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An effective decision making method for product acceptance

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Abstract

Acceptance sampling plans are practical tools for quality assurance applications involving quality contract on product orders. The sampling plans provide the vendor and buyer decision rules for product acceptance to meet the preset product quality requirement. As the rapid advancement of manufacturing technology, suppliers require their products to be of high quality with very low fraction of defectives often measured in parts per million. Unfortunately, traditional methods for calculating fraction of defectives no longer work since any sample of reasonable size probably contains no defective product items. In this paper, we introduce an effective sampling plan based on process capability index C_{pk} to deal with product acceptance determination for low fraction of defectives. The proposed new sampling plan is developed based on the exact sampling distribution rather than approximation. Practitioners can use the proposed method to determine the number of required inspection units, the critical acceptance value, and make reliable decisions in product acceptance.

Keywords: Acceptance sampling plans; Critical acceptance values; Decision making; Fraction of defectives; Process capability indices

1. Introduction

Acceptance sampling plans are practical tools for quality assurance applications. Sampling plans provide the vendor and buyer general decision rules for product acceptance while meeting their needs for product quality. A well-designed sampling plan can effectively reduce the difference between the actual supply quantity and order quantity. Acceptance sampling plans state the required sample size for product inspection and the decision making rule for product sentencing. The criteria used for measuring the performance of an acceptance sampling plan is usually based on the operating characteristic (OC) curve quantifying the risk. The OC curve plots the probability of accepting the lot against actual lot fraction defective, which displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected.

The vendors or suppliers usually look at a specific level of product quality, which would yield a high probability of acceptance. For example, the vendor might be interested in the 0.95 probability of acceptance for a certain product quality level. This indicates the level of process fallout that could be experienced with 95% chances that the lots are accepted. The consumer, on the other hand, would look at the product with the acceptable quality level (AQL). The AQL presents the poorest level of quality for the vendor's process that the consumer would consider acceptable as a process average. The consumer would seek a sampling procedure with OC curve providing a high probability of acceptance at the AQL. The consumer would also look at the other end of the OC curve for products with quality worse than AQL. Thus, the consumer may establish a lot tolerance percent

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defective (LTPD). The LTPD is the poorest quality level that the consumer is willing to accept. The consumer demands the sampling plan to have a low probability of accepting the product with a defect level as high as the LTPD.

Acceptance sampling plans basically consist of a required sample size for inspection and an acceptance criterion. Clearly, such sampling scheme involves risks that the sample will not adequately reflect the product quality conditions. Type I error (α) is the probability, for a given sampling plan, of rejecting the product that has a defect level equal to the AQL. The producer suffers when this occurs because a product with acceptable quality is rejected. Thus, α is called the producer's risk with values commonly ranging from 0.01 to 0.10. Type II error (β) is the probability, for a given sampling plan, of accepting the product with defect level equal to the LTPD. The consumer suffers when this occurs, because product with unacceptable quality is accepted. Thus, β is called the consumer's risk with values typically ranging from 0.01 to 0.10. While any two points on the OC curve could be used to construct the sampling plan, it is customary in the industry to use the AQL and LTPD for this purpose.

There are a number of different ways to classify acceptance sampling plans. One major classification is by attributes and variables. When a quality characteristic is measurable on a continuous scale and is known to have a ution of a specified type, it may be appropriate to use variables sampling plans rather than attributes sampling plans for product acceptance applications. The primary advantage of variables sampling plans is that the same operating characteristic curve can be obtained with a smaller sample size than would be required using an attributes sampling plan. That is, a variables sampling plan that has the same protection as an attributes acceptance sampling plan would require less sampling. The precise measurements required by a variables plan would probably cost more than the simple classification of items required by an attributes plan. However, the reduction in sample size may be more than offset this exact expense. Such savings may be especially marked if the inspection is destructive and item is expensive (see, e.g., [1–3]). The basic concepts and models of statistically based on variables sampling plans were introduced by Jennett and Welch [4]. Lieberman and Resnikoff [5] developed extensive tables and OC curves for various AQLs for MIL-STD-414 sampling plan. Owen [6] considered variables sampling plans based on the normal distribution, and developed sampling plans for various levels of probabilities of type I error when the standard deviation is unknown. Das and Mitra [7] have investigated the effect of non-normality on the performance of the sampling plans. Bender [8] considered sampling plans for assuring the percent defective in the case of the product quality characteristics obeying a normal distribution with unknown standard deviation, and presented a procedure using iterative computer program calculating the non-central t-distribution. Govindaraju and Soundararajan [9] developed variables sampling plans that match the OC curves of MIL-STD-105D. Suresh and Ramanathan [10] developed a sampling plan based on a more general symmetric family of distributions. As for the attributes sampling plans, Guenther [11] developed a systematic search procedure, which can be used with published tables of binomial, hypergeometric, and Poisson distributions to obtain the desired acceptance sampling plans. Stephens [12] provided a closed form solution for single sample acceptance sampling plans using a normal approximation to the binomial distribution. Hailey [13] presented a computer program to obtain single sampling plans with a minimum sample size based on either the Poisson or binomial distribution. Hald [14] gave a systematic exposition of the existing statistical theory of lot-by-lot sampling inspection by attributes and provided some tables for the sampling plans. Comparisons between variables sampling plans and attributes sampling plans were investigated by Kao [15] and Hamaker [16], who concluded that the expected sample size required by variables sampling is smaller than those for comparable attributes sampling plans.

As the rapid advancement of manufacturing technology, suppliers require their products to be of high quality with a very low fraction of defectives. The required fraction of defectives is often lower than 0.01%, and is measured in parts per million (PPM). Unfortunately, traditional methods for calculating the fraction nonconforming no longer work since any sample of reasonable size will probably contain no defective product items. An alternative method of measuring the fraction of defectives is to use process capability indices, which are functions of manufacturing specification and actual process mean and standard deviation. In this paper, we introduce an effective acceptance sampling plan for lot sentencing based on the most popular index C_{pk} as a quality benchmark for product acceptance, specifically for normally distributed processes with low fraction of defectives.

2. Process capability indices and product quality

In recent years, process capability indices (PCIs) including C_p, C_{pu}, C_{pl}, and C_{pk} have received substantial research attention in quality assurance and statistical literatures (see [17–21] for more details). The indices have become popular unitless measures on whether a process is capable of reproducing items meeting the quality requirement preset by the product designer. Based on analyzing the PCIs, a production department can identify and improve a poor process so that the product quality can be enhanced and the requirements of the customers can be satisfied. Those indices are defined in the following:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma},$$

$$C_{pu} = \frac{\mathrm{USL} - \mu}{3\sigma}, \quad C_{pl} = \frac{\mu - \mathrm{LSL}}{3\sigma},$$

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\},$$

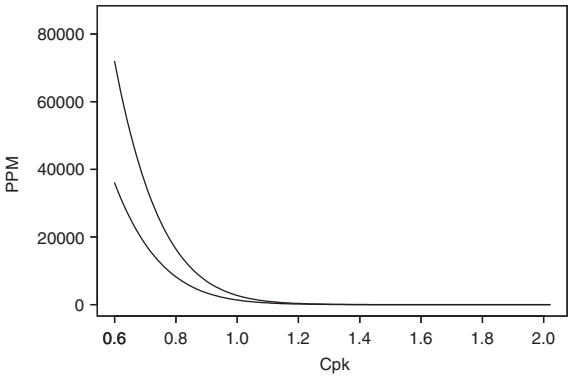

Fig. 1. The bounds on nonconforming units in PPM versus C_{pk}.

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation (overall process variation). While C_p and C_{pk} are appropriate measures for processes with two-sided specifications (which require both USL and LSL), C_{pu} and C_{pl} have been designed specifically for processes with one-sided specification limit (which require only USL or LSL). The C_{pu} index measures the capability of a smaller-the-better process with an upper specification limit USL, whereas the C_{pl} index measures the capability of a larger-the-better process with a lower specification limit LSL.

Process yield has long been the most common and standard criteria used in the manufacturing industries for judging process performance. Process yield is currently defined as the percentage of the processed product units passing the inspections. For normally distributed processes with one-sided specification limit USL or LSL, the process yield is

$$P(X < \mathrm{USL}) = P\left(\frac{X-\mu}{\sigma} < \frac{\mathrm{USL}-\mu}{\sigma}\right) = P(Z < 3C_{pu}) = \Phi(3C_{pu}), \quad (1)$$

Table 2
Some minimum capability requirements of C_{pk} for existing, new, and special processes

C_{pk} Index value	Production process types
1.33	Existing processes
1.50	New processes, or existing processes on safety,strength, or critical parameters
1.67	New processes on safety, strength, or critical parameters

$$P(X > \mathrm{LSL}) = P\left(\frac{X-\mu}{\sigma} > \frac{\mathrm{LSL}-\mu}{\sigma}\right) = 1 - \Phi(-3C_{pl}) = \Phi(3C_{pl}), \quad (2)$$

where Z follows the standard normal distribution $N(0, 1)$ with the cumulative distribution function $\Phi(x) = (2\pi)^{-1/2}\int_{-\infty}^{x}\exp(-t^2/2)\,dt$. It follows from (1) and (2) that bounds on process yield for a fixed value of C_{pk} are given by Boyles [22] and Kotz and Lovelace [18]

$$2\Phi(3C_{pk}) - 1 \leqslant p \leqslant \Phi(3C_{pk}). \quad (3)$$

The upper and lower bounds of nonconforming units in PPM are plotted in Fig. 1 as a function of C_{pk}. Table 1 displays some values of C_{pk} index along with the corresponding upper and lower bounds on nonconforming units in PPM. In a purchasing contract, a minimum C_{pk} value is usually specified. If the prescribed minimum C_{pk} fails to be met, the process is determined to be incapable. Otherwise, the process is determined to be capable.

Montgomery [3] recommended some minimum capability requirements for processes runs under some designated quality conditions, as summarized in Table 2. In particular, $C_{pk} \geqslant 1.33$ for existing processes, and $C_{pk} \geqslant 1.50$ for new processes; $C_{pk} \geqslant 1.50$ also for existing processes on safety, strength, or critical parameter, and $C_{pk} \geqslant 1.67$ for new processes on safety, strength, or critical parameter. Finley [23] also found that required C_{pk} values on all critical supplier processes are 1.33 or higher, with C_{pk} equals 1.67 or higher

Table 1
Index values and the corresponding PPM of nonconformities

C_{pk}	Lower bound	Upper bound	C_{pk}	Lower bound	Upper bound
0.60	35930	71861	1.33	33	66
0.70	17864	35729	1.40	13	27
0.80	8198	16395	1.45	6.807	13.614
0.90	3467	6934	1.50	3.398	6.795
1.00	1350	2700	1.60	0.793	1.587
1.10	483	967	1.67	0.272	0.544
1.20	159	318	1.70	0.170	0.340
1.24	100	200	1.80	0.033	0.067
1.25	88	177	1.90	0.006	0.012
1.30	48	96	2.00	0.001	0.002

preferred. Many companies have recently adopted criteria for evaluating their processes that include process capability objectives more stringent than those in Table 2. Motorola's "Six Sigma" program essentially requires the process capability at least 2.0 to accommodate the possible 1.5σ process shift, with no more than 3.4 PPM of nonconformities (see [24]).

On the other hand, in current practice a process is called "Inadequate" if $C_{pk} < 1.00$; it indicates that the process is not adequate with respect to the production tolerances (specifications), either process variation (σ^2) needs to be reduced or process mean (μ) needs to be shifted closer to the target value T. A process is called "Capable" if $1.00 \leqslant C_{pk} < 1.33$; it indicates that caution needs to be taken regarding process distribution, some process control is required. A process is called "Satisfactory" if $1.33 \leqslant C_{pk} < 1.50$; it indicates that process quality is satisfactory, material substitution may be allowed, and no stringent quality control is required. A process is called "Excellent" if $1.50 \leqslant C_{pk} < 2.00$; it indicates that process quality exceeds "Satisfactory". Finally, a process is called "Super" if $C_{pk} \geqslant 2.00$ (see also [25]).

3. Sampling distribution of the estimated C_{pk}

Utilizing the identity $\min\{a, b\} = (a+b)/2 - |a-b|/2$, the definition of C_{pk} index can be alternatively written as

$$C_{pk} = \frac{d - |\mu - M|}{3\sigma},$$

where $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half length of the specification interval, $M = (\mathrm{USL} + \mathrm{LSL})/2$ is the mid-point between the lower and the upper specification limits. In practice, the process mean μ and the process variance σ^2 are unknown. The natural estimator $\hat{C}_{pk}$ defined below can be obtained by replacing the process mean μ and the process standard deviation σ by their sample estimators $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S = [\sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)]^{1/2}$. We note that the process must be demonstrably stable (under statistical control) in order to produce a reliable estimate of process capability.

$$\hat{C}_{pk} = \frac{d - |\bar{X} - M|}{3S} = \left\{1 - \frac{|\bar{X} - M|}{d}\right\} \hat{C}_p. \tag{4}$$

Under normality assumption, Kotz et al. [26] obtained the rth moment, and in particular the first two moments of $\hat{C}_{pk}$. Numerous methods for constructing approximate confidence intervals of C_{pk} have been proposed in the literature. Examples include [27–33]. Kotz and Johnson [20] presented a thorough review for the development of process capability indices for the period of 1992–2000. For the estimated $\hat{C}_{pk}$ defined in (4), $\hat{C}_p$ is distributed as $(n-1)^{1/2} C_p (\chi^{-1}_{n-1})$, $n^{1/2}|\bar{X} - M|/\sigma$ is distributed as the folded normal distribution with parameter $n^{1/2}|\mu - M|/\sigma$. Therefore, the distribution of $\hat{C}_{pk}$ is a mixture of χ^{-1}_{n-1} and folded normal distributions [34]. The probability density function of $\hat{C}_{pk}$ can be obtained as the following [35], where $D = (n-1)^{1/2} d/\sigma$, $a = [(n-1)/n]^{1/2}$.

$$f_{\hat{C}_{pk}}(y) = \begin{cases} 4A_n \sum_{\ell=0}^{\infty} P_\ell(\lambda) B_\ell \times \frac{D^{n+2\ell}}{a^{2\ell+1}} \int_0^{\infty} (1-yz)^{2\ell} z^{n-1} \\ \quad \times \exp\left\{-\frac{D^2}{18a^2}(a^2z^2 + 9(1-yz)^2)\right\} \mathrm{d}z, \quad y \leqslant 0, \\ 4A_n \sum_{\ell=0}^{\infty} P_\ell(\lambda) B_\ell \times \frac{D^{n+2\ell}}{a^{2\ell+1}} \int_0^{1/y} (1-yz)^{2\ell} z^{n-1} \\ \quad \times \exp\left\{-\frac{D^2}{18a^2}(a^2z^2 + 9(1-yz)^2)\right\} \mathrm{d}z, \quad y > 0, \end{cases}$$

$$P_\ell(\lambda) = \frac{\mathrm{e}^{-(\lambda/2)}(\lambda/2)^\ell}{\ell!}, \quad A_n = \frac{1}{3^{n-1} 2^{n/2} \Gamma((n-1)/2)},$$

$$B_\ell = \frac{1}{2^\ell \Gamma((2\ell+1)/2)}.$$

Using the integration technique similar to that presented in [36], we obtained an exact form of the cumulative distribution function of the natural estimator $\hat{C}_{pk}$, which can be expressed in terms of a mixture of the chi-square and the normal distributions

$$F_{\hat{C}_{pk}}(y) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n} - t)^2}{9ny^2}\right) \times [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})]\,\mathrm{d}t \tag{5}$$

for $y > 0$, where $b = d/\sigma$, $\xi = (\mu - M)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution with degree of freedom $n-1$, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$. It is noted that we would obtain an identical equation if we substitute ξ by $-\xi$ into Eq. (5) for fixed values of y and n.

4. Designing C_{pk} acceptance sampling plans

Consider a sampling plan to control the lot or process fraction of defectives. Since the quality characteristic is variable, the lower specification limit (LSL) and the upper specification limit (USL) can be used to define the acceptable values of this parameter. It is easy to design a sampling plan with a specified OC curve. Let (AQL, $1-\alpha$) and (LTPD, β) be the two points on the OC curve of interest. Note that AQL and LTPD are levels of the product fraction of defectives that correspond to acceptable and rejectable quality levels. To determine whether a given process is capable, we can consider the testing hypothesis as

$\mathrm{H}_0 : p = \mathrm{AQL}$ (process is capable),
$\mathrm{H}_1 : p = \mathrm{LTPD}$ (process is not capable).

As indicated earlier, PCIs are a function of process parameters (μ and σ) and manufacturing specifications (USL, LSL and M). It measures the ability of the process to reproduce product units that meet the prescribed specifications. The C_{pk} index is an appropriate measure of progress for quality

improvement paradigms in which reduction of variability is the guiding principle and process yield is the primary measure of success. Therefore, the C_{pk} index can be used as a quality benchmark for product acceptance. That is, the null hypothesis with fraction of defectives, $H_0 : p = \text{AQL}$, is equivalent to test process capability with $H_0 : C_{pk} \geqslant C_{\text{AQL}}$, where C_{AQL} is the level of acceptable quality for C_{pk} index correspond to the lot or process fraction of defectives AQL (in PPM) as $\Phi^{-1}(1-(\text{AQL}/2)\times 10^{-6})/3$. For instance, if the vender's fraction of defectives $p = \text{AQL}$ is less than 66 PPM, then the probability of accepting the product is desired to be greater than $100(1-\alpha)\%$. On the other hand, if the vender's fraction of defectives $p = \text{LTPD}$ is more than 2700 PPM, then the probability of accepting the product is set to be no more than $100\beta\%$. From the relationship between the index value and fraction of defectives, we could obtain the equivalent $C_{\text{AQL}} = 1.33$ and the $C_{\text{LTPD}} = 1.00$ based on C_{pk}. Therefore, the required inspection sample size n and critical acceptance value c_0 for the sampling plan are the solutions to the following two nonlinear simultaneous equations:

$$\Pr\{\text{Accepting the product} \mid \text{fraction of defectives } p = \text{AQL}\} \geqslant 1-\alpha, \tag{6}$$

$$\Pr\{\text{Accepting the product} \mid \text{fraction of defectives } p = \text{LTPD}\} \leqslant \beta. \tag{7}$$

For processes with target value sett to the mid-point of the specification limits (i.e. $T = M$), the index may be rewritten as $C_{pk} = (d/\sigma - |\xi|)/3$, where $\xi = (\mu - M)/\sigma$. As noted earlier, the sampling distribution of $\hat{C}_{pk}$ is expressed in terms of a mixture of the chi-square and the normal distributions. Given $C_{pk} = C$, $b = d/\sigma$ can be expressed as $b = 3C + |\xi|$. Thus, the probability of accepting the product can be expressed as

$$\begin{aligned}\pi_A(C_{pk}) &= P(\hat{C}_{pk} \geqslant c_0 | C_{pk}) \\ &= \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nc_0^2}\right) \\ &\quad \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))\,\mathrm{d}t. \end{aligned} \tag{8}$$

Accordingly, from expression (8), Eqs. (9) and (10) can be rewritten as

$$\begin{aligned}1-\alpha \leqslant \int_0^{b_1\sqrt{n}} G\left(\frac{(n-1)(b_1\sqrt{n}-t)^2}{9nc_0^2}\right) \\ \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))\,\mathrm{d}t, \end{aligned} \tag{9}$$

$$\begin{aligned}\beta \geqslant \int_0^{b_2\sqrt{n}} G\left(\frac{(n-1)(b_2\sqrt{n}-t)^2}{9nc_0^2}\right) \\ \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))\,\mathrm{d}t, \end{aligned} \tag{10}$$

where $b_1 = 3C_{\text{AQL}} + |\xi|$ and $b_2 = 3C_{\text{LTPD}} + |\xi|$, $C_{\text{AQL}} > C_{\text{LTPD}}$. We note that the required sample size n is the smallest possible value of n satisfying Eqs. (9) and (10), and determining the $\lceil n \rceil$ as sample size, where $\lceil n \rceil$ means the least integer greater than or equal to n.

4.1. Critical acceptance value c_0 versus sample size n and Parameter ξ

Since the process parameters μ and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - M)/\sigma$ is also unknown, which has to be estimated in real applications. Such an approach introduces additional sampling errors from estimating ξ in finding the critical acceptance values and the required sample sizes. To eliminate the need for estimating the distribution characteristic parameter ξ, we perform extensive calculations to investigate the behavior of the critical acceptance value c_0 and sample size n for various parameters. Fig. 2(a) plots the required sample size n against ξ for $C_{\text{AQL}} = 1.33, 1.50, 1.67, 2.00$ and $C_{\text{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$. Fig. 2(b) plots the critical acceptance value c_0 against ξ value for $C_{\text{AQL}} = 1.33, 1.50, 1.67, 2.00$ and $C_{\text{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$. The pattern of the sample sizes and critical values are consistent for cases with different α and β risks.

Note that parameter values we investigated, $\xi = 0(0.05)3.00$, cover a wide range of applications with process capability $C_{pk} \geqslant 0$ and the calculation results are identical if ξ is replaced by $-\xi$. We find that the critical value c_0 (i) is increasing in ξ, (ii) reaches its maximum at $\xi = 1.00$ in all cases, and (iii) stays the same for $\xi \geqslant 1.00$ for all C_{AQL} (with an accuracy up to 10^{-4}). Further, we find that the critical acceptance value c_0 reaches its maximum at $\xi = 0.50$ and stays the same for $\xi \geqslant 0.50$ as the sample size $n \geqslant 30$ (and for $n \geqslant 100$, $\xi = 0.35$ with accuracy up to 10^{-3}). Hence, we may solve Eqs. (9) and (10) with $\xi = 1.00$ to obtain the conservative critical acceptance value and sample size without having to estimate the parameter ξ. This approach ensures that the decisions made based on those critical acceptance values are reliable.

4.2. Solving the nonlinear simultaneous equations

In order to illustrate how we solve the above two nonlinear simultaneous Eqs. (9) and (10), let

$$\begin{aligned}S_1(n, c_0) &= \int_0^{b_1\sqrt{n}} G\left(\frac{(n-1)(b_1\sqrt{n}-t)^2}{9nc_0^2}\right) \\ &\quad \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))\,\mathrm{d}t - (1-\alpha), \end{aligned} \tag{11}$$

$$\begin{aligned}S_2(n, c_0) = \int_0^{b_2\sqrt{n}} G\left(\frac{(n-1)(b_2\sqrt{n}-t)^2}{9nc_0^2}\right) \\ \times (\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n}))\,\mathrm{d}t - \beta. \end{aligned} \tag{12}$$

For $C_{\text{AQL}} = 1.33$ and $C_{\text{LTPD}} = 1.00$, Figs. 3(a)–(b) and 4(a)–(b) display the surface and contour plots of Eqs. (11) and (12), respectively, with α-risk $= 0.05$ and β-risk $= 0.10$.

Figs. 5(a)–(b) display the surface and contour plots of Eqs. (11) and (12) simultaneously with α-risk $= 0.05$ and β-risk $= 0.10$, respectively. From Fig. 5(b), we can see that the interaction of $S_1(n, c_0)$ and $S_2(n, c_0)$ contour curves at

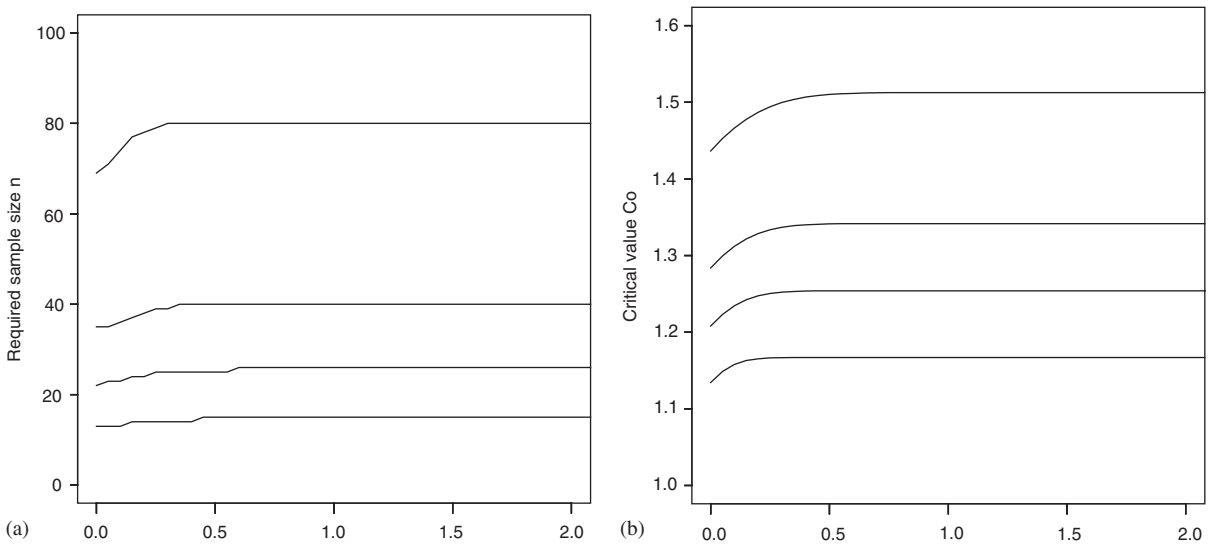

Fig. 2. (a) The plot of the required sample size n versus ξ for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$, $\alpha = 0.05$, $\beta = 0.05$. (from bottom to top in plot). (b) The plot of the critical acceptance value c_0 versus ξ for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$, $\alpha = 0.05$, $\beta = 0.05$. (from bottom to top in plot).

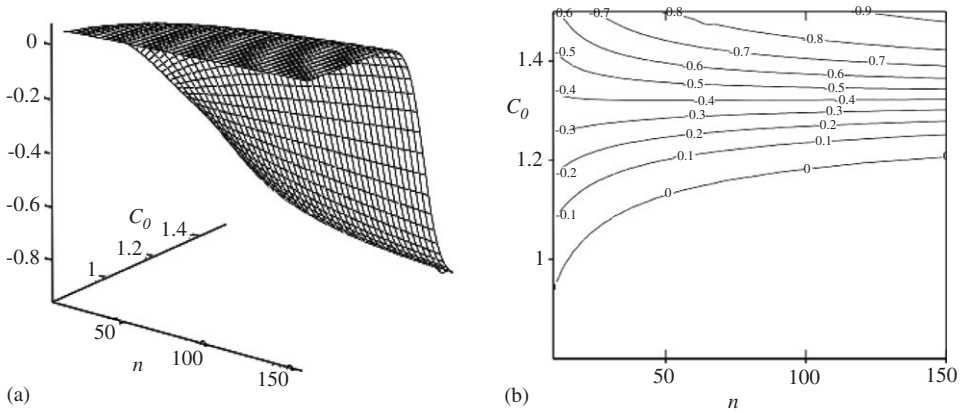

Fig. 3. (a) Surface plot of $S_1(n, c_0)$. (b) Contour plot of $S_1(n, c_0)$.

level 0 is $(n, c_0) = (62, 1.1477)$, which is the solution to nonlinear simultaneous equations (9) and (10). That is, in this case, the minimum required sample size $n = 62$ and critical acceptance value $c_0 = 1.1477$ of the sampling plan are based on the capability index C_{pk}.

For practical application purposes, we calculate and tabulate the critical acceptance values (c_0) and required sample sizes (n) for the sampling plans, with commonly used α, β, C_{AQL} and C_{LTPD}. Table 3 displays (n, c_0) values for producer's α-risk $= 0.01, 0.025, 0.05, 0.075, 0.10$ and buyer's β-risk $= 0.01, 0.025, 0.05, 0.075, 0.10$, with various benchmarking quality levels, $(C_{\mathrm{AQL}}, C_{\mathrm{LPTD}}) =$ (1.33, 1.00), (1.50, 1.33), (1.67, 1.33) and (2.00, 1.67). For example, if the benchmarking quality level $(C_{\mathrm{AQL}}, C_{\mathrm{LPTD}})$ is set to (1.33, 1.00) with producer's α-risk $= 0.01$ and buyer's β-risk $= 0.05$, then the corresponding sample size and critical acceptance value can be obtained as $(n, c_0) = (112, 1.1372)$. The lot will be accepted if the 112 inspected product items yield measurements with $\hat{C}_{pk} \geqslant 1.1372$.

From the results, we observe that the larger of the risks which the producer or customer would suffer, the smaller sample size n is required for inspection. This phenomenon can be interpreted intuitively, as if we would want the chance of wrongly concluding bad products as good, or good products as bad, to be smaller, we would need more sample information to make the judgment. Further, for fixed α, β risks and C_{LTPD}, the required sample sizes become smaller

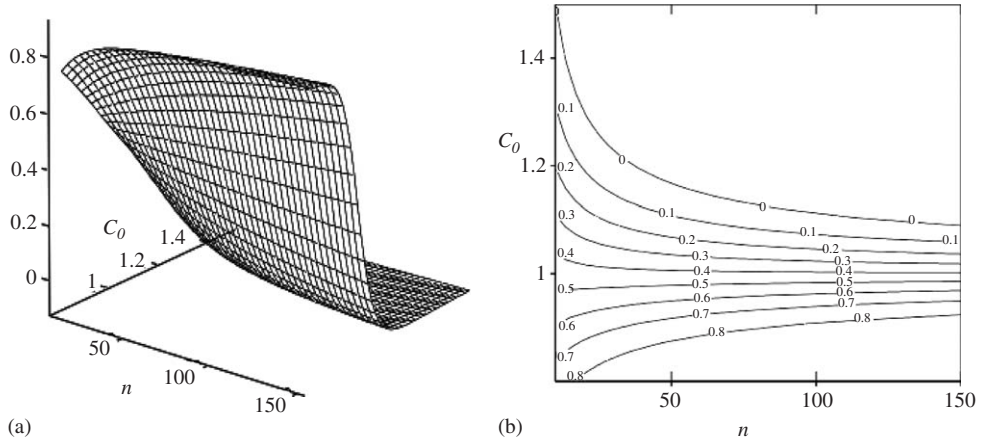

Fig. 4. (a) Surface plot of $S_2(n, c_0)$. (b) Contour plot of $S_2(n, c_0)$.

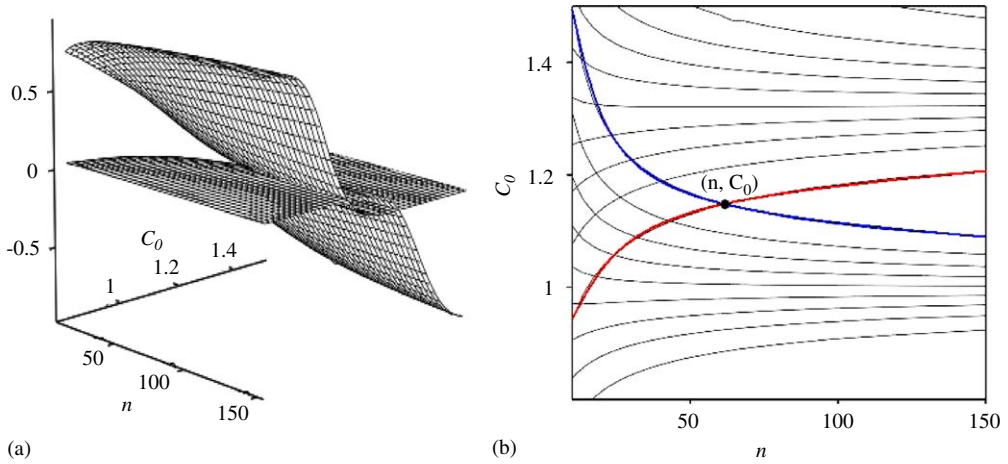

Fig. 5. (a) Surface plot of S_1 and S_2. (b) Contour plot of S_1 and S_2.

when the value of C_{AQL} becomes larger. This can also be explained by the same reasoning, as the judgment will be more correct with a larger value of difference between the C_{AQL} and C_{LTPD}.

5. Sampling procedure and decision making

Selection of a meaningful critical value for capability test requires specification of an acceptable quality level (AQL) and a lot tolerance percent defective (LTPD) for the C_{pk} value. The AQL is simply a standard against which to judge the product. It is hoped that the vendor's process will operate at a fallout level that is considerably better than the AQL. Both the producer and the consumer will lay down their requirements in the contract: the former demands that not too many "good" lots shall be rejected by the sampling inspection, while the latter demands that not too many "bad" lots shall be accepted. A sampling plan attempt will be chosen to meet these somewhat opposing requirements.

Thus, in order to judge whether a given process meets the capability requirement, we can first determine the specified value of the capability requirement C_{AQL} and C_{LTPD} (or fraction of defectives AQL and LTPD), and the α-risk, β-risk. That is, if product process capability is $C_{pk} = C_{\mathrm{AQL}}$ (in high quality), the probability of acceptance must be greater than $1-\alpha$. If producer's capability is only $C_{pk} = C_{\mathrm{LTPD}}$ (in low quality), consumer accept the product with probability no more than β. By checking Table 3, we may obtain the sample size n and the critical value c_0 based on given values of α-risk, β-risk, AQL and LTPD. If the estimated C_{pk} value is greater than the critical value c_0, then the consumer accepts the product. Otherwise, we do not have sufficient information to conclude that the process meets the present

Table 3
Required sample sizes (n) and critical acceptance values (c_0) for various α- and β-risks with selected C_{AQL}, C_{LPTD}

α	β	$C_{AQL}=1.33$ $C_{LTPD}=1.00$		$C_{AQL}=1.50$ $C_{LTPD}=1.33$		$C_{AQL}=1.67$ $C_{LTPD}=1.33$		$C_{AQL}=2.00$ $C_{LTPD}=1.67$	
		n	c_0	n	c_0	n	c_0	n	c_0
0.01	0.01	158	1.1645	834	1.4149	232	1.4994	358	1.8345
	0.025	132	1.1509	704	1.4077	195	1.4853	301	1.8207
	0.05	112	1.1372	600	1.4005	166	1.4712	256	1.8069
	0.075	100	1.1271	537	1.3952	148	1.4607	229	1.7967
	0.10	91	1.1186	491	1.3907	135	1.4519	209	1.7881
0.025	0.01	136	1.1790	713	1.4222	200	1.5144	307	1.8489
	0.025	113	1.1655	593	1.4151	165	1.5033	254	1.8352
	0.05	94	1.1517	498	1.4078	139	1.4860	213	1.8212
	0.075	83	1.1414	441	1.4024	122	1.4754	188	1.8108
	0.10	75	1.1327	399	1.3977	110	1.4663	170	1.8019
0.05	0.01	119	1.1937	616	1.4297	174	1.5294	266	1.8635
	0.025	97	1.1805	505	1.4227	142	1.5157	218	1.8501
	0.05	80	1.1669	418	1.4154	117	1.5016	180	1.8362
	0.075	70	1.1566	366	1.4099	102	1.4908	157	1.8257
	0.10	62	1.1477	328	1.4052	91	1.4816	141	1.8167
0.075	0.01	108	1.2047	557	1.4352	158	1.5407	241	1.8743
	0.025	87	1.1919	451	1.4284	128	1.5273	195	1.8613
	0.05	71	1.1785	369	1.4123	104	1.5134	159	1.8476
	0.075	62	1.1683	320	1.4158	90	1.5028	138	1.8372
	0.10	55	1.1595	285	1.4110	80	1.4937	123	1.8282
0.10	0.01	100	1.2140	513	1.4399	146	1.5502	223	1.8836
	0.025	80	1.2016	412	1.4333	117	1.5373	179	1.8709
	0.05	65	1.1886	334	1.4263	95	1.5238	145	1.8576
	0.075	56	1.1787	288	1.4209	81	1.5134	124	1.8473
	0.10	49	1.1700	254	1.4161	72	1.5043	110	1.8384

capability requirement. In this case, the consumer will reject the product. For the proposed sampling plan to be practical and convenient to use, a step-by-step procedure is provided below.

Step 1: Decide process capability requirements (i.e. C_{AQL} and C_{LTPD}), α-risk, the chance of wrongly rejecting a capable process, and the β-risk, the chance of wrongly concluding a bad lot as good one.

Step 2: Check Table 3 to find the critical acceptance value and the required number of product units for inspection, (n, c_0), based on given values of α-risk, β-risk, C_{AQL} and C_{LTPD}.

Step 3: Calculate the value of $\hat{C}_{pk}$ (sample estimator) from the n inspected sample data.

Step 4: Make a decision to accept the entire products if the estimated $\hat{C}_{pk}$ value is greater than the critical value c_0 ($\hat{C}_{pk} > c_0$). Otherwise, we reject the entire products.

5.1. Application example

To illustrate how the sampling plan can be applied to the actual data collected from the process, we present a case study on the liquid-crystal module (LCM) manufacturing process. The LCM is one of the key components used in many high-tech electronic commercial devices, such as the cellular phone, the PDA (personal digital assistant), the digital watch, pocket calculator, automobile accessory visual displays, and many others. Three key components make the LCM functions properly. Those include the liquid crystal display, the back lighting, and the peripheral (interface) system. The mounting technology for the chip on glass (COG) makes the exposed particle overturned, with the side of circuits facing downward. Then, the electricity conduction is joined between the IC and panel of the liquid-crystal display through the mounting material. For the main bonding

Table 4
Sample data of 80 observations (unit: μm)

1.28	−5.12	6.75	−7.34	9.50	5.70	9.40	1.09	1.32	−5.59
−4.73	3.14	0.38	8.36	−6.88	−7.06	3.47	−4.42	3.34	4.55
2.84	10.25	5.72	−0.11	6.59	−3.31	−8.18	3.71	4.38	3.25
−4.70	−3.45	1.07	−1.58	2.45	7.02	−7.28	4.48	1.28	−2.54
2.58	−5.98	4.50	4.66	−6.75	1.19	−2.11	−2.34	−7.46	5.92
2.93	−2.44	−5.51	2.63	2.04	−2.19	1.40	−2.53	−4.14	−1.93
4.93	−0.17	9.70	3.47	4.86	1.02	−2.06	2.90	5.50	1.06
−4.86	4.75	8.25	6.12	4.63	−5.15	4.11	4.90	−4.74	4.03

process, the bonding precision is an essential process parameter.

We investigated a particular model of the LCM product, with the upper and the lower specification limits on the bonding precision are set to USL = 15 μm, LSL = −15 μm, and the target value is set to $T = M = 0$. If the characteristic data do not fall within the tolerance (LSL, USL), the lifetime or reliability of the LCM will be discounted. In the contract, the C_{AQL} and C_{LTPD} are set to 1.33 and 1.00 with α-risk = 0.05 and β-risk = 0.05. Therefore, the problem is the determination of critical acceptance values and the inspected sample sizes that provide the desired levels of protection for both producer and consumer. The sampling plan must provide a probability of at least $1 - \alpha$ of accepting the lot if the lot proportion defective is at the $C_{\mathrm{AQL}} = 1.33$ (which is equivalent to no more than 66 PPM fraction of defectives for the product), and also provide a probability of no more than β accepting the lot if the lot proportion defective is at the $C_{\mathrm{LTPD}} = 1.00$ (which is equivalent to 2700 PPM fraction of defectives for the product).

First, we find the acceptance critical values and inspected sample sizes of sampling plan $(n, c_0) = (80, 1.1669)$ from Table 4 The required sample of product items for inspection are taken from the process randomly, using microscope visually for inspection, the observations measurement are displayed in Table 4. Based on those data, the calculation shows the following. Therefore, the consumer would "reject" the entire products since the sample estimator from the inspection, 0.968, is smaller than the critical acceptance value 1.1669.

$$\bar{X} = 0.959, \quad S = 4.835,$$

$$\hat{C}_{pk} = \frac{d - |\bar{X} - M|}{3S} = 0.968.$$

5.2. Comments on existing methods

We note that if existing sampling plans are applied here, it is almost certain that any sample of 80 LCMs taken from the process will contain zero defective items. All the products therefore will be accepted, which obviously provide no protection to the buyer at all.

6. Conclusions

Process capability indices are useful management tools, particularly in the manufacturing industry, which provide common quantitative measures of manufacturing capability and production quality. Most supplier certification manuals include a discussion of process capability analysis and describe the recommended procedure for computing a process capability index. In spite of the introductions of many process capability indices, the C_{pk} index remains the most popular one because it provides quantitative measures of process yield and upper bound on product fraction of defectives. Acceptance sampling plans are practical tools for quality assurance applications. It provides the buyer and the vendor a decision rule for product sentencing to meet their needs. Since the sampling cannot guarantee that the defective items in a lot will be sampled and inspected, then the sampling involves risks of not adequately reflecting the quality conditions of the lot. Such a risk is even more significant as the rapid advancement of the manufacturing technology and stringent customers demand is enforced. Particularly, when the product fraction of defectives is very low and measured in parts per million (PPM), the required number of inspection items must be enormously large in order to adequately reflect the actual lot quality. In this paper, we developed an effective sampling plan based on process capability index C_{pk}, to deal with product acceptance problem for processes with very low fraction of defectives. The proposed sampling plan provides a feasible inspection policy, which can be applied to products with very low fraction of defectives where classical sampling plans cannot be applied. The proposed new sampling plan is developed based on the exact sampling distribution rather than approximation. We developed a method to determine the sample size required for inspection and the corresponding acceptance criterion, to provide the desired levels of protection to both producers and consumers. We tabulated the required sample size n and the corresponding critical acceptance value c_0 for various α-risks, β-risks, and the capability requirements AQL, LTPD. The results obtained in this paper are useful to the practitioners in making reliable decisions. For illustrative purpose, we demonstrated the proposed method by presenting a case study on liquid-crystal

module (LCM) manufacturing process to evaluate the process performance.

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