Contradirectional two-wave mixing with partially coherent waves in photorefractive crystals

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We investigate contradirectional two-wave mixing with partially coherent waves in photorefractive crystals in the nondepleted pump regime. Equations governing the propagation of the self-coherence function and the mutual-coherence function of the signal wave and the pump wave are derived and simulated numerically. Numerical solutions of these equations are in excellent agreement with the experimental measurements.
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Two-wave mixing in photorefractive crystals is an interesting and useful nonlinear-optical phenomenon for many applications such as image amplification, laser wave cleanup, spatial light modulators, thresholding, and power-limiting devices. Most of the theoretical study in this area has addressed wave mixing with monochromatic waves. Two-wave mixing with partially coherent waves has been studied for the case of transmission grating interaction,[1–3] in which the optical path difference between the two interacting waves remains approximately the same as the two waves propagate codirectionally through the photorefractive medium, especially when the incident angles of the two waves are close. In the case of reflection grating interaction the optical path difference between the two interacting waves varies significantly as the two waves propagate contradirectionally through the photorefractive medium. As a result, the latter case is quite different from the former and is much more complicated to analyze. In this Letter we present a theoretical analysis and experimental investigation for the case of reflection grating interaction in the nondepleted pump regime. The absorption effect is also included.

In the case of contradirectional two-wave mixing in a purely diffusive photorefractive medium the coupled-wave equations for the slowly varying amplitudes $\mathbf{E}_1(z,t)$ and $\mathbf{E}_2(z,t)$ can be written as[1]

$$\frac{\partial \mathbf{E}_1}{\partial z} + \frac{1}{v}\frac{\partial \mathbf{E}_1}{\partial t} = \frac{\gamma}{2}\frac{\mathbf{Q}\cdot\mathbf{E}_2}{\mathbf{I}_1+\mathbf{I}_2} - \frac{\alpha}{2}\mathbf{E}_1\,, \tag{1}$$

$$\frac{\partial \mathbf{E}_2}{\partial z} - \frac{1}{v}\frac{\partial \mathbf{E}_2}{\partial t} = \frac{\gamma}{2}\frac{\mathbf{Q}^*\cdot\mathbf{E}_1}{\mathbf{I}_1+\mathbf{I}_2} + \frac{\alpha}{2}\mathbf{E}_2\,, \tag{2}$$

where γ is the intensity coupling constant, α is the intensity absorption coefficient, v is the group velocity, and $\mathbf{E}_1\exp(-i\omega t + ikz)$ and $\mathbf{E}_2\exp(-i\omega t - ikz)$ are the coupled quasi-monochromatic waves that interact through a dynamic photorefractive grating $\delta n(z,t) \propto \mathbf{Q}\exp(2ikz) + \text{c.c.}$ The dynamics of the photorefractive grating can be written as

$$\tau\frac{\partial \mathbf{Q}}{\partial t} + \mathbf{Q} = \mathbf{E}_1{\mathbf{E}_2}^*\,, \tag{3}$$

where τ is the relaxation time constant.

The temporal behavior of each wave's complex amplitude can be modeled with a stationary random process, with coherence time $\delta\omega^{-1}$ being substantially less than the relaxation time of the material, i.e., $\delta\omega\tau \gg 1$.[4] When the optical path difference of the two waves is smaller than the coherence length of the source laser wave, a dynamic photorefractive grating is recorded in the medium. Its position and profile are nearly temporally constant. Then, as an approximation, we can replace the dynamic grating amplitude $\mathbf{Q}$ in Eqs. (1) and (2) with its ensemble average $\mathbf{Q} \approx \langle\mathbf{Q}\rangle = \langle\mathbf{E}_1{\mathbf{E}_2}^*\rangle$.

Since the complex amplitudes $\mathbf{E}_1(z,t)$ and $\mathbf{E}_2(z,t)$ are stationary random processes, we can define some of their ensemble averages as $\Gamma_{12}(z,\Delta t) \equiv \langle\mathbf{E}_1(z,t_1){\mathbf{E}_2}^*(z,t_2)\rangle$, $\Gamma_{11}(z,\Delta t) \equiv \langle\mathbf{E}_1(z,t_1){\mathbf{E}_1}^*(z,t_2)\rangle$, and $\Gamma_{22}(z,\Delta t) \equiv \langle\mathbf{E}_2(z,t_1){\mathbf{E}_2}^*(z,t_2)\rangle$, where $\Delta t = t_1 - t_2$. We refer to Γ_{11} and Γ_{22} as the self-coherence of the signal wave $\mathbf{E}_1(z,t)$ and the pump wave $\mathbf{E}_2(z,t)$, respectively, and Γ_{12} as the mutual coherence between the signal wave and the pump wave. With this notation we can immediately write $\mathbf{Q} = \Gamma_{12}(z,0)$, $I_1 = \Gamma_{11}(z,0)$, and $I_2 = \Gamma_{22}(z,0)$. Equations (1)–(3) can therefore be reduced to a system for these average values[5]:

$$\frac{\partial\Gamma_{12}(z,\Delta t)}{\partial z} = -\frac{2}{v}\frac{\partial\Gamma_{12}(z,\Delta t)}{\partial\Delta t} + \frac{\gamma}{2}\frac{\Gamma_{12}(z,0)}{I_1+I_2} \times[\Gamma_{11}(z,\Delta t) + \Gamma_{22}(z,\Delta t)]\,, \tag{4}$$

$$\frac{\partial\Gamma_{11}(z,\Delta t)}{\partial z} = \frac{\gamma}{2}\frac{\Gamma_{12}(z,0)}{I_1+I_2}{\Gamma_{12}}^*(z-\Delta t) + \frac{{\Gamma_{12}}^*(z,0)}{I_1+I_2} \times\Gamma_{12}(z,\Delta t) - \alpha\Gamma_{11}(z,\Delta t)\,, \tag{5}$$

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$$\frac{\partial \Gamma_{22}(z, \Delta t)}{\partial z} = \frac{\gamma}{2} \frac{\Gamma_{12}(z, 0)}{I_1 + I_2} \Gamma_{12}^*(z, -\Delta t) + \frac{\gamma}{2} \frac{\Gamma_{12}^*(z, 0)}{I_1 + I_2} \times \Gamma_{12}(z, \Delta t) + \alpha \Gamma_{22}(z, \Delta t) . \quad (6)$$

Note that Eqs. (4)–(6) are a set of self-consistent partial differential equations governing the propagation of the mutual-coherence and self-coherence functions of the two waves. We can solve them numerically as an initial-value problem, if we have the complete boundary conditions at either one of the two boundaries that the two waves are incident upon. In general, we know only the self-coherence functions and the mutual-coherence function of the two waves before they enter the medium. In the case of pump depletion, either wave has changed significantly as it reaches the second boundary. Therefore the complete boundary conditions are unavailable at either boundary. In the case of the nondepleted pump, we can assume that the pump wave passes through the photorefractive medium unaffected by the weak signal wave. In this case we obtain the complete boundary conditions at the boundary where the signal wave enters the medium. We then can solve Eqs. (4)–(6) for the self-coherence function of the signal wave and the mutual-coherence function of the two waves.

To determine the boundary conditions in the nondepleted pump regime, we assume that both the signal wave and the pump wave are derived from the same source wave, as is shown in Fig. 1. If the source wave has a Gaussian line shape with a linewidth of Δv, then the normalized self-coherence function of the source wave can be written as

$$\Gamma_s(\delta t) = \exp\left[-\left(\frac{\pi \Delta v \delta t}{2\sqrt{\ln 2}}\right)^2\right]. \quad (7)$$

At the signal-wave incident plane $z = 0$ the intensity ratio between the signal wave and the pump wave is assumed to be β. In our simulation the intensity of the pump wave is taken to be 1. Then the boundary conditions at $z = 0$ can be written as

$$\Gamma_{12}(z = 0, \Delta t) \equiv \sqrt{\beta}\, \Gamma_s(\Delta t + \delta t) , \quad (8)$$

$$\Gamma_{11}(z = 0, \Delta t) \equiv \beta\, \Gamma_s(\Delta t) , \quad (9)$$

$$\Gamma_{22}(z = 0, \Delta t) \equiv \Gamma_s(\Delta t) , \quad (10)$$

where δt is the time delay between the two waves.

In Fig. 2 we show the mutual-coherence function $\Gamma_{12}(z, 0)$ of the two waves and the self-coherence function $\Gamma_{11}(z, 0)$ of the signal wave as a function of the position z inside the photorefractive medium for two coupling constants γ of 3 and 7. At the signal entrance plane $z = 0$ the time delay δt is 0 s, and the input intensity ratio β is 10^{-4}. The linewidth Δv of the source is 10 GHz, and the index refraction of the photorefractive medium is 2.3. As a result of the coupling, part of the pump wave branches off in the direction of the signal wave, retaining its temporal profile. Therefore the mutual-coherence function will increase with coupling constant γ. For a small coupling constant the mutual-coherence function eventually decreases to zero as z increases because of the relatively rapid increase of the time delay of the two waves. Thus, as z increases, the coupling between the two waves decreases, and the signal intensity approaches a certain limit that is much less than the pump intensity. For a large coupling constant the mutual-coherence function increases as z increases because of relatively strong coupling. Thus, as z increases, the coupling between the two waves increases, and the signal intensity increases exponentially. In other words, for a large coupling constant the interaction length of the two waves can be much longer than the coherent length of the source wave.

The above theory is validated experimentally. Referring to Fig. 1, we consider two partially coherent waves, obtained by splitting an argon laser wave with a linewidth of 1.83 GHz. The signal and pump waves are contradirectionally incident upon a $KNbO_3$:Co crystal ($\gamma = 3.3\ cm^{-1}$, $\alpha = 0.5\ cm^{-1}$, and thickness $d = 0.72$ cm). The optical path difference of the two waves at the signal-wave incident plane $z = 0$ was set to be $\Delta L = L_2 - L_1$. To monitor the mutual coherence between the signal wave and the pump wave at the output plane $z = d$, we employ another reference

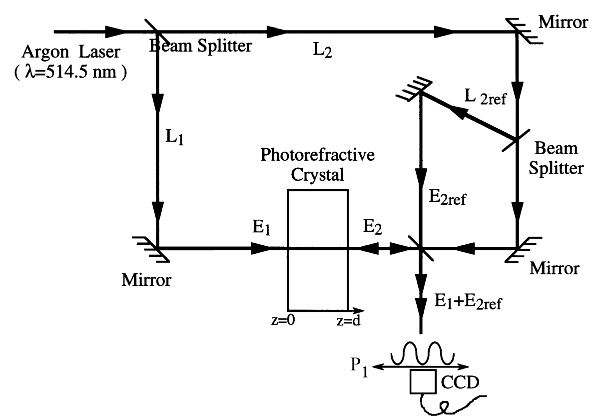

Fig. 1. Schematic of the two-wave-mixing configuration used in our calculations and experiments. The distances L_1 and L_2 are the optical path lengths of the signal wave and the pump wave from the laser source to the signal-wave incident plane $z = 0$, respectively. L_{2ref} is the optical path length of reference wave E_{2ref} from the laser source to the signal output plane $z = d$.

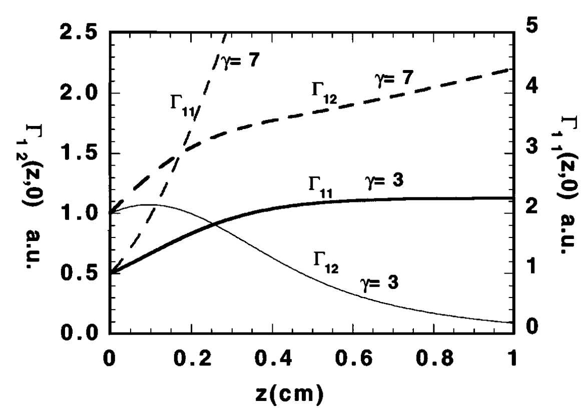

Fig. 2. Mutual-coherence function $\Gamma_{12}(z, 0)$ and the self-coherence function of the signal wave $\Gamma_{11}(z, 0)$ as a function of z for coupling constants $\gamma = 3$ (solid curves) and $\gamma = 7$ (dashed curves).

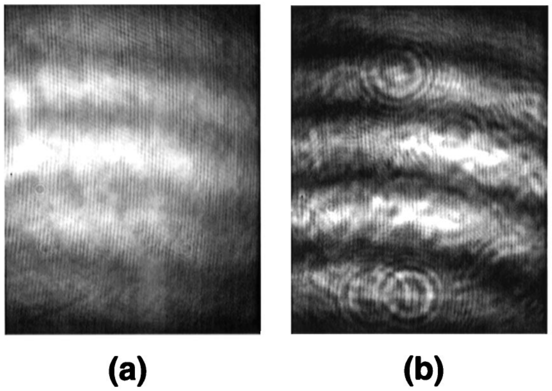

Fig. 3. Interference patterns of the signal wave and the reference wave at the output plane P1 (a) without and (b) with coupling. Note the increase of fringe visibility owing to the coupling.

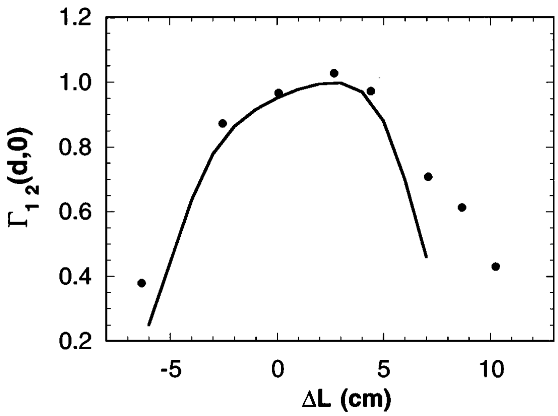

Fig. 4. Mutual coherence $\Gamma_{12}(d, 0)$ as a function of the optical path difference ΔL.

wave $E_{2\mathrm{ref}}$ that was split from the pump wave E_2. The optical path difference of waves E_1 and $E_{2\mathrm{ref}}$ was adjusted to be the same as that of waves E_1 and E_2 at the output plane $z = d$. Using a simple homodyne technique, we observed the interference fringes generated by waves E_1 and $E_{2\mathrm{ref}}$ with a CCD camera at the output plane P1. The normalized mutual coherence $\Gamma_{12}(d, 0)/[\Gamma_{11}(d, 0)\Gamma_{22}(d, 0)]^{1/2}$ can be estimated as $(I_{\max} - I_{\min})/(4\sqrt{I_1 I_2})$, where $(I_{\max} - I_{\min})$ is the amplitude of the fringes. In our experiment we monitored the interference pattern with and without pump beam E_2. Figure 3 shows photographs taken with a normalized mutual coherence $\Gamma_{12}(0, 0) \approx 0.43$ at $z = 0$ ($\Delta L = 4$ cm) and an intensity ratio $\beta = 0.00151$. The measured normalized mutual coherence increases from 0.19 to 0.7 at $z = d$. We then measured the normalized mutual coherence as a function of the optical path difference. Figure 4 shows the data

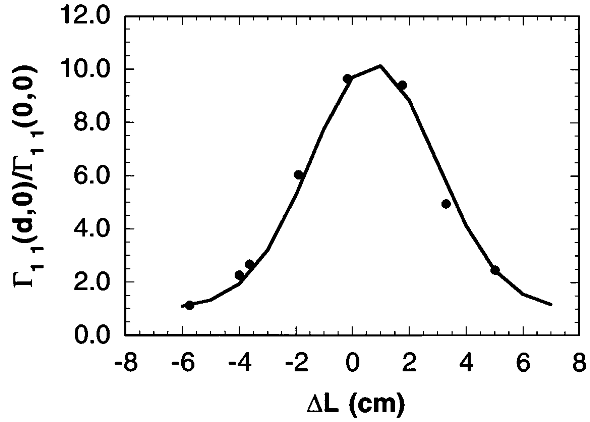

Fig. 5. Intensity gain of the signal wave at $z = d$ plane as a function of the path difference ΔL.

obtained from this measurement (filled circles) and the theoretical curve for the same parameters. Note that the normalized mutual coherence is close to 1 for small optical path differences and decreases quickly to zero as the optical path difference increases beyond certain values. The intensity gain of the signal wave was also measured. Figure 5 shows the measurement of the intensity gain (filled circles) of the signal wave at the $z = d$ plane as a function of the optical path difference $\mathbf{\Delta L}$, along with the theoretical curve for the same parameters. Excellent agreement between theory and experiment has been achieved in the case of the signal-wave intensity gain.

In conclusion, we have studied contradirectional two-wave mixing with partially coherent waves in photorefractive crystals both theoretically and experimentally. A set of partial differential equations has been derived to describe the propagation of the mutual-coherence and self-coherence functions of the two waves in the nondepleted pump regime. Excellent agreement has been achieved between theory and experiment.

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5. See, for example, J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985), Chap. 5.