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Accuracy Analysis of the Percentile Method for Estimating Non Normal Manufacturing Quality

Chien-Wei Wu [a] , W. L. Pearn [b] , C. S. Chang [c] & H. C. Chen [b]

[a] Department of Industrial Engineering & Systems Management , Feng Chia University , Taiwan

[b] Department of Industrial Engineering & Management , National Chiao Tung University , Taiwan

[c] Department of Industrial Engineering & Management , Cheng Kuo Institute of Technology , Taiwan

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Accuracy Analysis of the Percentile Method for Estimating Non Normal Manufacturing Quality

CHIEN-WEI WU[1], W. L. PEARN[2], C. S. CHANG[3], AND H. C. CHEN[2]

[1]Department of Industrial Engineering & Systems Management,
Feng Chia University, Taiwan
[2]Department of Industrial Engineering & Management,
National Chiao Tung University, Taiwan
[3]Department of Industrial Engineering & Management,
Cheng Kuo Institute of Technology, Taiwan

Vännman (1995) proposed a superstructure $C_p(u, v)$ of capability indices for processes with normal distributions, which include C_p, C_{pk}, C_{pm}, and C_{pmk} as special cases. Pearn and Chen (1997) considered a generalization of $C_p(u, v)$, called $C_{Np}(u, v)$, to cover processes with non normal distributions. Pearn and Chen (1997) also proposed a sample percentile estimator for the generalization $C_{Np}(u, v)$. In this article, we investigate the performance of the sample percentile estimator. We perform some simulation study, which covers the normal distribution and various non normal distributions including the uniform distribution, chi-square distribution, student's t distributions, F distribution, beta distribution, gamma distribution, Weibull distribution, lognormal distribution, triangular distribution, and Laplace distribution, with selected parameter values. Extensive simulation results, comparisons, and analysis are provided.

Keywords Flexible capability indices; Non normal distributions; Relative bias; Sample percentile estimator; Simulation.

Mathematics Subject Classification Primary 62G05; Secondary 62P30.

1. Introduction

Process capability indices $C_p(u, v)$, which include the two basic indices C_p and C_{pk} (Kane, 1986), and the two more-advanced indices C_{pm} and C_{pmk} (Chan et al., 1988; Pearn et al., 1992) as special cases, have been proposed in the

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Address correspondence to Chien-Wei Wu, Department of Industrial Engineering and Systems Management, Feng Chia University, 100, Wenhwa Road, Seatwen, Taichung 40724, Taiwan; E-mail: cweiwu@fcu.edu.tw

manufacturing industry to provide numerical measures on process potential and process performance. The superstructure $C_p(u, v)$ has been defined as the following (see Vännman, 1995):

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}},$$

where μ is the process mean, σ is the process standard deviation, $d = (USL - LSL)/2$ is half length of the specification interval $[LSL, USL]$, $m = (USL + LSL)/2$ is the mid-point between the upper and the lower specification limits, T is the target value, and $u, v \geq 0$. By setting $u, v = 0$ and 1, we obtain the four indices, $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$, and $C_p(1, 1) = C_{pmk}$. These four indices, have been investigated extensively by Kane (1986), Chan et al. (1988), Pearn et al. (1992, 1998), Kotz et al. (1993), and many others. For thorough discussions of different capability indices and their statistical properties, see, e.g., the books by Kotz and Johnson (1993) and Kotz and Lovelace (1998), and the review article with discussion by Kotz and Johnson (2002). The index C_p only considers the process variability σ, thus provides no sensitivity on process departure from the target value at all. The index C_{pk} takes the process mean into consideration but it can fail to distinguish between on-target processes from off-target processes. The index C_{pm} takes proximity of process mean from the target value into account, and is more sensitive to process departure than C_p and C_{pk}. The index C_{pmk} adds an additional term $(\mu - T)^2$ in the definition, as a penalty to the process quality due to the departure of process mean from the target value. This additional penalty will be more sensitive to departure and therefore is able to distinguish better between off-target and on-target processes.

The formulae for these indices are easy to understand and straightforward to apply. In practice, sample data must be collected to calculate these indices since the process mean μ and process variance σ^2 are usually unknown, while the sample mean $\overline{X} = \sum_{i=1}^{n} X_i/n$, and the sample variance $S^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2/(n-1)$, are the conventional estimators of process mean μ and process variance σ^2. For normal distributions, these estimators based on $\overline{X}$ and S^2 are quite stable and reliable. However, for non normal distributions, they become highly unstable since the distribution of the sample variance, S^2, is sensitive to departures from normality. Somerville and Montgomery (1996–97) presented an extensive study to illustrate how poorly the normally based capability indices perform as a predictor of process fallout when the process is non normally distributed. If the normally based capability indices are still used to deal with non normal process data, the values of the capability indices are incorrect and might misrepresent the actual product quality.

Clements (1989) proposed a conceptually simple method for calculating C_p and C_{pk}, for any shape of distribution, using the Pearson family of curves. Pearn and Kotz (1994–95) adopted Clements' method to modify normality-based capability indices into non normality Pearson-based capability indices for the C_{pm} and C_{pmk} indices. The advantage of Clements' method is that it is easy to understand and apply. But somewhat limited as not all data distributions can be described adequately with a Pearson or Johnson curve. Chang and Lu (1994) also applied Clements' approach and proposed a percentile method for calculating PCIs without assuming normal distribution. Moreover, Pearn and Chen (1997) extended their method to the generalization $C_{Np}(u, v)$ and constructed a superstructure for the estimators of $C_{Np}(u, v)$, namely, $\widehat{C}_{Np}(u, v)$. Although they proposed these estimators, they did not show how good it is, i.e., relative bias. In this article, we first give a brief introduction to the flexible capability index and then introduce their estimators based on the percentile method in Sec. 3. Subsequently, in Sec. 4, an extensive simulation study was conducted to examine the performance of the estimated $C_{Np}(u, v)$ for 11 distributions (which cover processes with normal and various non normal distributions). In Sec. 5, the performance analysis of the percentile estimators $\widehat{P}_{0.135}$, $\widehat{M}$, $\widehat{P}_{99.865}$ and the estimators of $C_p(u, v)$ are presented based on the relative bias. In Sec. 6, an illustrative example is presented for demonstrating how we apply the percentile method to the actual data taken from a factory. Finally, some concluding remarks are made in Sec. 7.

2. Flexible Capability Index $C_{Np}(u, v)$

The indices $C_p(u, v)$ are appropriate for processes with normal distributions, but have been shown to be inappropriate for processes with non normal distributions. Pearn and Chen (1997) considered the following generalization of $C_p(u, v)$, called $C_{Np}(u, v)$, to cover processes with non normal distributions.

$$C_{Np}(u, v) = \frac{d - u|M - m|}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + v(M - T)^2}},$$

where P_α is the α-percentile, M is the median of the distribution, m and T are defined as before. In developing the generalization $C_{Np}(u, v)$, Pearn and Chen (1997) replaced the process mean μ by the process median M (a more robust measure for process central tendency), and the process standard deviation σ by $(P_{99.865} - P_{0.135})/6$ in the definition of the original indices $C_p(u, v)$. We note that $P_{99.865} - P_{0.135}$ covers a probability of 99.73% for any distributions. In the special case where the underlying distribution is normal, then $M = \mu$ and $P_{99.865} - P_{0.135} = 6\sigma$. Clearly, the generalization of $C_{Np}(u, v)$ will reduce to $C_p(u, v)$. By setting $u, v = 0$ and 1, we obtain $C_{Np}(0, 0) = C_{Np}$, $C_{Np}(1, 0) = C_{Npk}$, $C_{Npm}(0, 1) = C_{Npm}$, and $C_{Np}(1, 1) = C_{Npmk}$, which can be expressed as the following:

$$C_{Np} = \frac{USL - LSL}{P_{99.865} - P_{0.135}},$$

$$C_{Npk} = \min\left\{\frac{USL - M}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2}}, \frac{M - LSL}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2}}\right\},$$

$$C_{Npm} = \frac{USL - LSL}{6\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + (M - T)^2}},$$

$$C_{Npmk} = \min\left\{\frac{USL - M}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + (M - T)^2}}, \frac{M - LSL}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + (M - T)^2}}\right\}.$$

Pearn and Chen (1997) showed that the generalization $C_{Np}(u, v)$ is more consistent and more accurate than the original indices $C_p(u, v)$ in measuring process

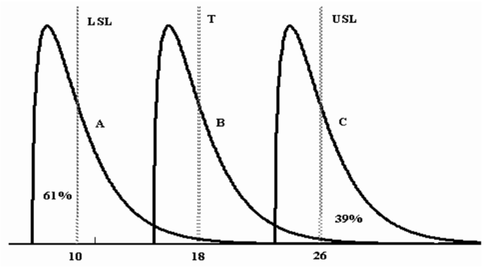

Figure 1. Distribution plots of processes A, B, and C.

capability for non normal distributions. To compare the original indices $C_p(u, v)$ and the generalization $C_{Np}(u, v)$, we consider an example of three processes A, B, and C, depicted in Fig. 1. All three processes are distributed as chi-square distributions with 3 degrees of freedom (heavily skewed with long tails). While process B is on-target ($\mu_B = T$), processes A and C are severely off-target ($\mu_A = LSL$ and $\mu_C = USL$). Table 1 displays the characteristics of processes A, B, and C, including the process mean, standard deviation, median and percentiles. The proportion of non-conformity for process A is 61%, which is significantly greater than that, 39%, for process C. But, both processes A and C obtain the same original index values, thus the original index $C_p(u, v)$ inconsistently measures the process capability. On the other hand, the generalization $C_{Np}(u, v)$ clearly differentiates A and C by giving small value to A and larger value to C. Table 2 summarizes the value of two generalize indices, $C_p(u, v)$ and $C_{Np}(u, v)$, for the three processes A, B, and C. Furthermore, for processes distributed as Weibull, the result is also the same. In fact, for Weibull distributions $W(\alpha, \beta)$ with $\alpha = 3$ and $\beta = 1.1$, the percentage comparisons, 61% versus 39%, displayed in Fig. 1 will be replaced by 62% versus 38%.

Table 1
Characteristics of processes A, B, C

Process	μ	σ	$P_{0.135}$	M	$P_{99.865}$
A	10.00	2.45	7.73	9.37	22.63
B	18.00	2.45	15.73	17.37	30.63
C	26.00	2.45	23.00	25.37	34.63

Table 2
A comparison between $C_p(u, v)$ and $C_{Np}(u, v)$ for processes A, B, C

Process	C_p	C_{pk}	C_{pm}	C_{pmk}	C_{Np}	C_{Npk}	C_{Npm}	C_{Npmk}
A	1.09	0.00	0.27	0.00	1.07	−0.08	0.30	−0.02
B	1.09	1.09	1.09	1.09	1.07	0.99	1.04	0.96
C	1.09	0.00	0.27	0.00	1.07	0.08	0.34	0.03

3. Percentile Estimator of $C_{Np}(u, v)$

Pearn and Chen (1997) considered a sample percentile estimator to calculate the index $C_{Np}(u, v)$. The estimator essentially applies the sample percentile method proposed by Chang and Lu (1994) for calculating $P_{99.865}$, $P_{0.135}$, and the median M. The estimator can be expressed as:

$$\widehat{C}_{Np}(u, v) = \frac{d - u\left|\widehat{M} - m\right|}{3\sqrt{\left(\frac{\widehat{P}_{99.865}-\widehat{P}_{0.135}}{6}\right)^2 + v\left(\widehat{M} - T\right)^2}},$$

$$\widehat{P}_{99.865} = X_{(R_1)} + \left\{\left(\frac{99.865n + 0.135}{100}\right) - R_1\right\} \times \left\{X_{(R_1+1)} - X_{(R_1)}\right\},$$

$$\widehat{P}_{0.135} = X_{(R_2)} + \left\{\left(\frac{0.135n + 99.865}{100}\right) - R_2\right\} \times \left\{X_{(R_2+1)} - X_{(R_2)}\right\},$$

$$\widehat{M} = X_{(R3)} + \left\{\frac{n+1}{2} - R_3\right\} \times \left\{X_{(R_3+1)} - X_{(R_3)}\right\},$$

$$R_1 = \left[\frac{99.865n + 0.135}{100}\right],$$

$$R_2 = \left[\frac{0.135n + 99.865}{100}\right], \quad R_3 = \left[\frac{n+1}{2}\right].$$

In this setting, the notation $[R]$ is defined as the greatest integer less than or equal to the number R, and $X_{(i)}$ is defined as the ith order statistic. The sample percentile estimator is easy to understand and straightforward to apply. Since the calculation does not require any knowledge of the process distribution, then the estimator may be used on any process regardless of whether the underlying distribution is normal or non normal. Several other alternatives for estimating the percentile points and those indices have been proposed, which are described below. Those alternatives are either inefficient, or require a large amount of data. Therefore, those alternatives are inferior to the percentile estimator (which is applicable for any sample size), and are not appropriate for factory applications.

Clements Method. If the underlying process distribution is of Pearson type, Clements (1989) proposed a method for estimating capability indices, C_{Np} and C_{Npk}. The method first utilizes estimates of the mean $(\overline{X})$, standard deviation (S), skewness (sk), and kurtosis (ku), to determine the type of the Pearson distribution curve. The method then utilizes the tables provided by Gruska et al. (1989) for percentages of the family of Pearson curves as a function of skewness and kurtosis. Pearn and Kotz (1994–95) extended the method to the other two indices, C_{Npm} and C_{Npmk}. The proposed method, however, requires that the underlying process distribution be Pearson type, and a large number of tables provided by Gruska et al. (1989).

Zwick Method. For non normal process distributions, Zwick (1995) proposed a method to fit continuous probability distributions for given data. The method essentially calculates the uniform order statistic medians for the inverse or the percentile function. Traditional method, the maximal likelihood estimate (MLE), modified moment estimator (MME) are used for estimating the parameters.

The percentile points $\widehat{P}_{99.865}$, $\widehat{P}_{0.135}$, and the sample median $\widehat{M}$, are then calculated from the fitted continuous probability distribution function.

Schneider et al. Method. Schneider et al. (1995) pointed out that percentiles are not statistically efficient estimates, and proposed to take a sample of size $n \geq 1/0.00135$ then use maximum and minimum data values for the percentiles $\widehat{P}_{99.865}$ and $\widehat{P}_{0.135}$. This method is obviously not efficient as it requires a large amount of data, although may be efficient in supplier certification process where enough data are usually available.

Sarkar and Pal Method. If the underlying process distribution is of Extreme Value distribution type whose parameter α and θ can be estimated from the sample, Sarkar and Pal (1997) proposed a method for estimating the one-sided capability index $C_{PU} = (USL - \widehat{M})/(\widehat{P}_{99.865} - \widehat{M})$, where $\widehat{P}_{99.865}$ and $\widehat{M}$ can be calculated as the following: $\widehat{P}_{99.865} = \alpha - \theta \ln[-\ln(0.99865)] = \alpha + 0.3665\,\theta$, $\widehat{M} = \alpha - \theta \ln[-\ln(0.5)] = \alpha + 6.607\,\theta$. If the underlying process distribution is not of extreme value distribution type, the calculation can be misleading.

4. Empirical Distribution of Percentile Estimator

The exact distribution of the percentile estimator, $\widehat{C}_{Np}(u, v)$, is mathematically intractable even under normality assumption. We therefore use simulation technique to investigate the accuracy of the estimator $\widehat{C}_{Np}(u, v)$ based on the relative bias for a wide variety of distributions. The relative bias was defined as the bias of the estimated $E[\widehat{C}_{Np}(u, v)]$ divided by the true value of $C_{Np}(u, v)$, $\{E[\widehat{C}_{Np}(u, v)] - C_{Np}(u, v)\}/C_{Np}(u, v)$. It cannot only provide a measure of the magnitude of the bias but also facilitate interpretation of the bias on an appropriate scale. The distributions in our investigation including: (1) the normal distribution $N(\mu, \sigma^2)$; (2) the uniform distribution $U(a, b)$; (3) the Laplace distribution $L(a, b)$; (4) the Student's t distribution $t(r)$; (5) the chi-square distribution $\chi^2(r)$; (6) the F distribution $F(r_1, r_2)$; (7) the beta distribution $B(\alpha, \beta)$; (8) the gamma distribution $G(\alpha, \beta)$; (9) the Weibull distribution $W(\alpha, \beta)$; (10) the lognormal distribution $LN(\mu, \sigma^2)$; and (11) the Triangular distribution $T(a, b, c)$, with some selected parameter values.

1. Normal distribution, $N(\mu, \sigma^2)$, with probability density function $f(x) = [(2\pi)^{1/2}\sigma]^{-1} \exp[-(x-\mu)^2/(2\sigma^2)]$, for $-\infty < x < \infty$, $\sigma > 0$. Parameters are set to $\mu = 0$, and $\sigma = 1$.
2. Uniform distribution, $U(a, b)$, with probability density function $f(x) = 1/(b-a)$, for $a \leq x \leq b$, $-\infty < a < b < \infty$. Parameters are set to $a = 0$, $b = 1$.
3. Laplace distribution, $L(a, b)$, with probability density function $f(x) = \exp(-|x-a|/b)/(2b)$, for all x. Parameters are set to $a = 0$, $b = 1$.
4. Student's t distribution, $t(r)$, with probability density function $f(x) = \Gamma[(r+1)/2)][(\pi r)^{1/2}\Gamma(r/2)]^{-1}(1 + x^2/r)^{-(r+1)/2}$, for $-\infty < x < \infty$. Degrees of freedom are set to $r = 4, 5, 6, 7, 8$.
5. Chi-square distribution, $\chi^2(r)$, with probability density function $f(x) = 2^{-r/2}x^{(r/2)-1}\exp(-x/2)/\Gamma(r/2)$, for $0 < x < \infty$. Degrees of freedom are set to $r = 3, 4, 5, 6, 7$.

(6) F distribution $F(r_1, r_2)$, with probability density function $f(x) = \Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} x^{r_1/2-1}\{\Gamma(r_1/2)\Gamma(r_2/2)[1 + (r_1/r_2)\,x]^{(r_1+r_2)/2}\}^{-1}$, for $x > 0$. Degree of freedoms are set to $r_1 = 10$, and $r_2 = 10, 20, 30, 40, 50$.

(7) Beta distribution $B(\alpha, \beta)$, with probability density function $f(x) = x^{\alpha-1}\,(1 - x)^{\beta-1}\,\Gamma(\alpha + \beta)/[\Gamma(\alpha)\Gamma(\beta)]$, for $0 \le x \le 1$. Parameters are set to $\alpha = 3$, $\beta = 2.0, 2.5, 3, 3.5, 4.0$.

(8) Gamma distribution $G(\alpha, \beta)$, with probability density function $f(x) = \beta^{-\alpha} x^{\alpha-1} e^{-x/\beta}/\Gamma(\alpha)$, for $x > 0$. Parameters are set to $\alpha = 1.5, 2.0, 2.5, 3.0, 3.5$, and $\beta = 1$.

(9) Weibull distribution $W(\alpha, \beta)$, with probability density function $f(x) = \alpha\beta x^{\beta-1}\exp(-\alpha x^{\beta})$, for $x \ge 0$. Parameters are set to $\alpha = 1$, $\beta = 1.4, 1.6, 1.8, 2.0, 2.2$.

(10) Lognomal distribution $LN(\mu, \sigma^2)$, with probability density function $f(x) = \exp[-(\ln x - \mu)^2/(2\sigma^2)]/[(2\pi)^{1/2}\sigma x]$, for $x \ge 0$. Parameters are set to $\mu = 0$, $\sigma^2 = l$, $1/4$, $1/9$, $1/16$, $1/25$.

(11) Triangular distribution $T(a, b, c)$, with probability density function $f(x) = 2(x - a)/[(b - a)(c - a)]$, for $a \le x \le c$, $f(x) = 2(b - x)/[(b - a)(b - c)]$, for $c < x \le b$, $a < c < b$. Parameters are set to $a = 0$, $b = 1$, $c = 1/4$, $1/3, 1/2, 3/4, 5/6$.

Figures 2–9 display the probability density function (pdf) plots for simulated distributions. Figure 2 plots the probability density function for standard normal, uniform $U(0, 1)$, Laplace $L(0, 1)$, and Student's $t(4)$ distributions. Figure 3 plots the chi-square distributions $\chi^2(r)$ with selected degrees of freedom $\gamma = 3, 4, 5, 6, 7$ (from left to right in plot). Figure 4 plots the F distributions $F(r_1, r_2)$ with selected degrees of freedom parameters $\gamma_1 = 10$ and $\gamma_2 = 10, 20, 30, 40, 50$ (from bottom to top in plot). Figure 5 plots the beta distributions $B(\alpha, \beta)$ with parameters $\alpha = 3$ and $\beta = 2.0, 2.5, 3.0, 3.5, 4.0$ (from left to right in plot). Figure 6 plots the gamma distributions $G(\alpha, \beta)$ with parameter $\alpha = 1.5, 2.0, 2.5, 3.0, 3.5$, and $\beta = 1$ (from left to right in plot). Figure 7 plots the Weibull distributions $W(\alpha, \beta)$ with parameters $\alpha = 1$ and $\beta = 1.4, 1.6, 1.8, 2.0, 2.2$ (from left to right in plot). Figure 8 plots the lognormal distributions $LN(\mu, \sigma^2)$ for parameters $\mu = 0$, and $\sigma^2 = 1, 1/4, 1/9, 1/16$,

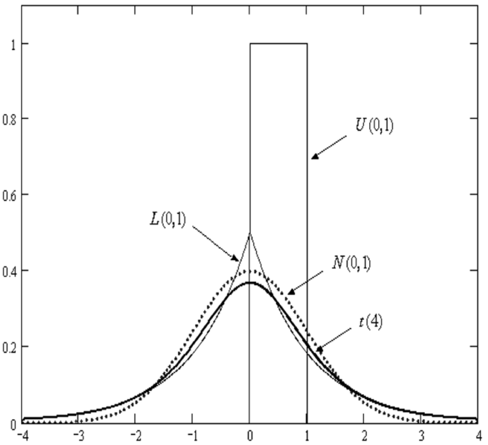

Figure 2. The pdf of normal, uniform, Laplace, and t distributions.

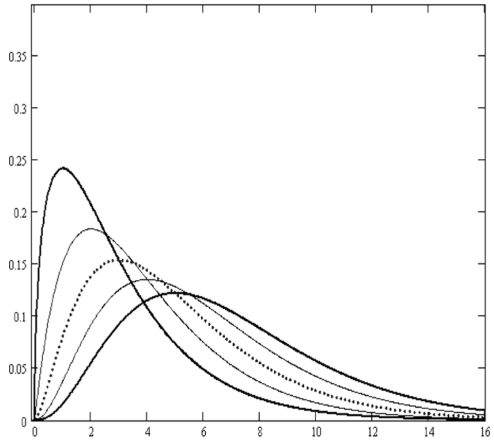

Figure 3. The pdf of $\chi^2(r)$ with $r = 3, 4, 5, 6, 7$ (from left to right).

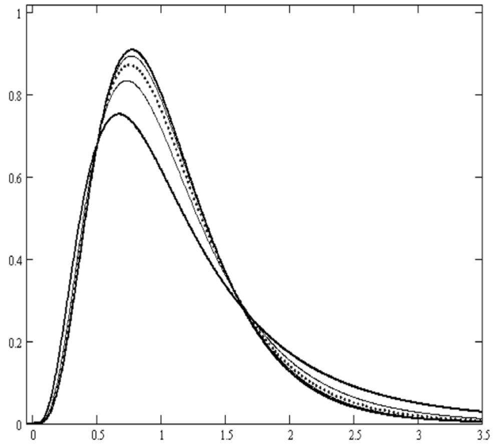

Figure 4. The pdf of $F(r_1, r_2)$ with $r_1 = 10$, and $r_2 = 10, 20, 30, 40, 50$ (from bottom to top).

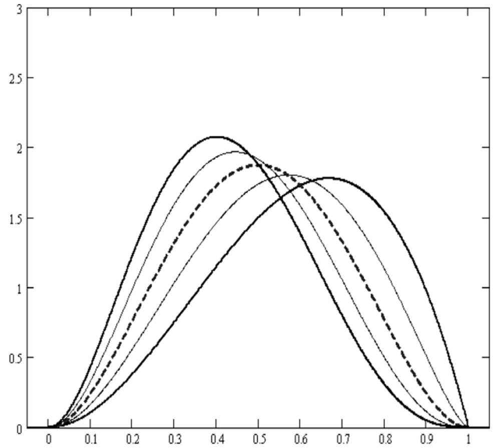

Figure 5. The pdf of $B(\alpha, \beta)$ with $\alpha = 3$, $\beta = 2.0, 2.5, 3, 3.5, 4.0$ (from left to right).

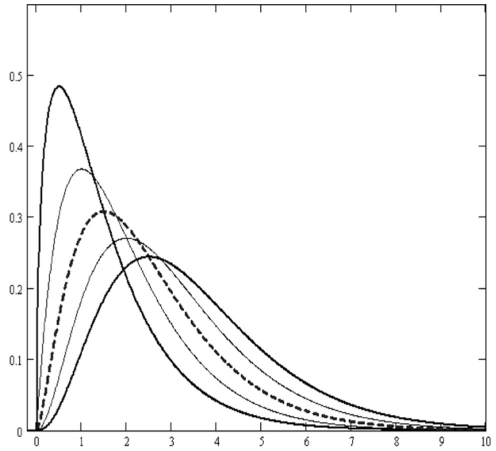

Figure 6. The pdf of $G(\alpha, \beta)$ with $\alpha = 1.5, 2.0, 2.5, 3.0, 3.5$, and $\beta = 1$ (from left to right).

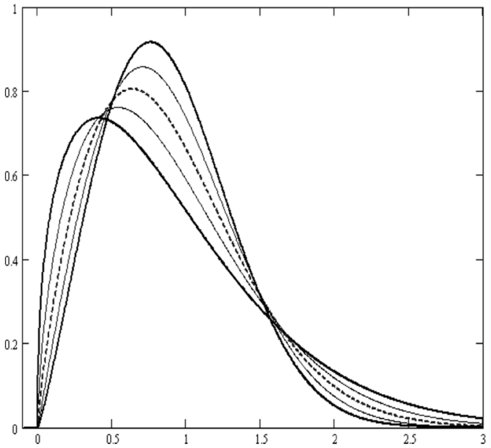

Figure 7. The pdf of $W(\alpha, \beta)$ with $\alpha = 1$, $\beta = 1.4, 1.6, 1.8, 2.0, 2.2$ (from left to right).

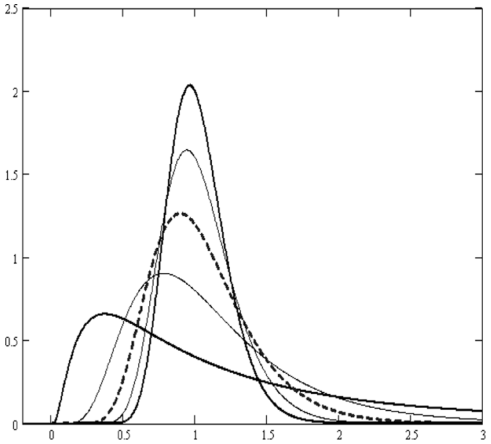

Figure 8. The pdf of $LN(\mu, \sigma^2)$ with $\mu = 0$, $\sigma^2 = l, 1/4, 1/9, 1/16, 1/25$ (from bottom to top).

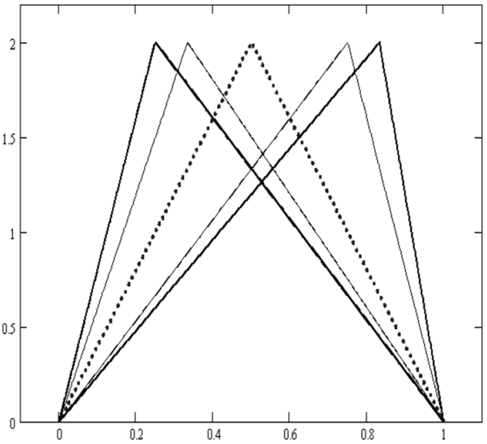

Figure 9. The pdf of $T(a, b, c)$ with $a = 0$, $b = 1$, $c = 1/4$, $1/3$, $1/2$, $3/4$, $5/6$ (from left to right).

1/25 (from bottom to top in plot). Figure 9 plots the triangular distributions $T(a, b, c)$ for parameters $a = 0$, $b = 1$ and $c = 1/4, 1/3, 1/2, 3/4, 5/6$ (from left to right in plot).

We have chosen those distributions with selected parameter values to represent slight, moderate, and severe departures from normality, as those distributions are known to have significantly different tail behaviors, which greatly influence the process capability calculations. The simulation is carried out using 15,000,000 random numbers generated from the uniform distribution $U(0, 1)$, applying AS183 generator (Wichmann and Hill, 1987) with multiple seeds using IBM RISC/6000 workstation.

5. Accuracy Analysis of Percentile Estimator

Parameter values used in our simulation study have been described earlier. We first perform the simulation for the three percentile estimators, $\widehat{P}_{0.135}$, $\widehat{M}$, $\widehat{P}_{99.865}$, to investigate the accuracy and calculate the relative bias. Next, an extensive simulation study was conducted to examine the performance of the estimated $C_{Np}(u, v)$ for the 11 distributions (covering normal distribution and various non normal distributions), in terms of the relative bias, $\{E[\widehat{C}_{Np}(u, v)] - C_{Np}(u, v)\}/C_{Np}(u, v)$, for various values of d/σ and $|(\mu - T)/\sigma|$.

5.1. *Accuracy Result for* $\widehat{P}_{0.135}$, $\widehat{M}$, $\widehat{P}_{99.865}$

For simplicity of the presentation, detailed results are presented here only for some selected parameters. Tables 3–11 display the performance results of the three percentile estimators, $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, from the simulation for all 11 distributions, in terms of the relative bias $(E[\widehat{P}_\alpha] - P_\alpha)/P_\alpha$. The sample sizes in the simulation are set to n = 10, 20, 30, 50, 100, 300, 500, 1,000, and 3,000. It is clear that for the three percentile estimators $\widehat{P}_{0.135}, \widehat{M}, \widehat{P}_{99.865}$, the relative bias decreases as the sample size n increases. This can be understood since the three percentile estimators are consistent and asymptotically unbiased, hence become

Table 3
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for normal, uniform, and Laplace distributions

	$N(0, 1)$			$U(0, 1)$			$L(0, 1)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)
10	−48.9	∗∗∗	−48.9	6705.1	0.0	−9.1	−62.4	∗∗∗	−62.4
20	−38.1	∗∗∗	−38.1	3510.7	0.0	−4.8	−51.3	∗∗∗	−51.3
30	−32.4	∗∗∗	−32.5	2378.4	0.0	−3.2	−44.8	∗∗∗	−44.8
40	−28.7	∗∗∗	−28.7	1800.4	0.0	−2.4	−40.3	∗∗∗	−40.3
50	−25.9	∗∗∗	−25.9	1445.2	0.0	−2.0	−36.8	∗∗∗	−36.8
100	−18.0	∗∗∗	−18.0	728.2	0.0	−1.0	−26.2	∗∗∗	−26.3
200	−11.4	∗∗∗	−11.5	365.9	0.0	−0.5	−16.9	∗∗∗	−16.9
300	−8.3	∗∗∗	−8.4	243.5	0.0	−0.3	−12.3	∗∗∗	−12.4
500	−5.6	∗∗∗	−5.7	146.0	0.0	−0.2	−8.2	∗∗∗	−8.4
1000	−3.3	∗∗∗	−3.4	73.6	0.0	−0.1	−5.0	∗∗∗	−5.1
3000	−1.3	∗∗∗	−1.4	25.0	0.0	0.0	−2.0	∗∗∗	−2.1

more accurate as the sample size increases. If the relative bias is negative, it indicates that $\widehat{P}_\alpha$ underestimates the parameter. On the other hand, if the relative bias is positive it indicates that $\widehat{P}_\alpha$ overestimates the parameter. For the relative bias of $\widehat{M}$ marked "∗∗∗" in Tables 3–4, it indicates that the value does not exist because M is 0. From Tables 3–11, we observe that the relative bias of the three percentile estimators all exceed 80% for sample size n no greater than 50, except for normal, t, chi-square, and the Laplace distributions. It is noted that the relative bias is greater than 100% for $U(0, 1)$, $\chi^2(3)$, $G(1.5, 1)$, and $W(1, 1.4)$ when the sample size n is 300. Further, the relative bias of the three percentile estimators, are greater than 10% for $U(0, 1)$, $\chi^2(3)$, $G(1.5, 1)$, $W(1, 1.4)$, $W(1, 1.6)$, and $W(1, 1.8)$ when the sample size n is 3,000. Thus, for non normal distributions, the three percentile estimators are indeed highly inaccurate.

5.2. *Accuracy Result for* $\widehat{C}_{\mathrm{Np}}(u, v)$

Tables 12(a)–22 display the results from the simulation for the relative bias of $\widehat{C}_{Np}(u, v)$, for all 11 distributions with various values of $d/\sigma = 2, 3, 4, 5$, and $|(\mu - T)/\sigma| = 0, 0.5, 1.0, 1.5$, and 2.0. Clearly, the relative bias decreases as the sample size n increases. It is noted that for the relative bias of $\widehat{C}_{Np}(u, v)$ marked "∗∗∗" in Tables 12(a)–22, it indicates that the value is meaningless because $C_{Np}(u, v)$ is 0. For instance, if $d/\sigma = 2$ and $(\mu - T)/\sigma = 2$, then the value of relative bias is meaningless since $C_{Npk} = 0$ and $C_{Npmk} = 0$ for Student's t, normal, uniform, Laplace distributions. For most cases, direction of the relative bias of $\widehat{C}_{Np}(u, v)$ is positive except for the uniform, beta, and triangular distributions. For beta distribution B (3.0, 3.0) and triangular distribution T (0, 1, 1/2), the bias of $\widehat{C}_{Npk}$, and $\widehat{C}_{Npmk}$ are negative for large n when the process is on-target. For uniform distribution, U (0, 1), the bias of $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$ and $\widehat{C}_{Npmk}$ are negative when the process is on-target. Since the probability that the process is (or detected to be) on target is zero, the relative bias can be regarded as positive.

Table 4
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for t distribution, $t(\gamma)$ with $\gamma = 4(1)8$

	$t(4)$			$t(5)$			$t(6)$			$t(7)$			$t(8)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$	$\widehat{P}_{99.865}$ (%)
10	−67.3	∗∗∗	−67.4	−63.8	∗∗∗	−63.8	−61.4	∗∗∗	−61.4	−59.5	∗∗∗	−59.6	−58.2	∗∗∗	−58.2
20	−57.5	∗∗∗	−57.5	−53.7	∗∗∗	−53.7	−51.1	∗∗∗	−51.1	−49.2	∗∗∗	−49.2	−47.7	∗∗∗	−47.8
30	−51.3	∗∗∗	−51.4	−47.6	∗∗∗	−47.6	−45.0	∗∗∗	−45.0	−43.1	∗∗∗	−43.2	−41.8	∗∗∗	−41.8
40	−46.8	∗∗∗	−46.8	−43.2	∗∗∗	−43.2	−40.7	∗∗∗	−40.7	−38.9	∗∗∗	−38.9	−37.6	∗∗∗	−37.6
50	−13.1	∗∗∗	−13.2	−39.7	∗∗∗	−39.7	−37.3	∗∗∗	−37.3	−35.6	∗∗∗	−35.6	−34.3	∗∗∗	−34.4
100	−31.3	∗∗∗	−31.4	−28.7	∗∗∗	−28.7	−26.9	∗∗∗	−26.9	−25.6	∗∗∗	−25.6	−24.6	∗∗∗	−24.6
200	−19.6	∗∗∗	−19.6	−18.1	∗∗∗	−18.1	−17.1	∗∗∗	−17.1	−16.3	∗∗∗	−16.3	−15.7	∗∗∗	−15.7
300	−13.5	∗∗∗	−13.5	−12.7	∗∗∗	−12.8	−12.2	∗∗∗	−12.2	−11.6	∗∗∗	−11.7	−11.2	∗∗∗	−11.3
500	−8.6	∗∗∗	−8.6	−8.2	∗∗∗	−8.4	−8.0	∗∗∗	−8.0	−7.6	∗∗∗	−7.8	−7.4	∗∗∗	−7.5
1000	−5.8	∗∗∗	−5.9	−5.4	∗∗∗	−5.5	−5.2	∗∗∗	−5.2	−4.8	∗∗∗	−4.9	−4.6	∗∗∗	−4.7
3000	−2.5	∗∗∗	−2.6	−2.3	∗∗∗	−2.3	−2.2	∗∗∗	−2.2	−2.0	∗∗∗	−2.1	−1.9	∗∗∗	−2.0

Table 5
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for chi-square distribution, χ_γ^2 with $\gamma = 3, 4, 5, 6$, and 7

	χ_1^2			χ_4^2			χ_5^2			χ_6^2			χ_7^2		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	1658.1	4.5	−51.4	787.1	3.2	−48.2	497.0	2.5	−45.7	361.0	2.0	−43.7	284.1	1.7	−42.0
20	982.7	2.2	−42.0	503.2	1.6	−39.3	331.2	1.2	−37.1	246.9	1.0	−35.4	197.8	0.8	−34.0
30	722.4	1.5	−36.6	385.8	1.0	−34.2	259.7	0.8	−32.3	196.4	0.7	−30.7	158.8	0.6	−29.4
40	579.9	1.1	−32.9	318.6	0.8	−30.6	217.7	0.6	−28.9	166.1	0.5	−27.5	135.2	0.4	−26.3
50	487.4	0.9	−30.0	273.5	0.6	−27.9	189.0	0.5	−26.3	145.2	0.4	−25.0	118.7	0.3	−23.9
100	280.7	0.4	−21.4	166.9	0.3	−19.8	118.9	0.2	−18.7	93.1	0.2	−17.8	77.0	0.2	−17.0
200	157.6	0.2	−13.7	97.9	0.2	−12.7	71.3	0.1	−12.0	56.5	0.1	−11.4	47.1	0.1	−10.9
300	110.6	0.1	−10.0	70.0	0.1	−7.3	51.4	0.1	−8.8	40.9	0.1	−8.3	34.2	0.1	−8.0
500	70.4	0.1	−6.8	45.5	0.1	−6.3	33.8	0.0	−5.9	27.0	0.0	−5.6	22.7	0.0	−5.4
1000	37.5	0.0	−4.1	25.1	0.0	−3.8	18.9	0.0	−3.6	15.3	0.0	−3.4	12.9	0.0	−3.3
3000	13.1	0.0	−1.7	9.2	0.0	−1.6	7.1	0.0	−1.5	5.8	0.0	−1.4	4.9	0.0	−1.3

Table 6
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for F distribution, $F(10, \gamma_2)$ with $\gamma_2 10(10)50$

	$F(10, 10)$			$F(10, 20)$			$F(10, 30)$			$F(10, 40)$			$F(10, 50)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	216.3	3.6	−62.5	197.8	2.3	−51.9	191.3	1.9	−47.6	187.7	1.7	−45.4	185.6	1.6	−44.0
20	149.3	1.7	−53.7	139.1	1.1	−43.4	135.4	0.9	−39.4	133.3	0.8	−37.3	132.0	0.8	−36.0
30	119.7	1.1	−48.0	112.4	0.7	−38.3	109.7	0.6	−34.6	108.2	0.6	−32.7	107.3	0.5	−31.5
40	101.9	0.8	−43.8	96.2	0.5	−34.6	94.1	0.5	−31.2	92.8	0.4	−29.4	92.1	0.4	−28.3
50	89.4	0.7	−40.4	84.9	0.4	−31.8	83.0	0.4	−28.5	81.9	0.3	−26.8	81.3	0.3	−25.8
100	58.0	0.3	−29.5	55.5	0.2	−22.9	54.6	0.2	−20.5	53.9	0.2	−19.2	53.6	0.2	−18.4
200	35.3	0.2	−18.7	34.2	0.1	−14.7	33.7	0.1	−13.2	33.3	0.1	−12.3	33.1	0.1	−11.8
300	25.4	0.1	−13.2	24.8	0.1	−10.6	24.5	0.1	−9.6	24.2	0.1	−9.0	24.1	0.1	−8.6
500	16.6	0.1	−8.6	16.5	0.0	−7.1	16.3	0.0	−6.4	16.1	0.0	−6.0	16.0	0.0	−5.8
1000	9.2	0.0	−5.7	9.4	0.0	−4.5	9.4	0.0	−4.0	9.2	0.0	−3.7	9.2	0.0	−3.6
3000	3.1	0.0	−2.4	3.6	0.0	−1.9	3.6	0.0	−1.6	3.5	0.0	−1.5	3.5	0.0	−1.5

Table 7
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for beta distribution, $B(3, \beta)$ with $\beta = 2.0(0.5)4.0$

	$B(3.0, 2.0)$			$B(3.0, 2.5)$			$B(3.0, 3.0)$			$B(3.0, 3.5)$			$B(3.0, 4.0)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	297.1	−0.5	−11.1	305.4	−0.2	-14.5	311.7	0.0	−17.4	316.8	0.2	−19.8	320.8	0.3	−21.8
20	212.8	−0.2	−7.3	217.4	−0.1	-10.0	220.9	0.0	−12.3	223.6	0.1	−14.3	225.8	0.2	−16.0
30	172.7	−0.2	−5.7	175.9	−0.1	-7.9	178.4	0.0	−9.9	180.3	0.1	−11.7	181.8	0.1	−13.2
40	147.9	−0.1	−4.7	150.4	−0.1	-6.7	152.3	0.0	−8.5	153.7	0.0	−10.0	154.9	0.1	−11.4
50	130.3	−0.1	−4.1	132.4	0.0	-5.9	133.9	0.0	−7.5	135.1	0.0	−8.9	136.1	0.1	−10.1
100	85.2	0.0	−2.5	86.3	0.0	-3.7	87.1	0.0	−4.9	87.8	0.0	−5.9	88.3	0.0	−6.8
200	52.4	0.0	−1.5	52.9	0.0	-2.3	53.4	0.0	−3.0	53.7	0.0	−3.6	54.0	0.0	−4.2
300	38.1	0.0	−1.1	38.5	0.0	-1.6	38.8	0.0	−2.2	39.0	0.0	−2.7	39.2	0.0	−3.1
500	25.3	0.0	−0.7	25.6	0.0	-1.1	25.8	0.0	−1.5	25.9	0.0	−1.8	26.0	0.0	−2.1
1000	14.4	0.0	−0.4	14.5	0.0	-0.6	14.6	0.0	−0.8	14.7	0.0	−1.0	14.8	0.0	−1.2
3000	5.5	0.0	−0.1	5.5	0.0	-0.2	5.6	0.0	−0.3	5.6	0.0	−0.4	5.6	0.0	−0.5

Table 8
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$, for gamma distribution, $G(\alpha, 1)$ with $\alpha = 1.5(0.5)3.5$

	$G(1.5, 1)$			$G(2.0, 1)$			$G(2.5, 1)$			$G(3.0, 1)$			$G(3.5, 1)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	1658.1	4.5	−51.4	787.1	3.2	−48.2	497.0	2.5	−45.7	361.0	2.0	−43.7	284.1	1.7	−42.0
20	982.7	2.2	−42.0	503.2	1.6	−39.3	331.2	1.2	−37.1	246.9	1.0	−35.4	197.8	0.8	−34.0
30	722.4	1.5	−36.6	385.8	1.0	−34.2	259.7	0.8	−32.3	196.4	0.7	−30.7	158.8	0.6	−29.4
40	579.9	1.1	−32.9	318.6	0.8	−30.6	217.7	0.6	−28.9	126.1	0.5	−27.5	135.2	0.4	−26.3
50	487.4	0.9	−30.0	273.5	0.6	−27.9	189.0	0.5	−26.3	145.2	0.4	−25.0	118.7	0.3	−23.9
100	280.7	0.4	−21.4	167.0	0.3	−19.9	119.0	0.2	−18.7	93.1	0.2	−17.8	77.0	0.2	−17.0
200	157.6	0.2	−13.7	97.9	0.2	−12.8	71.3	0.1	−12.0	56.5	0.1	−11.4	47.1	0.1	−10.9
300	110.5	0.1	−10.1	70.0	0.1	−9.3	51.4	0.1	−8.8	40.9	0.1	−8.3	34.2	0.1	−8.0
500	70.4	0.1	−6.8	45.6	0.1	−6.3	33.8	0.0	−5.9	27.0	0.0	−5.6	22.7	0.0	−5.4
1000	37.6	0.0	−4.1	25.1	0.0	−3.8	18.9	0.0	−3.6	15.3	0.0	−3.4	12.9	0.0	−3.3
3000	13.3	0.0	−1.7	9.3	0.0	−1.6	7.1	0.0	−1.5	5.8	0.0	−1.4	4.9	0.0	−1.3

Table 9
Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$ for Weibull distribution, $W(1, \beta)$ with $\beta = 1.4(0.2)2.2$

	$W(1, 1.4)$			$W(1, 1.6)$			$W(1, 1.8)$			$W(1, 2.0)$			$W(1, 2.2)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	1893.5	3.2	−45.2	1230.0	2.2	−41.2	876.6	1.5	−37.8	665.9	1.1	−35.0	530.2	0.7	−32.5
20	1126.3	1.5	−36.3	769.5	1.1	−32.8	569.3	0.7	−30.0	445.1	0.5	−27.5	362.6	0.4	−25.5
30	826.7	1.0	−31.4	580.5	0.7	−28.2	438.3	0.5	−25.7	348.1	0.3	−23.6	287.1	0.2	−21.7
40	662.2	0.8	−28.0	473.5	0.5	−25.1	362.4	0.4	−22.8	290.8	0.2	−20.9	241.9	0.2	−19.2
50	555.4	0.6	−25.4	402.6	0.4	−22.8	311.2	0.3	−20.7	251.6	0.2	−18.9	210.6	0.1	−17.4
100	317.3	0.3	−18.0	238.7	0.2	−16.0	189.7	0.2	−14.5	156.6	0.1	−13.2	133.3	0.1	−12.1
200	176.8	0.2	−11.5	136.8	0.1	−10.3	110.9	0.1	−9.3	93.0	0.1	−8.4	80.1	0.0	−7.7
300	123.7	0.1	−8.4	96.8	0.1	−7.5	79.1	0.0	−6.8	66.8	0.0	−6.2	57.8	0.0	−5.7
500	78.4	0.1	−5.7	62.2	0.0	−5.1	51.4	0.0	−4.6	43.7	0.0	−4.2	38.1	0.0	−3.8
1000	41.8	0.0	−3.4	33.7	0.0	−3.1	28.2	0.0	−2.7	24.2	0.0	−2.5	21.3	0.0	−2.3
3000	14.9	0.0	−1.4	12.2	0.0	−1.2	10.3	0.0	−1.1	8.9	0.0	−1.0	8.0	0.0	−0.9

Table 10

Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$ for lognormal, $LN(0, \sigma^2)$ with $\sigma = 1, 1/2, 1/3, 1/4$, and $1/5$

	$LN(0, 1)$			$LN(0, 1/4)$			$LN(0, 1/9)$			$LN(0, 1/16)$			$LN(0, 1/25)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	407.9	8.7	−72.2	117.1	2.1	−49.8	66.1	0.9	−37.5	45.8	0.5	−30.0	35.0	0.3	−24.9
20	254.6	4.1	−62.9	83.1	1.0	−41.5	48.6	0.4	−30.6	34.2	0.3	−24.2	26.4	0.2	−20.0
30	194.1	2.7	−56.7	67.6	0.7	−36.5	40.2	0.3	−26.7	28.5	0.2	−21.0	22.1	0.1	−17.3
40	160.0	2.0	−52.0	58.1	0.5	−33.0	34.9	0.2	−24.0	24.9	0.1	−18.8	19.3	0.1	−15.4
50	137.1	1.6	−48.1	51.3	0.4	−30.2	31.1	0.2	−21.9	22.2	0.1	−17.1	17.3	0.1	−14.0
100	82.9	0.8	−35.4	33.9	0.2	−21.8	20.9	0.1	−15.6	15.1	0.1	−12.1	11.8	0.0	−9.9
200	47.2	0.4	−22.5	21.0	0.1	−14.0	13.1	0.1	−10.0	9.5	0.0	−7.8	7.4	0.0	−6.3
300	32.5	0.3	−15.8	15.3	0.1	−10.2	9.5	0.0	−7.3	6.9	0.0	−5.7	5.4	0.0	−4.6
500	19.0	0.2	−10.2	10.1	0.0	−6.8	6.4	0.0	−4.9	4.6	0.0	−3.8	3.6	0.0	−3.1
1000	6.6	0.1	−6.9	5.9	0.0	−4.3	3.7	0.0	−3.0	2.7	0.0	−2.3	2.1	0.0	−1.9
3000	−3.1	0.0	−3.0	2.3	0.0	−1.8	1.4	0.0	−1.2	1.1	0.0	−0.9	0.8	0.0	−0.8

Table 11

Relative bias of $\widehat{P}_{0.135}$, $\widehat{M}$, and $\widehat{P}_{99.865}$ for triangular distribution, $T(0, 1, c)$ with $c = 1/4, 1/3, 1/2, 3/4$, and $5/6$

	$T(0, 1, 1/4)$			$T(0, 1, 1/3)$			$T(0, 1, 1/2)$			$T(0, 1, 3/4)$			$T(0, 1, 5/6)$		
n	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)	$\widehat{P}_{0.135}$ (%)	$\widehat{M}$ (%)	$\widehat{P}_{99.865}$ (%)
10	641.0	1.9	−21.0	639.8	1.5	−19.8	639.4	0.0	−17.1	639.4	−1.2	−12.0	639.4	-1.3	−9.9
20	435.8	1.0	−14.3	435.8	0.8	−13.5	435.7	0.0	−11.6	435.7	−0.6	−8.2	435.7	-0.6	−6.7
30	343.1	0.7	−11.3	343.1	0.6	−10.6	343.0	0.0	−9.2	343.0	−0.4	−6.4	343.0	-0.4	−5.2
40	287.6	0.5	−9.5	287.6	0.4	−8.9	287.6	0.0	−7.7	287.6	−0.3	−5.4	287.6	-0.3	−3.8
50	249.4	0.4	−8.2	249.4	0.3	−7.7	249.4	0.0	−6.7	249.4	−0.3	−4.7	249.4	-0.3	−3.8
100	155.9	0.2	−5.2	155.9	0.2	−4.8	155.9	0.0	−4.2	155.9	−0.1	−2.9	155.9	-0.1	−2.4
200	92.8	0.1	−3.1	92.8	0.1	−2.9	92.8	0.0	−2.5	92.8	−0.1	−1.8	92.8	-0.1	−1.4
300	66.8	0.1	−2.2	66.8	0.1	−2.1	66.8	0.0	−1.8	66.8	0.0	−1.3	66.8	0.0	−1.0
500	43.8	0.0	−1.5	43.8	0.0	−1.4	43.8	0.0	−1.2	43.8	0.0	−0.8	43.8	0.0	−0.7
1000	24.3	0.0	−0.8	24.3	0.0	−0.8	24.3	0.0	−0.7	24.3	0.0	−0.5	24.3	0.0	−0.4
3000	9.1	0.0	−0.3	9.1	0.0	−0.3	9.1	0.0	−0.2	9.1	0.0	−0.2	9.1	0.0	−0.1

Table 12(a)

Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for Student's t distribution, $t(4)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	257.3	257.3	257.3	257.3	257.3	271.4	255.0	257.3	257.4	* * *	206.6	142.8	72.6	40.0	24.7	176.3	152.3	84.5	54.6	* * *
	3	257.3	257.3	257.3	257.3	257.3	230.7	255.9	257.3	257.3	257.3	206.6	142.8	72.6	40.0	24.7	186.4	148.5	78.6	44.9	29.8
	4	257.3	257.3	257.3	257.3	257.3	237.3	256.3	257.3	257.3	257.3	206.6	142.8	72.6	40.0	24.7	191.5	146.9	76.6	42.9	27.2
	5	257.3	257.3	257.3	257.3	257.3	241.3	256.5	257.3	257.3	257.3	206.6	142.8	72.6	40.0	24.7	194.5	146.0	75.6	42.1	26.4
20	2	163.2	163.2	163.2	163.2	163.2	141.7	163.0	163.2	163.3	* * *	148.2	105.6	57.4	33.1	20.9	129.1	109.8	62.3	39.7	* * *
	3	163.2	163.2	163.2	163.2	163.2	148.9	163.1	163.2	163.2	163.2	148.2	105.6	57.4	33.1	20.9	135.5	108.1	59.9	35.3	23.3
	4	163.2	163.2	163.2	163.2	163.2	152.5	163.1	163.2	163.2	163.2	148.2	105.6	57.4	33.1	20.9	138.6	107.4	59.1	44.4	22.1
	5	163.2	163.2	163.2	163.2	163.2	154.6	163.1	163.2	163.2	163.2	148.2	105.6	57.4	33.1	20.9	140.6	107.0	58.7	34.1	21.7
30	2	126.7	126.7	126.7	126.7	126.7	111.5	126.7	126.8	126.8	* * *	119.6	87.6	49.7	29.4	18.8	105.2	90.0	52.6	33.4	* * *
	3	126.7	126.7	126.7	126.7	126.7	116.6	127.7	126.8	126.8	126.8	119.6	87.6	49.7	29.4	18.8	110.0	89.0	51.2	30.7	20.3
	4	126.7	126.7	126.7	126.7	126.7	119.1	126.7	126.7	126.8	126.8	119.6	87.6	49.7	29.4	18.8	112.4	88.6	50.7	30.2	19.2
	5	126.7	126.7	126.7	126.7	126.7	120.6	126.7	126.7	126.7	126.7	119.6	87.6	49.7	29.4	18.8	113.8	88.4	50.5	30.3	19.3
50	2	91.6	91.6	91.6	91.6	91.6	81.5	91.6	91.6	91.7	* * *	88.8	67.7	40.5	20.6	15.9	79.0	68.8	41.9	26.8	* * *
	3	91.6	91.6	91.6	91.6	91.6	84.9	91.6	91.6	91.6	91.6	88.8	67.7	40.5	20.6	15.9	82.3	68.4	41.2	25.3	16.8
	4	91.6	91.6	91.6	91.6	91.6	86.6	91.6	91.6	91.6	91.6	88.8	67.7	40.5	20.6	15.9	83.9	68.2	41.0	25.0	16.4
	5	91.6	91.6	91.6	91.6	91.6	87.6	91.6	91.6	91.6	91.6	88.8	67.7	40.5	20.6	15.9	84.9	68.1	40.8	24.9	16.2
100	2	56.6	56.6	56.6	56.6	56.6	50.7	56.6	56.6	56.6	* * *	55.8	44.7	28.4	17.8	11.8	50.0	45.0	29.0	18.8	* * *
	3	56.6	56.6	56.6	56.6	56.6	52.7	56.6	56.6	56.6	56.6	55.8	44.7	28.4	17.8	11.8	51.9	44.9	28.7	18.2	12.1
	4	56.6	56.6	56.6	56.6	56.6	53.6	56.6	56.6	56.6	56.6	55.8	44.7	28.4	17.8	11.8	52.9	44.8	28.6	18.0	11.9
	5	56.6	56.6	56.6	56.6	56.6	54.2	56.6	56.6	56.6	56.6	55.8	44.7	28.4	17.8	11.8	53.4	44.8	28.5	18.0	11.9
300	2	21.5	21.5	21.5	21.5	21.5	18.8	21.5	21.5	21.5	* * *	21.3	18.0	12.2	7.9	5.2	18.7	18.0	12.3	8.1	* * *
	3	21.5	21.5	21.5	21.5	21.5	19.7	21.5	21.5	21.5	21.5	21.3	18.0	12.2	7.9	5.2	19.6	18.0	12.2	8.0	5.3
	4	21.5	21.5	21.5	21.5	21.5	20.1	21.5	21.5	21.5	21.5	21.3	18.0	12.2	7.9	5.2	20.0	18.0	12.2	7.9	5.3
	5	21.5	21.5	21.5	21.5	21.5	20.4	21.5	21.5	21.5	21.5	21.3	18.0	12.2	7.9	5.2	20.3	18.0	12.2	7.9	5.3
1000	2	8.4	8.4	8.4	8.4	8.4	7.1	8.4	8.4	8.5	* * *	8.4	7.2	5.1	3.4	2.3	7.1	7.3	5.1	3.5	* * *
	3	8.4	8.4	8.4	8.4	8.4	7.6	8.4	8.4	8.4	8.4	8.4	7.2	5.1	3.4	2.3	7.5	7.3	5.1	3.5	2.3
	4	8.4	8.4	8.4	8.4	8.4	7.8	8.4	8.4	8.4	8.4	8.4	7.2	5.1	3.4	2.3	7.7	7.2	5.1	3.5	2.3
	5	8.4	8.4	8.4	8.4	8.4	7.9	8.4	8.4	8.4	8.4	8.4	7.2	5.1	3.4	2.3	7.9	7.2	5.1	3.5	2.3
3000	2	3.4	3.4	3.4	3.4	3.4	2.7	3.5	3.5	3.5	* * *	3.4	3.0	2.2	1.4	1.0	2.7	3.0	2.2	1.5	* * *
	3	3.4	3.4	3.4	3.4	3.4	3.0	3.5	3.5	3.5	3.5	3.4	3.0	2.2	1.4	1.0	3.0	3.0	2.2	1.5	1.0
	4	3.4	3.4	3.4	3.4	3.4	3.1	3.4	3.4	3.5	3.5	3.4	3.0	2.2	1.4	1.0	3.1	3.0	2.2	1.5	1.0
	5	3.4	3.4	3.4	3.4	3.4	3.2	3.4	3.4	3.4	3.4	3.4	3.0	2.2	1.4	1.0	3.2	3.0	2.2	1.5	1.0

Table 12(b)
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for t distribution, $t(7)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	175.3	175.3	175.3	175.3	175.3	139.7	172.0	175.3	175.4	∗∗∗	133.2	98.5	52.4	28.6	17.4	106.7	106.5	65.9	46.4	∗∗∗
	3	175.3	175.3	175.3	175.3	175.3	151.6	173.3	175.3	175.3	175.4	133.2	98.5	52.4	28.6	17.4	115.6	103.3	59.2	34.5	23.7
	4	175.3	175.3	175.3	175.3	175.3	157.5	173.9	175.3	175.3	175.3	133.2	98.5	52.4	28.6	17.4	120.0	101.9	56.9	32.1	20.5
	5	175.3	175.3	175.3	175.3	175.3	161.1	174.2	175.3	175.3	175.3	133.2	98.5	52.4	28.6	17.4	122.5	101.2	55.8	31.1	19.5
20	2	110.8	110.8	110.8	110.8	110.8	90.9	110.4	110.9	110.9	∗∗∗	96.9	71.4	39.2	22.2	13.9	79.4	75.5	44.9	30.3	∗∗∗
	3	110.8	110.8	110.8	110.8	110.8	97.5	110.6	110.8	110.8	110.9	96.9	71.4	39.2	22.2	13.9	85.2	73.9	42.0	24.9	16.9
	4	110.8	110.8	110.8	110.8	110.8	100.8	110.6	110.8	110.8	110.8	96.9	71.4	39.2	22.2	13.9	88.1	73.2	41.1	23.9	15.4
	5	110.8	110.8	110.8	110.8	110.8	102.8	110.7	110.8	110.8	110.8	96.9	71.4	39.2	22.2	13.9	89.9	72.8	40.6	23.4	14.9
30	2	86.2	86.2	86.2	86.2	86.2	71.6	86.1	86.2	86.3	∗∗∗	78.9	58.8	33.1	19.2	12.1	65.5	61.3	36.6	24.3	∗∗∗
	3	86.2	86.2	86.2	86.2	86.2	76.5	86.1	86.2	86.2	86.2	78.9	58.8	33.1	19.2	12.1	70.0	60.3	34.8	20.9	14.1
	4	86.2	86.2	86.2	86.2	86.2	78.9	86.2	86.2	86.2	86.2	78.9	58.8	33.1	19.2	12.1	72.2	59.9	34.3	20.2	13.1
	5	86.2	86.2	86.2	86.2	86.2	80.4	86.2	86.2	86.2	86.2	78.9	58.8	33.1	19.2	12.1	73.6	59.6	30.4	19.9	12.7
50	2	62.6	62.6	62.6	62.6	62.6	52.7	62.6	62.6	62.7	∗∗∗	59.4	45.2	26.4	15.6	10.0	49.9	46.5	28.2	18.5	∗∗∗
	3	62.6	62.6	62.6	62.6	62.6	56.0	62.6	62.6	62.6	62.6	59.4	45.2	26.4	15.6	10.0	53.1	46.0	27.3	16.6	11.1
	4	62.6	62.6	62.6	62.6	62.6	57.6	62.6	62.6	62.6	62.6	59.4	45.2	26.4	15.6	10.0	54.7	45.7	27.0	16.2	10.5
	5	62.6	62.6	62.6	62.6	62.6	58.6	62.6	62.6	62.6	62.6	59.4	45.2	26.4	15.6	10.0	55.6	45.6	26.8	16.0	10.4
100	2	39.2	39.2	39.2	39.2	39.2	33.1	39.2	39.2	39.2	∗∗∗	38.1	29.9	18.3	11.1	7.2	32.2	30.4	19.0	12.4	∗∗∗
	3	39.2	39.2	39.2	39.2	39.2	35.1	39.2	39.2	39.2	39.2	38.1	29.9	18.3	11.1	7.2	34.2	30.2	18.7	11.6	7.7
	4	39.2	39.2	39.2	39.2	39.2	36.1	39.2	39.2	39.2	39.2	38.1	29.9	18.3	11.1	7.2	35.1	30.1	18.5	11.4	7.5
	5	39.2	39.2	39.2	39.2	39.2	36.8	39.2	39.2	39.2	39.2	38.1	29.9	18.3	11.1	7.2	35.7	30.1	18.5	11.3	7.4
300	2	15.6	15.6	15.6	15.6	15.6	12.7	15.7	15.7	15.7	∗∗∗	15.4	12.6	8.1	5.1	3.3	15.1	12.7	8.3	5.5	∗∗∗
	3	15.6	15.6	15.6	15.6	15.6	13.7	15.7	15.7	15.7	15.7	15.4	12.6	8.1	5.1	3.3	13.5	12.7	8.2	5.2	3.5
	4	15.6	15.6	15.6	15.6	15.6	14.2	15.6	15.6	15.7	15.7	15.4	12.6	8.1	5.1	3.3	14.0	12.6	8.2	5.2	3.4
	5	15.6	15.6	15.6	15.6	15.6	14.5	15.6	15.6	15.6	15.6	15.4	12.6	8.1	5.1	3.3	14.3	12.6	8.2	5.1	3.4
1000	2	6.1	6.1	6.1	6.1	6.1	4.6	6.1	6.1	6.1	∗∗∗	6.1	5.0	3.3	2.1	1.4	4.6	5.1	3.4	2.3	∗∗∗
	3	6.1	6.1	6.1	6.1	6.1	5.1	6.1	6.1	6.1	6.1	6.1	5.0	3.3	2.1	1.4	5.1	5.1	3.4	2.2	1.5
	4	6.1	6.1	6.1	6.1	6.1	5.4	6.1	6.1	6.1	6.1	6.1	5.0	3.3	2.1	1.4	5.3	5.1	3.4	2.1	1.4
	5	6.1	6.1	6.1	6.1	6.1	5.5	6.1	6.1	6.1	6.1	6.1	5.0	3.3	2.1	1.4	5.5	5.0	3.3	2.1	1.4
3000	2	2.5	2.5	2.5	2.5	2.5	1.6	2.5	2.5	2.5	∗∗∗	2.4	2.0	1.4	0.9	0.6	1.6	2.1	1.4	0.9	∗∗∗
	3	2.5	2.5	2.5	2.5	2.5	1.9	2.5	2.5	2.5	2.5	2.4	2.0	1.4	0.9	0.6	1.9	2.1	1.4	0.9	0.6
	4	2.5	2.5	2.5	2.5	2.5	2.0	2.5	2.5	2.5	2.5	2.4	2.0	1.4	0.9	0.6	2.0	2.1	1.4	0.9	0.6
	5	2.5	2.5	2.5	2.5	2.5	2.1	2.5	2.5	2.5	2.5	2.4	2.0	1.4	0.9	0.6	2.1	2.1	1.4	0.9	0.6

Table 13(a)

Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for chi-square distribution, $\chi^2(3)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	154.7	154.7	154.7	154.7	154.7	142.2	144.8	153.2	152.2	147.5	94.7	108.0	55.6	24.9	13.3	91.2	106.0	63.2	29.9	18.5
	3	154.7	154.7	154.7	154.7	154.7	146.87	148.4	153.9	153.6	153.2	94.7	108.0	55.6	24.9	13.3	92.5	106.7	59.8	27.0	14.4
	4	154.7	154.7	154.7	154.7	154.7	148.9	150.1	154.1	154.0	153.9	94.7	108.0	55.6	24.9	13.3	93.1	107.0	58.35	26.3	13.9
	5	154.7	154.7	154.7	154.7	154.7	150.1	150.1	154.3	154.2	154.1	94.7	108.0	55.6	24.9	13.3	93.4	107.3	57.9	25.9	13.7
20	2	95.3	95.3	95.3	95.3	95.3	91.3	91.7	94.2	93.6	90.2	70.1	75.0	39.9	19.7	11.0	69.1	74.4	43.1	22.3	14.3
	3	95.3	95.3	95.3	95.3	95.3	92.7	93.0	94.7	94.5	94.2	70.1	75.0	39.9	19.7	11.0	69.5	74.7	41.7	20.8	11.7
	4	95.3	95.3	95.3	95.3	95.3	94.9	94.8	94.7	70.1	75.0	70.1	75.0	39.9	19.7	11.0	69.6	74.7	41.2	20.4	11.4
	5	95.3	95.3	95.3	95.3	95.3	93.8	95.0	94.9	94.9	70.1	70.1	75.0	39.9	19.7	11.0	69.7	74.8	40.9	20.2	11.3
30	2	73.4	73.4	73.4	73.4	73.4	71.6	71.6	72.7	72.2	69.9	57.7	60.4	33.0	16.9	9.6	57.6	60.3	35.0	18.6	12.0
	3	73.4	73.4	73.4	73.4	73.4	72.3	72.3	73.0	72.9	72.7	57.7	60.4	33.0	16.9	9.6	57.7	60.3	34.1	17.6	10.1
	4	73.4	73.4	73.4	73.4	73.4	72.6	72.6	73.1	73.1	73.0	57.7	60.4	33.0	16.9	9.6	57.7	60.4	33.8	17.3	9.9
	5	73.4	73.4	73.4	73.4	73.4	72.8	72.8	73.2	73.2	73.1	57.7	60.4	33.0	16.9	9.6	57.7	60.4	33.6	17.2	9.8
50	2	53.0	53.0	53.0	53.0	53.0	52.5	52.3	52.5	52.2	50.8	44.1	45.4	25.7	13.6	7.9	44.5	45.5	26.7	14.6	9.3
	3	53.0	53.0	53.0	53.0	53.0	52.7	52.5	52.7	52.6	52.5	44.1	45.4	25.7	13.6	7.9	44.3	45.4	26.3	14.0	8.2
	4	53.0	53.0	53.0	53.0	53.0	52.8	52.7	52.8	52.8	52.7	44.1	45.4	25.7	13.6	7.9	44.2	45.4	26.1	13.9	8.1
	5	53.0	53.0	53.0	53.0	53.0	52.8	52.7	52.8	52.8	52.8	44.1	45.4	25.7	13.6	7.9	44.2	45.4	26.0	13.8	8.0
100	2	33.3	33.3	33.3	33.3	33.3	33.3	33.1	33.0	32.9	32.1	29.0	29.5	17.5	9.6	5.7	29.3	29.6	17.9	10.1	6.3
	3	33.3	33.3	33.3	33.3	33.3	33.3	33.1	33.1	33.1	33.0	29.0	29.5	17.5	9.6	5.7	29.2	29.6	17.7	9.8	5.8
	4	33.3	33.3	33.3	33.3	33.3	33.3	33.2	33.2	33.1	33.1	29.0	29.5	17.5	9.6	5.7	29.1	29.5	17.7	9.7	5.8
	5	33.3	33.3	33.3	33.3	33.3	33.3	33.2	33.2	33.2	33.2	29.0	29.5	17.5	9.6	5.7	29.1	29.5	17.6	9.7	5.7
300	2	13.9	13.9	13.9	13.9	13.9	13.9	13.8	13.8	13.8	13.8	12.5	12.7	7.9	4.5	2.7	12.6	12.7	8.0	4.6	2.9
	3	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.8	12.5	12.7	7.9	4.5	2.7	12.6	12.7	8.0	4.5	2.7
	4	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	12.5	12.7	7.9	4.5	2.7	12.6	12.7	8.0	4.5	2.7
	5	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	12.5	12.7	7.9	4.5	2.7	12.6	12.7	8.0	4.5	2.7
1000	2	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	4.9	5.0	3.2	1.9	1.1	5.0	5.0	3.2	1.9	1.2
	3	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	4.9	5.0	3.2	1.9	1.1	5.0	5.0	3.2	1.9	1.1
	4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	4.9	5.0	3.2	1.9	1.1	5.0	5.0	3.2	1.9	1.1
	5	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	4.9	5.0	3.2	1.9	1.1	5.0	5.0	3.2	1.9	1.1
3000	2	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.2	2.2	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	3	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	4	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	5	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5

Table 13(b)

Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for chi-square distribution, $\chi^2(6)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	130.4	130.4	130.4	130.4	130.4	113.5	122.5	129.4	128.8	124.6	84.4	85.6	47.0	22.5	12.4	75.7	86.1	57.0	31.5	30.9
	3	130.4	130.4	130.4	130.4	130.4	119.5	125.4	129.9	129.7	129.5	84.4	85.6	47.0	22.5	12.4	78.7	85.9	52.4	26.2	15.3
	4	130.4	130.4	130.4	130.4	130.4	122.4	126.8	130.0	130.0	129.9	84.4	85.6	47.0	22.5	12.4	80.3	85.8	50.7	24.8	14.0
	5	130.4	130.4	130.4	130.4	130.4	124.0.	127.5	130.1	130.1	130.0	84.4	85.6	47.0	22.5	12.4	81.1	85.8	49.8	24.1	13.5
20	2	80.7	80.7	80.7	80.7	80.7	73.7	78.2	80.0	79.6	79.7	62.2	59.8	32.8	16.9	9.7	57.8	60.7	37.1	21.4	19.7
	3	80.7	80.7	80.7	80.7	80.7	76.2	79.1	80.3	80.2	80.0	62.2	59.8	32.8	16.9	9.7	59.4	60.4	35.2	18.7	11.3
	4	80.7	80.7	80.7	80.7	80.7	77.4	79.5	80.4	80.4	80.3	62.2	59.8	32.8	16.9	9.7	60.1	60.2	34.4	18.0	10.6
	5	80.7	80.7	80.7	80.7	80.7	78.0	79.8	80.5	80.5	80.4	62.2	59.8	32.8	16.9	9.7	60.5	60.1	34.1	17.7	10.3
30	2	62.3	62.3	62.3	62.3	62.3	58.4	61.2	61.9	61.5	59.5	51.1	48.3	26.9	14.2	8.3	48.6	49.2	29.5	17.1	15.1
	3	62.3	62.3	62.3	62.3	62.3	59.8	61.6	62.1	62.0	61.9	51.1	48.3	26.9	14.2	8.3	49.5	48.9	28.3	15.4	9.4
	4	62.3	62.3	62.3	62.3	62.3	60.4	61.8	62.1	62.1	62.1	51.1	48.3	26.9	14.2	8.3	49.9	48.7	27.9	15.0	8.9
	5	62.3	62.3	62.3	62.3	62.3	60.8	61.9	62.2	62.2	62.1	51.1	48.3	26.9	14.2	8.3	50.2	48.6	27.6	14.8	8.7
50	2	45.1	45.1	45.1	45.1	45.1	43.4	44.7	44.8	44.6	43.3	39.0	36.4	20.7	11.3	6.7	38.0	37.1	22.1	12.9	10.7
	3	45.1	45.1	45.1	45.1	45.1	44.0	44.8	44.9	44.9	44.8	39.0	36.4	20.7	11.3	6.7	38.4	36.8	21.5	11.9	7.3
	4	45.1	45.1	45.1	45.1	45.1	44.3	44.9	45.0	44.9	44.9	39.0	36.4	20.7	11.3	6.7	38.5	36.7	21.3	11.3	7.0
	5	45.1	45.1	45.1	45.1	45.1	44.4	44.9	45.0	45.0	45.0	39.0	36.4	20.7	11.3	6.7	38.6	36.7	21.1	11.6	6.9
100	2	28.4	28.4	28.4	28.4	28.4	28.0	28.3	28.2	28.1	27.4	25.6	23.8	14.0	7.8	4.7	25.5	24.1	14.6	8.6	6.6
	3	28.4	28.4	28.4	28.4	28.4	28.1	28.3	28.3	28.3	28.2	25.6	23.8	14.0	7.8	4.7	25.6	24.0	14.3	8.1	5.0
	4	28.4	28.4	28.4	28.4	28.4	28.2	28.3	28.3	28.3	28.3	25.6	23.8	14.0	7.8	4.7	25.6	24.0	14.2	8.0	4.9
	5	28.4	28.4	28.4	28.4	28.4	28.2	28.3	28.3	28.3	28.3	25.6	23.8	14.0	7.8	4.7	25.6	23.9	14.2	8.0	4.8
300	2	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.7	11.1	10.4	6.3	3.6	2.2	11.2	10.4	6.5	3.9	2.9
	3	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.1	10.4	6.4	3.7	2.3
	4	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.1	10.4	6.4	3.7	2.3
	5	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.1	10.4	6.4	3.7	2.3
1000	2	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.4	4.1	2.5	1.5	0.9	4.4	4.1	2.6	1.6	1.2
	3	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.4	4.1	2.5	1.5	0.9	4.4	4.1	2.6	1.5	1.0
	4	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.4	4.1	2.5	1.5	0.9	4.4	4.1	2.6	1.5	0.9
	5	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.4	4.1	2.5	1.5	0.9	4.4	4.1	2.6	1.5	0.9
3000	2	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.9	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.0	0.6	0.5
	3	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.0	0.6	0.4
	4	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.0	0.6	0.4
	5	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.0	0.6	0.4

Table 14(a)
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for F distribution, $F(10, 10)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	278.5	278.5	278.5	278.5	278.5	267.7	271.9	277.6	277.0	274.0	178.3	202.3	87.2	40.7	22.8	177.8	204.2	92.2	42.8	22.3
	3	278.5	278.5	278.5	278.5	278.5	271.6	274.3	278.0	277.8	277.6	178.3	202.3	87.2	40.7	22.8	178.0	203.5	90.0	41.6	22.7
	4	278.5	278.5	278.5	278.5	278.5	273.5	275.4	278.1	278.1	278.0	178.3	202.3	87.2	40.7	22.8	178.1	203.2	89.1	41.3	22.7
	5	278.5	278.5	278.5	278.5	278.5	274.5	276.0	278.2	278.2	278.1	178.3	202.3	87.2	40.7	22.8	178.1	203.0	88.7	41.1	22.7
20	2	174.3	174.3	174.3	174.3	174.3	171.7	172.5	173.6	173.2	171.0	132.5	141.4	69.2	35.0	20.2	133.2	142.5	71.4	36.1	20.5
	3	174.3	174.3	174.3	174.3	174.3	172.6	173.2	173.9	173.8	173.6	132.5	141.4	69.2	35.0	20.2	133.0	142.1	70.4	35.5	20.3
	4	174.3	174.3	174.3	174.3	174.3	173.1	173.5	174.1	174.0	173.9	132.5	141.4	69.2	35.0	20.2	132.8	141.9	70.1	35.3	20.2
	5	174.3	174.3	174.3	174.3	174.3	173.4	173.7	174.1	174.1	174.1	132.5	141.4	69.2	35.0	20.2	132.8	141.8	69.9	35.2	20.2
30	2	135.0	135.0	135.0	135.0	135.0	134.2	134.2	134.5	134.2	132.7	109.2	114.1	59.7	31.3	18.3	110.1	114.9	61.0	32.0	18.7
	3	135.0	135.0	135.0	135.0	135.0	134.4	134.5	134.7	134.6	134.5	109.2	114.1	59.7	31.3	18.3	109.8	114.6	60.4	31.6	18.4
	4	135.0	135.0	135.0	135.0	135.0	134.6	134.6	134.8	134.8	134.7	109.2	114.1	59.7	31.3	18.3	109.6	114.5	60.2	31.5	18.4
	5	135.0	135.0	135.0	135.0	135.0	134.6	134.7	134.8	134.8	134.8	109.2	114.1	59.7	31.3	18.3	109.5	114.4	60.1	31.4	18.4
50	2	97.6	97.6	97.6	97.6	97.6	97.5	97.3	97.3	97.1	96.2	83.4	85.8	48.4	26.4	15.7	84.1	86.2	49.0	26.8	15.9
	3	97.6	97.6	97.6	97.6	97.6	97.5	97.4	97.4	97.4	97.3	83.4	85.8	48.4	26.4	15.7	83.8	86.0	48.7	26.5	15.8
	4	97.6	97.6	97.6	97.6	97.6	97.6	97.5	97.5	97.5	97.4	83.4	85.8	48.4	26.4	15.7	83.7	86.0	48.6	26.5	15.7
	5	97.6	97.6	97.6	97.6	97.6	97.6	97.5	97.5	97.5	97.5	83.4	85.8	48.4	26.4	15.7	83.6	85.9	48.6	26.4	15.7
100	2	61.0	61.0	61.0	61.0	61.0	61.1	60.9	60.9	60.8	60.4	54.6	55.5	34.0	19.4	11.7	54.9	55.7	34.2	19.5	11.8
	3	61.0	61.0	61.0	61.0	61.0	61.1	61.0	60.9	60.9	60.9	54.6	55.5	34.0	19.4	11.7	54.8	55.6	34.1	19.4	11.7
	4	61.0	61.0	61.0	61.0	61.0	61.1	61.0	61.0	61.0	61.0	54.6	55.5	34.0	19.4	11.7	54.7	55.6	34.1	19.4	11.7
	5	61.0	61.0	61.0	61.0	61.0	61.1	61.0	61.0	61.0	61.0	54.6	55.5	34.0	19.4	11.7	54.7	55.6	34.1	19.4	11.7
300	2	24.7	24.7	24.7	24.7	24.7	24.7	24.6	24.6	24.6	24.5	22.8	23.1	15.3	9.0	5.5	22.9	23.1	15.4	9.1	5.5
	3	24.7	24.7	24.7	24.7	24.7	24.7	24.6	24.6	24.6	24.6	22.8	23.1	15.3	9.0	5.5	22.8	23.1	15.3	9.0	5.5
	4	24.7	24.7	24.7	24.7	24.7	24.7	24.6	24.6	24.6	24.6	22.8	23.1	15.3	9.0	5.5	22.8	23.1	15.3	9.0	5.5
	5	24.7	24.7	24.7	24.7	24.7	24.7	24.6	24.6	24.6	24.6	22.8	23.1	15.3	9.0	5.5	22.8	23.1	15.3	9.0	5.5
1000	2	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.7	9.2	9.2	6.4	3.9	2.4	9.2	9.3	6.4	3.9	2.4
	3	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.2	9.2	6.4	3.9	2.4	9.2	9.3	6.4	3.9	2.4
	4	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.2	9.2	6.4	3.9	2.4	9.2	9.3	6.4	3.9	2.4
	5	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.2	9.2	6.4	3.9	2.4	9.2	9.3	6.4	3.9	2.4
3000	2	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.7	3.8	2.7	1.6	1.0	3.7	3.8	2.7	1.7	1.0
	3	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.7	3.8	2.7	1.6	1.0	3.7	3.8	2.7	1.7	1.0
	4	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.7	3.8	2.7	1.6	1.0	3.7	3.8	2.7	1.7	1.0
	5	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.7	3.8	2.7	1.6	1.0	3.7	3.8	2.7	1.7	1.0

Table 14(b)
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for F distribution, $F(10, 40)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	149.9	149.9	149.9	149.9	149.9	132.9	142.3	149.0	148.4	144.7	100.0	100.2	53.0	25.4	14.1	91.4	101.2	62.7	33.8	30.8
	3	149.9	149.9	149.9	149.9	149.9	138.9	145.1	149.4	149.3	149.0	100.0	100.2	53.0	25.4	14.1	94.5	100.8	58.3	28.9	16.8
	4	149.9	149.9	149.9	149.9	149.9	141.8	146.4	149.5	149.5	149.4	100.0	100.2	53.0	25.4	14.1	95.9	100.6	56.6	27.6	15.6
	5	149.9	149.9	149.9	149.9	149.9	143.5	147.1	149.6	149.6	149.5	100.0	100.2	53.0	25.4	14.1	96.8	100.5	55.8	27.0	15.1
20	2	93.6	93.6	93.6	93.6	93.6	86.8	91.3	93.0	92.6	89.9	73.8	70.5	38.1	19.7	11.4	69.6	71.6	42.2	23.8	20.4
	3	93.6	93.6	93.6	93.6	93.6	89.2	92.2	93.2	93.2	93.0	73.8	70.5	38.1	19.7	11.4	71.1	71.2	40.3	21.4	12.8
	4	93.6	93.6	93.6	93.6	93.6	90.3	92.5	93.3	93.3	93.2	73.8	70.5	38.1	19.7	11.4	71.8	71.0	39.6	20.7	12.2
	5	93.6	93.6	93.6	93.6	93.6	91.0	92.8	93.4	93.4	93.3	73.8	70.5	38.1	19.7	11.4	72.2	70.9	39.3	20.4	11.9
30	2	72.5	72.5	72.5	72.5	72.5	68.8	71.5	72.1	71.8	70.0	60.7	57.1	31.6	16.8	9.9	58.4	58.1	34.1	19.5	15.9
	3	72.5	72.5	72.5	72.5	72.5	70.1	71.9	72.3	72.2	72.1	60.7	57.1	31.6	16.8	9.9	59.2	57.7	32.9	17.9	10.8
	4	72.5	72.5	72.5	72.5	72.5	70.7	72.1	72.4	72.3	72.3	60.7	57.1	31.6	16.8	9.9	59.6	57.6	32.5	17.5	10.4
	5	72.5	72.5	72.5	72.5	72.5	71.1	72.2	72.4	72.4	72.4	60.7	57.1	31.6	16.8	9.9	59.8	57.5	32.3	17.3	10.2
50	2	52.6	52.6	52.6	52.6	52.6	51.1	52.3	52.4	52.2	51.1	46.3	43.2	24.7	13.5	8.1	45.4	43.9	26.0	15.0	11.6
	3	52.6	52.6	52.6	52.6	52.6	51.6	52.4	52.5	52.5	52.4	46.3	43.2	24.7	13.5	8.1	45.7	43.6	25.4	14.1	8.6
	4	52.6	52.6	52.6	52.6	52.6	51.9	52.5	52.6	52.5	52.5	46.3	43.2	24.7	13.5	8.1	45.9	43.5	25.2	13.9	8.4
	5	52.6	52.6	52.6	52.6	52.6	52.1	52.5	52.6	52.6	52.6	46.3	43.2	24.7	13.5	8.1	46.0	43.4	25.0	13.8	8.3
100	2	33.2	33.2	33.2	33.2	33.2	32.9	33.1	33.1	33.0	32.4	30.4	28.3	16.8	9.5	5.8	30.3	28.6	17.4	10.2	7.4
	3	33.2	33.2	33.2	33.2	33.2	33.0	33.2	33.1	33.1	33.1	30.4	28.3	16.8	9.5	5.8	30.3	28.5	17.1	9.8	6.0
	4	33.2	33.2	33.2	33.2	33.2	33.1	33.2	33.2	33.2	33.1	30.4	28.3	16.8	9.5	5.8	30.4	28.4	17.0	9.7	5.9
	5	33.2	33.2	33.2	33.2	33.2	33.1	33.2	33.2	33.2	33.2	30.4	28.3	16.8	9.5	5.8	30.4	28.4	17.0	9.6	5.9
300	2	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.8	13.8	13.7	13.0	12.2	7.6	4.4	2.7	13.1	12.3	7.8	4.6	3.2
	3	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.8	13.0	12.2	7.6	4.4	2.7	13.1	12.3	7.7	4.5	2.8
	4	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.0	12.2	7.6	4.4	2.7	13.1	12.3	7.7	4.5	2.8
	5	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.0	12.2	7.6	4.4	2.7	13.1	12.3	7.7	4.5	2.8
1000	2	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.2	4.9	3.1	1.8	1.1	5.2	4.9	3.1	1.9	1.3
	3	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.2	4.9	3.1	1.8	1.1	5.2	4.9	3.1	1.9	1.2
	4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.2	4.9	3.1	1.8	1.1	5.2	4.9	3.1	1.8	1.1
	5	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.2	4.9	3.1	1.8	1.1	5.2	4.9	3.1	1.8	1.1
3000	2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.1	1.9	7.3	0.7	0.5	2.1	2.0	1.3	0.8	0.5
	3	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.1	1.9	7.3	0.7	0.5	2.1	2.0	1.3	0.8	0.5
	4	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.1	1.9	7.3	0.7	0.5	2.1	1.9	1.3	0.8	0.5
	5	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.1	1.9	7.3	0.7	0.5	2.1	1.9	1.3	0.8	0.5

Table 15(a)
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for beta distribution, $B(3, 2.0)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	61.2	61.2	61.2	61.2	61.2	55.3	61.1	61.0	60.9	52.5	30.2	9.0	2.1	0.9	0.4	26.4	10.6	3.0	1.7	13.3
	3	61.2	61.2	61.2	61.2	61.2	57.3	61.1	61.1	61.1	61.0	30.2	9.0	2.1	0.9	0.4	27.7	10.0	2.6	1.2	0.7
	4	61.2	61.2	61.2	61.2	61.2	58.3	61.1	61.1	61.1	61.1	30.2	9.0	2.1	0.9	0.4	28.3	9.7	2.4	1.0	0.6
	5	61.2	61.2	61.2	61.2	61.2	58.9	61.1	61.1	61.1	61.1	30.2	9.0	2.1	0.9	0.4	28.7	9.5	2.3	1.0	0.5
20	2	34.8	34.8	34.8	34.8	34.8	31.5	34.7	34.7	34.6	28.7	21.2	5.6	1.4	0.6	0.3	18.6	6.3	1.9	1.1	8.2
	3	34.8	34.8	34.8	34.8	34.8	32.6	34.7	34.7	34.7	34.7	21.2	5.6	1.4	0.6	0.3	19.5	6.0	1.6	0.8	0.5
	4	34.8	34.8	34.8	34.8	34.8	33.1	34.8	34.8	34.7	34.7	21.2	5.6	1.4	0.6	0.3	19.9	5.9	1.6	0.7	0.4
	5	34.8	34.8	34.8	34.8	34.8	33.5	34.8	34.8	34.8	34.8	21.2	5.6	1.4	0.6	0.3	20.2	5.8	1.5	0.7	0.4
30	2	25.7	25.7	25.7	25.7	25.7	23.3	25.7	25.7	25.6	21.2	17.2	4.2	1.1	0.5	0.2	15.2	4.7	1.4	0.8	5.8
	3	25.7	25.7	25.7	25.7	25.7	24.1	25.7	25.7	25.7	25.7	17.2	4.2	1.1	0.5	0.2	15.8	4.5	1.3	0.6	0.4
	4	25.7	25.7	25.7	25.7	25.7	24.5	25.7	25.7	25.7	25.7	17.2	4.2	1.1	0.5	0.2	16.2	4.4	1.2	0.5	0.3
	5	25.7	25.7	25.7	25.7	25.7	24.8	25.7	25.7	25.7	25.7	17.2	4.2	1.1	0.5	0.2	16.4	4.4	1.2	0.5	0.3
50	2	17.8	17.8	17.8	17.8	17.8	16.2	17.7	17.7	17.7	14.8	13.0	2.9	0.8	0.3	0.2	11.6	3.3	1.0	0.6	3.7
	3	17.8	17.8	17.8	17.8	17.8	16.7	17.7	17.7	17.7	17.7	13.0	2.9	0.8	0.3	0.2	12.0	3.1	1.0	0.4	0.2
	4	17.8	17.8	17.8	17.8	17.8	17.0	17.7	17.7	17.7	17.7	13.0	2.9	0.8	0.3	0.2	12.3	3.1	0.9	0.4	0.2
	5	17.8	17.8	17.8	17.8	17.8	17.1	17.8	17.7	17.7	17.7	13.0	2.9	0.8	0.3	0.2	12.4	3.1	0.8	0.4	0.2
100	2	10.7	10.7	10.7	10.7	10.7	9.8	10.6	10.6	10.6	9.3	8.4	1.8	0.5	0.2	0.1	7.6	2.0	0.6	0.3	2.1
	3	10.7	10.7	10.7	10.7	10.7	10.1	10.6	10.6	10.6	10.6	8.4	1.8	0.5	0.2	0.1	7.9	1.9	0.5	0.3	0.2
	4	10.7	10.7	10.7	10.7	10.7	10.2	10.7	10.6	10.6	10.6	8.4	1.8	0.5	0.2	0.1	8.0	1.9	0.5	0.2	0.1
	5	10.7	10.7	10.7	10.7	10.7	10.3	10.7	10.7	10.7	10.6	8.4	1.8	0.5	0.2	0.1	8.1	1.9	0.5	0.2	0.1
300	2	4.4	4.4	4.4	4.4	4.4	4.2	4.4	4.4	4.4	3.8	3.7	0.8	0.2	0.1	0.1	3.5	0.8	0.2	0.1	0.6
	3	4.4	4.4	4.4	4.4	4.4	4.2	4.4	4.4	4.4	4.4	3.7	0.8	0.2	0.1	0.1	3.6	0.8	0.2	0.1	0.1
	4	4.4	4.4	4.4	4.4	4.4	4.3	4.4	4.4	4.4	4.4	3.7	0.8	0.2	0.1	0.1	3.6	0.8	0.2	0.1	0.1
	5	4.4	4.4	4.4	4.4	4.4	4.3	4.4	4.4	4.4	4.4	3.7	0.8	0.2	0.1	0.1	3.7	0.8	0.2	0.1	0.1
1000	2	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.2	1.4	0.3	0.1	0.0	0.0	1.4	0.3	0.1	0.0	−0.1
	3	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	0.3	0.1	0.0	0.0	1.4	0.3	0.1	0.0	0.0
	4	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	0.3	0.1	0.0	0.0	1.4	0.3	0.1	0.0	0.0
	5	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	0.3	0.1	0.0	0.0	1.4	0.3	0.1	0.0	0.0
3000	2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.3	0.5	0.1	0.0	0.0	0.0	0.5	0.1	0.0	0.0	−0.2
	3	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.1	0.0	0.0	0.0	0.5	0.1	0.0	0.0	0.0
	4	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.1	0.0	0.0	0.0	0.5	0.1	0.0	0.0	0.0
	5	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.1	0.0	0.0	0.0	0.5	0.1	0.0	0.0	0.0

Table 15(b)

Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for beta distribution, $B(3, 3.5)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	68.6	68.6	68.6	68.6	68.6	42.9	63.6	68.3	68.4	63.3	37.4	36.4	25.3	14.3	8.4	19.7	40.69	41.0	36.9	380.7
	3	68.6	68.6	68.6	68.6	68.6	51.5	65.6	68.5	68.5	68.5	37.4	36.4	25.3	14.3	8.4	25.6	39.1	33.3	22.0	16.8
	4	68.6	68.6	68.6	68.6	68.6	55.8	66.4	68.5	68.6	68.6	37.4	36.4	25.3	14.3	8.4	28.6	38.3	30.6	19.0	12.6
	5	68.6	68.6	68.6	68.6	68.6	58.4	66.9	68.5	68.6	68.6	37.4	36.4	25.3	14.3	8.4	30.3	37.9	29.3	17.6	11.2
20	2	39.9	39.9	39.9	39.9	39.9	24.5	38.6	39.8	39.7	36.5	26.5	24.0	15.3	8.7	5.3	13.9	27.4	22.7	19.6	192.9
	3	39.9	39.9	39.9	39.9	39.9	29.7	39.1	39.8	39.8	39.8	26.5	24.0	15.3	8.7	5.3	18.2	26.0	19.1	12.5	9.5
	4	39.9	39.9	39.9	39.9	39.9	32.2	39.3	39.8	39.8	39.8	26.5	24.0	15.3	8.7	5.3	20.3	25.4	17.8	11.0	7.4
	5	39.9	39.9	39.9	39.9	39.9	34.8	39.5	39.8	39.8	39.8	26.5	24.0	15.3	8.7	5.3	21.5	25.1	17.2	10.3	6.7
30	2	29.8	29.8	29.8	29.8	29.8	18.3	29.4	29.7	29.7	27.5	21.6	18.7	11.6	6.6	4.1	11.5	21.4	16.3	13.7	129.2
	3	29.8	29.8	29.8	29.8	29.8	22.1	29.5	29.8	29.8	29.7	21.6	18.7	11.6	6.6	4.1	14.9	20.3	14.0	9.1	6.9
	4	29.8	29.8	29.8	29.8	29.8	24.1	29.6	29.8	29.8	29.8	21.6	18.7	11.6	6.6	4.1	16.6	19.9	13.2	8.1	5.5
	5	29.8	29.8	29.8	29.8	29.8	25.2	29.7	29.8	29.8	29.8	21.6	18.7	11.6	6.6	4.1	17.6	19.6	12.8	7.7	5.0
50	2	20.8	20.8	20.8	20.8	20.8	12.7	20.7	20.8	20.7	19.3	16.3	13.6	8.2	4.7	2.9	8.8	15.3	10.9	8.9	77.5
	3	20.8	20.8	20.8	20.8	20.8	15.4	20.8	20.8	20.8	20.8	16.3	13.6	8.2	4.7	2.9	11.3	14.7	9.6	6.1	4.6
	4	20.8	20.8	20.8	20.8	20.8	16.8	20.8	20.8	20.8	20.8	16.3	13.6	8.2	4.7	2.9	12.6	14.4	9.1	5.6	3.8
	5	20.8	20.8	20.8	20.8	20.8	17.6	20.8	20.8	20.8	20.8	16.3	13.6	8.2	4.7	2.9	13.3	14.2	8.9	5.3	3.5
100	2	12.7	12.7	12.7	12.7	12.7	7.6	12.6	12.6	12.6	11.3	10.6	8.6	5.1	2.9	1.8	5.8	9.4	6.3	4.9	38.3
	3	12.7	12.7	12.7	12.7	12.7	9.3	12.6	12.6	12.6	12.6	10.6	8.6	5.1	2.9	1.8	7.4	9.1	5.7	3.6	2.6
	4	12.7	12.7	12.7	12.7	12.7	10.1	12.6	12.6	12.6	12.6	10.6	8.6	5.1	2.9	1.8	8.2	8.9	5.5	3.3	2.2
	5	12.7	12.7	12.7	12.7	12.7	10.6	12.6	12.6	12.6	12.6	10.6	8.6	5.1	2.9	1.8	8.7	8.9	5.4	3.2	2.1
300	2	5.3	5.3	5.3	5.3	5.3	3.0	5.3	5.3	5.3	5.4	4.7	3.7	2.2	1.3	0.8	2.4	4.0	2.6	1.9	13.1
	3	5.3	5.3	5.3	5.3	5.3	3.7	5.3	5.3	5.3	5.3	4.7	3.7	2.2	1.3	0.8	3.2	3.9	2.4	1.5	1.1
	4	5.3	5.3	5.3	5.3	5.3	4.1	5.3	5.3	5.3	5.3	4.7	3.7	2.2	1.3	0.8	3.6	3.8	2.3	1.4	0.9
	5	5.3	5.3	5.3	5.3	5.3	4.4	5.3	5.3	5.3	5.3	4.7	3.7	2.2	1.3	0.8	3.8	3.8	2.3	1.4	0.9
1000	2	2.0	2.0	2.0	2.0	2.0	1.1	2.0	2.0	2.0	2.7	1.8	1.4	0.8	0.5	0.3	0.9	1.5	1.0	0.7	4.8
	3	2.0	2.0	2.0	2.0	2.0	1.4	2.0	2.0	2.0	2.0	1.8	1.4	0.8	0.5	0.3	1.2	1.5	0.9	0.6	0.4
	4	2.0	2.0	2.0	2.0	2.0	1.5	2.0	2.0	2.0	2.0	1.8	1.4	0.8	0.5	0.3	1.4	1.4	0.9	0.5	0.4
	5	2.0	2.0	2.0	2.0	2.0	1.6	2.0	2.0	2.0	2.0	1.8	1.4	0.8	0.5	0.3	1.4	1.4	0.9	0.5	0.3
3000	2	0.7	0.7	0.7	0.7	0.7	0.5	0.8	0.8	0.8	1.5	0.7	0.5	0.3	0.2	0.1	0.4	0.6	0.4	0.3	2.1
	3	0.7	0.7	0.7	0.7	0.7	0.6	0.8	0.8	0.8	0.8	0.7	0.5	0.3	0.2	0.1	0.5	0.6	0.3	0.2	0.2
	4	0.7	0.7	0.7	0.7	0.7	0.6	0.7	0.7	0.8	0.8	0.7	0.5	0.3	0.2	0.1	0.6	0.6	0.3	0.2	0.1
	5	0.7	0.7	0.7	0.7	0.7	0.6	0.7	0.7	0.7	0.7	0.7	0.5	0.3	0.2	0.1	0.6	0.6	0.3	0.2	0.1

Table 16(a)
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for gamma distribution, $G(1.5, 1)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	154.7	154.7	154.7	154.7	154.7	142.2	144.8	153.2	152.2	147.5	94.7	108.0	55.6	24.9	13.3	91.2	106.0	63.2	29.9	18.5
	3	154.7	154.7	154.7	154.7	154.7	146.8	148.4	153.9	153.6	153.2	94.7	108.0	55.6	24.9	13.3	92.5	106.7	59.8	27.0	14.4
	4	154.7	154.7	154.7	154.7	154.7	148.9	150.1	154.1	154.0	153.9	94.7	108.0	55.6	24.9	13.3	93.1	107.1	58.5	26.3	13.9
	5	154.7	154.7	154.7	154.7	154.7	150.1	151.0	154.3	154.2	154.1	94.7	108.0	55.6	24.9	13.3	93.4	107.3	57.9	25.9	13.7
20	2	95.3	95.3	95.3	95.3	95.3	91.3	91.7	94.2	93.6	90.3	70.1	75.0	39.9	19.7	11.0	69.1	74.4	43.1	22.3	14.3
	3	95.3	95.3	95.3	95.3	95.3	92.7	93.0	94.7	94.5	94.2	70.1	75.0	39.9	19.7	11.0	69.5	74.7	41.7	20.8	11.7
	4	95.3	95.3	95.3	95.3	95.3	93.4	93.6	94.9	94.8	94.7	70.1	75.0	39.9	19.7	11.0	69.6	74.8	41.2	20.4	11.4
	5	95.3	95.3	95.3	95.3	95.3	93.8	94.0	95.0	94.9	94.9	70.1	75.0	39.9	19.7	11.0	69.7	74.8	40.9	20.2	11.3
30	2	73.4	73.4	73.4	73.4	73.4	71.6	71.6	72.7	72.7	69.9	57.7	60.4	33.0	16.9	9.6	57.6	60.3	35.0	18.6	12.0
	3	73.4	73.4	73.4	73.4	73.4	72.3	72.3	73.0	72.9	72.7	57.7	60.4	33.0	16.9	9.6	57.7	60.3	34.1	17.6	10.1
	4	73.4	73.4	73.4	73.4	73.4	72.6	72.6	73.1	73.1	73.0	57.7	60.4	33.0	16.9	9.6	57.7	60.4	33.8	17.3	9.9
	5	73.4	73.4	73.4	73.4	73.4	72.8	72.8	73.2	73.2	73.1	57.7	60.4	33.0	16.9	9.6	57.7	60.4	33.6	17.2	9.8
50	2	53.0	53.0	53.0	53.0	53.0	52.5	52.3	52.5	52.2	50.8	44.1	45.4	25.7	13.6	7.9	44.5	45.5	26.7	14.6	9.3
	3	53.0	53.0	53.0	53.0	53.0	52.7	52.5	52.7	52.7	52.5	44.1	45.4	25.7	13.6	7.9	44.3	45.4	26.3	14.0	8.2
	4	53.0	53.0	53.0	53.0	53.0	52.8	52.7	52.8	52.8	52.7	44.1	45.4	25.7	13.6	7.9	44.2	45.4	26.1	13.9	8.1
	5	53.0	53.0	53.0	53.0	53.0	52.8	52.7	52.8	52.8	52.8	44.1	45.4	25.7	13.6	7.9	44.2	45.4	26.0	13.8	8.0
100	2	33.3	33.3	33.3	33.3	33.3	33.3	33.1	33.0	32.9	32.1	29.0	29.5	17.5	9.6	5.7	29.4	29.6	17.9	10.0	6.3
	3	33.3	33.3	33.3	33.3	33.3	33.3	33.1	33.1	33.1	33.0	29.0	29.5	17.5	9.6	5.7	29.2	29.6	17.7	9.8	5.8
	4	33.3	33.3	33.3	33.3	33.3	33.3	33.2	33.25	33.25	33.1	29.0	29.5	17.5	9.6	5.7	29.1	29.5	17.7	9.7	5.8
	5	33.3	33.3	33.3	33.3	33.3	33.3	33.2	33.2	33.2	33.2	29.0	29.5	17.5	9.6	5.7	29.1	29.5	17.6	9.7	5.7
300	2	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.8	13.8	13.6	12.5	12.7	8.0	4.5	2.7	12.6	12.7	8.0	4.6	2.9
	3	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.8	12.5	12.7	8.0	4.5	2.7	12.6	12.7	8.0	4.6	2.8
	4	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	12.5	12.7	8.0	4.5	2.7	12.6	12.7	8.0	4.5	2.7
	5	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	12.5	12.7	8.0	4.5	2.7	12.6	12.7	8.0	4.5	2.7
1000	2	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.0	5.0	3.2	1.9	1.1	5.0	5.0	3.3	1.9	1.2
	3	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.0	5.0	3.2	1.9	1.1	5.0	5.0	3.3	1.9	1.1
	4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.0	5.0	3.2	1.9	1.1	5.0	5.0	3.3	1.9	1.1
	5	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.0	5.0	3.2	1.9	1.1	5.0	5.0	3.3	1.9	1.1
3000	2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	3	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	4	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5
	5	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.0	2.0	1.3	0.8	0.5	2.0	2.0	1.3	0.8	0.5

Table 16(b)

Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for gamma distribution, $G(3.0, 1)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	130.4	130.4	130.4	130.4	130.4	113.5	122.5	129.4	128.8	124.6	84.4	85.6	47.0	22.5	12.4	75.7	86.1	57.0	31.5	30.9
	3	130.4	130.4	130.4	130.4	130.4	119.5	125.4	129.9	129.7	129.5	84.4	85.6	47.0	22.5	12.4	78.8	85.9	52.4	26.2	15.3
	4	130.4	130.4	130.4	130.4	130.4	122.4	126.8	130.0	130.0	129.9	84.4	85.6	47.0	22.5	12.4	80.3	85.8	50.7	24.8	14.0
	5	130.4	130.4	130.4	130.4	130.4	124.0	127.5	130.1	130.1	130.0	84.4	85.6	47.0	22.5	12.4	81.1	85.8	49.8	24.1	13.5
20	2	80.7	80.7	80.7	80.7	80.7	73.7	78.2	80.0	79.6	76.7	62.2	59.8	32.8	16.9	9.7	57.8	60.7	37.7	21.4	19.7
	3	80.7	80.7	80.7	80.7	80.7	76.2	79.1	80.3	80.2	80.0	62.2	59.8	32.8	16.9	9.7	59.4	60.4	35.2	18.7	11.3
	4	80.7	80.7	80.7	80.7	80.7	77.4	79.5	80.4	80.4	80.3	62.2	59.8	32.8	16.9	9.7	60.1	60.2	34.4	18.0	15.6
	5	80.7	80.7	80.7	80.7	80.7	78.0	79.8	80.5	80.5	80.4	62.2	59.8	32.8	16.9	9.7	60.5	60.1	34.1	17.7	10.3
30	2	62.3	62.3	62.3	62.3	62.3	58.4	61.2	61.9	61.5	59.5	51.5	48.3	26.9	14.2	8.3	48.6	49.2	29.5	17.1	15.1
	3	62.3	62.3	62.3	62.3	62.3	59.8	61.6	62.1	62.0	61.9	51.5	48.3	26.9	14.2	8.3	19.5	48.9	28.3	15.4	9.4
	4	62.3	62.3	62.3	62.3	62.3	60.4	61.8	62.1	62.1	62.1	51.5	48.3	26.9	14.2	8.3	50.0	48.7	27.9	15.0	8.9
	5	62.3	62.3	62.3	62.3	62.3	60.8	61.9	62.2	62.2	62.1	51.5	48.3	26.9	14.2	8.3	50.2	48.6	27.6	14.8	8.7
50	2	45.1	45.1	45.1	45.1	45.1	43.4	44.7	44.8	44.6	43.3	39.0	36.4	20.7	11.3	6.7	38.0	37.1	22.1	12.9	10.7
	3	45.1	45.1	45.1	45.1	45.1	44.0	44.8	44.9	44.9	44.8	39.0	36.4	20.7	11.3	6.7	38.4	36.9	21.5	11.9	7.3
	4	45.1	45.1	45.1	45.1	45.1	44.3	44.9	45.0	44.9	44.9	39.0	36.4	20.7	11.3	6.7	38.5	36.7	21.3	11.7	7.0
	5	45.1	45.1	45.1	45.1	45.1	44.4	44.9	45.0	45.0	45.0	39.0	36.4	20.7	11.3	6.7	38.6	36.7	21.1	11.6	6.9
100	2	28.4	28.4	28.4	28.4	28.4	28.0	28.3	28.2	28.1	27.4	25.6	23.8	14.0	7.8	4.7	25.5	24.1	14.6	8.6	6.6
	3	28.4	28.4	28.4	28.4	28.4	28.1	28.3	28.3	28.3	28.2	25.6	23.8	14.0	7.8	4.7	25.6	24.0	14.3	8.1	5.0
	4	28.4	28.4	28.4	28.4	28.4	28.2	28.3	28.3	28.3	28.3	25.6	23.8	14.0	7.8	4.7	25.6	24.0	14.2	8.0	4.9
	5	28.4	28.4	28.4	28.4	28.4	28.2	28.3	28.3	28.3	28.3	25.6	23.8	14.0	7.8	4.7	25.6	24.0	14.2	8.0	4.8
300	2	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.7	11.1	10.4	6.3	3.6	2.2	11.2	10.4	6.4	3.9	2.9
	3	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.2	10.4	6.4	3.7	2.3
	4	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.2	10.4	6.4	3.7	2.3
	5	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.1	10.4	6.3	3.6	2.2	11.2	10.4	6.4	3.7	2.3
1000	2	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	6.5	6.1	3.8	2.2	1.4	4.4	4.1	2.6	1.6	1.2
	3	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	6.5	6.1	3.8	2.2	1.4	4.4	4.1	2.6	1.5	1.0
	4	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	6.5	6.1	3.8	2.2	1.4	4.4	4.1	2.6	1.5	0.9
	5	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.6	6.5	6.1	3.8	2.2	1.4	4.4	4.1	2.6	1.5	0.9
3000	2	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.9	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.1	0.6	0.5
	3	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.1	0.6	0.4
	4	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.1	0.6	0.4
	5	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.6	1.0	0.6	0.4	1.7	1.6	1.1	0.6	0.4

Table 17(a)
Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for Weibull distribution, $W(1, 1.4)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	121.1	121.1	121.1	121.1	121.1	105.7	112.1	120.1	119.4	115.3	74.2	79.5	44.9	20.8	11.2	67.2	78.5	54.7	29.2	26.0
	3	121.1	121.1	121.1	121.1	121.1	111.3	115.4	120.6	120.4	120.1	74.2	79.5	44.9	20.8	11.2	69.7	78.9	50.3	24.3	13.8
	4	121.1	121.1	121.1	121.1	121.1	113.9	117.7	120.7	120.7	120.6	74.2	79.5	44.9	20.8	11.2	70.9	79.1	48.6	23.0	12.6
	5	121.1	121.1	121.1	121.1	121.1	115.4	117.8	120.8	120.8	120.7	74.2	79.5	44.9	20.8	11.2	71.6	79.2	47.7	22.4	12.2
20	2	74.6	74.6	74.6	74.6	74.6	68.5	71.5	73.8	73.3	70.3	55.0	55.4	30.8	15.4	8.7	51.8	55.6	35.0	19.8	17.1
	3	74.6	74.6	74.6	74.6	74.6	70.7	72.6	74.2	74.1	73.8	55.0	55.4	30.8	15.4	8.7	52.9	55.5	33.1	17.3	10.2
	4	74.6	74.6	74.6	74.6	74.6	71.7	73.1	74.3	74.3	74.2	55.0	55.4	30.8	15.4	8.7	53.5	55.5	32.4	16.6	9.6
	5	74.6	74.6	74.6	74.6	74.6	72.3	73.4	74.4	74.3	74.3	55.0	55.4	30.8	15.4	8.7	53.8	55.5	32.0	16.3	9.3
30	2	57.5	57.5	57.5	57.5	57.5	54.2	56.0	57.0	56.6	54.5	45.4	44.8	25.0	13.0	7.4	43.7	45.2	27.6	15.8	13.2
	3	57.5	57.5	57.5	57.5	57.5	55.4	56.6	57.3	57.2	57.0	45.4	44.8	25.0	13.0	7.4	44.3	45.1	26.4	14.1	8.5
	4	57.5	57.5	57.5	57.5	57.5	56.0	56.8	57.3	57.3	57.3	45.4	44.8	25.0	13.0	7.4	44.6	45.0	26.0	13.7	8.0
	5	57.5	57.5	57.5	57.5	57.5	56.3	57.0	57.4	57.4	57.3	45.4	44.8	25.0	13.0	7.4	44.8	45.0	25.7	13.5	7.8
50	2	41.6	41.6	41.6	41.6	41.6	40.3	41.1	41.3	41.0	39.7	34.9	33.8	19.1	10.2	6.0	34.3	34.3	20.5	11.8	9.4
	3	41.6	41.6	41.6	41.6	41.6	40.8	41.3	41.4	41.4	41.3	34.9	33.8	19.1	10.2	6.0	34.5	34.1	19.9	10.9	6.6
	4	41.6	41.6	41.6	41.6	41.6	41.0	41.4	41.5	41.5	41.4	34.9	33.8	19.1	10.2	6.0	34.6	34.0	19.7	10.6	6.3
	5	41.6	41.6	41.6	41.6	41.6	41.1	41.4	41.5	41.5	41.5	34.9	33.8	19.1	10.2	6.0	34.7	34.0	19.5	10.5	6.2
100	2	26.2	26.2	26.2	26.2	26.2	26.0	26.1	26.1	25.9	25.2	23.1	22.2	12.9	7.0	4.2	23.2	22.4	13.5	7.8	5.8
	3	26.2	26.2	26.2	26.2	26.2	26.1	26.1	26.1	26.1	26.1	23.1	22.2	12.9	7.0	4.2	23.1	22.3	13.2	7.3	4.5
	4	26.2	26.2	26.2	26.2	26.2	26.1	26.2	26.2	26.2	26.1	23.1	22.2	12.9	7.0	4.2	23.1	22.3	13.1	7.2	4.3
	5	26.2	26.2	26.2	26.2	26.2	26.2	26.2	26.2	26.2	26.2	23.1	22.2	12.9	7.0	4.2	23.1	22.2	13.0	7.2	4.3
300	2	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.0	11.0	10.8	10.1	9.7	5.8	3.3	2.0	10.2	9.8	6.0	3.5	2.5
	3	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.0	10.1	9.7	5.8	3.3	2.0	10.1	9.7	5.9	3.4	2.1
	4	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	10.1	9.7	5.8	3.3	2.0	10.1	9.7	5.9	3.3	2.0
	5	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	10.1	9.7	5.8	3.3	2.0	10.1	9.7	5.9	3.3	2.0
1000	2	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.0	3.8	2.3	1.3	0.8	4.0	3.9	2.4	1.4	1.1
	3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.0	3.8	2.3	1.3	0.8	4.0	3.8	2.4	1.4	0.9
	4	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.0	3.8	2.3	1.3	0.8	4.0	3.8	2.4	1.4	0.8
	5	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.0	3.8	2.3	1.3	0.8	4.0	3.8	2.4	1.3	0.8
3000	2	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.8	1.6	1.5	0.9	0.5	0.3	1.6	1.5	1.0	0.6	0.5
	3	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.5	0.9	0.5	0.3	1.6	1.5	1.0	0.6	0.4
	4	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.5	0.9	0.5	0.3	1.6	1.5	1.0	0.6	0.3
	5	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.6	1.5	0.9	0.5	0.3	1.6	1.5	0.9	0.5	0.3

Table 17(b)

Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for Weibull distribution, $W(1, 2.0)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	95.9	95.9	95.9	95.9	95.9	74.7	89.4	95.5	95.5	93.7	59.5	57.2	35.0	17.9	10.1	46.2	59.5	47.8	32.8	62.2
	3	95.9	95.9	95.9	95.9	95.9	82.1	91.8	95.7	95.7	95.6	59.5	57.2	35.0	17.9	10.1	50.8	58.6	41.8	23.6	15.5
	4	95.9	95.9	95.9	95.9	95.9	85.6	93.0	95.7	95.8	95.7	59.5	57.2	35.0	17.9	10.1	53.0	58.2	39.6	21.4	13.0
	5	95.9	95.9	95.9	95.9	95.9	87.7	93.6	95.8	95.8	95.8	59.5	57.2	35.0	17.9	10.1	54.3	58.0	38.5	20.4	12.1
20	2	58.4	58.4	58.4	58.4	58.4	47.7	56.6	58.2	58.0	56.3	43.6	39.5	23.0	12.3	7.2	35.4	41.6	28.8	19.5	34.3
	3	58.4	58.4	58.4	58.4	58.4	51.4	57.3	58.3	58.3	58.2	43.6	39.5	23.0	12.3	7.2	38.3	40.8	26.0	15.1	10.0
	4	58.4	58.4	58.4	58.4	58.4	53.2	57.6	58.3	58.3	58.3	43.6	39.5	23.0	12.3	7.2	39.6	40.4	25.0	14.0	8.7
	5	58.4	58.4	58.4	58.4	58.4	54.3	57.8	58.3	58.3	58.3	43.6	39.5	23.0	12.3	7.2	40.5	40.2	24.5	13.5	8.2
30	2	44.8	44.8	44.8	44.8	44.8	37.7	44.1	44.7	44.5	43.3	35.9	31.7	18.2	9.9	5.9	30.1	33.4	21.9	14.7	24.1
	3	44.8	44.8	44.8	44.8	44.8	40.2	44.4	44.7	44.7	44.7	35.9	31.7	18.2	9.9	5.9	32.1	32.8	20.2	11.7	7.8
	4	44.8	44.8	44.8	44.8	44.8	41.4	44.5	44.8	44.7	44.7	35.9	31.7	18.2	9.9	5.9	33.1	32.5	19.5	11.0	6.9
	5	44.8	44.8	44.8	44.8	44.8	42.1	44.6	44.8	44.8	44.8	35.9	31.7	18.2	9.9	5.9	33.7	32.3	19.2	10.7	6.6
50	2	32.2	32.2	32.2	32.2	32.2	28.2	32.0	32.1	32.0	31.2	27.5	23.8	13.6	7.5	4.6	24.0	25.0	15.7	10.3	15.4
	3	32.2	32.2	32.2	32.2	32.2	29.6	32.1	32.2	32.2	32.1	27.5	23.8	13.6	7.5	4.6	25.2	24.5	14.7	8.6	5.7
	4	32.2	32.2	32.2	32.2	32.2	30.3	32.1	32.2	32.2	32.2	27.5	23.8	13.6	7.5	4.6	25.8	24.3	14.4	8.2	5.1
	5	32.2	32.2	32.2	32.2	32.2	30.7	32.2	32.2	32.2	32.2	27.5	23.8	13.6	7.5	4.6	26.2	24.2	14.2	8.0	5.0
100	2	20.2	20.2	20.2	20.2	20.2	18.5	20.2	20.2	20.1	19.6	18.2	15.6	8.9	5.0	3.1	16.7	16.1	9.9	6.3	8.4
	3	20.2	20.2	20.2	20.2	20.2	19.1	20.2	20.2	20.2	20.2	18.2	15.6	8.9	5.0	3.1	17.2	15.9	9.4	5.5	3.6
	4	20.2	20.2	20.2	20.2	20.2	19.4	20.2	20.2	20.2	20.2	18.2	15.6	8.9	5.0	3.1	17.4	15.8	9.3	5.3	3.4
	5	20.2	20.2	20.2	20.2	20.2	19.6	20.2	20.2	20.2	20.2	18.2	15.6	8.9	5.0	3.1	17.6	15.7	9.2	5.3	3.3
300	2	8.6	8.6	8.6	8.6	8.6	8.3	8.6	8.6	8.6	8.5	8.0	6.8	4.0	2.3	1.4	7.8	7.0	4.3	2.7	3.3
	3	8.6	8.6	8.6	8.6	8.6	8.4	8.6	8.6	8.6	8.6	8.0	6.8	4.0	2.3	1.4	7.9	6.9	4.2	2.5	1.6
	4	8.6	8.6	8.6	8.6	8.6	8.5	8.6	8.6	8.6	8.6	8.0	6.8	4.0	2.3	1.4	7.9	6.9	4.1	2.4	1.5
	5	8.6	8.6	8.6	8.6	8.6	8.5	8.6	8.6	8.6	8.6	8.0	6.8	4.0	2.3	1.4	7.9	6.9	4.1	2.4	1.5
1000	2	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.5	3.1	2.7	1.6	0.9	0.6	3.1	2.7	1.7	1.1	1.3
	3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.1	2.7	1.6	0.9	0.6	3.1	2.7	1.6	1.0	0.6
	4	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.1	2.7	1.6	0.9	0.6	3.1	2.7	1.6	1.0	0.6
	5	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.1	2.7	1.6	0.9	0.6	3.1	2.7	1.6	0.9	0.6
3000	2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.5	1.2	1.1	0.6	0.4	0.2	1.2	1.1	0.7	0.5	0.6
	3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.1	0.6	0.4	0.2	1.2	1.1	0.7	0.4	0.3
	4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.1	0.6	0.4	0.2	1.2	1.1	0.7	0.4	0.3
	5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.1	0.6	0.4	0.2	1.2	1.1	0.6	0.4	0.2

Table 18(a)
Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for lognormal distribution, $LN(0, 1)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	452.0	452.0	452.0	452.0	452.0	447.4	448.8	450.3	449.3	444.8	243.9	329.8	110.8	51.7	29.0	250.3	334.4	110.9	49.2	20.4
	3	452.0	452.0	452.0	452.0	452.0	449.1	449.9	451.0	450.8	450.3	243.9	329.8	110.8	51.7	29.0	248.0	332.8	110.9	50.5	27.0
	4	452.0	452.0	452.0	452.0	452.0	449.9	450.5	451.3	451.2	451.0	243.9	329.8	110.8	51.7	29.0	246.9	332.0	110.9	50.9	27.9
	5	452.0	452.0	452.0	452.0	452.0	450.3	450.8	451.5	451.4	451.3	243.9	329.8	110.8	51.7	29.0	246.2	331.6	110.9	51.1	28.2
20	2	267.9	267.9	267.9	267.9	267.9	267.9	267.9	266.8	266.7	262.8	180.8	219.4	93.6	46.5	26.8	184.2	220.8	93.7	45.4	23.0
	3	267.9	267.9	267.9	267.9	267.9	267.9	267.9	267.2	267.2	266.7	180.8	219.4	93.6	46.5	26.8	182.9	220.3	93.7	46.0	25.9
	4	267.9	267.9	267.9	267.9	267.9	267.9	267.9	267.4	267.4	267.2	180.8	219.4	93.6	46.5	26.8	182.4	220.1	93.7	46.2	26.3
	5	267.9	267.9	267.9	267.9	267.9	267.9	267.9	267.5	267.6	267.4	180.8	219.4	93.6	46.5	26.8	182.0	219.9	93.7	46.3	26.4
30	2	201.6	201.6	201.6	201.6	201.6	201.9	200.9	200.8	200.3	198.1	148.6	173.0	82.4	42.4	24.8	150.7	173.6	82.5	41.8	22.4
	3	201.6	201.6	201.6	201.6	201.6	201.8	201.2	201.1	201.0	200.8	148.6	173.0	82.4	42.4	24.8	149.9	173.4	82.5	42.1	24.2
	4	201.6	201.6	201.6	201.6	201.6	201.7	201.3	201.2	201.2	201.1	148.6	173.0	82.4	42.4	24.8	149.6	173.3	82.5	42.2	24.5
	5	201.6	201.6	201.6	201.6	201.6	201.7	201.3	201.2	201.3	201.2	148.6	173.0	82.4	42.4	24.8	149.4	173.2	82.5	42.3	24.6
50	2	141.0	141.0	141.0	141.0	141.0	141.4	140.7	140.6	140.3	139.1	112.9	126.5	67.8	36.5	21.7	113.9	126.6	67.8	36.1	20.3
	3	141.0	141.0	141.0	141.0	141.0	141.2	140.8	140.8	140.7	140.6	112.9	126.5	67.8	36.5	21.7	113.5	126.5	67.8	36.3	21.3
	4	141.0	141.0	141.0	141.0	141.0	141.2	140.9	140.9	140.8	140.8	112.9	126.5	67.8	36.5	21.7	113.4	126.5	67.8	36.3	21.5
	5	141.0	141.0	141.0	141.0	141.0	141.2	140.9	140.9	140.9	140.9	112.9	126.5	67.8	36.5	21.7	113.3	126.5	67.8	36.4	21.5
100	2	85.0	85.0	85.0	85.0	85.0	85.1	84.8	84.8	84.7	84.1	73.1	79.1	48.1	27.3	16.5	73.4	79.1	48.0	27.1	15.8
	3	85.0	85.0	85.0	85.0	85.0	85.1	84.9	84.9	84.8	84.8	73.1	79.1	48.1	27.3	16.5	73.3	79.1	48.0	27.2	16.4
	4	85.0	85.0	85.0	85.0	85.0	85.1	84.9	84.9	84.9	84.9	73.1	79.1	48.1	27.3	16.5	73.2	79.1	48.0	27.2	16.4
	5	85.0	85.0	85.0	85.0	85.0	85.0	84.9	84.9	84.9	84.9	73.1	79.1	48.1	27.3	16.5	73.2	79.1	48.1	27.2	16.5
300	2	33.0	33.0	33.0	33.0	33.0	33.0	33.0	32.9	32.9	32.8	30.0	31.6	21.7	13.0	7.9	30.0	31.6	21.7	12.9	7.7
	3	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	32.9	30.0	31.6	21.7	13.0	7.9	30.0	31.6	21.7	12.9	7.9
	4	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	30.0	31.6	21.7	13.0	7.9	30.0	31.6	21.7	12.9	7.9
	5	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	30.0	31.6	21.7	13.0	7.9	30.0	31.6	21.7	12.9	7.9
1000	2	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	11.9	12.4	9.1	5.7	3.5	11.9	12.4	9.1	5.7	3.5
	3	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	11.9	12.4	9.1	5.7	3.5	11.9	12.4	9.1	5.7	3.5
	4	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	11.9	12.4	9.1	5.7	3.5	11.9	12.4	9.1	5.7	3.5
	5	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	11.9	12.4	9.1	5.7	3.5	11.9	12.4	9.1	5.7	3.5
3000	2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.1	4.8	5.0	3.8	2.4	1.5	4.8	5.0	3.8	2.4	1.5
	3	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	4.8	5.0	3.8	2.4	1.5	4.8	5.0	3.8	2.4	1.5
	4	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	4.8	5.0	3.8	2.4	1.5	4.8	5.0	3.8	2.4	1.5
	5	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	4.8	5.0	3.8	2.4	1.5	4.8	5.0	3.8	2.4	1.5

Table 18(b)

Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for lognormal distribution, $LN(0, 1/16)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	126.6	126.6	126.6	126.6	126.6	105.1	120.4	126.0	125.7	121.8	86.2	79.0	44.0	22.3	12.7	72.6	81.9	55.7	35.0	54.0
	3	126.6	126.6	126.6	126.6	126.6	112.5	122.8	126.3	126.2	126.1	86.2	79.0	44.0	22.3	12.7	77.3	80.8	50.2	27.2	17.2
	4	126.6	126.6	126.6	126.6	126.6	116.2	123.8	126.4	126.4	126.3	86.2	79.0	44.0	22.3	12.7	79.6	80.3	48.2	25.3	15.1
	5	126.6	126.6	126.6	126.6	126.6	118.3	124.4	126.4	126.4	126.4	86.2	79.0	44.0	22.3	12.7	81.0	80.0	47.2	24.5	14.3
20	2	78.6	78.6	78.6	78.6	78.6	68.3	77.0	78.3	78.0	75.3	63.2	55.4	30.8	16.5	9.7	55.5	57.6	35.8	22.5	30.9
	3	78.6	78.6	78.6	78.6	78.6	71.9	77.6	78.4	78.4	78.3	63.2	55.4	30.8	16.5	9.7	58.0	56.8	35.5	18.8	12.0
	4	78.6	78.6	78.6	78.6	78.6	73.6	77.9	78.5	78.5	78.4	63.2	55.4	30.8	16.5	9.7	59.3	56.4	32.6	17.9	11.0
	5	78.6	78.6	78.6	78.6	78.6	74.6	78.0	78.5	78.5	78.5	63.2	55.4	30.8	16.5	9.7	60.1	56.2	32.2	17.5	10.6
30	2	60.8	60.8	60.8	60.8	60.8	54.2	60.1	60.5	60.3	58.5	51.8	44.9	25.2	13.8	8.3	46.4	46.5	28.3	17.7	22.3
	3	60.8	60.8	60.8	60.8	60.8	56.5	60.4	60.6	60.6	60.5	51.8	44.9	25.2	13.8	8.3	48.3	45.9	26.8	15.3	9.8
	4	60.8	60.8	60.8	60.8	60.8	57.6	60.5	60.7	60.7	60.6	51.8	44.9	25.2	13.8	8.3	49.2	45.6	26.3	14.7	9.1
	5	60.8	60.8	60.8	60.8	60.8	58.2	60.5	60.7	60.7	60.7	51.8	44.9	25.2	13.8	8.3	49.8	45.5	26.0	14.5	8.8
50	2	44.0	44.0	44.0	44.0	44.0	40.4	43.8	43.8	43.7	42.6	39.4	34.0	19.4	10.9	6.6	36.4	34.9	21.1	13.1	14.9
	3	44.0	44.0	44.0	44.0	44.0	41.6	43.9	43.9	43.9	43.8	39.4	34.0	19.4	10.9	6.6	37.4	34.6	20.3	11.7	7.5
	4	44.0	44.0	44.0	44.0	44.0	42.2	43.9	43.9	43.9	43.9	39.4	34.0	19.4	10.9	6.6	38.0	34.4	20.0	11.4	7.1
	5	44.0	44.0	44.0	44.0	44.0	42.6	43.9	43.9	43.9	43.9	39.4	34.0	19.4	10.9	6.6	38.3	34.3	19.9	11.3	7.0
100	2	27.7	27.7	27.7	27.7	27.7	26.3	27.6	27.6	27.5	26.9	25.8	22.3	13.1	7.5	4.7	24.6	22.7	13.9	8.6	8.6
	3	27.7	27.7	27.7	27.7	27.7	26.8	27.6	27.6	27.6	27.6	25.8	22.3	13.1	7.5	4.7	25.0	22.5	13.5	7.9	5.1
	4	27.7	27.7	27.7	27.7	27.7	27.0	27.6	27.6	27.6	27.6	25.8	22.3	13.1	7.5	4.7	25.2	22.4	13.4	7.8	4.9
	5	27.7	27.7	27.7	27.7	27.7	27.1	27.6	27.6	27.6	27.6	25.8	22.3	13.1	7.5	4.7	25.3	22.4	13.3	7.7	4.8
300	2	11.6	11.6	11.6	11.6	11.6	11.4	11.6	11.6	11.5	11.4	11.4	11.6	11.6	11.5	11.4	11.0	9.8	6.1	3.8	3.5
	3	11.6	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.0	9.7	6.0	3.6	2.3
	4	11.6	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.0	9.7	6.0	3.6	2.3
	5	11.6	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.5	11.6	11.6	11.6	11.6	11.0	9.7	6.0	3.5	2.2
1000	2	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.3	3.8	2.4	1.4	0.9	4.3	3.8	2.4	1.5	1.4
	3	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.3	3.8	2.4	1.4	0.9	4.3	3.8	2.4	1.5	1.0
	4	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.3	3.8	2.4	1.4	0.9	4.3	3.8	2.4	1.4	0.9
	5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.3	3.8	2.4	1.4	0.9	4.3	3.8	2.4	1.4	0.9
3000	2	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.9	1.7	1.5	1.0	0.6	0.4	1.7	1.5	1.0	0.6	0.4
	3	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.5	1.0	0.6	0.4	1.7	1.5	1.0	0.6	0.4
	4	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.5	1.0	0.6	0.4	1.7	1.5	1.0	0.6	0.4
	5	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.5	1.0	0.6	0.4	1.7	1.5	1.0	0.6	0.4

Table 19(a)
Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for triangular distribution, $T(0, 1, 1/4)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	58.7	58.7	58.7	58.7	58.7	40.6	50.6	55.8	53.6	35.1	27.9	31.2	23.3	12.0	6.6	16.9	31.5	34.7	25.3	45.3
	3	58.7	58.7	58.7	58.7	58.7	47.0	53.7	57.2	56.7	55.9	27.9	31.2	23.3	12.0	6.6	20.7	31.4	29.4	17.2	11.2
	4	58.7	58.7	58.7	58.7	58.7	50.0	55.1	57.7	57.5	57.2	27.9	31.2	23.3	12.0	6.6	22.6	31.4	27.5	15.3	9.0
	5	58.7	58.7	58.7	58.7	58.7	51.8	55.9	57.9	57.8	57.7	27.9	31.2	23.3	12.0	6.6	23.7	31.3	26.5	14.4	8.2
20	2	32.4	32.4	32.4	32.4	32.4	23.0	29.4	30.9	29.6	19.4	19.0	19.4	13.5	7.4	4.2	12.1	20.4	19.0	14.1	25.4
	3	32.4	32.4	32.4	32.4	32.4	26.3	30.5	31.6	31.3	30.9	19.0	19.4	13.5	7.4	4.2	14.5	20.0	16.4	10.0	6.8
	4	32.4	32.4	32.4	32.4	32.4	27.9	31.1	31.9	31.7	31.6	19.0	19.4	13.5	7.4	4.2	15.7	19.9	15.5	9.0	5.6
	5	32.4	32.4	32.4	32.4	32.4	28.8	31.4	32.0	31.9	31.9	19.0	19.4	13.5	7.4	4.2	16.4	19.8	15.0	8.5	5.1
30	2	23.6	23.6	23.6	23.6	23.6	17.3	22.0	22.5	21.7	14.8	15.1	14.8	10.0	5.5	3.2	10.2	15.9	13.5	10.1	17.8
	3	23.6	23.6	23.6	23.6	23.6	19.5	22.6	23.0	22.8	22.5	15.1	14.8	10.0	5.5	3.2	11.9	15.5	11.9	7.3	5.0
	4	23.6	23.6	23.6	23.6	23.6	20.5	22.8	23.2	23.1	23.0	15.1	14.8	10.0	5.5	3.2	12.7	15.3	11.3	6.6	4.2
	5	23.6	23.6	23.6	23.6	23.6	21.1	23.0	23.3	23.2	23.2	15.1	14.8	10.0	5.5	3.2	13.2	15.2	11.0	6.3	3.9
50	2	15.9	15.9	15.9	15.9	15.9	12.3	15.2	15.3	14.8	10.7	11.1	10.5	6.8	3.8	2.3	8.2	11.4	8.9	6.6	11.2
	3	15.9	15.9	15.9	15.9	15.9	13.5	15.5	15.6	15.5	15.3	11.1	10.5	6.8	3.8	2.3	9.2	11.0	7.9	4.9	3.3
	4	15.9	15.9	15.9	15.9	15.9	14.1	15.6	15.7	15.6	15.6	11.1	10.5	6.8	3.8	2.3	9.7	10.9	7.6	4.5	2.8
	5	15.9	15.9	15.9	15.9	15.9	14.5	15.7	15.7	15.7	15.7	11.1	10.5	6.8	3.8	2.3	10.0	10.8	7.4	4.3	2.7
100	2	9.3	9.3	9.3	9.3	9.3	7.8	9.1	9.0	8.7	6.6	7.1	6.4	4.0	2.3	1.4	5.9	6.9	5.0	3.6	5.8
	3	9.3	9.3	9.3	9.3	9.3	8.3	9.2	9.1	9.1	9.0	7.1	6.4	4.0	2.3	1.4	6.3	6.7	4.6	2.8	1.9
	4	9.3	9.3	9.3	9.3	9.3	8.6	9.2	9.2	9.2	9.1	7.1	6.4	4.0	2.3	1.4	6.5	6.7	4.4	2.6	1.6
	5	9.3	9.3	9.3	9.3	9.3	8.7	9.2	9.2	9.2	9.2	7.1	6.4	4.0	2.3	1.4	6.6	6.6	4.3	2.5	1.6
300	2	3.8	3.8	3.8	3.8	3.8	3.6	3.7	3.7	3.6	3.6	3.1	2.7	1.7	0.9	0.6	3.0	2.9	2.0	1.4	2.1
	3	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.7	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.8	1.1	0.8
	4	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.7	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.8	1.1	0.7
	5	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.7	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.8	1.0	0.6
1000	2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.1	1.0	0.6	0.4	0.2	1.2	1.1	0.7	0.5	0.8
	3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.4	0.2	1.2	1.0	0.7	0.4	0.3
	4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.4	0.2	1.1	1.0	0.7	0.4	0.3
	5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.4	0.2	1.1	1.0	0.6	0.4	0.2
3000	2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.3	0.2	0.4
	3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.3	0.2	0.1
	4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.3	0.2	0.1
	5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.2	0.2	0.1

Table 19(b)
Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$, and $\widehat{C}_{N_{pmk}}$ for triangular distribution, $T(0, 1, 3/4)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{N_p}$					$\widehat{C}_{N_{pk}}$					$\widehat{C}_{N_{pm}}$					$\widehat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	58.7	58.7	58.7	58.7	58.7	40.6	50.6	55.7	53.5	34.5	27.9	31.3	23.3	12.0	6.5	16.9	31.4	34.6	25.2	45.0
	3	58.7	58.7	58.7	58.7	58.7	46.9	53.6	57.1	56.7	55.8	27.9	31.3	23.3	12.0	6.5	20.8	31.3	29.3	17.1	11.2
	4	58.7	58.7	58.7	58.7	58.7	50.0	55.0	57.6	57.4	57.1	27.9	31.3	23.3	12.0	6.5	22.6	31.3	27.4	15.2	9.0
	5	58.7	58.7	58.7	58.7	58.7	51.8	55.8	57.9	57.8	57.6	27.9	31.3	23.3	12.0	6.5	23.7	31.2	26.4	14.3	8.2
20	2	32.4	32.4	32.4	32.4	32.4	23.0	29.4	30.8	29.5	18.9	19.0	19.4	13.5	7.5	4.2	12.1	20.3	18.9	14.0	25.0
	3	32.4	32.4	32.4	32.4	32.4	26.3	30.5	31.5	31.3	30.8	19.0	19.4	13.5	7.5	4.2	14.5	20.0	16.4	9.9	6.7
	4	32.4	32.4	32.4	32.4	32.4	27.9	31.0	31.8	31.7	31.5	19.0	19.4	13.5	7.5	4.2	15.7	19.8	15.4	8.9	5.5
	5	32.4	32.4	32.4	32.4	32.4	28.8	31.3	32.0	31.9	31.8	19.0	19.4	13.5	7.5	4.2	16.4	19.7	15.0	8.5	5.1
30	2	23.5	23.5	23.5	23.5	23.5	17.3	21.9	22.4	21.6	14.4	15.1	14.8	9.9	5.5	3.2	10.2	15.8	13.5	10.0	17.5
	3	23.5	23.5	23.5	23.5	23.5	19.5	22.6	23.0	22.8	22.4	15.1	14.8	9.9	5.5	3.2	11.9	15.4	11.8	7.2	4.9
	4	23.5	23.5	23.5	23.5	23.5	20.5	22.8	23.1	23.1	23.0	15.1	14.8	9.9	5.5	3.2	12.8	15.2	11.2	6.6	4.1
	5	23.5	23.5	23.5	23.5	23.5	21.1	23.0	23.2	23.2	23.1	15.1	14.8	9.9	5.5	3.2	13.2	15.1	10.9	6.3	3.8
50	2	15.9	15.9	15.9	15.9	15.9	12.3	15.2	15.2	14.7	10.4	11.2	10.5	6.8	3.8	2.3	8.2	11.3	8.9	6.5	11.0
	3	15.9	15.9	15.9	15.9	15.9	13.5	15.5	15.5	15.4	15.2	11.2	10.5	6.8	3.8	2.3	9.2	11.0	7.9	4.9	3.3
	4	15.9	15.9	15.9	15.9	15.9	14.1	15.6	15.7	15.6	15.5	11.2	10.5	6.8	3.8	2.3	9.7	10.8	7.6	4.5	2.8
	5	15.9	15.9	15.9	15.9	15.9	14.5	15.7	15.7	15.7	15.7	11.2	10.5	6.8	3.8	2.3	10.0	10.8	7.4	4.3	2.6
100	2	9.3	9.3	9.3	9.3	9.3	7.8	9.1	9.0	8.7	6.7	7.1	6.4	4.0	2.3	1.4	5.8	6.9	5.0	3.6	5.9
	3	9.3	9.3	9.3	9.3	9.3	8.3	9.1	9.1	9.1	9.0	7.1	6.4	4.0	2.3	1.4	6.3	6.7	4.6	2.8	1.9
	4	9.3	9.3	9.3	9.3	9.3	8.6	9.2	9.2	9.2	9.1	7.1	6.4	4.0	2.3	1.4	6.5	6.6	4.4	2.6	1.7
	5	9.3	9.3	9.3	9.3	9.3	8.7	9.2	9.2	9.2	9.2	7.1	6.4	4.0	2.3	1.4	6.6	6.6	4.3	2.5	1.6
300	2	3.8	3.8	3.8	3.8	3.8	3.6	3.7	3.6	3.6	2.8	3.1	2.7	1.7	0.9	0.6	3.0	2.9	2.0	1.4	2.0
	3	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.6	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.8	1.1	0.7
	4	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.7	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.8	1.0	0.6
	5	3.8	3.8	3.8	3.8	3.8	3.7	3.7	3.7	3.7	3.7	3.1	2.7	1.7	0.9	0.6	3.0	2.8	1.7	1.0	0.6
1000	2	1.3	1.3	1.3	1.3	1.3	1.4	1.3	1.2	1.2	0.9	1.1	1.0	0.6	0.3	0.2	1.2	1.0	0.7	0.4	0.5
	3	1.3	1.3	1.3	1.3	1.3	1.4	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.3	0.2	1.2	1.0	0.6	0.4	0.2
	4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.3	0.2	1.2	1.0	0.6	0.4	0.2
	5	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.1	1.0	0.6	0.3	0.2	1.2	1.0	0.6	0.3	0.2
3000	2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.3	0.4	0.4	0.2	0.1	0.1	0.5	0.4	0.2	0.1	0.1
	3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.2	0.1	0.1
	4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.2	0.1	0.1
	5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.2	0.1	0.1	0.4	0.4	0.2	0.1	0.1

Table 20
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for normal distribution, $N(0, 1)$

$\|(\mu - T)/\sigma\|$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	110.3	110.3	110.3	110.3	110.3	79.4	106.2	110.2	110.4	* * *	75.0	60.4	35.1	19.3	11.6	52.7	66.8	50.0	40.4	* * *
	3	110.3	110.3	110.3	110.3	110.3	89.7	107.8	110.2	110.3	110.3	75.0	60.4	35.1	19.3	11.6	60.1	64.3	42.5	26.3	19.2
	4	110.3	110.3	110.3	110.3	110.3	94.8	108.5	110.2	110.3	110.3	75.0	60.4	35.1	19.3	11.6	63.9	63.2	40.1	23.5	15.4
	5	110.3	110.3	110.3	110.3	110.3	97.9	108.9	110.2	110.3	110.3	75.0	60.4	35.1	19.3	11.6	66.1	62.2	38.8	22.3	14.1
20	2	67.9	67.9	67.9	67.9	67.9	49.8	67.2	67.9	68.0	* * *	54.6	42.2	23.9	13.5	8.3	39.1	46.1	30.5	23.3	* * *
	3	67.9	67.9	67.9	67.9	67.9	55.9	67.5	67.9	67.9	67.9	54.6	42.2	23.9	13.5	8.3	44.3	44.6	27.2	16.7	12.0
	4	67.9	67.9	67.9	67.9	67.9	58.9	67.6	67.9	67.9	67.9	54.6	42.2	23.9	13.5	8.3	46.9	43.9	26.1	15.4	10.2
	5	67.9	67.9	67.9	67.9	67.9	60.7	67.7	67.9	67.9	67.9	54.6	42.2	23.9	13.5	8.3	48.4	43.5	25.6	14.9	9.5
30	2	52.3	52.3	52.3	52.3	52.3	38.7	52.1	52.3	52.3	* * *	44.7	34.1	19.3	11.0	6.9	32.4	36.8	23.5	17.3	* * *
	3	52.3	52.3	52.3	52.3	52.3	43.2	52.2	52.3	52.3	52.3	44.7	34.1	19.3	11.0	6.9	36.5	35.7	21.4	13.1	9.3
	4	52.3	52.3	52.3	52.3	52.3	45.5	52.2	52.3	52.3	52.3	44.7	34.1	19.3	11.0	6.9	38.6	35.3	20.7	12.3	8.1
	5	52.3	52.3	52.3	52.3	52.3	46.8	52.2	52.3	52.3	52.3	44.7	34.1	19.3	11.0	6.9	39.8	35.0	20.4	11.9	7.7
50	2	37.6	37.6	37.6	37.6	37.6	28.1	37.6	37.7	37.7	* * *	33.9	25.8	14.7	8.5	5.4	24.9	27.3	17.0	12.1	* * *
	3	37.6	37.6	37.6	37.6	37.6	31.3	37.6	37.7	37.7	37.7	33.9	25.8	14.7	8.5	5.4	27.9	26.7	15.9	9.7	6.8
	4	37.6	37.6	37.6	37.6	37.6	32.9	37.6	37.7	37.7	37.7	33.9	25.8	14.7	8.5	5.4	29.4	26.4	15.5	9.2	6.1
	5	37.6	37.6	37.6	37.6	37.6	33.8	37.6	37.7	37.7	37.7	33.9	25.8	14.7	8.5	5.4	30.3	26.3	15.3	9.0	5.8
100	2	23.6	23.6	23.6	23.6	23.6	17.4	23.6	23.6	23.6	* * *	22.1	16.9	9.8	5.8	3.7	16.1	17.5	10.9	7.5	* * *
	3	23.6	23.6	23.6	23.6	23.6	19.5	23.6	23.6	23.6	23.6	22.1	16.9	9.8	5.8	3.7	18.1	17.3	10.4	6.3	4.4
	4	23.6	23.6	23.6	23.6	23.6	20.5	23.6	23.6	23.6	23.6	22.1	16.9	9.8	5.8	3.7	19.1	17.2	10.2	6.1	4.0
	5	23.6	23.6	23.6	23.6	23.6	21.1	23.6	23.6	23.6	23.6	22.1	16.9	9.8	5.8	3.7	19.7	17.1	10.1	6.0	3.9
300	2	9.8	9.8	9.8	9.8	9.8	6.7	9.8	9.8	9.8	* * *	9.5	7.4	4.4	2.7	1.7	6.3	7.6	4.7	3.2	* * *
	3	9.8	9.8	9.8	9.8	9.8	7.7	9.8	9.8	9.8	9.8	9.5	7.4	4.4	2.7	1.7	7.4	7.5	4.6	2.8	1.9
	4	9.8	9.8	9.8	9.8	9.8	8.3	9.8	9.8	9.8	9.8	9.5	7.4	4.4	2.7	1.7	7.9	7.5	4.5	2.8	1.8
	5	9.8	9.8	9.8	9.8	9.8	8.6	9.8	9.8	9.8	9.8	9.5	7.4	4.4	2.7	1.7	8.2	7.4	4.5	2.7	1.8
1000	2	3.8	3.8	3.8	3.8	3.8	2.1	3.8	3.8	3.8	* * *	3.7	2.9	1.8	1.1	0.7	2.0	3.0	1.9	1.3	* * *
	3	3.8	3.8	3.8	3.8	3.8	2.7	3.8	3.8	3.8	3.8	3.7	2.9	1.8	1.1	0.7	2.6	2.9	1.8	1.1	0.8
	4	3.8	3.8	3.8	3.8	3.8	2.9	3.8	3.8	3.8	3.8	3.7	2.9	1.8	1.1	0.7	2.9	2.9	1.8	1.1	0.7
	5	3.8	3.8	3.8	3.8	3.8	3.1	3.8	3.8	3.8	3.8	3.7	2.9	1.8	1.1	0.7	3.0	2.9	1.8	1.1	0.7
3000	2	1.5	1.5	1.5	1.5	1.5	0.5	1.5	1.5	1.5	* * *	1.4	1.1	0.7	0.4	0.3	0.5	1.2	0.7	0.5	* * *
	3	1.5	1.5	1.5	1.5	1.5	0.9	1.5	1.5	1.5	1.5	1.4	1.1	0.7	0.4	0.3	0.8	1.2	0.7	0.5	0.3
	4	1.5	1.5	1.5	1.5	1.5	1.0	1.5	1.5	1.5	1.5	1.4	1.1	0.7	0.4	0.3	1.0	1.2	0.7	0.4	0.3
	5	1.5	1.5	1.5	1.5	1.5	1.1	1.5	1.5	1.5	1.5	1.4	1.1	0.7	0.4	0.3	1.1	1.2	0.7	0.4	0.3

Table 21
Relative bias (%) of $\widehat{C}_{Np}$, $\widehat{C}_{Npk}$, $\widehat{C}_{Npm}$, and $\widehat{C}_{Npmk}$ for uniform distribution, $U(0, 1)$

$\lvert(\mu - T)/\sigma\rvert$		$\widehat{C}_{Np}$					$\widehat{C}_{Npk}$					$\widehat{C}_{Npm}$					$\widehat{C}_{Npmk}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	25.0	25.0	25.0	25.0	25.0	0.6	18.9	24.5	24.9	∗∗∗	−3.9	10.6	17.8	11.7	6.9	−19.4	14.5	38.8	46.0	∗∗∗
	3	25.0	25.0	25.0	25.0	25.0	8.8	21.3	24.7	25.0	24.9	−3.9	10.6	17.8	11.7	6.9	−14.2	12.9	28.3	23.1	19.4
	4	25.0	25.0	25.0	25.0	25.0	12.8	22.4	24.8	25.0	25.0	−3.9	10.6	17.8	11.7	6.9	−11.6	12.3	24.8	18.5	132.2
	5	25.0	25.0	25.0	25.0	25.0	15.2	23.0	24.9	25.0	25.0	−3.9	10.6	17.8	11.7	6.9	−10.1	11.9	23.0	16.6	11.1
20	2	11.1	11.1	11.1	11.1	11.1	−5.1	9.1	11.0	11.0	∗∗∗	−4.5	6.0	9.2	5.9	3.6	−16.8	9.9	20.5	23.3	∗∗∗
	3	11.1	11.1	11.1	11.1	11.1	0.3	9.9	11.1	11.1	11.1	−4.5	6.0	9.2	5.9	3.6	−12.7	8.3	14.8	11.7	10.2
	4	11.1	11.1	11.1	11.1	11.1	3.0	10.3	11.1	11.1	11.1	−4.5	6.0	9.2	5.9	3.6	−10.7	7.7	13.0	9.4	6.9
	5	11.1	11.1	11.1	11.1	11.1	4.6	10.4	11.1	11.1	11.1	−4.5	6.0	9.2	5.9	3.6	−9.5	7.3	12.0	8.4	5.8
30	2	7.1	7.1	7.1	7.1	7.1	−5.8	6.3	7.1	7.1	∗∗∗	−4.0	4.4	6.3	4.0	2.5	−14.6	7.9	13.8	15.6	∗∗∗
	3	7.1	7.1	7.1	7.1	7.1	−1.5	6.6	7.1	7.1	7.1	−4.0	4.4	6.3	4.0	2.5	−11.1	6.5	10.0	7.8	6.9
	4	7.1	7.1	7.1	7.1	7.1	0.7	6.8	7.1	7.1	7.1	−4.0	4.4	6.3	4.0	2.5	−9.3	5.9	8.8	6.3	4.7
	5	7.1	7.1	7.1	7.1	7.1	1.9	6.9	7.1	7.1	7.1	−4.0	4.4	6.3	4.0	2.5	−8.3	5.6	8.1	5.6	4.0
50	2	4.2	4.2	4.2	4.2	4.2	−5.8	4.0	4.2	4.1	∗∗∗	−3.1	2.9	3.8	2.4	1.5	−11.8	5.7	8.4	9.4	∗∗∗
	3	4.2	4.2	4.2	4.2	4.2	4.0	4.2	4.2	4.2	4.2	−3.1	2.9	3.8	2.4	1.5	−8.9	4.6	6.1	4.7	4.2
	4	4.2	4.2	4.2	4.2	4.2	4.1	4.2	4.2	4.2	4.2	−3.1	2.9	3.8	2.4	1.5	−7.4	4.1	5.3	3.8	2.9
	5	4.2	4.2	4.2	4.2	4.2	4.1	4.2	4.2	4.2	4.2	−3.1	2.9	3.8	2.4	1.5	−6.6	3.8	4.9	3.4	2.4
100	2	2.0	2.0	2.0	2.0	2.0	−4.9	2.0	2.0	2.1	∗∗∗	−1.9	1.7	1.9	1.2	0.8	−8.4	3.3	4.2	4.7	∗∗∗
	3	2.0	2.0	2.0	2.0	2.0	−2.6	2.0	2.0	2.0	2.0	−1.9	1.7	1.9	1.2	0.8	−6.3	2.6	3.1	2.4	2.2
	4	2.0	2.0	2.0	2.0	2.0	−1.5	2.0	2.0	2.0	2.0	−1.9	1.7	1.9	1.2	0.8	−5.2	2.3	2.7	1.9	1.5
	5	2.0	2.0	2.0	2.0	2.0	−0.8	2.0	2.0	2.0	2.0	−1.9	1.7	1.9	1.2	0.8	−4.5	2.2	2.5	1.7	1.2
300	2	0.7	0.7	0.7	0.7	0.7	−.3.3	0.7	0.7	0.7	∗∗∗	−0.8	0.6	0.6	0.4	0.3	−4.7	1.2	1.4	1.5	∗∗∗
	3	0.7	0.7	0.7	0.7	0.7	−2.0	0.7	0.7	0.7	0.7	−0.8	0.6	0.6	0.4	0.3	−3.4	0.9	1.0	0.8	0.7
	4	0.7	0.7	0.7	0.7	0.7	−1.3	0.7	0.7	0.7	0.7	−0.8	0.6	0.6	0.4	0.3	−2.7	0.8	0.9	0.6	0.5
	5	0.7	0.7	0.7	0.7	0.7	−0.9	0.7	0.7	0.7	0.7	−0.8	0.6	0.6	0.4	0.3	−2.3	0.8	0.8	0.6	0.4
1000	2	0.2	0.2	0.2	0.2	0.2	−2.0	0.2	0.2	0.2	∗∗∗	−0.3	0.2	0.2	0.1	0.1	−2.4	0.3	0.4	0.4	∗∗∗
	3	0.2	0.2	0.2	0.2	0.2	−1.3	0.2	0.2	0.2	0.2	−0.3	0.2	0.2	0.1	0.1	−1.7	0.3	0.3	0.2	0.2
	4	0.2	0.2	0.2	0.2	0.2	−0.9	0.2	0.2	0.2	0.2	−0.3	0.2	0.2	0.1	0.1	−1.3	0.2	0.2	0.2	0.1
	5	0.2	0.2	0.2	0.2	0.2	−0.7	0.2	0.2	0.2	0.2	−0.3	0.2	0.2	0.1	0.1	−1.1	0.2	0.2	0.2	0.1
3000	2	0.1	0.1	0.1	0.1	0.1	−1.2	0.1	0.1	0.0	∗∗∗	−0.1	0.0	0.1	0.0	0.0	−1.3	0.1	0.1	0.1	∗∗∗
	3	0.1	0.1	0.1	0.1	0.1	−0.8	0.1	0.1	0.1	0.1	−0.1	0.0	0.1	0.0	0.0	−0.9	0.1	0.1	0.1	0.0
	4	0.1	0.1	0.1	0.1	0.1	−0.6	0.1	0.1	0.1	0.1	−0.1	0.0	0.1	0.0	0.0	−0.7	0.1	0.1	0.1	0.0
	5	0.1	0.1	0.1	0.1	0.1	−0.4	0.1	0.1	0.1	0.1	−0.1	0.0	0.1	0.0	0.0	−0.6	0.1	0.1	0.1	0.0

Table 22
Relative bias (%) of $\widehat{C}_{N_p}$, $\widehat{C}_{N_{pk}}$, $\widehat{C}_{N_{pm}}$ and $\widehat{C}_{N_{pmk}}$ for Laplace distribution, $L(0, 1)$

$\lvert(\mu - T)/\sigma\rvert$		$\hat{C}_{N_p}$					$\hat{C}_{N_{pk}}$					$\hat{C}_{N_{pm}}$					$\hat{C}_{N_{pmk}}$				
n	d/σ	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0	0.0	0.5	1.0	1.5	2.0
10	2	207.6	207.6	207.6	207.6	207.6	176.9	205.9	207.6	207.7	***	171.5	113.6	57.3	31.5	19.4	147.4	120.4	66.4	43.3	***
	3	207.6	207.6	207.6	207.6	207.6	187.2	206.6	207.6	207.7	207.7	171.5	113.6	57.3	31.5	19.4	155.4	117.7	61.8	35.4	23.6
	4	207.6	207.6	207.6	207.6	207.6	192.3	206.9	207.6	207.7	207.7	171.5	113.6	57.3	31.5	19.4	159.4	116.5	60.3	33.9	21.5
	5	207.6	207.6	207.6	207.6	207.6	195.4	207.1	207.6	207.7	207.7	171.5	113.6	57.3	31.5	19.4	161.8	115.9	59.6	33.2	20.8
20	2	124.2	124.2	124.2	124.2	124.2	108.6	124.1	124.2	124.3	***	115.0	80.8	43.8	25.2	15.8	100.7	83.5	47.1	29.8	***
	3	124.2	124.2	124.2	124.2	124.2	113.8	124.1	124.2	124.2	124.2	115.0	80.8	43.8	25.2	15.8	105.5	82.4	45.5	26.7	17.6
	4	124.2	124.2	124.2	124.2	124.2	116.4	124.1	124.2	124.2	124.2	115.0	80.8	43.8	25.2	15.8	107.8	82.0	44.9	26.1	16.7
	5	124.2	124.2	124.2	124.2	124.2	117.9	124.1	124.2	124.2	124.2	115.0	80.8	43.8	25.2	15.8	109.3	81.7	44.6	25.8	16.4
30	2	94.0	94.0	94.0	94.0	94.0	83.1	94.0	94.0	94.0	***	89.8	65.6	37.2	21.9	13.9	79.4	67.1	39.0	24.6	***
	3	94.0	94.0	94.0	94.0	94.0	86.7	94.0	94.0	94.0	94.0	89.8	65.6	37.2	21.9	13.9	82.9	66.5	38.1	22.8	15.0
	4	94.0	94.0	94.0	94.0	94.0	88.5	94.0	94.0	94.0	94.0	89.8	65.6	37.2	21.9	13.9	84.6	66.2	37.8	22.4	14.5
	5	94.0	94.0	94.0	94.0	94.0	89.6	94.0	94.0	94.0	94.0	89.8	65.6	37.2	21.9	13.9	85.7	66.1	37.6	22.3	14.3
50	2	66.3	66.3	66.3	66.3	66.3	59.2	66.3	66.3	66.4	***	64.8	49.5	29.5	17.9	11.6	57.8	50.1	30.5	19.3	***
	3	66.3	66.3	66.3	66.3	66.3	61.6	66.3	66.3	66.3	66.3	64.8	49.5	29.5	17.9	11.6	60.1	49.9	30.3	18.4	12.1
	4	66.3	66.3	66.3	66.3	66.3	62.7	66.3	66.3	66.3	66.3	64.8	49.5	29.5	17.9	11.6	61.3	49.7	29.8	18.2	11.8
	5	66.3	66.3	66.3	66.3	66.3	63.5	66.3	66.3	66.3	66.3	64.8	49.5	29.5	17.9	11.6	62.0	49.7	29.8	18.1	11.7
100	2	40.4	40.4	40.4	40.4	40.4	36.2	40.4	40.4	40.4	***	40.0	32.0	20.3	12.7	8.4	35.8	32.3	20.7	13.3	***
	3	40.4	40.4	40.4	40.4	40.4	37.6	40.4	40.4	40.4	40.4	40.0	32.0	20.3	12.7	8.4	37.2	32.2	20.5	12.9	8.6
	4	40.4	40.4	40.4	40.4	40.4	38.3	40.4	40.4	40.4	40.4	40.0	32.0	20.3	12.7	8.4	37.9	32.1	20.4	12.8	8.5
	5	40.4	40.4	40.4	40.4	40.4	38.7	40.4	40.4	40.4	40.4	40.0	32.0	20.3	12.7	8.4	38.3	32.1	20.4	12.8	8.5
300	2	16.1	16.1	16.1	16.1	16.1	14.2	16.1	16.1	16.1	***	16.0	13.5	9.1	5.9	4.0	14.1	13.5	9.2	6.1	***
	3	16.1	16.1	16.1	16.1	16.1	14.8	16.1	16.1	16.1	16.1	16.0	13.5	9.1	5.9	4.0	14.7	13.5	9.2	6.0	4.1
	4	16.1	16.1	16.1	16.1	16.1	15.1	16.1	16.1	16.1	16.1	16.0	13.5	9.1	5.9	4.0	15.1	13.5	9.2	6.0	4.0
	5	16.1	16.1	16.1	16.1	16.1	15.3	16.1	16.1	16.1	16.1	16.0	13.5	9.1	5.9	4.0	15.3	13.5	9.2	6.0	4.0
1000	2	6.2	6.2	6.2	6.2	6.2	5.2	6.2	6.2	6.2	***	6.1	5.3	3.7	2.4	1.7	5.2	5.3	3.7	2.5	***
	3	6.2	6.2	6.2	6.2	6.2	5.5	6.2	6.2	6.2	6.2	6.1	5.3	3.7	2.4	1.7	5.5	5.3	3.7	2.5	1.7
	4	6.2	6.2	6.2	6.2	6.2	5.7	6.2	6.2	6.2	6.2	6.1	5.3	3.7	2.4	1.7	5.7	5.3	3.7	2.5	1.7
	5	6.2	6.2	6.2	6.2	6.2	5.8	6.2	6.2	6.2	6.2	6.1	5.3	3.7	2.4	1.7	5.8	5.3	3.7	2.4	1.7
3000	2	2.4	2.4	2.4	2.4	2.4	1.9	2.4	2.4	2.5	***	2.4	2.1	1.5	1.0	0.7	1.9	2.1	1.5	1.0	***
	3	2.4	2.4	2.4	2.4	2.4	2.1	2.4	2.4	2.4	2.4	2.4	2.1	1.5	1.0	0.7	2.1	2.1	1.5	1.0	0.7
	4	2.4	2.4	2.4	2.4	2.4	2.2	2.4	2.4	2.4	2.4	2.4	2.1	1.5	1.0	0.7	2.2	2.1	1.5	1.0	0.7
	5	2.4	2.4	2.4	2.4	2.4	2.2	2.4	2.4	2.4	2.4	2.4	2.1	1.5	1.0	0.7	2.2	2.1	1.5	1.0	0.7

Table 23
Example data taken from Polansky (1998)

0.684	0.347	0.527	0.572	0.231	0.388	0.221	0.577	0.660	0.685
0.261	0.345	0.376	0.779	0.624	0.198	0.425	0.513	0.415	0.335
0.557	0.274	0.401	0.447	0.359	0.461	0.486	0.517	0.202	0.388
0.107	0.838	0.414	0.721	0.193	0.423	0.637	0.350	0.390	0.350
0.393	0.626	0.159	0.559	0.321	0.175	0.330	0.503	0.570	0.348
0.512	0.178	0.140	0.408	0.571	0.488	0.271	0.653	0.235	0.543
0.587	0.136	0.520	0.119	0.486	0.330	0.553	0.460	0.422	0.286
0.478	0.374	0.432	0.185	0.550	0.325	0.259	0.221	0.147	0.543
0.290	0.319	0.229	0.165	0.598	0.179	0.199	0.306	0.553	0.374
0.191	0.234	0.715	0.572	0.376	0.648	0.206	0.172	0.641	0.591

For all eight distributions excluding normal, uniform, and Laplace distributions, each distribution has five combinations of parameters (only two combinations are tabulated in the tables). The results show that the relative bias of $\widehat{C}_{Np}(u, v)$ do not make much difference when the sample size n is exceeds 100 for the same distribution. For all 11 distributions, the relative bias of $\widehat{C}_{Np}(u, v)$ do not make much difference when the sample size n exceeds 500, and down to 10% except for lognormal distributions when sample size n is higher than 1,000. Further, when the sample size n reaches 3,000, the relative bias of $\widehat{C}_{Np}(u, v)$ is down to 2.5% except for lognormal, F, and t distributions.

6. An Illustrative Example

The example was taken from Polansky (1998) for demonstrating how we apply the introduced percentile method to estimate the process capability in real applications. Consider a process with specification limits $USL = 1.030$, $LSL = 0$, and $T = m = 0.515$, the data shown in Table 23 are a sample of size 100 from a stable process. The mean and the standard deviation of the data are calculated as $\hat{\mu} = 0.406$ and $\hat{\sigma} = 0.173$. Thus, the estimators of C_p and C_{pk} can be obtained as

$$\widehat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{1.030 - 0}{6(0.173)} = 0.992,$$

$$\widehat{C}_{pk} = \min\left\{\frac{USL - \hat{\mu}}{6\hat{\sigma}}, \frac{\hat{\mu} - LSL}{6\hat{\sigma}}\right\} = \min\left\{\frac{1.030 - 0.406}{6(0.173)}, \frac{0.406 - 0}{6(0.173)}\right\} = 0.782.$$

However, the data appear non normal and a Shapiro-Wilk test for normality confirms this with a p-value of 0.0121. Thus, to assess the capability of the process using normal-based estimators could be misleading. Estimators based on the percentile method for estimating the capability of the non normal process are considered in this study. Based on the sample percentile estimators $\widehat{P}_{99.865} = 0.8301$, $\widehat{P}_{0.1355} = 0.1086$, and $\widehat{M} = 0.3915$, we have

$$\widehat{C}_{Np} = \widehat{C}_{Np}(0, 0) = \frac{0.515}{3\sqrt{\left(\frac{0.8301 - 0.1086}{6}\right)^2}} = 1.428, \quad \text{and}$$

$$\widehat{C}_{Npk} = \widehat{C}_{Np}(1,0) = \min\left\{ \frac{1.030 - 0.3915}{3\sqrt{\left(\frac{0.8301-0.1086}{6}\right)^2}}, \frac{0.3915 - 0}{3\sqrt{\left(\frac{0.8301-0.1086}{6}\right)^2}} \right\} = 1.085.$$

Therefore, there are differences between these two estimators of C_p and C_{pk} indices. They indicate that $\widehat{C}_p$ and $\widehat{C}_{pk}$ may not be reliable indications of the true process capability and that the process is likely to have better capability based on the percentile estimators than the normal-based estimates.

7. Concluding Remarks

Process capability indices $C_p(u, v)$ for normal processes, which include C_p, C_{pk}, C_{pm}, and C_{pmk} as special cases, have been widely discussed as a means of summarizing process performance relative to a set of specification limits. Pearn and Chen (1997) considered a generalization of $C_p(u, v)$, called $C_{Np}(u, v)$, to cover processes with non normal distributions. Pearn and Chen (1997) also applied the popular percentile method to propose an estimator for the generalization $C_{Np}(u, v)$. However, the accuracy of the sample percentile estimator is unproved. Therefore, whether the PCIs can truly reflect the performance of a process is questionable. In this article, an extensive simulation study was conducted to examine performance of this percentile estimator for 11 distributions (which cover processes with normal and various non normal distributions for selected parameter values). Furthermore, the accuracy analysis of the percentile estimators $\widehat{P}_{0.135}$, $\widehat{M}$, $\widehat{P}_{99.865}$ and the estimators of $C_p(u, v)$ are presented and summarized based on the relative bias.

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