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Group decision-making based on concepts of ideal and anti-ideal points in a fuzzy environment

Ming-Shin Kuo[a], Gwo-Hshiung Tzeng[b,c,*], Wen-Chih Huang[d]

[a] Department of River and Harbor Engineering, National Taiwan Ocean University, P.O. Box 7-630 Keelung, Keelung 20299, Taiwan
[b] Department of Business Administration, Kainan University, No.1, Kainan Road, Luchu, Taoyuan County 338, Taiwan
[c] Institute of Management of Technology, National Chiao Tung University, Ta-Hsuch Road, Hsinchu 300, Taiwan
[d] Department of River and Harbor Engineering, National Taiwan Ocean University, P.O. Box 7-107 Keelung, Keelung 20299, Taiwan

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Abstract

Decision-making problems (location selection) often involve a complex decision-making process in which multiple requirements and uncertain conditions have to be taken into consideration simultaneously. In evaluating the suitability of alternatives, quantitative/qualitative assessments are often required to deal with uncertainty, subjectiveness and imprecise data, which are best represented with fuzzy data. This paper presents a new method of analysis of multicriteria based on the incorporated efficient fuzzy model and concepts of positive ideal and negative ideal points to solve decision-making problems with multi-judges and multicriteria in real-life situations. As a result, effective decisions can be made on the basis of consistent evaluation results. Finally, this paper uses a numerical example of location selection to demonstrate the applicability of this method, with its simplicity in both concept and computation. The results show that this method can be implemented as an effective decision aid in selecting location or decision-making problems.

Keywords: Fuzzy multicriteria decision; Multiple criteria decision-making (MCDM); TOPSIS, Fuzzy sets; Location selection

1. Introduction

Multicriteria evaluation often requires the decision makers to provide qualitative/quantitative assessments for determining the performance of each alternative with respect to each criterion, and the relative importance of evaluation criteria with respect to the overall objective of the problems. These problems will usually result in uncertain, imprecise, indefinite and subjective data being present, which makes the decision-making process complex and challenging. In other words, decision-making often occurs in a fuzzy environment where the information available is imprecise/uncertain [2,11,17,27,28,31], and these problems bring about much torment among decision makers in the decision-making process. In the last few years, numerous studies attempting to handle this uncertainty, imprecision, and subjectiveness have been carried out basically by means of fuzzy set theory, as fuzzy set theory might provide

* Corresponding author at: Institute of Management of Technology, National Chiao Tung University, Ta-Hsuch Road, Hsinchu 300, Taiwan.
E-mail addresses: mingshin.a@mail.ntou.edu.tw (M.-S. Kuo), ghtzeng@cc.nctu.edu.tw, ghtzeng@mail.knu.edu.tw (G.-H. Tzeng).

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the flexibility needed to represent the imprecision or vague information resulting from a lack of knowledge or information [1,4,6,16,22]. Therefore, the application of fuzzy set theory to multicriteria evaluation methods under the framework of utility theory has proven to be an effective approach [5,15,32]. The overall utility of the alternatives with respect to all criteria is often represented by a fuzzy number, which is named the fuzzy utility and is often referred to by fuzzy multicriteria evaluation methods. The ranking of the alternatives is based on the comparison of their corresponding fuzzy utilities [5,6,29,32]. Fuzzy multicriteria evaluation methods are used widely in fields such as information project selection [20,21], material selection [23], and many other areas of management decision problems [8,9,24,27,28] and strategy selection problems [7,10,13,25].

Li [18] proposed a simple and efficient fuzzy model for dealing with multi-judges and multicriteria decision-making problems in a fuzzy environment and suggested a level weighted fuzzy preference relation for comparing or ranking sets. This method can avoid an immediately defuzzified process when it can provide a precise solution. In addition, the technique of ideal and anti-ideal points is easily used to find the best alternative, considering that the chosen alternative should simultaneously have the shortest distance from the positive ideal point and the longest distance from the negative ideal point [9,19,27]. The ideal point is composed of all best criteria values attainable, and the anti-ideal point is composed of all worst criteria values attainable. This technique can also obtain the gap between the ideal alternative and each alternative, and the ranking order of alternatives, so it can be used widely in many fields. For example, Chen [8] extended the TOPSIS for group decision-making in a fuzzy environment. Liang [19] incorporated the fuzzy set theory and the basic concepts of positive ideal and negative ideal to expand multicriteria decision-making in a fuzzy environment. Deng [12] incorporated fuzzy pairwise comparison and the basic concepts of positive ideal and negative ideal points to expand multicriteria decision-making in a fuzzy environment. Yeh et al. [28] proposed a fuzzy multicriteria decision-making method based on concepts of positive ideal and negative ideal points to evaluate bus companies' performance. Yeh and Deng's [29] comparison of fuzzy utilities of the decision alternatives for determining their ranking plays a critical role in fuzzy multicriteria analysis. Based on above concepts from the studies, despite their applicability to any decision-making problems, typical fuzzy multiple criteria analysis requires the comparison of fuzzy numbers. This comparison process can be quite complex and produce unreliable and/or reliable results, as it may: (1) involve considerable computations, (2) produce inconsistency via respective fuzzy ranking methods, and (3) generate counter-intuitive ranking outcomes for similar fuzzy utilities [28,29].

In this paper, on the basis of the above concepts from the literature, fuzzy multicriteria decision-making problems are mainly discussed, and a novel multicriteria decision-making method, which may reflect both subjective judgment and objective information in real-life situations, is proposed. The proposed method is based on the incorporated efficient fuzzy model [18] and concepts of positive ideal and negative ideal points for solving decision-making problems with multi-judges and multicriteria in a fuzzy environment. It will efficiently grasp the ambiguity existing in the available information as well as the essential fuzziness in human judgment and preference: linguistic variables are used to assess the ratings of each alternative with respect to each criterion. In addition, the importance weights of each criterion can be obtained by either direct assignment or indirectly using pairwise comparisons [8]. Here, in this paper it is suggested that the decision makers use the fuzzy AHP (analytic hierarchy process) to determine the weightings with respect to various subjective criteria. Finally, this study will use an example of a distribution center (DC) location selection problem to illustrate the proposed method, as this problem is complex and difficult in a real-life environment. Through this case, we can demonstrate that the method proposed for solving the multicriteria decision-making problem for selecting the DC location is a good means of evaluation, and it appears to be more appropriate.

The remainder of this paper is organized as follows. Section 2 introduces the new technique method based on the incorporated efficient fuzzy model and concepts of positive ideal and negative ideal points. In Section 3, an illustrative example applying the fuzzy multicriteria decision-making method from Section 2 for potential alternatives of feasible DC locations is presented, after which we discuss and show how the new technique method of this paper is effective. Finally, conclusions are presented in Section 5.

2. New technique of the ideal and anti-ideal points for group decision-making

In this section, this study will provide interesting results on group decision-making and multicriteria decision-making with the help of fuzzy sets theory, and it will propose a new method based on the incorporated efficient fuzzy model [18] and concepts of positive ideal and negative ideal points for solving multicriteria decision-making

Table 1
Linguistic variables for the ratings

Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

problems. This method employs some basic concepts and definitions of fuzzy sets and fuzzy operations, as shown in **Appendix A**. The procedures of calculation are delineated as follows.

In this paper, the ratings of qualitative criteria are considered as linguistic variables, and those linguistic variables can be expressed in positive triangular fuzzy numbers, as in Table 1. The importance weights of each criterion can be obtained either by direct assignment or indirectly using pairwise comparisons [8,14]. Here, this paper suggests that the decision makers use fuzzy AHP to determine the weightings with respect to various subjective criteria, due to the evaluators always perceiving the weights with the judge's own subjective evaluation and exact or precise weight for a specified criterion not being given [9]. Buckley [3] considered a fuzzy positive reciprocal matrix $\tilde{A} = [\tilde{a}_{jk}]$, and used a geometric mean technique to define the fuzzy geometric mean and fuzzy weights of each criterion as follows:

$$\tilde{e}_j = \left(\tilde{a}_{j1}(\cdot)\tilde{a}_{j2}(\cdot)\cdots(\cdot)\tilde{a}_{jn}\right)^{1/n}, \qquad \tilde{w}_j = \tilde{e}_j(\cdot)(\tilde{e}_1(+)\tilde{e}_2(+)\cdots(+)\tilde{e}_n)^{-1}; \tag{1}$$

where $\tilde{a}_{jn}$ is the fuzzy comparison value of criterion j with respect to criterion n, $\tilde{e}_j$ is the geometric mean of fuzzy comparison values of criterion j with respect to each criterion, $\tilde{w}_j$ is the fuzzy weight of the jth criterion, which can be indicated by a synthetic triangular fuzzy number $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

Assuming that a decision group has K judges, then the weight of each criterion to calculate in Eq. (1), and the rating of alternatives with respect to each criterion can be calculated as follows:

$$\tilde{x}_{ij} = \frac{1}{K}\left[\tilde{x}_{ij}^{1}(+)\tilde{x}_{ij}^{2}(+)\cdots(+)\tilde{x}_{ij}^{K}\right] = \frac{1}{K}\sum_{k=1}^{K}\tilde{x}_{ij}^{k} \tag{2}$$

where $\tilde{x}_{ij}^{k}$ is the rating of the kth judge.

Assume an evaluation problem has m possible alternatives and n criteria with which alternative performances are measured. As stated above, a decision-making problem of a fuzzy multicriteria group decision can be concisely expressed in matrix format as follows:

$$\tilde{\mathbf{D}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} = \left[\tilde{x}_{ij}\right]_{m\times n},$$

$$\tilde{\boldsymbol{w}} = \left[\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n\right] \tag{3}$$

where $\tilde{x}_{ij}, \forall i, j$, is the fuzzy rating of possible alternatives $A_i, i = 1, 2, \ldots, m$, with respect to criterion C_j, and $\tilde{w}_j$ is the fuzzy weight of criterion $C_j, j = 1, 2, \ldots, n$, and these can be indicated by a synthetic triangular fuzzy number, $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ and $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$. We must use the normalization method to transform the various criteria scales into a comparable scale, due to the various criteria scales not being comparable scales in an anti-normalization environment, and moreover to ensure compatibility between the evaluations of objective criteria and linguistic ratings of subjective criteria. We used the linear scale transformation given by Hsu and Chen [14] to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix which we denote by $\tilde{\mathbf{R}}$:

$$\tilde{\mathbf{R}} = \left[\tilde{r}_{ij}\right]_{m\times n},$$
$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \quad \frac{b_{ij}}{c_j^*}, \quad \frac{c_{ij}}{c_j^*}\right), \quad j \in B,$$
$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \quad \frac{a_j^-}{b_{ij}}, \quad \frac{a_j^-}{a_{ij}}\right), \quad j \in C,$$
$$c_j^* = \max_i c_{ij} \quad \text{if } j \in B,$$
$$a_j^- = \min_i a_{ij} \quad \text{if } j \in C, \tag{4}$$

where B is the benefit criteria set, C is the cost criteria set. The normalization method mentioned above is used to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0,1].

After performance normalization of various criteria scales, we can define the positive ideal point A^* and negative ideal point A^- using Eq. (5) as follows:

$$A^* = \left(\tilde{r}_1^*, \tilde{r}_1^*, \ldots, \tilde{r}_n^*\right), \qquad A^- = \left(\tilde{r}_1^-, \tilde{r}_1^-, \ldots, \tilde{r}_n^-\right),$$
$$\text{where} \quad \tilde{r}_j^* = \max_i(\tilde{r}_{ij}), \quad j = 1, 2, \ldots, n \quad \text{and} \quad \tilde{r}_j^- = \min_i(\tilde{r}_{ij}), \quad j = 1, 2, \ldots, n. \tag{5}$$

From Eq. (5), the Hamming distance between the possible alternative A_i and the positive ideal point A^* or the negative ideal point A^- can be calculated respectively using

$$\tilde{d}_{ij}^* = \left(\tilde{r}_j^* - \tilde{r}_{ij}\right); \qquad \tilde{d}_{ij}^- = \left(\tilde{r}_{ij} - \tilde{r}_j^-\right), \quad i = 1, 2, \ldots, m; \qquad j = 1, 2, \ldots, n. \tag{6}$$

Therefore, we will obtain the positive ideal matrix $\tilde{\mathbf{H}}^*$ and negative ideal matrix $\tilde{\mathbf{H}}^-$ after each distance is obtained, denoted by

$$\tilde{\mathbf{H}}^* = \left[\tilde{d}_{ij}^*\right]_{m\times n}, \qquad \tilde{\mathbf{H}}^- = \left[\tilde{d}_{ij}^-\right]_{m\times n} \tag{7}$$

where $\tilde{d}_{ij}^*$ and $\tilde{d}_{ij}^-$ are triangular fuzzy numbers denoted by $\tilde{d}_{ij}^* = (l_{ij}^*, m_{ij}^*, r_{ij}^*)$ and $\tilde{d}_{ij}^- = (l_{ij}^-, m_{ij}^-, r_{ij}^-)$.

Considering the different levels of importance of each criterion, we can construct both the positive ideal weighted fuzzy evaluation values $\tilde{p}_{ij}^*$ and negative ideal weighted fuzzy evaluation values $\tilde{p}_{ij}^-$ from each possible alternative A_i with respect to criterion C_j, where $\tilde{p}_{ij}^*$ and $\tilde{p}_{ij}^-$ are the fuzzy numbers with parabolic membership functions in the form of

$$\begin{aligned}\tilde{p}_{ij}^k &= \tilde{d}_{ij}^k(\cdot)\tilde{w}_j \\ &= (\delta_{1ij}^k, \delta_{2ij}^k, \delta_{3ij}^k/\gamma_{ij}^k/\Delta_{1ij}^k, \Delta_{2ij}^k, \Delta_{3ij}^k),\end{aligned} \tag{8}$$

where

$$\delta_{1ij}^k = (m_{ij}^k - l_{ij}^k)(w_{j2} - w_{j1}), \quad \delta_{2ij}^k = w_{j1}(m_{ij}^k - l_{ij}^k) + l_{ij}^k(w_{j2} - w_{j1}), \quad \delta_{3ij}^k = w_{j1}l_{ij}^k,$$
$$\Delta_{1ij}^k = (r_{ij}^k - m_{ij}^k)(w_{j3} - w_{j2}), \quad \Delta_{2ij}^k = w_{j3}(r_{ij}^k - m_{ij}^k) + (w_{j3} - w_{j2})r_{ij}^k, \quad \Delta_{3ij}^k = w_{j3}r_{ij}^k,$$
$$\gamma_{ij}^k = w_{j2}m_{ij}^k, \quad k = *, -; \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n.$$

We can define each final fuzzy evaluation value of the positive ideal $\tilde{p}_i^*$ and negative ideal $\tilde{p}_i^-$ by means of extended addition and scalar multiplication through the criteria as follows:

$$\tilde{p}_i^k = \sum_{j=1}^{n} \tilde{p}_{ij}^k, \quad k = *, - \tag{9}$$

with a parabolic membership function in the form of $(\delta_{1i}^k, \delta_{2i}^k, \delta_{3i}^k/\gamma_i^k/\Delta_{1i}^k, \Delta_{2i}^k, \Delta_{3i}^k)$, where

$$\delta_{gi}^k = \sum_{j=1}^{n} \delta_{gij}^k, \quad g = 1, 2, 3; k = {}^*, -,$$

$$\Delta_{gi}^k = \sum_{j=1}^{n} \Delta_{gij}^k, \quad g = 1, 2, 3; k = {}^*, -,$$

$$\gamma_i^k = \sum_{j=1}^{n} \gamma_{ij}^k.$$

We can calculate the extended difference value of $\tilde{p}_i^-(-)\tilde{p}_i^*$ between each final fuzzy evaluation value $\tilde{p}_i^*$ of the positive ideal and each final fuzzy evaluation value $\tilde{p}_i^-$ of the negative ideal. Finally, this paper will calculate the closeness coefficient values CC_i for the final positive ideal fuzzy evaluation value and negative ideal fuzzy evaluation value of each alternative by means of the fuzzy preference relation R. The closeness coefficient of each alternative is calculated as

$$CC_i = \mu_R(\tilde{p}_i^-(-)\tilde{p}_i^*, 0) = \begin{cases} \dfrac{\beta^+}{(\beta^+ + \beta^-)}, & \text{for } \delta_{3i}^- - \Delta_{3i}^* < 0, \Delta_{3i}^- - \delta_{3i}^* \geq 0, \gamma_i^- \geq \gamma_i^*, \\ \dfrac{\lambda^+}{(\lambda^+ + \lambda^-)}, & \text{for } \delta_{3i}^- - \Delta_{3i}^* \leq 0, \Delta_{3i}^- - \delta_{3i}^* > 0, \gamma_i^- \leq \gamma_i^*, \\ 0.5, & \text{for } \delta_{3i}^- - \Delta_{3i}^* = 0, \Delta_{3i}^- - \delta_{3i}^* = 0, \gamma_i^- = \gamma_i^*, \\ 1, & \text{for } \delta_{3i}^- - \Delta_{3i}^* \geq 0, \Delta_{3i}^- - \delta_{3i}^* > 0, \gamma_i^- \geq \gamma_i^*, \\ 0, & \text{for } \delta_{3i}^- - \Delta_{3i}^* < 0, \Delta_{3i}^- - \delta_{3i}^* \leq 0, \gamma_i^- \leq \gamma_i^*, \end{cases} \tag{10}$$

where

$$\begin{aligned} \beta^+ &= \left[\frac{1}{4}(\Delta_{1i}^- - \delta_{1i}^*) - \frac{1}{3}(\Delta_{2i}^- + \delta_{2i}^*) + \frac{1}{2}(\Delta_{3i}^- - \delta_{3i}^*)\right] \\ &\quad + \left[\frac{1}{4}(\delta_{1i}^- - \Delta_{1i}^*)(1 - \mu_1^4) + \frac{1}{3}(\delta_{2i}^- + \Delta_{2i}^*)(1 - \mu_1^3) + \frac{1}{2}(\delta_{3i}^- - \Delta_{3i}^*)(1 - \mu_1^2)\right], \\ \beta^- &= -\left[\frac{1}{4}(\delta_{1i}^- - \Delta_{1i}^*)\mu_1^4 + \frac{1}{3}(\delta_{2i}^- + \Delta_{2i}^*)\mu_1^3 + \frac{1}{2}(\delta_{3i}^- - \Delta_{3i}^*)\mu_1^2\right], \\ \mu_1 &= \frac{\left[-(\delta_{2i}^- + \Delta_{2i}^*) + \sqrt{(\delta_{2i}^- + \Delta_{2i}^*)^2 - 4(\delta_{1i}^- - \Delta_{1i}^*)(\delta_{3i}^- - \Delta_{3i}^*)}\right]}{\left[2(\delta_{1i}^- - \Delta_{1i}^*)\right]}, \\ \lambda^+ &= \frac{1}{4}(\Delta_{1i}^- - \delta_{1i}^*)\mu_2^4 + \frac{1}{3}(-\Delta_{2i}^- - \delta_{2i}^*)\mu_2^3 + \frac{1}{2}(\Delta_{3i}^- - \delta_{3i}^*)\mu_2^2, \\ \lambda^- &= -\left[\frac{1}{4}(\delta_{1i}^- - \Delta_{1i}^*) + \frac{1}{3}(\delta_{2i}^- + \Delta_{2i}^*) + \frac{1}{2}(\delta_{3i}^- - \Delta_{3i}^*)\right] \\ &\quad - \left[\frac{1}{4}(\Delta_{1i}^- - \delta_{1i}^*)(1 - \mu_2^4) - \frac{1}{3}(\Delta_{2i}^- + \delta_{2i}^*)(1 - \mu_2^3) + \frac{1}{2}(\Delta_{3i}^- - \delta_{3i}^*)(1 - \mu_2^2)\right], \\ \mu_2 &= \frac{\left[(\Delta_{2i}^- + \delta_{2i}^*) - \sqrt{(-\Delta_{2i}^- - \delta_{2i}^*)^2 - 4(\Delta_{1i}^- - \delta_{1i}^*)(\Delta_{3i}^- - \delta_{3i}^*)}\right]}{\left[2(\Delta_{1i}^- - \delta_{1i}^*)\right]}. \end{aligned}$$

Obviously, an alternative A_i becomes closer to the positive ideal point and farther from the final negative ideal point as the CC_i membership value approaches 1. As a result, a compromise satisfactory solution can be found, so the closeness coefficient value of each alternative for the positive ideal point and negative ideal point can also be considered, while maintaining the objectivity with regard to the criteria of ups and downs of alternatives. Therefore, according to the closeness coefficient values, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

In sum, an algorithm of the multi-person multicriteria decision-making under uncertainty presented above is summarized below:

(1) obtain the decision matrix of the fuzzy ratings of possible alternatives with respect to criteria and the weights of criteria, and construct the fuzzy decision matrix $\tilde{\mathbf{D}}$ and the fuzzy weight vector $\tilde{\boldsymbol{w}}$ as expressed in Eq. (3);
(2) construct the normalized fuzzy decision matrix, as expressed in Eq. (4);
(3) determine the positive ideal point and negative ideal point from the normalized fuzzy decision matrix as shown in Eq. (5);
(4) calculate the Hamming distances between each alternative and the positive ideal and negative ideal points, using Eq. (6), and construct the positive ideal matrix $\tilde{\mathbf{H}}^*$ and negative ideal matrix $\tilde{\mathbf{H}}^-$ as shown in Eq. (7);
(5) calculate the positive ideal weighted fuzzy evaluation values and negative ideal weighted fuzzy evaluation values, using Eq. (8);
(6) calculate the final fuzzy evaluation values of the positive ideal and negative ideal from each alternative, using Eq. (9);
(7) calculate the closeness coefficients for each alternative using Eq. (10);
(8) according to the closeness coefficients, the ranking order of all alternatives can be determined.

3. Numerical example

In this section, we present the theoretical case of a distribution center (DC) location selection problem, and this case can simulate complex and difficult selection problems to agree with the real-life environment. In general, the selection of the best DC location for an industrial organization or conglomerate with many decision factors or criteria is a multicriteria decision-making problem. Besides, these factors or criteria often have qualitative/quantitative characteristics in the real-life environment. The values for the qualitative criteria are often imprecisely defined for the decision makers in many situations. Therefore, the selection of a DC location is a complex and difficult problem in the real-life environment.

We assume that a logistic company desires to select a suitable city for establishing a new DC. The hierarchical structure of this decision problem is shown in Table 2. The evaluation is done by a committee of five judges $D_1, D_2, \ldots, D_5$. First, we search for three possible alternatives A_1, A_2 and A_3 to remain for further evaluation after preliminary screening. The company considers six criteria for selecting the most suitable possible alternatives. The six estimation criteria are considered as follows:

(1) benefit criteria: (a) expansion possibility (C_1), (b) availability of acquirement material (C_2), (c) closeness to demand market (C_3), (d) human resources (C_4), (e) square measure of area (C_5).
(2) cost criterion: (a) investment cost (C_6).

The proposed method is now applied to solve this problem. The computational procedure is summarized as follows:

Step 1. Judges' subjective judgments develop the fuzzy criteria weights with respect to aspects by using the fuzzy geometric mean method, as in Eq. (1) above;
Step 2. Judges use the linguistic variables (shown in Table 1) to evaluate the ratings of alternatives with respect to each criterion and we present them in Table 2;
Step 3. Obtain the decision matrix of fuzzy ratings of possible alternatives with respect to criteria as in Eq. (2) above and the weights of criteria, and construct the fuzzy decision matrix $\tilde{\mathbf{D}}$ and the fuzzy weight matrix $\tilde{w}$ (shown in Table 3) as expressed in Eq. (3);
Step 4. Construct the fuzzy normalized decision matrix as in Eq. (4) above (shown in Table 4);
Step 5. Determine the positive ideal point and negative ideal point as expressed in Eq. (5);

$$\begin{aligned} A^* &= [(1, 1, 1), (0.512, 0.756, 1), (0.740, 0.920, 1), (0.673, 0.878, 1), \\ &\quad (0.673, 0.878, 1), (1, 1, 1)]; \\ A^- &= [(0.625, 0.625, 0.625), (0.317, 0.561, 0.805), (0.5, 0.7, 0.88), \\ &\quad (0.122, 0.306, 0.51), (0.122, 0.306, 0.51), (0.61, 0.61, 0.61)]. \end{aligned}$$

Table 2
The ratings of the three candidates by judges under all criteria

Criteria	Candidates	Decision makers				
		D_1	D_2	D_3	D_4	D_5
C_1	A_1			8 million		
	A_2			5 million		
	A_3			5.2 million		
C_2	A_1	MP	F	MG	MP	F
	A_2	F	F	MG	F	MG
	A_3	F	F	MG	MG	MG
C_3	A_1	MG	MG	G	F	MG
	A_2	G	MG	VG	G	G
	A_3	G	G	VG	G	G
C_4	A_1	P	MP	F	MP	MP
	A_2	MG	G	G	G	G
	A_3	MG	G	G	G	G
C_5	A_1	P	MP	F	MP	MP
	A_2	MG	MG	G	MG	MG
	A_3	MG	G	G	G	G
C_6	A_1			5 hectare		
	A_2			3.5 hectare		
	A_3			3.05 hectare		

Table 3
The fuzzy weight of criteria and fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
Weight	(0.164, 0.232, 0.327)	(0.080, 0.113, 0.159)	(0.096, 0.136, 0.191)	(0.130, 0.188, 0.274)	(0.095, 0.139, 0.202)	(0.137, 0.192, 0.273)
A_1	(8, 8, 8)	(2.6, 4.6, 6.6)	(5, 7, 8.8)	(1.2, 3, 5)	(1.2, 3, 5)	(5, 5, 5)
A_2	(5, 5, 5)	(5, 3.8, 5.8)	(7, 8.8, 9.8)	(6.6, 8.6, 9.8)	(5.4, 7.4, 9.2)	(3.5, 3.5, 3.5)
A_3	(5.2, 5.2, 5.2)	(4.2, 6.2, 8.2)	(7.4, 9.2, 10)	(6.6, 8.6, 9.8)	(6.6, 8.6, 9.8)	(3.05, 3.05, 3.05)

Table 4
The fuzzy normalized decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.625, 0.625, 0.625)	(0.317, 0.561, 0.805)	(0.500, 0.700, 0.880)	(0.122, 0.306, 0.510)	(0.122, 0.306, 0.510)	(1, 1, 1)
A_2	(1, 1, 1)	(0.463, 0.707, 0.951)	(0.700, 0.880, 0.980)	(0.673, 0.878, 1)	(0.551, 0.755, 0.939)	(0.7, 0.7, 0.7)
A_3	(0.962, 0.962, 0.962)	(0.512, 0.756, 1)	(0.740, 0.920, 1)	(0.673, 0.878, 1)	(0.673, 0.878, 1)	(0.61, 0.61, 0.61)

Step 6. Calculate the Hamming distances between each of the alternatives and the positive ideal and negative ideal points, respectively using Eqs. (6) and (7), and construct the positive ideal matrix and negative ideal matrix as shown in Tables 5 and 6;

Step 7. Calculate the weighted fuzzy evaluation values of the positive ideal and negative ideals, using Eq. (8), as shown in **Appendix B**;

Step 8. Calculate the final fuzzy evaluation values of the positive ideal and negative ideals, using Eq. (9);

$$\tilde{p}_1^* = (0.072, 0.192, 0.061/0.326/0.083, 0.502, 0.744);$$
$$\tilde{p}_2^* = (0.063, 0.107, -0.085/0.086/0.085, 0.404, 0.405);$$
$$\tilde{p}_3^* = (0.060, 0.102, -0.078/0.084/0.085, 0.403, 0.402);$$
$$\tilde{p}_1^- = (0.071, 0.113, -0.109/0.075/0.101, 0.467, 0.441);$$
$$\tilde{p}_2^- = (0.072, 0.189, 0.054/0.315/0.088, 0.518, 0.745);$$
$$\tilde{p}_3^- = (0.072, 0.190, 0.055/0.317/0.083, 0.498, 0.732).$$

Table 5
The fuzzy positive ideal matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1^*	(0.375, 0.375, 0.375)	(−0.292, 0.195, 0.683)	(−0.14, 0.22, 0.5)	(0.163, 0.571, 0.878)	(0.163, 0571, 0.878)	(0, 0, 0)
A_2^*	(0, 0, 0)	(−0.439, 0.049, 0.537)	(−0.24, 0.04, 0.3)	(−0.327, 0, 0.327)	(−0.265, 0.122, 0.449)	(0.3, 0.3, 0.3)
A_3^*	(0.038, 0.038, 0.038)	(−0.488, 0, 0.488)	(−0.26, 0, 0.26)	(−0.327, 0, 0.327)	(−0.327, 0, 0.327)	(0.39, 0.39, 0.39)

Table 6
The fuzzy negative ideal matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1^-	(0, 0, 0)	(−0.488, 0, 0.488)	(−0.38, 0, 0.38)	(−0.388, 0, 0.388)	(−0.388, 0, 0.388)	(0.39, 0.39, 0.39)
A_2^-	(0.375, 0.375, 0.375)	(−0.341, 0.146, 0.634)	(−0.18, 0.18, 0.48)	(0.163, 0.571, 0.878)	(0.041, 0.449, 0.816)	(0.09, 0.09, 0.09)
A_3^-	(0.337, 0.337, 0.337)	(−0.293, 0.196, 0.683)	(−0.14, 0.22, 0.5)	(0.163, 0.571, 0.878)	(0.163, 0.571, 0.878)	(0, 0, 0)

Step 9. Calculate the closeness coefficients of each of the alternatives using Eq. (10);

$$CC_1 = \mu_{\tilde{R}}\left(\tilde{p}_1^-(-)\tilde{p}_1^*, 0\right) = 0.0749;$$
$$CC_2 = \mu_{\tilde{R}}\left(\tilde{p}_2^-(-)\tilde{p}_2^*, 0\right) = 0.9240;$$
$$CC_3 = \mu_{\tilde{R}}\left(\tilde{p}_3^-(-)\tilde{p}_3^*, 0\right) = 0.9260.$$

Step 10. According to the closeness coefficient, the ranking order of all alternatives can be determined. In this case, the best selection is candidate A_3.

The closeness coefficient values of alternatives A_1, A_2 and A_3 are 0.0749, 0.9240 and 0.9260, respectively. Therefore, the ranking order of the three alternatives is $A_3 \succ A_2 \succ A_1$. The candidate A_3 is the best location for establishing a new DC. The proposed method was compared with other evaluation methods, and the results demonstrated that it is a good means of evaluation and appears to be more appropriate. We used the three methods of FSAW [6], FTOPSIS [8] and the efficient fuzzy model [18] to compare with the proposed method. In this case, we obtained the evaluation values of alternatives A_1, A_2 and A_3 as 0.6508, 0.8867 and 0.8832 by FSAW, respectively. Therefore, the ranking order of the three alternatives is $A_2 \succ A_3 \succ A_1$. The evaluation values of alternatives A_1, A_2 and A_3 were 0.2070, 0.6476 and 0.6475 by using the efficient fuzzy model, and the ranking order of the three alternatives is $A_2 \succ A_3 \succ A_1$. The results for the ranking order were $A_2 \succ A_3 \succ A_1$ by using FTOPSIS, which are similar to those from FSAW and the efficient fuzzy model, and the evaluation values of alternatives A_1, A_2 and A_3 were 0.1179, 0.1567 and 0.1556. We obtained the candidate A_2 as the best location using these methods. Therefore, this case shows that the proposed method produces satisfactory results. In general situations, it is obvious that the result obtained with this method coincides with the others presented in [7,9,18]. If the performances of two alternatives are close, then we can search for a best alternative from the candidates easily using the proposed method. Though the technique of positive ideal and negative ideal points easily produces satisfactory results which are composed of the overall best criteria values and overall worst criteria values attainable. The results above are based on the overall preferences of the evaluators and we can apply this method to individual evaluators according to their own preferences to select their ideal candidate (DC location).

4. Discussion

The algorithm presented in the previous section shows the practical advantages of the proposed method over other evaluation methods in terms of computational simplicity. To examine its results concerning rationality and discriminatory ability, the proposed method was compared with comparable evaluation methods, and we used one of the concepts discussed above. The examination shows that the proposed method always produces satisfactory results for all the cases in terms of rationality and discriminatory ability. It is noteworthy that the proposed method also gave satisfactory results if the closeness degree between the two alternatives was higher. To demonstrate how this method compares favorably with comparable methods, we present just five cases of degrees of closeness between the two alternatives (shown in **Appendix C**), and this paper shows the ranking results from the FSAW [6], FTOPSIS [8], efficient fuzzy model [18] and the proposed method. First, we assume that an evaluation problem has three possible alternatives and two criteria, where the performances of criterion C_1 with respect to each alternative are equivalent, and the performances of criterion C_2 with respect to two of the alternatives have a higher degree of closeness. We will subsequently assume the performances (C_1, C_2) of the two alternatives have a higher degree of closeness, as shown in **Appendix C**. Moreover, this study assumes that the weight value of each criterion is fixed, and the performance of each criterion is changeable. If the performance of alternative A is greater than the performance of alternative B, then the weighting performance of alternative (A) is certainly greater than the alternative (B).

In the case (a), let the performances of alternatives A_2 and A_3 under criteria C_1 and C_2 be close, then the close performances are defined by fuzzy reference sets. This paper found that the proposed method also gave satisfactory results, as the results of this method coincide with those from the performances ranking method. In the case (b), if the performances of the two alternatives under one criterion or all criteria are close, then the results of this method coincide with those from the performances ranking method and the efficient fuzzy model. In the case (c), if the performances of the two alternatives under one criterion are close, then the results of this method coincide with those from the performances ranking method and FTOPSIS. If the performances of the two alternatives under all criteria are quite close, then the results of this method are similar to those from the performances ranking method. In the case (d), if the performances of the alternatives under all criteria are close, then the results from all methods are equal except those from the efficient fuzzy model. In the case (e), if the performances of the two alternatives are close, then the results from this method coincide with those from FSAW, FTOPSIS and the performances ranking method. Lastly, this study found that all methods, except for the proposed method, gave unsatisfactory results for one case or more: in particular, when the performances of the two alternatives calculated by other methods are close, we obtained unsatisfactory results, but the results from this method coincide with those from the other methods in general situations. Therefore, the decision makers will have more confidence in this method if the alternative numbers are much larger or the performances of the alternatives are close.

In addition, the proposed method used the closeness coefficient values to rank all the alternatives, where the closeness coefficient value represents the utility degree size for each alternative. If the closeness coefficient value of an evaluation alternative is greater than 0.5, then this alternative has a higher utility degree. If the closeness coefficient value of an evaluation alternative is equal to 0.5, then this alternative has a moderate utility degree. If the closeness coefficient value of an evaluation alternative is smaller than 0.5, then this alternative has a lower utility degree. If the closeness coefficient value of an evaluation alternative is equal to 0, then this alternative has a much lower utility degree and it is a poorer alternative. If the closeness coefficient value of an evaluation alternative is equal to 1, then this alternative has a much higher utility degree and it is a better alternative. Therefore, the proposed method can see different degrees between each of the alternatives and the positive ideal and negative ideal points. If the closeness coefficient value of an evaluation alternative is getting on for 1, then this alternative is the only alternative which has the shortest distance to the ideal point and the farthest distance to the negative ideal point. We can see that the proposed method based on an aggregating function represents the relative degree of closeness to the ideal solution.

5. Conclusions

Fuzzy multicriteria analysis provides an effective framework for ranking competing alternatives in terms of their overall performance with respect to criteria. In this paper, we have presented an effective fuzzy multicriteria analysis method based on the incorporated efficient fuzzy model and concepts of positive ideal and negative ideal points to solve decision-making problems with multi-judges in the real-life environment, where judges are allowed to use fuzzy sets to evaluate the performance of alternatives and the importance of criteria. This method efficiently grasps the ambiguity existing in available information as well as the essential fuzziness in human judgment and preference, and it always produced satisfactory results for all the cases examined in terms of rationality and discriminatory ability. Furthermore, the technique of positive ideal and negative ideal points easily produces satisfactory results, and this technique can stimulate creativity and the invention of new methods and alternative techniques.

Although the proposed method presented in this paper is illustrated by a location selection problem, it can also be applied to problems such as information project selection, material selection and many other areas of management decision problems and strategy selection problems.

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Appendix A

The concepts and definitions of fuzzy sets and fuzzy operations are delineated as follows.

Definition 1. If A_i and A_j are both triangular fuzzy numbers in the form of (a_i, b_i, c_i) and $\left(a_j, b_j, c_j\right)$, convex fuzzy subsets of $\mathbb{R}$, note that $A_i(-)A_j$ is also a triangular fuzzy number in the form of (a, b, c), where $a = a_i - c_j$, $b = b_i - b_j$, and $c = c_i - a_j$. Then, $\mu_R(A_i, A_j)$ can be calculated using the following equation [18,30]:

$$\mu_R(A_i, A_j) = \begin{cases} 1, & \textit{for } a \geq 0, b \geq 0, c > 0, \\ 0, & \textit{for } a < 0, b \leq 0, c \leq 0, \\ 1 + \dfrac{a^3}{[(b-a)^2(a+4b+c) - 2a^3]}, & \textit{for } a < 0, b \geq 0, c \geq 0, \\ \dfrac{c^3}{[2c^3 - (c-b)^2(a+4b+c)]}, & \textit{for } a \leq 0, b \leq 0, c > 0, \\ 0.5, & \textit{for } a = b = c = 0. \end{cases}$$

Definition 2. Let D and W be triangular fuzzy numbers in the form of (d_1, d_2, d_3) and (w_1, w_2, w_3), respectively. The extended product of D and W, denoted by $D(\cdot)W = (\delta_1, \delta_2, \delta_3/\gamma/\Delta_1, \Delta_2, \Delta_3)$, is defined by the membership function $\mu_{D(\cdot)W}(x)$ [18,26]:

$$\mu_{D(\cdot)W}(x) = \begin{cases} \dfrac{-\delta_2 + \sqrt{\delta_2^2 - 4\delta_1(\delta_3 - x)}}{2\delta_1}, & \delta_3 \leq x \leq \gamma, \\ \dfrac{\Delta_2 - \sqrt{\Delta_2^2 - 4\Delta(\Delta_3 - x)}}{2\Delta_1}, & \gamma \leq x \leq \Delta_3, \\ 0, & \text{otherwise}, \end{cases}$$

where

$$\delta_1 = (w_2 - w_1)(d_2 - d_1), \quad \delta_2 = w_1(d_2 - d_1) + d_1(w_2 - w_1), \quad \delta_3 = w_1 d_1,$$
$$\Delta_1 = (w_3 - w_2)(d_3 - d_2), \quad \Delta_2 = w_3(d_3 - d_2) + d_3(w_3 - w_2), \quad \Delta_3 = w_3 d_3,$$
$$\gamma = w_2 d_2.$$

The fuzzy number with a parabolic membership function will be denoted by its parameters as $(\delta_1, \delta_2, \delta_3/\gamma/\Delta_1, \Delta_2, \Delta_3)$, and the proof of this is in [18,26].

Definition 3. If A_i and A_j are both fuzzy numbers with a parabolic membership function in the form of $(\delta_{i1}, \delta_{i2}, \delta_{i3}/\gamma_i/\Delta_{i1}, \Delta_{i2}, \Delta_{i3})$ and $\left(\delta_{j1}, \delta_{j2}, \delta_{j3}/\gamma_j/\Delta_{j1}, \Delta_{j2}, \Delta_{j3}\right)$, note that $A_i(-)A_j$ is also a fuzzy number with a parabolic membership function. Then, $\mu_R(A_i, A_j)$ can be calculated using the following equation [18]:

$$\mu_R(A_i(-)A_j, 0) = \begin{cases} \dfrac{\beta^+}{(\beta^+ + \beta^-)}, & \textit{for } \delta_{i3} - \Delta_{j3} < 0, \Delta_{i3} - \delta_{j3} \geq 0, \gamma_i \geq \gamma_j, \\ \dfrac{\lambda^+}{(\lambda^+ + \lambda^-)}, & \textit{for } \delta_{i3} - \Delta_{j3} \leq 0, \Delta_{i3} - \delta_{j3} > 0, \gamma_i \leq \gamma_j, \\ 0.5, & \textit{for } \delta_{i3} - \Delta_{j3} = 0, \Delta_{i3} - \delta_{j3} = 0, \gamma_i = \gamma_j, \\ 1, & \textit{for } \delta_{i3} - \Delta_{j3} \geq 0, \Delta_{i3} - \delta_{j3} > 0, \gamma_i \geq \gamma_j, \\ 0, & \textit{for } \delta_{i3} - \Delta_{j3} < 0, \Delta_{i3} - \delta_{j3} \leq 0, \gamma_i \leq \gamma_j, \end{cases}$$

where

$$\beta^+ = \left[\frac{1}{4}(\Delta_{i1} - \delta_{j1}) - \frac{1}{3}(\Delta_{i2} + \delta_{j2}) + \frac{1}{2}(\Delta_{i3} - \delta_{j3})\right] + \left[\frac{1}{4}(\delta_{i1} - \Delta_{j1})(1 - \mu_1^4) + \frac{1}{3}(\delta_{i2} + \Delta_{j2})(1 - \mu_1^3) + \frac{1}{2}(\delta_{i3} - \Delta_{j3})(1 - \mu_1^2)\right],$$

$$\beta^- = -\left[\frac{1}{4}(\delta_{i1} - \Delta_{j1})\mu_1^4 + \frac{1}{3}(\delta_{i2} + \Delta_{j2})\mu_1^3 + \frac{1}{2}(\delta_{i3} - \Delta_{j3})\mu_1^2\right],$$

$$\mu_1 = \frac{\left[-(\delta_{i2} + \Delta_{j2}) + \sqrt{(\delta_{i2} + \Delta_{j2})^2 - 4(\delta_{i1} - \Delta_{i1})(\delta_{i3} - \Delta_{j3})}\right]}{\left[2(\delta_{i1} - \Delta_{j1})\right]},$$

$$\lambda^+ = \frac{1}{4}(\Delta_{i1} - \delta_{j1})\mu_2^4 + \frac{1}{3}(-\Delta_{i2} - \delta_{j2})\mu_2^3 + \frac{1}{2}(\Delta_{i3} - \delta_{j3})\mu_2^2,$$

$$\lambda^- = -\left[\frac{1}{4}(\delta_{i1} - \Delta_{j1}) + \frac{1}{3}(\delta_{i2} + \Delta_{j2}) + \frac{1}{2}(\delta_{i3} - \Delta_{j3})\right] - \left[\frac{1}{4}(\Delta_{i1} - \delta_{j1})(1 - \mu_2^4) - \frac{1}{3}(\Delta_{i2} + \delta_{j2})(1 - \mu_2^3) + \frac{1}{2}(\Delta_{i3} - \delta_{j3})(1 - \mu_2^2)\right],$$

$$\mu_2 = \frac{\left[(\Delta_{i2} + \delta_{j2}) - \sqrt{(-\Delta_{i2} - \delta_{j2})^2 - 4(\Delta_{i1} - \delta_{j1})(\Delta_{i3} - \delta_{j3})}\right]}{\left[2(\Delta_{i1} - \delta_{j1})\right]}.$$

Definition 4. Consider two triangular fuzzy numbers $\tilde{a}_1 = (l_1, m_1, r_1)$ and $\tilde{a}_2 = (l_2, m_2, r_2)$. Their operational laws are as follows [6]:

1. $\tilde{a}_1(+)\tilde{a}_2 = (l_1, m_1, r_1)\,(+)\,(l_2, m_2, r_2) = (l_1 + l_2, m_1 + m_2, r_1 + r_2)$.
2. $\tilde{a}_1(-)\tilde{a}_2 = (l_1, m_1, r_1)\,(-)\,(l_2, m_2, r_2) = (r_1 - l_2, m_1 - m_2, l_1 - r_2)$.
3. $\tilde{a}_1(\cdot)\tilde{a}_2 = (l_1, m_1, r_1)\,(\cdot)\,(l_2, m_2, r_2) \cong (l_1 l_2, m_1 m_2, r_1 r_2)$.
4. $\lambda(\cdot)\tilde{a}_2 = (\lambda, \lambda, \lambda)\,(\cdot)\,(l_2, m_2, r_2) = (\lambda l_2, \lambda m_2, \lambda r_2)$.
5. $\tilde{a}_1^- = (l_1, m_1, r_1)^- \cong (1/l_1, 1/m_1, 1/r_1)$.

Appendix B

Calculate the weighted fuzzy evaluation values of positive ideal and negative ideals.

1. Calculate the positive ideal weighted fuzzy evaluation values of alternatives with respect to each criterion

$\tilde{p}_{11}^* = (0, 0.026, 0.062/0.087/0, 0.036, 0.123)$,
$\tilde{p}_{21}^* = (0, 0, 0/0/0, 0, 0)$,
$\tilde{p}_{31}^* = (0, 0.003, 0.006/0.009/0, 0.004, 0.013)$,
$\tilde{p}_{12}^* = (0.016, 0.029, -0.023/0.022/0.022, 0.109, 0.109)$,
$\tilde{p}_{22}^* = (0.016, 0.025, -0.035/0.006/0.022, 0.102, 0.085)$,
$\tilde{p}_{32}^* = (0.016, 0.023, -0.039/0/0.022, 0.100, 0.078)$,
$\tilde{p}_{13}^* = (0.014, 0.029, -0.013/0.030/0.015, 0.081, 0.096)$,
$\tilde{p}_{23}^* = (0.011, 0.017, -0.023/0.005/0.014, 0.066, 0.057)$,
$\tilde{p}_{33}^* = (0.010, 0.015, -0.025/0/0.014, 0.064, 0.050)$,
$\tilde{p}_{14}^* = (0.024, 0.063, 0.021/0.107/0.026, 0.159, 0.240)$,
$\tilde{p}_{24}^* = (0.019, 0.024, -0.042/0/0.028, 0.118, 0.089)$,
$\tilde{p}_{34}^* = (0.019, 0.024, -0.042/0/0.028, 0.118, 0.089)$,
$\tilde{p}_{15}^* = (0.018, 0.046, 0.016/0.079/0.019, 0.117, 0.117)$,
$\tilde{p}_{25}^* = (0.017, 0.025, -0.025/0.017/0.021, 0.094, 0.091)$,

Table C.1
Comparison results

(a)

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.673, 0.878,1)	1	(0.15, 0.45, 0.90)	3	0.70909	3	0.01228	3	0.37069	3	0.29163	3
A_2	(0.673, 0.878,1)	1	(0.4999, 0.7999, 0.9999)	2	0.84151	1	0.99102	2	0.42631	1	0.73324	2
A_3	(0.673, 0.878,1)	1	(0.50, 0.80,1)	1	0.84151	1	0.99103	1	0.42631	1	0.73326	1

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.45, 0.65, 0.85)	3	(0.15, 0.45, 0.90)	3	0.60898	3	0.01769	3	0.32596	3	0.27694	3
A_2	(0.5, 0.7, 0.9)	2	(0.4999, 0.7999, 0.9999)	2	0.76730	2	0.98232	2	0.39281	1	0.74694	2
A_3	(0.5001, 0.7001, 0.9001)	1	(0.50, 0.80, 1)	1	0.76731	1	0.98233	1	0.39281	1	0.74696	1

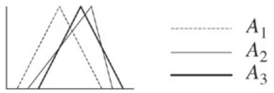
(b)

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.673, 0.878,1)	1	(0.15, 0.45, 0.90)	3	0.70909	3	0.01228	3	0.37069	3	0.29162	3
A_2	(0.673, 0.878,1)	1	(0.5001, 0.8, 0.9999)	1	0.84151	1	0.99104	1	0.42631	1	0.73329	1
A_3	(0.673, 0.878,1)	1	(0.50, 0.80,1)	2	0.84151	1	0.99103	2	0.42631	1	0.73327	2

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.45, 0.65, 0.85)	3	(0.15, 0.45, 0.90)	3	0.60898	3	0.02183	3	0.32596	3	0.27965	3
A_2	(0.451, 0.7, 0.949)	1	(0.499, 0.8, 0.999)	1	0.77208	2	0.96851	1	0.39638	2	0.73103	1
A_3	(0.45, 0.7, 0.95)	2	(0.5, 0.8, 1)	2	0.77218	1	0.96799	2	0.39648	1	0.73066	2

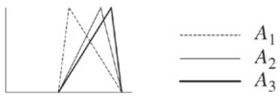
(continued on next page)

Table C.1 (*continued*)

(c)


	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.673, 0.878, 1)	1	(0.15, 0.45, 0.9)	3	0.70909	3	0.01265	3	0.37069	3	0.26672	3
A_2	(0.673, 0.878, 1)	1	(0.45, 0.9, 0.95)	1	0.84151	1	0.98956	2	0.42665	1	0.75896	1
A_3	(0.673, 0.878, 1)	1	(0.5, 0.8, 1)	2	0.84151	1	0.99045	1	0.42631	2	0.70350	2

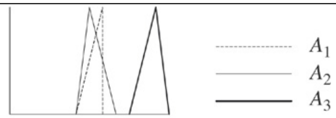
	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.45, 0.65, 0.85)	3	(0.15, 0.45, 0.9)	3	0.60898	3	0.00857	3	0.32596	3	0.19430	3
A_2	(0.55, 0.9, 0.95)	1	(0.45, 0.9, 0.95)	1	0.81875	2	0.98953	2	0.41663	2	0.83051	1
A_3	(0.6, 0.8, 1)	2	(0.5, 0.8, 1)	2	0.81910	1	0.99129	1	0.41670	1	0.75261	2

(d)


	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.673, 0.878, 1)	1	(0.7, 0.78, 1)	3	0.87078	3	0	3	0.43903	3	0.35242	3
A_2	(0.673, 0.878, 1)	1	(0.7, 0.9799, 1)	2	0.90409	2	0.5	1	0.45503	2	0.64748	2
A_3	(0.673, 0.878, 1)	1	(0.7, 0.98, 1)	1	0.90411	1	0.5	1	0.45504	1	0.64758	1

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.6, 0.75, 1)	3	(0.7, 0.78, 1)	3	0.83985	3	0	3	0.42554	3	0.26155	3
A_2	(0.6, 0.9499, 1)	2	(0.7, 0.9799, 1)	2	0.90722	2	0.99979	1	0.46764	2	0.73827	2
A_3	(0.6, 0.95, 1)	1	(0.7, 0.98, 1)	1	0.90725	1	0.99979	1	0.46780	1	0.73845	1

Table C.1 (*continued*)

(e)


	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.673, 0.878, 1)	1	(0.7, 0.77, 0.79)	2	0.83334	2	0	2	0.42023	2	0.24161	2
A_2	(0.673, 0.878, 1)	1	(0.7, 0.75, 0.8)	3	0.83171	3	0	2	0.41946	3	0.21933	3
A_3	(0.673, 0.878, 1)	1	(0.85, 0.95, 1)	1	0.92356	1	1	1	0.46407	1	0.76879	1

	C_1		C_2		FSAW [6]		Efficient fuzzy model [18]		FTOPSIS [8]		Proposed method	
Weight	(0.375, 0.511, 0.668)	Ranking [29]	(0.489, 0.5, 0.511)	Ranking [29]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(0.6, 0.7, 0.9)	3	(0.7, 0.75, 0.8)	3	0.74773	3	0	2	0.37819	3	0.02464	3
A_2	(0.6, 0.77, 0.79)	2	(0.7, 0.77, 0.79)	2	0.75906	2	0	2	0.38338	2	0.04634	2
A_3	(0.8, 0.9, 1)	1	(0.85, 0.95, 1)	1	0.94318	1	1	1	0.47280	1	0.96707	1

$\tilde{p}^*_{35} = (0.014, 0.017, -0.031/0/0.021, 0.087, 0.066),$
$\tilde{p}^*_{16} = (0, 0, 0/0/0, 0, 0),$
$\tilde{p}^*_{26} = (0, 0.017, 0.041/0.058/0, 0.024, 0.082),$
$\tilde{p}^*_{36} = (0, 0.021, 0.053/0.075/0, 0.032, 0.106).$

2. Calculate the negative ideal weighted fuzzy evaluation values of alternatives with respect to each criterion

$\tilde{p}^-_{11} = (0, 0, 0/0/0, 0, 0),$
$\tilde{p}^-_{21} = (0, 0.026, 0.062/0.087/0, 0.036, 0.123),$
$\tilde{p}^-_{31} = (0, 0.023, 0.055/0.078/0, 0.032, 0.110),$
$\tilde{p}^-_{12} = (0.016, 0.023, -0.039/0/0.022, 0.100, 0.078),$
$\tilde{p}^-_{22} = (0.016, 0.028, -0.027/0.017/0.022, 0.107, 0.101),$
$\tilde{p}^-_{32} = (0.016, 0.029, -0.023/0.022/0.022, 0.109, 0.109),$
$\tilde{p}^-_{13} = (0.015, 0.021, -0.036/0/0.021, 0.093, 0.073),$
$\tilde{p}^-_{23} = (0.014, 0.027, -0.017/0.024/0.017, 0.084, 0.092),$
$\tilde{p}^-_{33} = (0.014, 0.029, -0.013/0.030/0.015, 0.081, 0.096),$
$\tilde{p}^-_{14} = (0.022, 0.028, -0.050/0/0.033, 0.140, 0.106),$
$\tilde{p}^-_{24} = (0.024, 0.063, 0.021/0.107/0.026, 0.159, 0.240),$
$\tilde{p}^-_{34} = (0.024, 0.063, 0.021/0.107/0.026, 0.159, 0.240),$
$\tilde{p}^-_{15} = (0.017, 0.020, -0.037/0/0.024, 0.103, 0.078),$
$\tilde{p}^-_{25} = (0.018, 0.041, 0.004/0.062/0.023, 0.126, 0.165),$
$\tilde{p}^-_{35} = (0.018, 0.046, 0.016/0.079/0.019, 0.117, 0.177),$
$\tilde{p}^-_{16} = (0, 0.021, 0.053/0.075/0, 0.032, 0.106),$
$\tilde{p}^-_{26} = (0, 0.005, 0.012/0.017/0, 0.007, 0.025),$
$\tilde{p}^-_{36} = (0, 0, 0/0/0, 0, 0).$

Appendix C

See Table C.1.

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