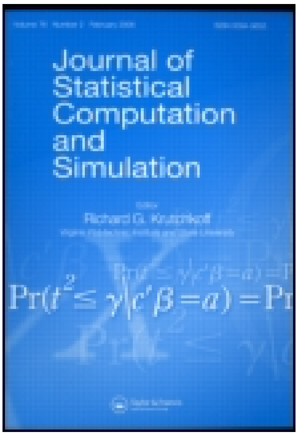

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Power and sample size determinations for the Wilcoxon signed-rank test

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The problem of calculating power and sample size for the Wilcoxon signed-rank test is discussed. The exact variance large-sample method is examined and explicit formulas are derived for observations from uniform, normal and Laplace distributions. Numerical results are presented to evaluate the exact variance procedure and compare its performance with two simplified approximations that have been suggested in the statistical literature. From the simulation results, it is evident that the exact variance approach is more accurate than the two approximate methods. To facilitate practical use, tabulated values of the estimated sample sizes are provided.

Keywords: Large-sample approximation; Nonparametric method; One-sample location problem

1. Introduction

The Wilcoxon signed-rank test is one of the most widely used nonparametric methods for the one-sample location problem. It provides an important alternative to the parametric t-test for giving robust results without the restriction of normality assumption in the population. Generally, the power function of the Wilcoxon signed-rank test is very difficult to express and only a few special cases have been examined, see Klotz [1] and Arnold [2] for shifts in normal and t-distributions, respectively. For the purpose of power and sample size calculations for the Wilcoxon signed-rank test, two simplified methods have been proposed in the statistical literature: Lehmann [3, p. 167] and Noether [4]. Both procedures are based on some approximate expressions for the asymptotic normal distribution of the Wilcoxon signed-rank statistic. Despite the extensive applicability in the planning of one-sample study, no research to date has compared these two formulas for their finite-sample properties. In fact, they yield markedly different results according to the findings presented in this article. More importantly, verification of the accuracy of their methods under a variety of different distributions would be useful.

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Furthermore, it is important to note that the approximations suggested by Lehmann [3] and Noether [4] rely on the assumption that the alternatives do not differ too much from the null hypothesis. It is of great interest to have a rule of thumb that indicates whether the location shift is small enough so that the results are valid. Unfortunately, no such general guideline is available. This serious disadvantage renders the limited use of their procedures in practical situations.

In order to improve the practical usefulness, this article aims to investigate exact variance large-sample solution to power and sample size determinations for the Wilcoxon signed-rank test. We concentrate on three prominent situations that represent typical light-, standard- and heavy-tailed distributions: uniform, normal and Laplace. For these three cases, analytic forms are derived explicitly and exploited thoroughly to assess the finite-sample adequacy of the exact variance method. Moreover, the characteristics of the simplified procedures considered in equation (4.32) of Lehmann [3] and Noether [4] are also examined in an attempt to provide some guidance in the choice of appropriate method for power and sample size calculations.

The next section presents some of the analytical justification and important details of the exact variance large-sample method, as well as two related simplified approaches. In section 3, Monte Carlo simulation studies are conducted to evaluate the exact variance procedure and compare its performance with the approximate approaches under the three prescribed symmetric distributions. The corresponding sample sizes needed to achieve the specified power levels are summarized. Finally, section 4 contains some final remarks.

2. Power and sample size calculations

Consider a random sample $X_1, \ldots, X_N$ from an arbitrary continuous and symmetric cumulative distribution $F(x-\theta)$, where θ is the unique median and mean if it exists. It is desirable to test the hypothesis $H_0\colon \theta = 0$ versus the alternative $H_1\colon \theta > 0$. We focus on the Wilcoxon signed-rank statistic W in the context of nonparametric methods defined as follows:

$$W = \sum_{i=1}^{N} \varphi(X_i) R(|X_i|),$$

where $\varphi(X_i) = 1$ if $X_i > 0$ and 0 otherwise, and $R(|X_i|)$ is the rank of $|X_i|$ among $|X_1|, \ldots, |X_N|$. It was shown in Theorem 2.5.1 of Hettmansperger [5, p. 47] that the mean μ and variance σ^2 of W are

$$\mu = Np_1 + \frac{N(N-1)}{2} p_2$$

and

$$\begin{aligned} \sigma^2 = {} & Np_1(1-p_1) + \frac{N(N-1)}{2} p_2(1-p_2) + 2N(N-1)(p_3 - p_1 p_2) \\ & + N(N-1)(N-2)(p_4 - p_2^2), \end{aligned} \tag{1}$$

respectively, where

$$p_1 = P(X_1 > 0) = F(\theta), \qquad p_2 = P(X_1 + X_2 > 0) = \int F(2\theta + x) f(x)\,\mathrm{d}x,$$

$$p_3 = P(X_1 + X_2 > 0, X_1 > 0) = (p_1^2 + p_2)/2,$$

$$p_4 = P(X_1 + X_2 > 0, X_1 + X_3 > 0) = \int \{F(2\theta + x)\}^2 f(x)\,\mathrm{d}x,$$

and $F(\cdot)$ and $f(\cdot)$ are the cumulative distribution function and probability density function of X_1 under the null hypothesis H_0: $\theta = 0$, respectively. Furthermore, Theorem 2.5.4 of Hettmansperger [5, p. 56] shows that $(W - \mu)/\sigma$ has an asymptotic standard normal distribution. Note that both μ and σ^2 are functions of (p_1, p_2, p_3, p_4) and consequently depend on the value of θ. For the case of $\theta = 0$, it can be easily obtained that μ and σ^2 given in equation (1) reduce to

$$\mu_0 = \frac{N(N+1)}{4} \quad \text{and} \quad \sigma_0^2 = \frac{N(N+1)(2N+1)}{24},$$

respectively. Therefore, the aforementioned test of location shift is carried out by rejecting H_0: $\theta = 0$ if the standardized value $(W - \mu_0)/\sigma_0$ is greater than z_α, where α is the specified significance level and z_α is the $100(1-\alpha)$th percentile of the standard normal distribution.

For the purpose of power and sample size calculations, we proceed to consider that $W \dot{\sim} N(\mu, \sigma^2)$ under H_1. Hence, given distribution $F(x - \theta)$ with location $\theta (>0)$ and sample size N, the statistical power achieved for testing hypothesis H_0: $\theta = 0$ versus the alternative H_1: $\theta > 0$ with specified significance level α is approximated by the probability

$$P\{W > \mu_0 + z_\alpha \sigma_0\} \doteq 1 - \Phi\left(\frac{z_\alpha \sigma_0 + \mu_0 - \mu}{\sigma}\right), \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. This process can be reversed to calculate the sample size needed to test the null hypothesis with specified significance level α and power $1 - \beta$. However, it usually involves an iterative process to find the solution because all the parameter values $(\mu_0, \sigma_0, \mu, \sigma)$ depend on the sample size N. More specifically, the resulting sample size, denoted by N_E, is the minimum N which satisfies the inequality

$$(\mu - \mu_0) \geq (z_\alpha \sigma_0 + z_\beta \sigma).$$

In general, there are no simple closed-form expressions for the preceding equations given in equation (1) except in some special cases. Essentially, the numerical computation requires the one-dimensional integration with respect to the probability density function $f(x)$. However, it is prudent to examine the exact variance large-sample method described earlier for some special $F(x - \theta)$ distributions that possesses potentially important implications. Hence, the following three cases ranging from light-tailed to heavy-tailed distributions are presented.

1. Uniform $(-1/2, 1/2)$:

$$p_1 = \frac{1}{2} + \theta, \quad p_2 = \frac{1}{2} + 2\theta(1-\theta) \text{ and } p_4 = \frac{1}{3} + 2\theta - \frac{8\theta^3}{3} \quad \text{for } \theta \geq \frac{1}{2}.$$

2. Standard normal $N(0, 1)$:

$$p_1 = \Phi(\theta), \quad p_2 = \Phi(\sqrt{2}\theta) \text{ and } p_4 = E[\{\Phi(2\theta + Z)\}^2], \quad \text{where } Z \sim N(0,1).$$

3. Laplace $(0, 1)$:

$$p_1 = 1 - \frac{1}{2}e^{-\theta}, \quad p_2 = 1 - \frac{1}{2}(1+\theta)e^{-2\theta} \text{ and } p_4 = 1 - \left(\frac{7}{12} + \theta\right)e^{-2\theta} - \frac{1}{12}e^{-4\theta}.$$

These explicit expressions are employed to illustrate the distinct features of the exact variance method in the subsequent section.

Along the same line of power and sample size calculations within the framework of Wilcoxon signed-rank test, two simplified approximations to the exact variance large-sample result have been proposed. Based on the asymptotic normal distribution described earlier, Lehmann [3, p. 167] and Hettmansperger [5, p. 60] suggested to approximate the power by

$$1 - \Phi\left(z_\alpha - \frac{N\theta f(0) + N(N-1)\theta f^*(0)}{\sigma_0}\right), \tag{3}$$

where $f(0)$ is $f(x)$ evaluated at $x = 0$, and $f^*(0) = \int f^2(x)\mathrm{d}x$. Correspondingly, Noether [4] proposed the power function

$$1 - \Phi\left(z_\alpha - (3N)^{1/2}\left(p_2 - \frac{1}{2}\right)\right). \tag{4}$$

Just as in the case of the exact variance method, both (3) and (4) can be applied to construct respective sample size estimates N_L and N_N required for testing the specified hypothesis with significance level α and power $1 - \beta$. Nevertheless, it is worthwhile to note that these two simplified procedures are valid for small values of θ. Failing to account for this nature may distort power analysis and lead to a poor choice of sample size. This phenomenon will be demonstrated in the following numerical illustrations.

3. Simulation study

As all the three approaches considered here use large-sample justifications, simulation studies are conducted to assess their adequacy for finite-sample and robustness for various configurations. For illustration, the three distributions of uniform, normal and Laplace are exploited as the bases for the numerical examinations.

To help clarify similarities and differences for the competing procedures in performing power and sample size calculations, the sample sizes (N_E, N_L, N_N) needed to achieve the power levels: 0.80, 0.90 and 0.95 are computed for the three methods defined in equations (2)–(4). We assume throughout the demonstration that type I error rate $\alpha = 0.05$. In each case, a total of six values of location shift are evaluated for $F(x - \theta)$ in terms of $\Delta = \theta/\sigma$. The results for the three power levels of 0.80, 0.90 and 0.95 are summarized in tables 1–3, respectively. SAS codes for the calculation of the exact variance large-sample method are available upon request.

Table 1. Sample size required to attain power level 0.80.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform $(-1/2, 1/2)$						
Exact variance	653	172	47	23	14	10
Lehmann	620	157	41	19	12	8
Noether	656	175	50	26	17	13
Standard normal						
Exact variance	649	164	42	20	12	9
Lehmann	649	163	42	19	11	8
Noether	652	167	45	23	15	12
Laplace (0, 1)						
Exact variance	419	109	31	16	11	8
Lehmann	412	103	26	11	6	4
Noether	422	112	34	19	14	12

Table 2. Sample size required to attain power level 0.90.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform $(-1/2, 1/2)$						
Exact variance	903	236	63	30	17	12
Lehmann	858	216	55	26	15	10
Noether	909	242	69	35	23	17
Standard normal						
Exact variance	898	225	57	26	15	11
Lehmann	898	225	57	26	15	10
Noether	903	231	63	32	21	17
Laplace (0, 1)						
Exact variance	580	150	42	21	14	10
Lehmann	571	143	36	16	9	6
Noether	585	156	47	27	19	16

Preliminary inspection of the tables yield the expected general relations: the computed sample size increases as the location shift decreases for all three approaches. Also, it is interesting to note that the ordering of sample size estimates is consistently $N_N > N_E \geq N_L$ except in the case of standard normal distribution for power level 0.95 where $N_N > N_L \geq N_E$ in table 3. However, the discrepancy between N_L and N_E in the last situation is never larger than 1. Although these values of sample sizes allow comparison of relative efficiencies of the methods, the magnitude of the sample size affects the accuracy of the asymptotic distribution and the resulting formula. A fair comparison among these approaches must adjust for this factor.

In order to identify the most reliable method, we need to evaluate their actual or simulated power with the nominal power for a given sample size. Hence, we unify the sample sizes in the following simulations by choosing the sample size N_E in tables 1–3 as the benchmark to re-calculate the nominal powers for all competing approaches. Accordingly, the nominal powers for the exact variance method are slightly greater than 0.8, 0.90 and 0.95 in tables 4–6, respectively.

Estimates of the true power associated with the given sample size and distribution configuration are then computed through Monte Carlo simulation of 10,000 independent data sets. For each replicate, N_E observations are generated from the selected distribution. Then the Wilcoxon signed-rank test statistic is computed and the simulated power is the proportion of the 10,000 replicates whose standardized test statistic values exceed the critical value z_α. The simulation results are presented in tables 4–6. The adequacy of the sample size formula

Table 3. Sample size required to attain power level 0.95.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform $(-1/2, 1/2)$						
Exact variance	1139	296	78	36	21	13
Lehmann	1084	273	70	32	19	13
Noether	1148	305	87	44	29	22
Standard normal						
Exact variance	1133	283	71	32	18	12
Lehmann	1134	284	72	33	19	12
Noether	1141	291	79	40	27	21
Laplace (0, 1)						
Exact variance	731	189	52	25	16	11
Lehmann	721	180	45	20	11	7
Noether	739	196	59	33	24	20

Table 4. Simulated power at specified sample size when nominal power of exact variance method is about 0.80.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform $(-1/2, 1/2)$						
Sample size	653	172	47	23	14	10
Nominal power						
Exact variance	0.8001	0.8012	0.8022	0.8111	0.8171	0.8458
Lehmann	0.8180	0.8332	0.8541	0.8733	0.8843	0.9028
Noether	0.7986	0.7958	0.7825	0.7686	0.7445	0.7273
Simulated power						
Error	0.8055	0.8064	0.8091	0.8123	0.8360	0.8593
Exact variance	−0.0054	−0.0052	−0.0069	−0.0012	−0.0189	−0.0135
Lehmann	0.0125	0.0268	0.0450	0.0610	0.0483	0.0435
Noether	−0.0069	−0.0106	−0.0266	−0.0437	−0.0915	−0.1320
Standard normal						
Sample size	649	164	42	20	12	9
Nominal power						
Exact variance	0.8001	0.8015	0.8001	0.8088	0.8074	0.8478
Lehmann	0.8005	0.8031	0.8074	0.8251	0.8395	0.8823
Noether	0.7985	0.7953	0.7762	0.7561	0.7195	0.7070
Simulated power						
Error	0.8054	0.7970	0.7990	0.8084	0.8235	0.8468
Exact variance	−0.0053	0.0045	0.0011	0.0004	−0.0161	0.0010
Lehmann	−0.0049	0.0061	0.0084	0.0167	0.0160	0.0355
Noether	−0.0069	−0.0017	−0.0228	−0.0523	−0.1040	−0.1398
Laplace (0, 1)						
Sample size	419	109	31	16	11	8
Nominal power						
Exact variance	0.8002	0.8003	0.8047	0.8079	0.8308	0.8185
Lehmann	0.8061	0.8208	0.8653	0.9097	0.9530	0.9726
Noether	0.7976	0.7907	0.7708	0.7409	0.7225	0.6755
Simulated power						
Error	0.8050	0.7954	0.8099	0.8117	0.8265	0.8196
Exact variance	−0.0048	0.0049	−0.0052	−0.0038	0.0043	−0.0011
Lehmann	0.0011	0.0254	0.0554	0.0980	0.1265	0.1530
Noether	−0.0074	−0.0047	−0.0391	−0.0708	−0.1040	−0.1441

Table 5. Simulated power at specified sample size when nominal power of exact variance method is about 0.90.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform $(-1/2, 1/2)$						
Sample size	903	236	63	30	17	12
Nominal power						
Exact variance	0.9002	0.9008	0.9014	0.9090	0.9011	0.9290
Lehmann	0.9127	0.9219	0.9322	0.9408	0.9351	0.9470
Noether	0.8986	0.8944	0.8777	0.8582	0.8138	0.7937
Simulated power						
Error	0.9047	0.8982	0.8928	0.8986	0.8862	0.9203
Exact variance	−0.0045	0.0026	0.0086	0.0104	0.0149	0.0087
Lehmann	0.0080	0.0237	0.0394	0.0422	0.0489	0.0267
Noether	−0.0061	−0.0038	−0.0151	−0.0404	−0.0724	−0.1266
Standard normal						
Sample size	898	225	57	26	15	11
Nominal power						
Exact variance	0.9003	0.9005	0.9024	0.9054	0.9036	0.9355
Lehmann	0.9002	0.9001	0.9013	0.9045	0.9064	0.9337
Noether	0.8986	0.8940	0.8762	0.8465	0.8007	0.7810

(*continued*)

Table 5. Continued.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Simulated power						
Error	0.9027	0.9011	0.8988	0.8940	0.8988	0.9163
Exact variance	−0.0024	−0.0006	0.0036	0.0114	0.0048	0.0192
Lehmann	−0.0025	−0.0010	0.0025	0.0105	0.0076	0.0174
Noether	−0.0041	−0.0071	−0.0226	−0.0475	−0.0981	−0.1353
Laplace (0, 1)						
Sample size	580	150	42	21	14	10
Nominal power						
Exact variance	0.9004	0.9005	0.9053	0.9073	0.9236	0.9212
Lehmann	0.9042	0.9131	0.9393	0.9609	0.9814	0.9898
Noether	0.8980	0.8913	0.8714	0.8364	0.8097	0.7590
Simulated power						
Error	0.8971	0.8937	0.9002	0.8990	0.9101	0.9059
Exact variance	0.0033	0.0068	0.0051	0.0083	0.0135	0.0153
Lehmann	0.0071	0.0194	0.0391	0.0619	0.0713	0.0839
Noether	0.0009	−0.0024	−0.0288	−0.0626	−0.1004	−0.1469

Table 6. Simulated power at specified sample size when nominal power of exact variance method is about 0.95.

$\Delta = \theta/\sigma$	0.1	0.2	0.4	0.6	0.8	1.0
Uniform (−1/2, 1/2)						
Sample size	1139	296	78	36	21	13
Nominal power						
Exact variance	0.9501	0.9502	0.9509	0.9538	0.9602	0.9537
Lehmann	0.9580	0.9631	0.9681	0.9701	0.9710	0.9612
Noether	0.9487	0.9449	0.9306	0.9085	0.8799	0.8212
Simulated power						
Error	0.9496	0.9496	0.9481	0.9431	0.9433	0.9277
Exact variance	0.0005	0.0006	0.0028	0.0107	0.0169	0.0260
Lehmann	0.0084	0.0135	0.0200	0.0270	0.0277	0.0335
Noether	−0.0009	−0.0047	−0.0175	−0.0346	−0.0634	−0.1065
Standard normal						
Sample size	1133	283	71	32	18	12
Nominal power						
Exact variance	0.9501	0.9503	0.9522	0.9559	0.9552	0.9599
Lehmann	0.9499	0.9494	0.9488	0.9493	0.9466	0.9507
Noether	0.9488	0.9452	0.9307	0.9055	0.8603	0.8114
Simulated power						
Error	0.9504	0.9459	0.9489	0.9455	0.9361	0.9449
Exact variance	−0.0003	0.0044	0.0033	0.0104	0.0191	0.0150
Lehmann	−0.0005	0.0035	−0.0001	0.0038	0.0105	0.0058
Noether	−0.0016	−0.0007	−0.0182	−0.0400	−0.0758	−0.1335
Laplace (0, 1)						
Sample size	731	189	52	25	16	11
Nominal power						
Exact variance	0.9500	0.9506	0.9531	0.9505	0.9571	0.9503
Lehmann	0.9523	0.9579	0.9716	0.9805	0.9902	0.9939
Noether	0.9482	0.9437	0.9259	0.8885	0.8533	0.7932
Simulated power						
Error	0.9491	0.9509	0.9491	0.9390	0.9405	0.9283
Exact variance	0.0009	−0.0003	0.0040	0.0115	0.0166	0.0220
Lehmann	0.0032	0.0070	0.0225	0.0415	0.0497	0.0656
Noether	−0.0009	−0.0072	−0.0232	−0.0505	−0.0872	−0.1351

is determined by the difference between the nominal power and simulated power. For ease of comparison, the discrepancy between nominal power and simulated power is also reported and is termed as error = nominal power − simulated power. In general, the absolute errors increase with decreasing nominal powers for all competing methods. The results suggest that the exact variance method performs consistently well with respect to all distribution and location specifications. For the two approximate procedures, the nominal powers of Lehmann's [3] approach tend to be higher than the simulated powers while Noether's [4] method shows the opposite pattern that the nominal powers are generally lower than the simulated powers. In addition, both approximations give accurate results for small values of $\Delta = 0.1$ and 0.2. However, Lehmann's method is slightly inferior. As expected, larger values of Δ appear to degrade the two approximate approaches, especially, this phenomenon is more pronounced for Noether's approximation. Their performances also vary with the structure of distribution. In the case of normal distribution, it is interesting to note that Lehmann's approximation maintains a reasonable agreement between the simulated power and the nominal power for all values of Δ. Nevertheless, Lehmann's approximation incurs substantially larger errors for heavy-tailed Laplace distribution than the light-tailed uniform distribution. Also, Noether's approximation seems to deteriorate progressively from light- to heavy-tailed cases. Overall, the exact variance large-sample method has a clear advantage over the two approximate counterparts.

4. Conclusion

The purpose of this article is to present power and sample size determinations for the Wilcoxon signed-rank test that have not previously been discussed in literature, especially, the exact variance large-sample method for obtaining accurate results. Analytical formulas are provided for the three prominent situations of light-, standard- and heavy-tailed distributions: uniform, normal and Laplace that researchers are likely to encounter with real data. In addition, we examine two approximations that are valid only for small values of location shift. Particular emphasis is devoted to the demonstration of their differences that arise in power function considerations. According to our findings, the accuracy of the two approximate approaches not only varies with the underlying distributions but also decreases considerably for $\Delta = \theta/\sigma > 0.2$. One clear advantage of the exact variance approach is that it circumvents the restriction of small values of location shift, however, the formulation and computation is slightly more involved. In summary, the exact variance large-sample method is recommended according to its remarkable accuracy under the range of distributions and location configurations considered here.

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