Remote preparation of qutrit states with biphotons

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We report an experimental demonstration of remote preparation of an arbitrary pure qutrit state. A qutrit is encoded in a two-photon (biphoton) polarization state. Prepared states are characterized by quantum state tomography. Experimental imperfection is discussed and evaluated by an experimentally obtained density matrix. The present scheme can develop into generation of an arbitrary two-photon triplet state and accordingly to an essential multiparty quantum communication protocol. Generalization to higher order which enables remote preparation of an arbitrary qudit state and several typical multiphoton entangled states is also shown.

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A two-level quantum system, or qubit, is known as the most basic unit of quantum-information processing. A great deal of theoretical and experimental knowledge for quantum-information technology based on qubits has been developed. However, recent theoretical progress showed that a natural extension to higher order, i.e., general d-level systems, or qudits, provide superiority over qubit systems in various situations. The simplest extension, a three-level system, is called a qutrit. It is known that a three-level encoding protocol for quantum key distribution provides better security than that achievable with two-level encoding protocols [1]. In the quantum bit commitment protocol, the known best security is achieved by utilizing three-level systems [2]. The Bell inequality for multilevel systems exhibits a larger deviation from classical theory than the case of two-level systems [3].

Experimental implementation of qutrit states has been realized using several degrees of freedom of photons. In particular, generation of an entangled qutrit state has been well developed using orbital angular momentum (OAM) [4] and energy-time correlation of photon pairs [5], and also two-photon (biphoton) polarization [6,7]. On the other hand, it is very important to arbitrarily control the qutrit state. However, this has been a somewhat difficult task even in the simplest case of a single qutrit state. Recently, state preparation of an arbitrary pure single-qutrit state using biphoton polarization was realized [8]. However, the method is somewhat complicated and requires careful control of an interferometric setup, resulting in a practical problem of fragility of the setup. Later, a simplified scheme was realized [9], but it requires a postselection technique and is still composed of an interferometric setup. In this paper, we propose and demonstrate a preparation method of an arbitrary pure single-qutrit state with biphoton polarization. The present scheme utilizes a remote state preparation (RSP) protocol [10,11] and thus can prepare a qutrit state remotely, unlike the previous methods [8,9]. It does not require any interferometric setup and the preparation procedure is extremely simple because it requires only two single-qubit projective measurements which can be easily implemented. Moreover, the scheme can develop into RSP of an arbitrary two-qubit triplet state by a slight modification, and accordingly can be applied to quantum secret sharing [12–14] which is an essential multiparty quantum communication protocol. Generalization to higher order is also discussed and it is shown that an arbitrary qudit state and several important multiphoton entangled states can be prepared.

Before going to the detailed procedure, we explain the polarization of a biphoton state [8,15]. Polarization of a two-photon state can be described using three basis states, e.g., $|HH\rangle$, $|HV\rangle$, and $|VV\rangle$, where H (V) denotes a photon with horizontal (vertical) polarization. Thus a qutrit state can be encoded in it by correspondence as $0\rightarrow HH$, $1\rightarrow HV$, and $2\rightarrow VV$. Note that the two photons involved are spatially and temporally indistinguishable, and thus a biphoton polarization state can be distinguished from a direct product of two single-photon states, e.g., $|H\rangle|V\rangle$, where the two photons are distinguishable in terms of spatial or temporal modes.

Here we show the detailed procedure of our scheme. The basic principle is succinctly described in the original work on RSP [10]. First we prepare a maximally entangled two-qutrit state:

$$(1/\sqrt{3})(|0\rangle_a|0\rangle_b+|1\rangle_a|1\rangle_b+|2\rangle_b|2\rangle_b). \qquad (1)$$

Distant parties Alice and Bob possess a qutrit of the modes a and b, respectively. Suppose Alice desires to remotely prepare a qutrit state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle+\gamma|2\rangle$ at Bob's site. Then what Alice should do is to measure her qutrit with a projection on $|\psi^*\rangle=\alpha^*|0\rangle+\beta^*|1\rangle+\gamma^*|2\rangle$ and to send Bob the result of whether her qutrit successfully projected on $|\psi^*\rangle$. Bob keeps his qutrit only when Alice's qutrit was projected on $|\psi^*\rangle$ and otherwise discards it. Then the state of Bob's qutrit becomes the desired one.

The maximally entangled qutrit state (1) can be prepared using noncollinear type-II spontaneous parametric down-conversion (SPDC) [16,17]. The schematic diagram of our experimental setup is shown in Fig. 1. The interaction Hamiltonian of the process is set as $\kappa(a_H^\dagger b_V^\dagger+a_V^\dagger b_H^\dagger)+\text{H.c.}$, where κ is the coupling constant and $a^\dagger$ ($b^\dagger$) is the creation

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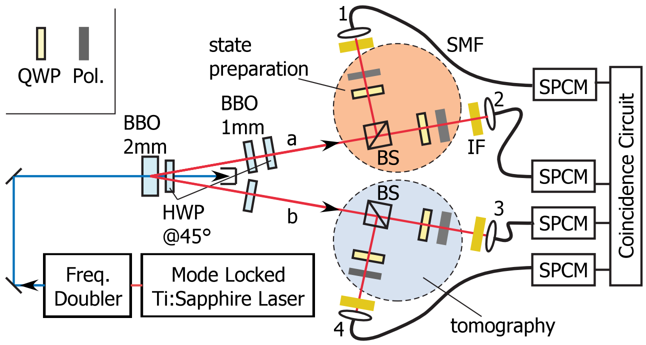

FIG. 1. (Color online) Schematic diagram of experimental setup; QWP, quarter-wave plate; Pol., polarizer; HWP, half-wave plate; BS, nonpolarizing 50-50 beam splitter; IF, 3-nm-bandwidth interference filter; SMF, single-mode fiber; SPCM, single-photon-counting module.

operator for photons in the spatial mode a (b). The subscript H (V) represents horizontal (vertical) polarization, and H.c. is the Hermitian conjugate. We focus on the four-photon process of SPDC where each of the two output modes contains two photons. Polarization of photons in mode a is rotated by 90° as $H \rightarrow V$ and $V \rightarrow H$ by inserting a half-wave plate. Accordingly, the output state becomes

$$(1/\sqrt{3})(|HH\rangle_a|HH\rangle_b + |HV\rangle_a|HV\rangle_b + |VV\rangle_a|VV\rangle_b), \quad (2)$$

which is the maximally entangled qutrit state (1) where the correspondence is as mentioned above.

Implementation of an arbitrary projection for a qutrit is a crucial part of the present scheme. In the present case of a biphoton qutrit, it can be easily realized compared with a time bin or OAM qutrit [18] in the following way. First we split a biphoton in mode a into two single photons using a nonpolarizing beam splitter (BS). Then each of the photons is projected on an appropriate polarization angle, which can easily be implemented by a quarter-wave plate and a polarizer. The polarization angles of the projection are determined as follows. Suppose we want to perform a projection on $|\psi\rangle = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VV\rangle$. The state is rewritten to be factorized as

$$\begin{aligned} &\alpha|HH\rangle + \beta|HV\rangle + \gamma|VV\rangle \\ &= [\alpha(a_H^\dagger)^2/\sqrt{2} + \beta a_H^\dagger a_V^\dagger + \gamma(a_V^\dagger)^2/\sqrt{2}]|0\rangle \\ &= (p_1 a_H^\dagger + q_1 a_V^\dagger)(p_2 a_H^\dagger + q_2 a_V^\dagger)|0\rangle. \quad (3)\end{aligned}$$

Then the polarization angles of the projection are determined to be $|\pi_1\rangle = p_1|H\rangle + q_1|V\rangle$ and $|\pi_2\rangle = p_2|H\rangle + q_2|V\rangle$. Successful projection on the desired biphoton state is assured by a coincidence detection of the projected photons. In this method, the probability that the biphoton in mode a is successfully projected onto a desired state $|\psi\rangle$ is expressed as $(1 + |\langle \pi_2|\pi_1\rangle|^2)/12$.[1] Note that this value depends on the desired state.

In the present scheme, the projection process is probabilistic. However, once the preparation process is successfully performed, a prepared state is exactly a biphoton state, that is, zero or one photon is not contained in the prepared state. Thus, ideally, unlike the previous work [8,9], the present scheme is postselection-free. However, in a real situation, loss of photons occurs because of imperfection of optical elements, which result in the presence of terms with zero or one photon. Nevertheless, the present scheme still has the effect of reducing such unwanted terms compared with the previous works [8,9], which leads to higher quality of the prepared state on the condition that postselection is forbidden. Thus it is useful even in practical situations.

Once a desired qutrit state is prepared, it should be characterized appropriately. This is performed by quantum state tomography (QST) [19]. QST is a method to reconstruct the density matrix of an experimentally obtained state from a set of projective measurements. For a qutrit state, it requires at least nine projective measurements. Implementation of each projective measurement is performed in the same way as mentioned above.

In our experiment, uv pulses with a central wavelength at 390 nm from a frequency-doubled mode-locked Ti:sapphire laser were incident on a 2-mm-thick β-barium borate (BBO) crystal as the pump source for SPDC. A half-wave plate and two auxiliary 1-mm-thick BBO crystals were used to compensate the spatial and temporal walk-off of down-converted photons. Photons in each output mode were filtered by a 3-nm-bandwidth interference filter, coupled into a single-mode fiber, and detected by single-photon-counting modules (Perkin Elmer SPCM-AQR-14-FC). Coincidence events of four detectors (two for state preparation and two for characterization of the prepared state) were recorded. Note that all the required measurements can be performed by combinations of single-qubit projective measurements, though biphoton polarization is presently treated.

To demonstrate the availability of the present scheme, we remotely prepared the states $|\psi_1\rangle = |HV\rangle$, $|\psi_2\rangle = (1/\sqrt{2})(|HH\rangle - |VV\rangle)$, and $|\psi_3\rangle = (1/\sqrt{3})(|HH\rangle + i|HV\rangle - |VV\rangle)$. For preparation of these states, the polarization angles of projection of two modes after the BS were set as $\{|H\rangle, |V\rangle\}$, $\{(1/\sqrt{2})(|H\rangle \pm |V\rangle)\}$, and $\{(1/\sqrt{2})(|H\rangle + e^{-i\pi/4}|V\rangle), (1/\sqrt{2})(|H\rangle + e^{-i3\pi/4}|V\rangle)\}$, respectively. The density matrices of the output states were measured by QST with the maximum likelihood method. The ratio of fourfold coincidence counts for tomographic measurement was about 10 counts per an hour on average.[2] The results are shown in Fig. 2. The values of fidelity with the desired states $\langle \psi_i|\rho_{\mathrm{expt}}|\psi_i\rangle$ ($i = 1,2,3$) were 0.93 ± 0.02, 0.89 ± 0.03, and 0.92 ± 0.02, respectively. The values of purity $\mathrm{Tr}[\rho_{\mathrm{expt}}^2]$ were 0.89 ± 0.03, 0.84 ± 0.05, and 0.91 ± 0.04, respectively. The errors were estimated by assuming Poissonian errors of the experimentally obtained count numbers.

Here we discuss the imperfection of the present setup. For

[1] If $|\pi_1\rangle \perp |\pi_2\rangle$, this probability (1/12) can be improved to 1/3 by utilizing a polarization beam splitter instead of a BS as in Refs. [6,7].

[2] The count ratio is determined by detection efficiency and loss of photons in the optical system. Only the latter is connected with the quality of the prepared state and therefore it is difficult to estimate the quality of a prepared state on the basis of the count ratio.

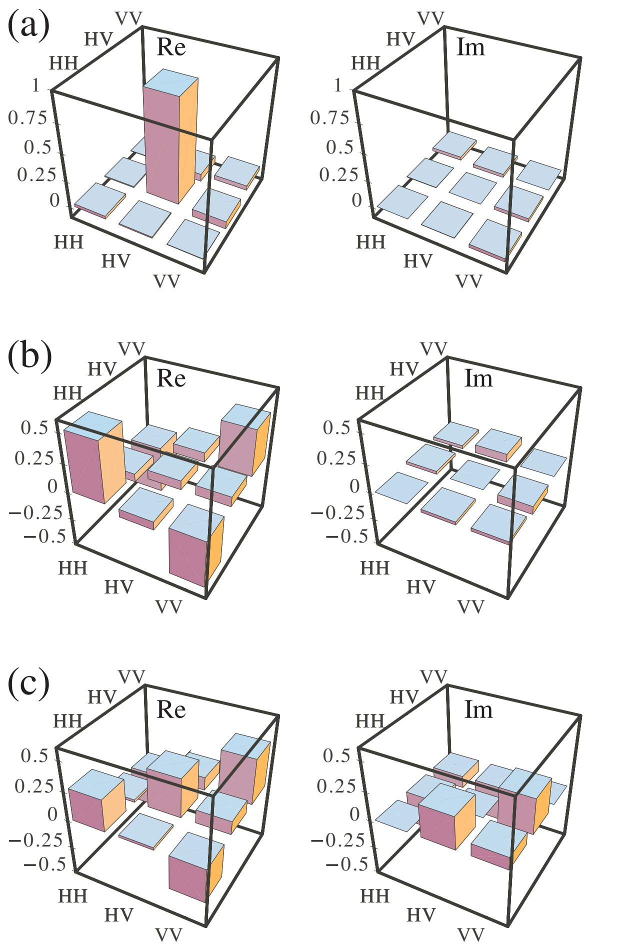

FIG. 2. (Color online) Experimentally obtained density matrices of the prepared qutrit states of (a) $|\psi_1\rangle = |HV\rangle$, (b) $|\psi_2\rangle = (1/\sqrt{2})(|HH\rangle - |VV\rangle)$, and (c) $|\psi_3\rangle = (1/\sqrt{3})(|HH\rangle + i|HV\rangle - |VV\rangle)$.

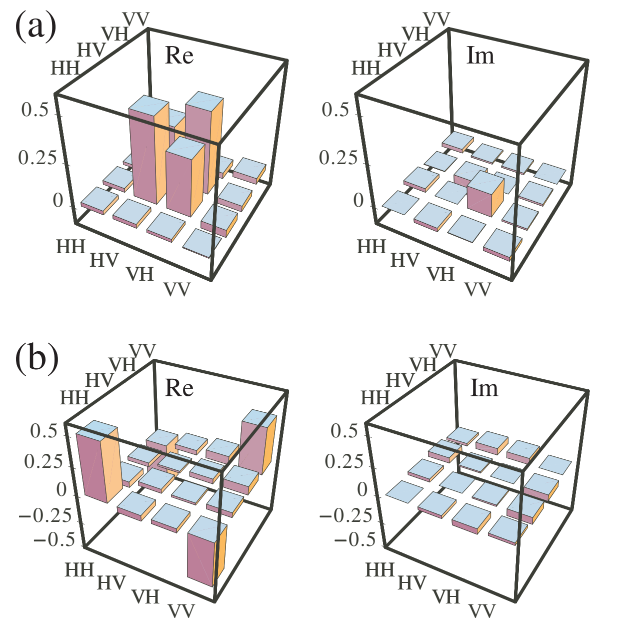

FIG. 3. (Color online) Experimentally obtained density matrices of the prepared two-qubit states of (a) $(1/\sqrt{2})(|H\rangle|V\rangle + |V\rangle|H\rangle)$ and (b) $(1/\sqrt{2})(|H\rangle|H\rangle - |V\rangle|V\rangle)$.

the tomographic measurement, not only a biphoton but also two distinguishable photons can contribute to the coincidence counts. Two such photons cannot be regarded as a qutrit in the present context, but should be treated as unwanted noise. Distinguishability between the two photons is caused by spatial or temporal mode mismatch between them. Especially in the present scheme, photons in the same path do not have temporal correlation with each other. Therefore when the pulse duration of the photons is longer than the coherence time determined by the interference filters, the two photons become partially distinguishable. We developed a simple method to detect and evaluate such partial distinguishability. Suppose we prepare the state $|HV\rangle$. If the photons pass through a BS and we select the case where each of the output ports contains one photon, these states are transformed into a two-qubit state $|H\rangle|V\rangle + |V\rangle|H\rangle$. On the other hand, a state of two distinguishable photons $|H\rangle|V\rangle$ is transformed into a two-qubit state $|H\rangle|V\rangle\langle H|\langle V| + |V\rangle|H\rangle\langle V|\langle H|$. Here we can see that distinguishability between the two photons induces decoherence of the two-qubit state obtained by the above procedure. Thus, the ratio of a true biphoton state within a measured state can be estimated by the mix of the two-photon state transformed from the original state. The experimentally obtained two-photon state from the data used to obtain the density matrix shown in Fig. 2(a) is shown in Fig. 3(a).[3] From this result, the estimated ratio of the true biphoton state was $\eta = 0.74 \pm 0.12$, which was calculated as $|\rho_{23}|/\sqrt{\rho_{22}\rho_{33}}$, where $\rho_{23} = \langle H|\langle V|\rho_{\mathrm{expt}}|V\rangle|H\rangle$, $\rho_{22} = \langle H|\langle V|\rho_{\mathrm{expt}}|H\rangle|V\rangle$, and $\rho_{33} = \langle V|\langle H|\rho_{\mathrm{expt}}|V\rangle|H\rangle$. Because the preparation process is also influenced by the same kind of imperfection, the ratio of a genuine biphoton state within a prepared state generally depends on the desired state. However, it can be shown that η gives the minimum value of the ratio for all the desired states. Thus we can conclude that biphoton states dominantly contributed to our measurement. Additionally, assuming that the photons of the experimentally generated resource state in modes a and b suffer the same degree of imperfection, the ratio of biphoton entanglement within the resource state in the context of Refs. [6,7] is expressed as $3\eta/(2+\eta)$ and thus presently 0.81 ± 0.10. This partial distinguishability can lower the quality of a prepared state. For example, given that $\eta = 0.74$ and other experimental conditions are perfect, the fidelity and the purity for preparation of $|\psi_2\rangle$ were calculated to be 0.86 and 0.76, respectively.[4] Another possible cause of imperfection is multiphoton contribution which is caused by the existence of more than two photons. However, in the present experiment, it is significantly small compared with the contribution of partial distinguishability.

As mentioned above, a biphoton state can be transformed into a two-qubit state after splitting by a BS and selecting the case where each of the output ports of the BS contains

[3]For the ability to reconstruct a two-qubit state, 16 projective measurements were performed for $|\psi_1\rangle$ and $|\psi_2\rangle$, while nine projective measurements were performed for $|\psi_3\rangle$.

[4]In general, the quality of a prepared state depends on the state itself and the measurement process.

one photon. The set of states which can be prepared from a biphoton state is that of triplet states. In fact, three triplet basis states $|\Psi^{+}\rangle=(1/\sqrt{2})(|H\rangle|V\rangle+|V\rangle|H\rangle)$ and $|\Phi^{\pm}\rangle=(1/\sqrt{2})(|H\rangle|H\rangle\pm|V\rangle|V\rangle)$ can be prepared by preparing biphoton states $|HV\rangle$, $|LR\rangle=(i/\sqrt{2})(|HH\rangle+|VV\rangle)$, and $|PM\rangle=(1/\sqrt{2})(|HH\rangle-|VV\rangle)$, respectively, where $|L\rangle=(1/\sqrt{2})(|H\rangle+i|V\rangle)$, $|R\rangle=(1/\sqrt{2})(|H\rangle-i|V\rangle)$, $|P\rangle=(1/\sqrt{2})(|H\rangle+|V\rangle)$, and $|M\rangle=(1/\sqrt{2})(|H\rangle-|V\rangle)$ [7]. Note that this feature is characteristic of a biphoton qutrit. To demonstrate the availability, we experimentally reconstructed the states $|\Psi^{+}\rangle$ and $|\Phi^{-}\rangle$ from the same data as used for reconstructing $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. The results are shown in Fig. 3. The values of fidelity with the desired states were 0.80 ± 0.06 and 0.87 ± 0.04, respectively. The values of tangle were 0.43 ± 0.15 and 0.61 ± 0.10, respectively, and thus the prepared states are highly entangled. The causes of imperfection of prepared two-qubit states are the same as in the above case of qutrit states. As an example of application of this feature, the present scheme can be utilized for quantum secret sharing which is an essential multiparty quantum communication protocol [12–14], according to the context of Ref. [13].

The present setup can be generalized by considering higher-order terms of SPDC. In general, the output state of the present setup can be written as [17,20]

$$|\phi\rangle=\frac{1}{\cosh^2\tau}\sum_{n=0}^{\infty}\tanh^n\tau|\phi_n\rangle, \tag{4}$$

$$|\phi_n\rangle=\frac{1}{\sqrt{n+1}}\sum_{m=0}^{n}|m,n-m\rangle_a|m,n-m\rangle_b, \tag{5}$$

where $|m,n\rangle_i$ represents m horizontally and n vertically polarized photons in mode i and τ is the interaction parameter of the nonlinear crystal. $|\phi_n\rangle$ is the term that contains n photons in each of the modes a and b. Now we focus on the state $|\phi_n\rangle$ for a certain n. This state can be extracted by detecting just n photons in Alice's site. A straightforward calculation shows that Alice's projection for her photons on $\sum_{i=0}^{n}c_i|i,n-i\rangle$ can prepare a state $\sum_{i=0}^{n}c_i^{*}|i,n-i\rangle$ in Bob's site in the same way shown above. The projection on an n-photon polarization state can be executed by splitting the n-photon state into n photons by $n-1$ BSs and performing single-qubit projective measurement on each of the photons. Thus, by encoding $(n+1)$-level states as $|i\rangle\rightarrow|i,n-i\rangle$ $(i=0,1,\ldots,n)$, an arbitrary qudit state $(d=n+1)$ can be prepared remotely. Moreover, this ability enables preparation of several typical multiphoton entangled states. First, as in Ref. [20], a photon-number-entangled state $(1/\sqrt{2})(|n,0\rangle+|0,n\rangle)$ can be prepared. The polarization angles of the required projections for the split photons are $\{(1/\sqrt{2})(|H\rangle+e^{i\pi(1+2k)/n}|V\rangle)\}$ $(k=0,1,\ldots,n-1)$. Second, this state is transformed into an n-photon Greenberger-Horne-Zeilinger state $(1/\sqrt{2})(|H\rangle\cdots|H\rangle+|V\rangle\cdots|V\rangle)$ by splitting the n-photon state into n photons with different paths by $n-1$ BSs. Third, by preparing the state $|n-1,1\rangle$ and splitting the n-photon state in the same way as above, an n-photon W state $(1/\sqrt{n})(|H\rangle\cdots|H\rangle|V\rangle+|H\rangle\cdots|V\rangle|H\rangle+\cdots+|V\rangle\cdots|H\rangle|H\rangle)$ can be prepared.

In conclusion, we have experimentally demonstrated a method for remotely preparing an arbitrary pure biphoton polarization qutrit state. Because the medium is a biphoton, it can develop into remote preparation of an arbitrary two-qubit triplet state, and accordingly enables application to quantum secret sharing. Higher-order generalization enables remote preparation of an arbitrary qudit state and accordingly several typical multiphoton entangled states. It is expected to help the study of multilevel or multipartite quantum systems and their application to quantum-information technology.

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