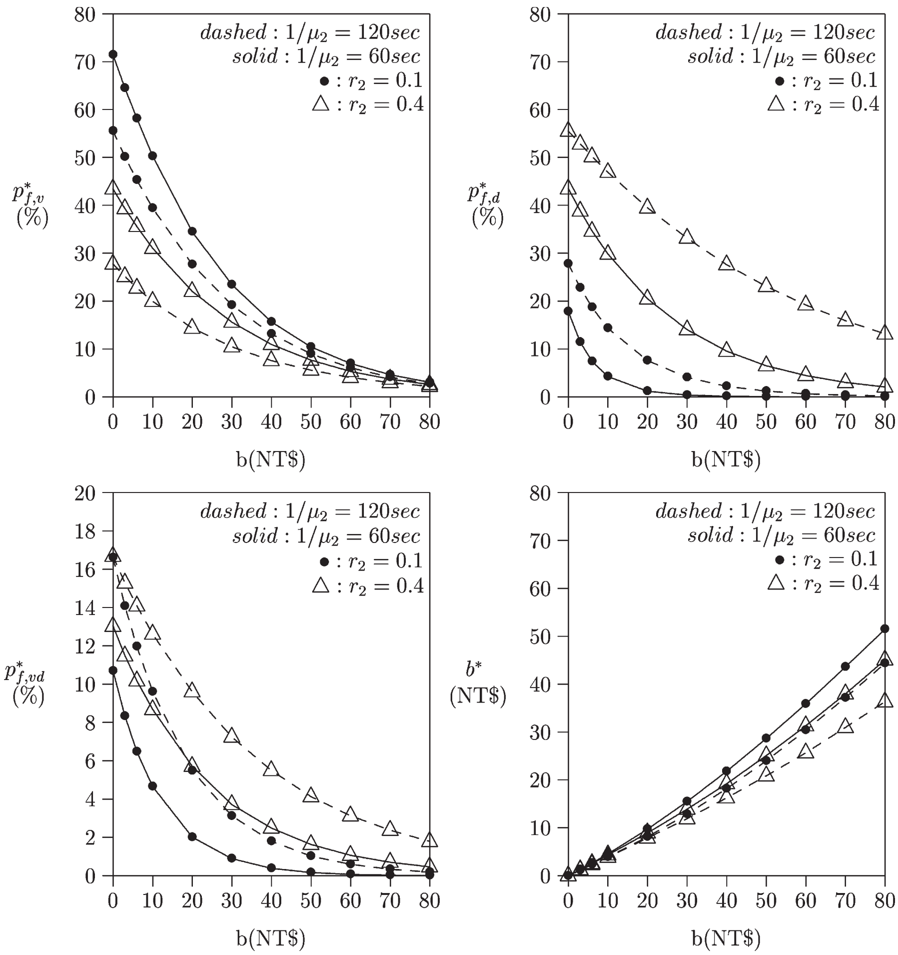

Fig. 5. Effect of b on $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$ (B = NT\$500, $\lambda_1 = \lambda_2 = 1/600$ s^{-1}, $1/\mu_1$ = 120 s, and r_1 = 0.2 NT/s).

Note that $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$ are equal to zero when $b \to \infty$. Fig. 5 also shows that the voice/data call termination probabilities decrease as b increases. However, if b is too large, it may not be good for user experience because calls cannot be made, even though there is still much remaining credit b. It is desirable to choose an appropriate value of "b," so that the voice/data call termination probabilities are lower than a threshold.

Let P^*_c be the probability that the calls terminate normally before the credit depletes. Note that

$$P^*_c = 1 - P^*_{f,v} - P^*_{f,d} - P^*_{f,vd}. \tag{9}$$

Assume that b^* is the remaining credit when the last call terminates. The expected value of b^* can be expressed as (10), shown at the bottom of the next page.

TABLE I
COMPARISON OF ANALYTIC AND SIMULATION MODELS (B = NT\$500, b = NT\$3.0, $\lambda_1 = 1/1200$ s^{-1}, $\lambda_2 = 1/1800$ s^{-1}, $\mu_1 = 1/180$ s^{-1}, $\mu_2 = 1/100$ s^{-1})

	$P^*_{f,v}$(%)		$P^*_{f,d}$(%)	
(r_1, r_2)	Simulation	Analytic	Simulation	Analytic
(0.2, 0.08)	75.66	75.65	8.63	8.59
(0.2, 0.3)	54.79	54.75	29.87	29.91
(0.2, 0.5)	43.64	43.76	41.45	41.33
(0.2, 0.8)	33.60	33.63	51.82	51.88

	$P^*_{f,vd}$(%)		E^*_b(NT$)	
(r_1, r_2)	Simulation	Analytic	Simulation	Analytic
(0.2, 0.08)	5.33	5.37	1.54	1.54
(0.2, 0.3)	7.46	7.47	1.52	1.52
(0.2, 0.5)	8.56	8.58	1.51	1.52
(0.2, 0.8)	9.68	9.61	1.52	1.51

IV. NUMERIC RESULT

This section investigates the performance of prepaid GSM and GPRS services based on the analytic model developed in the previous section. Simulation experiments have been conducted to validate the analytic results. Each simulation experiment was repeated 500 000 times to ensure stable results. To reflect the situation of prepaid service and GPRS service in Taiwan, R.O.C., the prepaid credit B is NT\$500, the charge rate of a voice call is NT\$0.2/s, and the charge rate of a data call ranges from NT\$0.08/s (for high usage) to NT\$0.8/s (for low usage). Table I compares the results of analytic and simulation models. The table shows that the analytic results are consistent with the simulation results for various charge rates r_1 and r_2.

A. *Effect of* b

Fig. 5 depicts the effect of b on $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$. The arrival rates of voice calls and data calls are both Poisson distributed with $\lambda_1 = \lambda_2 = 1/600\ \mathrm{s}^{-1}$. The arrival rates were chosen to emulate the behavior of a user with high usage. The voice call holding times are exponentially distributed with a mean value of 2 min (i.e., $1/\mu_1 = 120$ s), and the charge rate of a voice call is NT\$0.2/s. In Fig. 5, two cases are considered for the mean of the data call holding time: $1/\mu_2 = 60$ s and 120 s. In a GPRS network, the maximum transmission speed over the air is in the range of 40–53 kb/s [13]. Hence, the mean size of data transmission in our experiments corresponds to 300 and 600 kB.

The results in Fig. 5 indicate that $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$ decrease as the NMC threshold b increases. The decreasing rates are rapid when b is small, but they slow down as b increases. Smaller force-termination probabilities can provide better QoS to the subscriber. This improvement is obtained at the cost of increasing the NMC threshold that the subscriber is not allowed to use. Fig. 5 also shows that when b is less than NT\$30, b^* equals to about $b/2$. When b is chosen to be the total cost of a voice call and a data call, $P^*_{f,vd}$ can be reduced to 2% in our experiment.

B. *Effects of the Variance of Data Call Holding Times*

We have also studied the effects of the variance of data call holding time on $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$. The data call holding time is assumed to have a Gamma distribution. A Gamma distribution has the following density function:

$$f(t) = \frac{\mu^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\mu t} \qquad \text{for } t > 0 \tag{11}$$

where $\alpha(> 0)$ is the shape parameter, $\mu(> 0)$ is the scale parameter, and $\Gamma(q) = \int_{z=0}^{\infty} z^{q-1} e^{-z} dz$.

Let C_{x^*} be the coefficient of the variation of data call holding time. For the Gamma distribution, $C_{x^*} = (1/\sqrt{\alpha})$. A small C_{x^*} means that the variation is small and that the length of most calls are close to the mean length. When C_{x^*} is large, there are a few long calls and many short calls. In our experiment, C_{x^*} varies in the range of 10^{-2}–10^{2}. The arrival rates, call holding times, and charge rates are assumed to be the same as those

$$
\begin{aligned}
E[b^*] = {} & \left(\frac{1}{P_c^*}\right) \Bigg\{ P_{f,v} \int_{x=0}^{\frac{b}{r_1}} (b - r_1 x)\mu_1 e^{-\mu_1 x} dx + P_{f,d} \int_{y=0}^{\frac{b}{r_2}} (b - r_2 y)\mu_2 e^{-\mu_2 y} dy \\
& + P_{f,vd} \int_{x=0}^{\frac{b}{r_1}} \int_{y=0}^{\frac{b-r_1 x}{r_2}} (b - r_1 x - r_2 y)\mu_1 e^{-\mu_1 x} \mu_2 e^{-\mu_2 y} dy dx \Bigg\} \\
= {} & \left(\frac{1}{P_c^*}\right) \Bigg\{ P_{f,v} \left[b + \left(\frac{r_1}{\mu_1}\right) e^{-\mu_1\left(\frac{b}{r_1}\right)} - \frac{r_1}{\mu_1} \right] + P_{f,d} \left[b + \left(\frac{r_2}{\mu_2}\right) e^{-\mu_2\left(\frac{b}{r_2}\right)} - \frac{r_2}{\mu_2} \right] \\
& + P_{f,vd} \Bigg\{ b \left[1 - e^{-\left(\frac{b}{r_1}\right)\mu_1} \right] - \left[\frac{b\mu_1 e^{-\mu_2\left(\frac{b}{r_2}\right)}}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \left\{ 1 - e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} \right\} \\
& + b e^{-\mu_1\left(\frac{b}{r_1}\right)} - \left(\frac{r_1}{\mu_1}\right) \left[1 - e^{-\mu_1\left(\frac{b}{r_1}\right)} \right] + \left[\frac{r_1 \mu_1 e^{-\mu_2\left(\frac{b}{r_2}\right)}}{\mu_1 - \mu_2\left(\frac{r_1}{r_2}\right)} \right] \\
& \times \left\{ -\left(\frac{b}{r_1}\right) e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} + \left[\frac{1}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \left[1 - e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} \right] \right\} \\
& + \left[\frac{b\mu_1 e^{-\left(\frac{b}{r_2}\right)\mu_2}}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \left\{ 1 - e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} \right\} - \left[\frac{r_1\mu_1 e^{-\left(\frac{b}{r_2}\right)\mu_2}}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \\
& \times \left\{ -\left(\frac{b}{r_1}\right) e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} + \left[\frac{1}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \left\{ 1 - e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} \right\} \right\} \\
& - \left(\frac{r_2\mu_1}{\mu_2}\right) \left\{ \left(\frac{1}{\mu_1}\right) \left[1 - e^{-\left(\frac{b}{r_1}\right)\mu_1} \right] - \left[\frac{e^{-\left(\frac{b}{r_2}\right)\mu_2}}{\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2} \right] \left\{ 1 - e^{-\left[\mu_1 - \left(\frac{r_1}{r_2}\right)\mu_2\right]\left(\frac{b}{r_1}\right)} \right\} \right\} \Bigg\} \Bigg\}
\end{aligned}
\tag{10}
$$

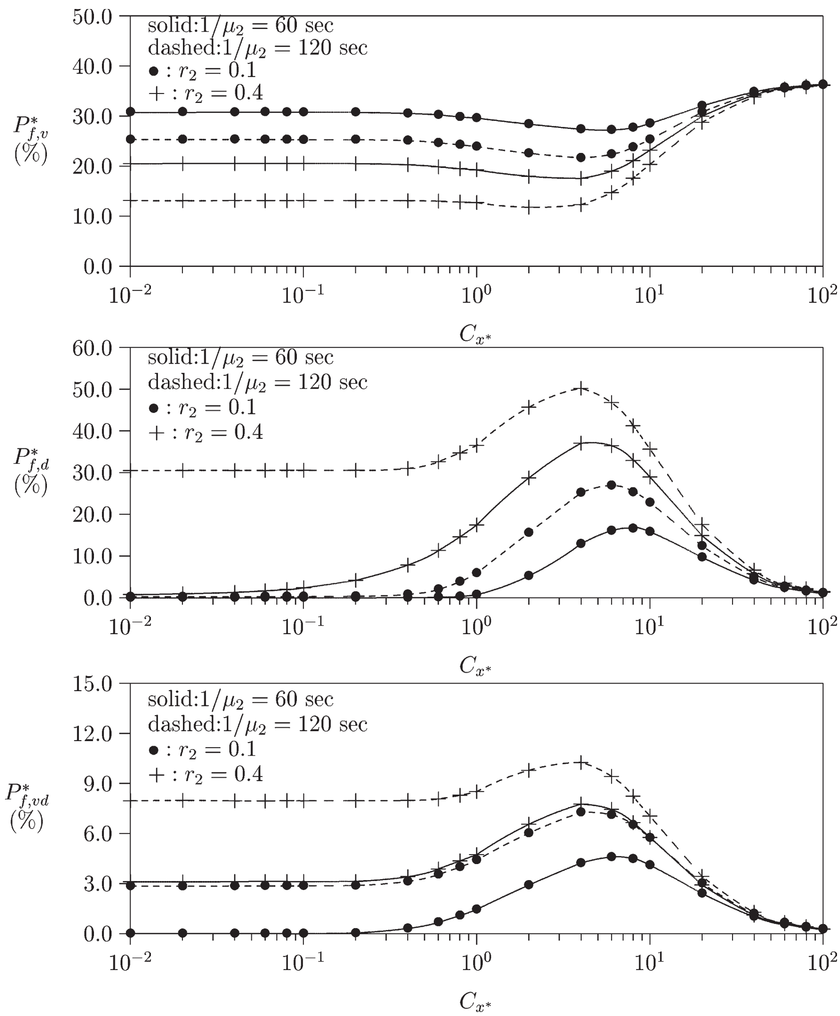

Fig. 6. Effect of C_{x^*} on $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$ ($B =$ NT\$500, $b =$ NT\$24, $\lambda_1 = \lambda_2 = 1/600\ \mathrm{s}^{-1}$, $1/\mu_1 = 120$ s, and $r_1 = 0.2$ NT/s).

in the previous experiment. The NMC threshold b is chosen to be NT$24.

Fig. 6 shows that when $C_{x^*} < 0.5$, the force-termination probabilities $P^*_{f,v}$, $P^*_{f,d}$, and $P^*_{f,vd}$ are insensitive to C_{x^*} but are sensitive to the charge of a data call. This is because when C_{x^*} is very small, most of the length of data calls are close to the mean value. The data calls with higher charge rate are more likely to be forced to terminate. Therefore, $P^*_{f,d}$ increases, and $P^*_{f,v}$ decreases as r_2 increases.

As C_{x^*} increases (i.e., $C_{x^*} > 0.5$), the number of long data calls whose charge is larger than the NMC threshold increases. As a result, more data calls are forced to terminate; $P^*_{f,d}$ and $P^*_{f,vd}$ increase as C_x increases. When C_{x^*} increases further (i.e., $C_{x^*} > 5$), the number of short data calls increase. More data calls complete normally, and $P^*_{f,d}$ and $P^*_{f,vd}$ decrease as C_x increases. For $C_{x^*} > 10^2$, $P^*_{f,d}$ and $P^*_{f,vd}$ are near zero. This is because most of the data calls are very short and can complete normally. Only the very long calls, whose number is small, may be forced to terminate. We come to the conclusion that NMC effect is more significant when C_{x^*} is large. Since the variance of voice call length remain the same, the probability that a voice call is forced to terminate [i.e., $(P^*_{f,v} + P^*_{f,vd})$] slightly increases as C_{x^*} increases. When $P^*_{f,vd}$ increases, $P^*_{f,v}$ decreases and vice versa.

V. Conclusion

This paper investigates the charging issues of an integrated GSM and GPRS prepaid service, where a single prepaid account provides the user both voice and data services. Based on the CAMEL network architecture, the call setup and charging procedures for GSM and GPRS have been illustrated. We propose an NMC algorithm to reduce the probability of terminating both ongoing voice and data calls, where no more new calls are admitted when the user credit is below a threshold. An analytic model has been developed to evaluate the performance of the NMC algorithm. Computer simulations have also been used to verify the analytic results. The numeric results indicate that the force-termination probability can be significantly reduced by choosing an appropriate threshold of the user credit. In addition, the force-termination probability of voice calls slightly increases as the call pattern of data calls becomes irregular.

As the need to rapidly roll out new services in mobile networks increases, our model can be easily extended to accommodate different real-time services. Our analytic method could provide guidelines to help operators to generate higher revenues.

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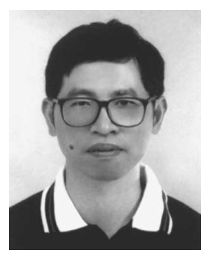

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