

# Development of optimised carrier sequence in MCU-based random-frequency PWM

H.C. Chen, Y.C. Chang and C.K. Huang

**Abstract:** The effect of the carrier sequences on the voltage spectrum of MCU(microcontroller)-based random-frequency pulse-width modulation (RFPWM) is presented. To optimise the carrier sequence efficiently, first all possible sequences are classified into groups according to the time-shift and time-reverse properties of the Fourier transform. Then, some evaluation indexes are proposed to obtain the optimal sequence quickly with the end of a personal computer (PC). Finally, some optimised carrier sequences with only two carrier frequencies are recommended. The experimental results also indicate that we should pay more attentions to the carrier sequences than to the selection of carrier frequency and on the numbers of carrier frequencies.

## 1 Introduction

Pulse-width-modulation (PWM) schemes are well known and used in applications of power electronics. The most popular scheme is the sinusoidal PWM (SPWM) scheme in which the switching signals are generated directly from a comparison of the sinusoidal signals and the same fixed-frequency carrier signal. We can easily obtain the desired fundamental voltage by changing the amplitude and frequency of the reference signals. Unfortunately, the harmonic voltages are usually concentrated at the integer multiples of the fixed carrier frequency. These undesired harmonics have adverse effects on the acoustic noise and electromagnetic interference (EMI).

To improve it, many random PWM (RPWM) schemes had been proposed in [1–5] and the most frequently seen scheme is the random-frequency PWM (RFPWM). Generally speaking, in the RFPWM, the carrier frequency is not fixed but varies with time and therefore the harmonic voltages can be widely and continuously distributed. With the development of semiconductors in the last decade, the MCUs had been widely used in digital control of variable-speed drives and consequently, some MCU-based RFPWM schemes have been implemented digitally [5–8].

Depending on the method of generation of the carrier frequency, the above MCU-based schemes can be divided into two groups: one looks up a given table tabulated in the read-only-memory (ROM) and the other calculates with the arithmetic rules—linear congruential method in [5, 6]—and the logical rules—pseudorandom bit sequences (PRBS) in [7, 8]. Briefly, the former takes the hardware resources and the latter takes the instruction time in software.

Ideally, the larger the number of the carrier frequencies, the smoother the voltage spectrum is [3]. In fact, we can find

that there are 47 485 different carrier frequencies in [5] and 32 768 in [6]. However, to limit the timer resolution and the closed-loop gain in an MCU-based system, too many carrier frequencies are not practical and not allowed for the digital implementation. To overcome the above obstacle, the optimised selection of the carrier frequency in the so-called ‘limited pool’ had been presented in [9, 10]. In [11], the authors turn their attention to the carrier sequence and find that different carrier sequences may result in different voltage spectra. A precomputed carrier-sequence table with a total of 1000 different frequencies has been offline optimised by the genetic algorithms (GA) [11] but it takes more than 4 kbytes in ROM space.

In [6], the so-called harmonics spread factor (HSF) is defined to quantify the spread-spectrum effect of random PWM. The common-multiple deviation (CMD) method in [9] and average harmonic weight (AHW) method in [10] are used to select the optimal switching frequency from only a certain number of practically achievable switching frequencies. However, all the above evaluation methods are based on the concept of statistical deviation.

In this paper, we propose a simple MCU-based RFPWM scheme which takes the minimum memory space and the calculation time. Instead of using enormous numbers of carrier frequencies as in [5–10], the proposed scheme looks up the optimised carrier sequence with only two different carrier frequencies. To optimise the carrier sequence efficiently, first all possible sequences are classified according to the time shift and time-reverse properties of Fourier transform. Then, some evaluated indexes are proposed to obtain the optimal sequence with the help of a personal computer. Finally, some optimised carrier sequences with the minimum number of carrier frequencies are also recommended. The experimental result also indicates that we should pay more attention to the carrier sequences than to selection of carrier frequency.

## 2 RFPWM tables

The power circuits and RFPWM scheme used are shown in Fig. 1. Three output voltages  $V_A$ ,  $V_B$  and  $V_C$  can be obtained by the switching signals  $S_a$ ,  $S_b$  and  $S_c$  which are determined from comparisons of the three control signals  $v_{cont,a}$ ,  $v_{cont,b}$  and  $v_{cont,c}$  and the unique carrier signal  $v_{tri}$ .

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doi:10.1049/iet-epa:20060140

Paper first received 22nd December 2005 and in final revised form 28th June 2006

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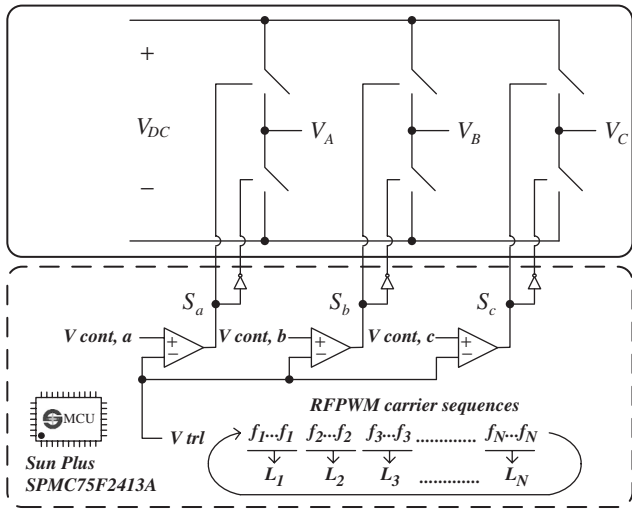


Fig. 1 Power circuits and RFPWM scheme

Undoubtedly, the different carrier sequences may result in the different carrier signals and therefore, the different output-voltage waveforms i.e. the different carrier sequence would lead to different voltage spectra. The following are the definitions of the proposed RFPWM tables.

Assume that there are  $N$  individual carrier members in the RFPWM table, namely  $f_1, f_2, \dots$  and  $f_N$  where  $f_{\min} \leq f_1 < f_2 < \dots < f_N \leq f_{\max}$  and that numbers in the table are  $L_1, L_2, \dots$  and  $L_N$ , respectively. Thus, the total length of the above RFPWM table is

$$L = L_1 + L_2 + \dots + L_N = \sum_{k=1}^N L_k \quad (1)$$

The maximum carrier frequency  $f_{\max}$  is selected taking into account the final duty resolution in digital implementation and the unavoidable derating of output power. The minimum carrier frequency  $f_{\min}$  is set by the constraints of the capability and the resulting current ripples.

In the RFPWM table, many random probability-distribution functions (pdf) are listed in [1]. In this paper, only uniform and trapezium ones are studied. Briefly, in uniform pdf the likelihood of appearance of each carrier frequency is identical but in the trapezium pdf each probability of appearance is proportional to its carrier frequency. Since the sum of all probabilities must be one, therefore the individual number  $L_k$  in the RFPWM table is

$$L_k = L \times \text{pdf}(f_k) = \begin{cases} \frac{L}{N} & \text{(uniform pdf)} \\ \frac{L \times f_k}{f_1 + f_2 + \dots + f_N} & \text{(trapezium pdf)} \end{cases} \quad (2)$$

With the above definitions of the RFPWM table, the number  $M$  of possible carrier sequences is calculated as

$$M = \frac{L!}{\prod_{k=1}^N (L_k!)} \quad (3)$$

To recognise each of the total  $N$  carrier members in the MCU-based RFPWM table, the required T-bit format for saving the carrier sequence is as follows, where  $\text{CEIL}(X)$  denotes the operator of rounding the element  $X$  to the nearest integer towards infinity:

$$T = \text{CEIL}\left(\frac{\log N}{\log 2}\right) \quad (4)$$

Therefore, we can obtain the total required memory space a  $B = L \times T$  in bits in order to store the whole carrier sequence in MCU. For  $N = 10$ , 4-bit values are enough to differentiate the individual carrier frequencies and the memory  $4L$  bits are required to store the carrier sequence.

It is known that the power loss can be divided into conducting loss and switching loss and that the latter is proportional to the average switching frequency. Because of the fixed carrier frequency in the conventional SPWM, the averaged switching frequency is equal to the fixed carrier frequency. However, since the carrier frequency is varying with time in the RFPWM scheme, the averaged switching frequency  $\bar{f}$  is dependent on the pdf of the carrier element.

$$\bar{f} = \begin{cases} \frac{N}{\frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_N}} & \text{(uniform pdf)} \\ \frac{f_1 + f_2 + \dots + f_N}{N} & \text{(trapezium pdf)} \end{cases} \quad (5)$$

In addition, since the table length  $L$  is finite, the practical carrier waveform obtained by looking up the table is repeated. The repeated periods of the carrier signal in the MCU-based RFPWM scheme are  $S = L/\bar{f}$  seconds.

Ideally, the larger the number  $N$  of the frequency elements, the better will be the voltage spectrum [1]. However, only certain carrier frequencies (i.e. limited pool in [6, 7]) are achievable owing to the timer resolution of the digital RFPWM implementation. In addition, for a closed-loop motor-drive system, the gain of the current controller depends on the sampling frequency (i.e. carrier frequency), i.e. not only the content of the RFPWM table but also the gains of each carrier element must be stored in memory, which takes much memory space in MCUs. It also means that an increasing number of carrier frequencies is not a good solution of the RFPWM scheme in MCU-based implementations.

### 3 Carrier-sequence optimisation

Since the number  $M$  of possible carrier sequences from (3) is enormous, we divide the optimisation steps into two steps to save searching-time. The first step classifies the possible carrier sequence into some groups according to the properties of the Fourier transform. The second step uses a proposed evaluating index to compare each voltage spectrum efficiently one by one using a PC.

To simplify the presentation, an example will be considered in the following sections where the RFPWM table contains only two carrier elements  $f_1 = 3$  kHz,  $f_2 = 4$  kHz (i.e.  $N = 2$ ) and  $L = 6$  with uniform pdf (i.e.  $L_1 = L_2 = 3$ ). There are 20 possible carrier sequences in this example from (3). All the possible carrier sequences are tabulated in Fig. 2. The carrier sequence 344334 means that the carrier signal  $v_{tri}$  repeats the sequence 3 kHz, 4 kHz, 4 kHz, 3 kHz, 3 kHz and 4 kHz.

#### 3.1 Classification

The following are the time-shift and time-reverse properties of the Fourier transform:

$$\left. \begin{aligned} x(t) &\leftrightarrow X(j\omega) \rightarrow |X(j\omega)| \\ x(t - t_0) &\leftrightarrow e^{-j\omega t_0} X(j\omega) \rightarrow |X(j\omega)| \\ x(-t) &\leftrightarrow X(-j\omega) \rightarrow |X(j\omega)| \end{aligned} \right\} \quad (6)$$

We can show from (6) that the carrier signal  $v_{tri}(t)$ , its time-reversal signal  $v_{tri}(-t)$  and time-shift signal  $v_{tri}(t - t_0)$  possess the same amplitude spectrum, i.e. the carrier

	Carrier Sequence	Waveform	Voltage Spectrum	Probability
A	434343 343434		Fig. 4a	0.1
B	444333 443334 433444 333444 334443 344433		Fig. 4b	0.3
C	443433 434334 343344 433443 334434 344343 334344 343443 434433 344334 443343 433434 433434		Fig. 4c	0.6

**Fig. 2** Classification of carrier sequences

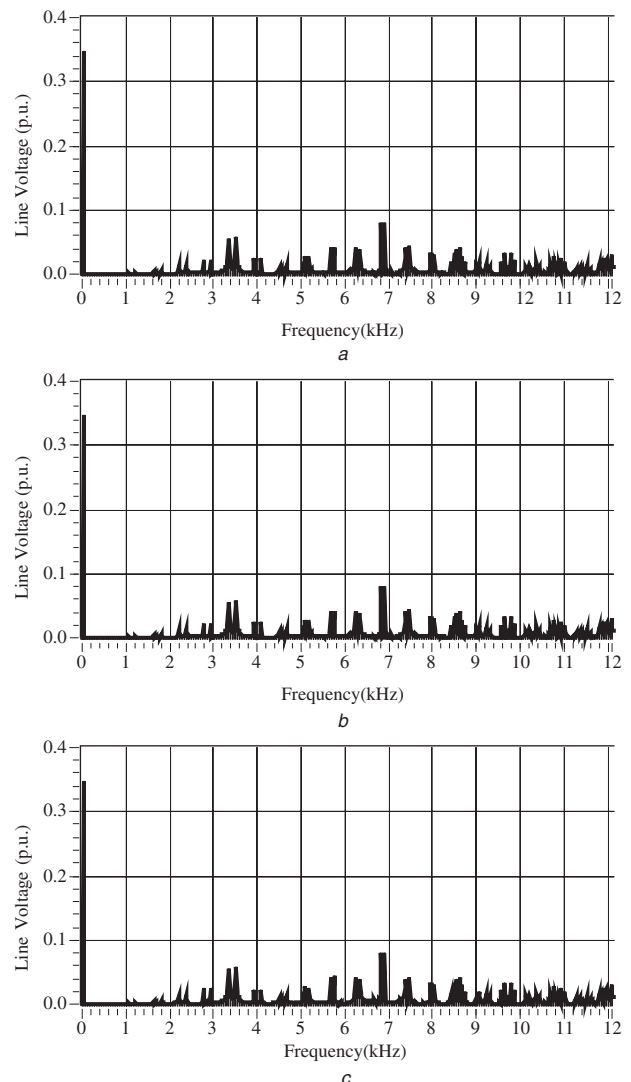
sequences 433443, 334434, 344343, 443433, 434334 and 343344 have the same amplitude spectra for the time-shift property of Fourier transform. The carrier sequences 443433 and 334344 possess the same amplitude spectrum for the time-reversal property. After running the discrete Fourier transform (DFT) of the output line-to-line voltage using a PC, the voltage spectra corresponding to the carrier sequences 434334, 334344 and 443433 with modulation index equal to 0.35 are plotted in Fig. 3. It shows that the output line-to-line voltages with the same carrier spectrum possess nearly identical voltage spectra in the RFPWM scheme. Since the voltage spectrum is dominated by the carrier spectra, the voltage spectrum corresponding to different modulation indexes are, in fact, similar. Therefore, only one modulation index is considered in the following sections.

In the above simple example, there are 20 carrier sequences and they can be classified into three carrier groups: A, B and C. The voltage spectra corresponding to the three groups are shown in Fig. 4. From Fig. 4, we can see that group B is preferred because its spectrum is smoother than the others, but the probability of group B is only 0.3. It means that when, an average 10 persons generate the above simple RFPWM table randomly, on average only three will obtain the optimal voltage spectrum B and the other seven will not; on average one gets the worst spectrum (A). Therefore, to obtain the optimal RFPWM voltage spectrum, we should pay attention not only to the selection of the carrier frequency [9, 10] but also to their carrier sequence in the RFPWM table [11].

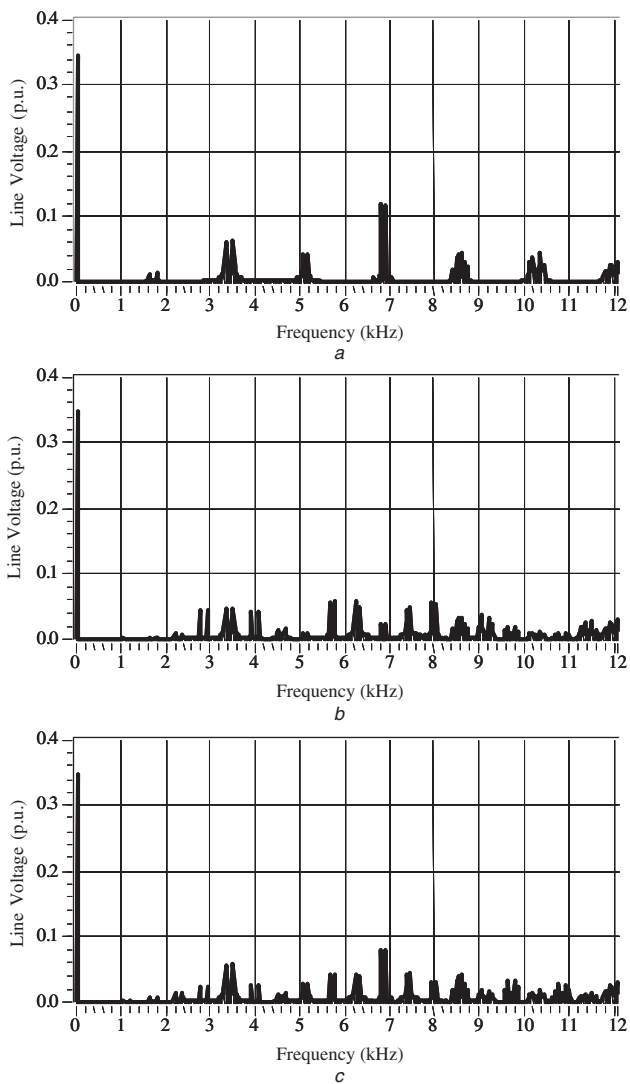
### 3.2 Evaluation index

Comparing the voltage spectra one by one and finding the best one is the most direct way to find the optimal carrier sequence. However, the number of classified groups increases rapidly with table length  $L$ . For example, the number of groups is up to 4752 when  $L_1 = L_2 = 10$  and  $L = 20$ . Therefore, an evaluation numerical index  $FI$  is proposed in this paper such that the comparison routine can be executed with the help of a PC.

Unlike the continuous spectrum using a Fourier transform, the DFT spectrum is discrete. Thus the index  $FI_n$  evaluates the envelope of the DFT result for the  $n$ th carrier



**Fig. 3** Output-voltage spectra with carrier sequences  
a 434334  
b 334344  
c 443433



**Fig. 4** Output voltage spectra  
*a* Group A  
*b* Group B  
*c* Group C

group over the frequency range  $f_B$  defined within the lowest frequency  $f_L$  and the highest frequency  $f_H$ . It is calculated by

$$FI_n(f_B, m) = \text{std}(X_{(1)} X_{(2)} \dots X_{(m)}) \quad (7)$$

where

$X_{(1)}$ : the largest values of the DFT spectrum  $X(f \in f_B)$

$X_{(2)}$ : the largest values of  $X(f \in f_B)$  except  $X_{(1)}$ :

$X_{(m)}$ : the largest values of  $X(f \in f_B)$  except  $X_{(1)} \dots X_{(m-1)}$

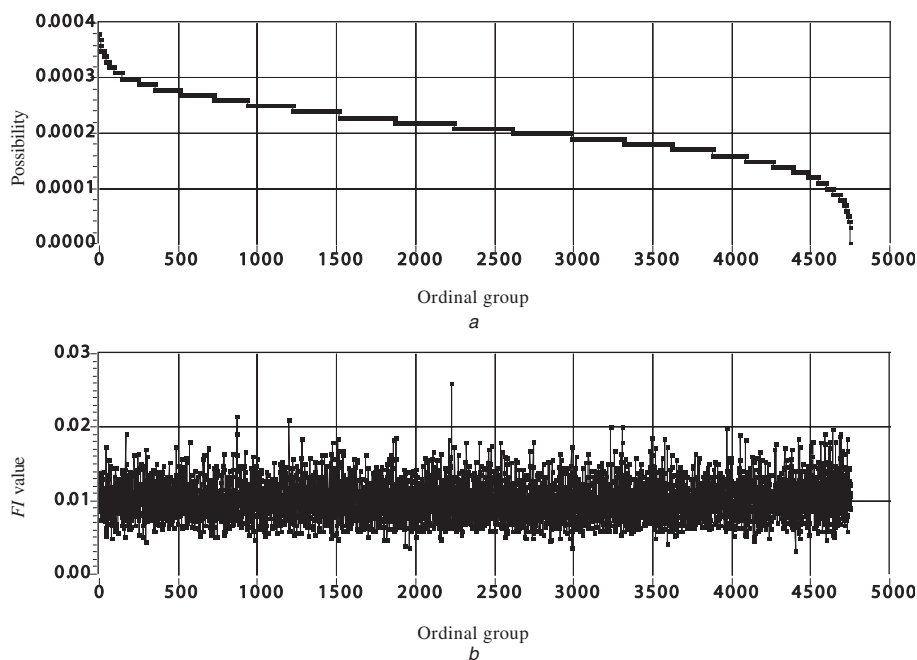
$\text{std}(\bullet)$ : the standard deviation operator of the  $m$  values  $X_{(1)} \dots X_{(m)}$ .

### 3.3 Optimal carrier sequence

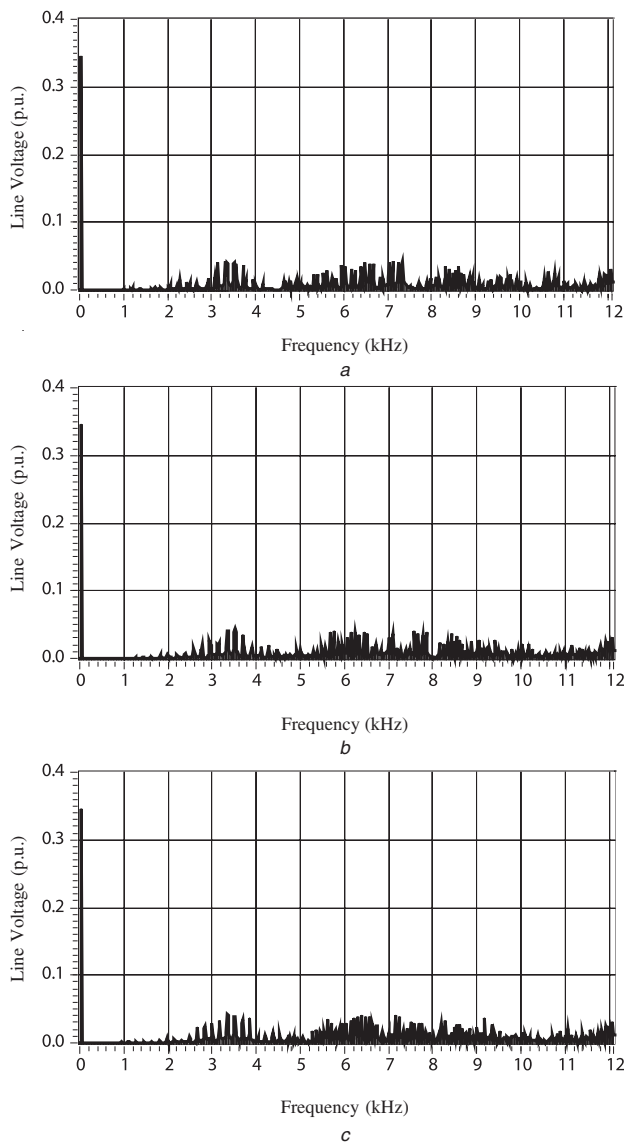
Using the above index, the optimal carrier sequence can be obtained by finding the minimum index value. Table 1 shows the optimal carrier sequences the cases  $L_1 = L_2 = 3 \dots 10$  where the index parameters are defined as  $f_L = 2$  kHz,  $f_H = 10$  kHz and  $m = 20$ . The possibilities of each carrier group for the example  $L_1 = L_2 = 10$  are plotted in Fig. 5a according to the descending series, and Fig. 5b shows the calculated index values. One can show that the index values are independent of the probability and that the

**Table 1: Recommended optimal carrier sequences**

$L$	$L_1, L_2$	Total carrier sequences	Total groups	Optimal carrier sequence	$FI$ value
6	3	20	3	333444	0.006594
8	4	70	8	33344434	0.005783
10	5	252	16	3333444434	0.005776
12	6	924	50	333444344334	0.004635
14	7	3432	133	33344333434444	0.003090
16	8	12870	440	3343443444334334	0.002566
18	9	48620	1387	333434444333443344	0.003089
20	10	184756	4752	33444433344344343334	0.003333



**Fig. 5** Probabilities with descending orders and calculated index values  
*a* Probabilities for the case  $L_1 = L_2 = 10$   
*b* Calculated index values



**Fig. 6** Output-voltage spectra with optimal carrier frequencies  
*a*  $L_1 = L_2 = 8$   
*b*  $L_1 = L_2 = 9$   
*c*  $L_1 = L_2 = 10$

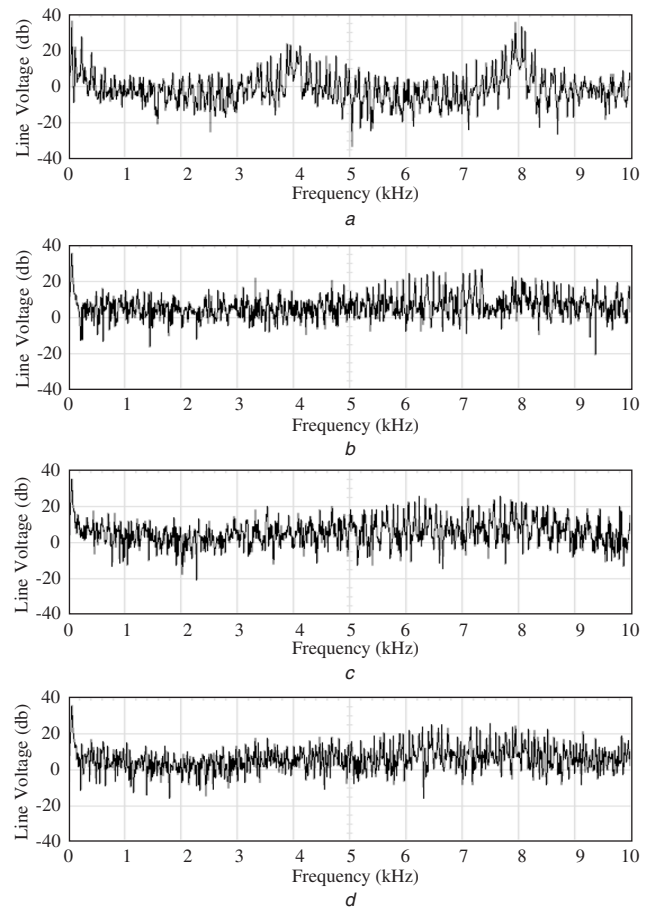
voltage spectrum thus has no relationship with their probability.

The individual voltage spectra with optimal carrier sequences for  $L_1 = L_2 = 8, 9$  and  $10$  are plotted in Fig. 6. The desired spectral distribution within the frequency band  $f_B$  can be seen from Fig. 6.

In our paper, the optimal carrier sequence is based on the optimal voltage spectrum. In the voltage-source inverter, the output-voltage waveforms are dependent on the switching signals and therefore have no relationship with the type of load. Therefore, no information about the load, the output power or the output current are presented in this paper.

#### 4 Experimental results

An MCU-based PWM inverter is implemented as shown in Fig. 1. The inverter in the solid-line region is a voltage-source inverter with a DC link voltage  $V_{DC} = 300$  V. All the PWM schemes in the dotted-line region, including the reference-signal generator, three comparators and the varying carrier frequency, are implemented in a 16-bit MCU SPMC75F2413A manufactured by SunPlus, Ltd. in Taiwan.



**Fig. 7** Output-voltage spectra  
*a* SPWM with fixed carrier frequency 4 kHz  
*b* RFPWM with recommended carrier sequences for  $L_1 = L_2 = 8$   
*c* RFPWM with recommended carrier sequences for  $L_1 = L_2 = 9$   
*d* RFPWM with recommended carrier sequences for  $L_1 = L_2 = 10$

Using the FFT function in digital oscilloscope, the spectrum corresponding to the conventional SPWM with fixed carrier frequency 4 kHz and the RFPWM with the recommended carrier sequences for  $L_1 = L_2 = 8, 9$  and  $10$  are plotted in Fig. 7. We can see that the spectrum over the desired frequency band 2–10 kHz in Fig. 7*a* is more concentrated than the others in Fig. 7*b, c* and *d*. Note that the dispersed voltage spectrum can also be obtained using only two carrier frequencies.

#### 5 Conclusions

In this paper, the effect of carrier-sequence order on the output-voltage spectrum is studied. In addition, a simple index for efficiently searching the optimal carrier sequence is proposed. Finally, some optimal showing orders are recommended. However, the optimal carrier sequences are dependent on the types of power circuits, the PWM scheme used, the evaluation formula for the index and its parameters. The experimental results also indicate that we should pay more attentions to the carrier sequences than to the selection of carrier frequency or on increasing the number of carrier frequencies.

#### 6 Acknowledgment

The authors acknowledge financial support from the Energy R&D Foundation provided by the Bureau of Energy, Ministry of Economic Affairs in Taiwan, ROC.



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