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Micro-genetic approach for surface meshing on a set of unorganized points

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Abstract

Surface meshing plays a crucial role in mesh generation. Usually, surface meshing methods in three dimensions generate meshes relying on prescribed patch interpolation. In this study, an approach of surface meshing directly on a set of unorganized points is developed, which consists of a mesh triangulation and a conversion scheme for primary triangular and quadrilateral surface meshes, a high order C^1 surface function reconstruction, and a micro-genetic algorithm (MGA) to smooth the meshes. Practical cases are given to demonstrate its successful performance and its versatility.

Keywords: Surface meshing; Unorganized points; C^1 surface function reconstruction; Micro-genetic algorithm

1. Introduction

Mesh generation is a critical step in a wide range of engineering practices, such as the study of scientific simulation by finite element analysis (FEA), of design automation with geometrical modeling, and of data presentation with graphics and visualization. It is well known that a quality mesh is imperative to the above study. In the engineering practices, meshes can be generated on the boundary and/or in the interior of the object, which correspond to surface meshing and volume meshing, respectively. Usually, a well-defined geometrical model is requested before the mesh generation. In this paper, we present a different approach that surface meshing is on a set of unorganized points in which the surface function is yet to be defined.

For surface meshing, most existing methods [29] generate the surface mesh based on pre-defined surface functions, either in parametric patches or in algebraic form, by using existing mesh generation schemes such as Delaunay Triangulation [20] or Advancing Front Methods [21]. However, in many real cases, the given data may just be a set of unorganized points, in which its surface function cannot be devised in a usual fashion. It is often encountered in the applications of biomechanics, which require a geometrical reconstruction from a set of sampling points that is extracted from a sequence of scanned images, e.g. histological sections in tomography. An immediate way to generate a surface mesh for this application is to triangulate the given points [1–4]. However, some sets of such sampling points can be scattered and irregular, and results in locally ill-posed meshes, which may not acceptable for finite element analysis. In this study, a posteriori approach is adopted to tackle this problem: the given points are first triangulated to generate the triangular mesh and/or convert to the quadrilateral mesh and then an additional procedure is introduced to enhance the quality of the mesh. For finite element meshes, the common used procedure for this enhancement is mesh smoothing.

The most popular mesh smoothing method is Laplacian smoothing [5], in which every internal grid node is reposi-

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tioned at the geometrical center of the adjacent nodes. Generally, for surface mesh smoothing, Laplacian smoothing is first employed on a parametric plane and then maps the result onto the physical surface domain. It is well known that the mapping between a reconstructed surface and its parametric plane for Laplacian smoothing strongly affects the resultant mesh quality. A given unorganized point set is usually lack of geometrical regularity in distribution. It cannot guarantee to form a proper primary mesh for the mapping of the Laplacian smoothing.

Optimization-based mesh smoothing technique is another way to smooth the finite element mesh, in which the new location of node is found by using the optimization algorithms [6–9]. Freitag [6] presented several articles described about the finite element mesh smoothing techniques, which contained smart Laplacian smoothing, optimization-based smoothing and the combination of both, in two-dimensional plane mesh and three-dimensional tetrahedral mesh. For the combination method, the smart Laplacian smoothing is used to adjust every internal node and is followed by the optimization-based algorithm, the steepest decent method [10], in only the poorest-quality elements [6]. The steepest decent method searches an optimal solution by a given initial step along a search direction, which is calculated from the gradients of the object function, i.e. mesh quality measurement. However, the calculating of gradients according to the real coordinates is inconvenient for this application, in which the surface function is yet to be defined. Besides, the search direction may let the nodes to deviate from the original surface.

Garimella et al. [8] presented a surface mesh quality optimization procedure that the nodes are repositioned based on element-based local parametric spaces. They employ the conjugate gradient method [10], whose search direction is estimated by computing the gradients of the mesh quality measurement with respect to the local parametric spaces, to reposition the nodes to enhance the mesh quality. It is beneficial that mesh smoothing based on local parametric spaces can remain the nodes close to the original surface. The calculation of the gradients of object function, which is based on the local parametric spaces and without the need of surface functions, is suitable for this application. However, two problems will be arisen: (1) the repositioned points are re-allocated on the planes of corresponding elements, that are not projected onto the original surface, and this will affect the geometrical accuracy of model. (2) The gradient search methods as a local search method may encounter local optimum problem.

In this study, we propose an innovation surface mesh smoothing procedures that mesh nodes were repositioned by using the micro-genetic algorithm (MGA) based on local reconstructed surface. The MGA [13–16] is similar to the genetic algorithms (GA) [11,12], which is a global search method that searches optimal solution by employing natural evolution without calculating search direction and step size. The MGA works with small population size and reaches new optimal regions much earlier than the conventional GA implementation [13]. It has been successfully applied to many fields [13–16]. Moreover, in order to ensure the geometrical accuracy of the analytical model, we projected the repositioned nodes onto the original surface based on an interpolation surface function [18], which is reconstructed from the primary triangular elements. The one drawback of our approaches is that the computational cost of MGA is larger than the gradient search methods [10] but it is feasible by applying parallel computation algorithms [19] to accelerate its computational efficiency. The computational efficiency will not be discussed in this study.

The outline for the rest of the paper is as follows. Section 2 presents surface mesh generation and surface function reconstruction algorithm. Section 3 describes the details and procedures of our proposed MGA approach. Numerical results and discussions are given in Section 4 and the conclusions in Section 5.

2. Mesh generation and surface function reconstruction

2.1. Mesh generation

The generation of surface mesh based on unorganized points set is necessary in some science and engineering fields, where geometrical data are often measured or generated at isolated and unorganized positions, such as mentioned earlier of the biomedical research. In this application, the given data are just the points set and the surface function is yet to be defined. Therefore, the surface mesh cannot be generated based on its surface function directly. For this application, a common way to reconstruct the surface model is to triangulate the given points [1–4]. The triangulation procedure aims at generating a primary triangular surface mesh as well as creating background triangular patches for the use in surface function reconstruction procedure. Furthermore, since the distribution of the given points may be irregular over the surface of the model, the primary triangular meshes always contain some ill-posed triangles. Therefore, some mesh cleanup operators [23] were introduced to improve the topological connectivity of the triangular meshes, and then MGA approach was applied to enhance the mesh quality further.

Once the primary triangular surface mesh was created, the quadrilateral surface mesh can be generated based on the triangular one. The conversion scheme [26–30] was employed to serve this purpose. It is a common and convenient way to generate an unstructured quadrilateral mesh. The quadrilateral mesh was created by a careful process to merge two adjoining triangles to form a quadrilateral element. However, the conversion scheme usually introduces plenty of ill-posed quadrilaterals. To improve the mesh quality, a two-stage procedure is required. First, mesh structure modification (topological improvement) operators [24,25], such as cleanup, edge swapping, node elimination and element collapse, were employed to refine the mesh connectivity. Then the mesh quality was further enhanced by applying the MGA approach.

2.2. Surface function reconstruction

In order to generate a proper surface mesh and ensure the geometrical accuracy of the analytical model, the surface function is necessary during the surface mesh smoothing procedures. In some previous articles, the mesh smoothing procedures were applied to reposition the internal nodes on the plane (or tangent plane) of the primary elements, which is C^0-continue [8,26]. It is based on an assumption that the primary surface elements were well matched to the original surface. However, in this study, since the given data points may be chosen irregularly, the primary surface elements, which were triangulated directly from the given data points, may not match the original surface well. Therefore, in this study, a C^1 continue surface function reconstruction algorithm was adopted to ensure the geometrical accuracy during the nodes repositioning.

There are many surface function reconstruction methods developed [18,31–36]. For a finite element analytical model, a C^1-continuous surface function is necessary for sufficient numerical accuracy. Here, a C^1 triangular patch interpolation method developed by Goodman and Said [18] was adopted to reconstruct the surface function. It is a simpler and efficient method for the surface function reconstruction. In this method, surface function is reconstructed by local cubic Bezier triangular patches. The gradients of vertices are necessary for this surface function reconstruction procedure. We adopted a local derivative estimation method, which is also developed by Goodman et al. [17], to calculate the gradients of vertices. Please refer to [17] and [18] for the details of the vertices gradients calculation and surface functions reconstruction, respectively.

The comparison of geometrical accuracy for the original C^0-continue and our enhanced C^1-continue surface can be shown as follows: 36 points and triangulation in Whelan [31] were chosen and two test functions were employed:

$$\begin{aligned} F_1(x,y) &= 0.75\exp(-((9x-2)^2+(9y-2)^2)/4) \\ &\quad + 0.75\exp(-(9x+1)^2/49-(9y+1)/10) \\ &\quad + 0.5\exp(-((9x-7)^2+(9y-3)^2)/4) \\ &\quad - 0.2\exp(-(9x-4)^2-(9y-7)^2) \\ F_2(x,y) &= (1.25+\cos(5.4y))/(6+6(3x-1)^2) \end{aligned}$$

The interpolated values of the test functions at a 25×25 uniform mesh points in a unit square were computed and the maximum and mean errors were listed in Table 1. The surface error, shown in Table 1, indicates that the C^1-continuous surface is more accuracy than the C^0-continue surface both on maximum and mean surface errors. Therefore, surface mesh smoothing based on the reconstructed C^1 surfaces is beneficial to enhance the geometrical accuracy of the analytical model.

Table 1
Surface errors of the test functions for C^0 and C^1 surface reconstruction

Test function	C^0 surface		C^1 surface	
	Maximum error	Mean error	Maximum error	Mean error
$F1(x,y)$	0.215184	0.027594	0.120067	0.022945
$F2(x,y)$	0.059769	0.008789	0.035420	0.005101

3. Genetic algorithm—micro-genetic approach

The genetic algorithm is one of the recent popular evolutionary algorithms. The GA begins with a set of initial individuals called the population that represent the candidate solutions, which were coded as a bit string to simulate the gene of nature, of the problem. At the beginning, several strings are created randomly to form an initial population. As per Darwin's theory of evolution, the population is evolved generation after generation to search an optimal solution and historical information is then exploited to speculate on new search points with expected performance during the iteration [11]. The GA works with the population constituted by the coding parameter set and searches for a global optimal solution instead of a local one.

The GA yields good results via three major genetic operators: selection, crossover, and mutation. First, the selection operator selects the fittest strings of the previous generation to be the parents of the new generation. This is used to preserve the better historical information to survive at the new generation. After the selection, the crossover operator generates new strings by crossing pair parents of the old. The new strings succeed and exchange the best information of parents to be the new individuals. To avoid the loss of some important genes and increase the variation of the individuals, mutation operator is imported to add new information occasionally.

3.1. Micro-genetic algorithm (MGA)

The standard genetic algorithm is successfully applied to many different applications [11,12]. However, one major drawback is that the iterative global searching of the algorithm is time consuming. It will be deteriorating when additional iterations are needed in the smoothing procedure. There are many approaches to tackle the problem. For the genetic algorithm per se, reduction of the population size is an effective one. For conventional GA, the usual choice of population size is based on the conception that larger population relates to better schema processing, lesser chance of premature convergence, and better optimal results [13]. This imposes a considerable loading on the computational time. To trade-off, the micro-genetic algorithm [13,16] is particularly adopted to accelerate the convergence of the conventional GA. The MGA is similar to the GA that proceeds with binary coded population and employs the selection and crossover operations to evolve population for generations, but with smaller population size than conventional GA. It had been reported that MGA reaches near optimal regions much earlier than the standard GA does [13].

It is well known that the GA works poorly with small population size due to insufficient information processing, which results in premature convergence to local optimal solutions. For the MGA, the best individual is passed to the new generation to ensure that the good individual is held, and it requires multiple convergences. The best individual of the old is remained and the others are randomly generated after each convergence. This operation is used to add new information and avoid premature convergence, and the mutation rate is set to zero.

In this study, an unorganized points set were triangulated to form a primary surface triangular mesh and/or converted the primary triangular mesh into quadrilateral mesh. The primary surface mesh was first refined by the mesh structure modification operators [23–25], and then the MGA mesh smoothing procedures were applied to enhance the surface mesh quality further. The procedures of our surface mesh smoothing and the MGA adopted in our scheme were summarized as follows:

Step 1. Input data: input the data of primary mesh, which includes surface function, node positions and element connectivity. If the surface function is not given, the gradients of nodes will estimated by the gradients estimation procedures [17].
Step 2. MGA mesh smoothing begins:
Step 3. Search the optimal solutions within each adjacent element by using steps 4–7 and then choose the best one to be the new position of the node.
Step 4. Initial population: the MGA requires multiple convergences. According to Ref. [13], a population size of five is chosen in each convergence. The best individual of the previous generation will be held. The others are generated randomly.
Step 5. Decode the strings and calculate their node positions based on the reconstructed surface function (Section 2.2). Calculate their fitness values and then carry the best string to the next generation.
Step 6. Select four strings (contains the best string) for reproduction by employing the roulette wheel strategy [11]. Generate four individuals by employing the crossover operator with probability of one [13].
Step 7. Check the convergence criterion. If it is not convergence, go to step 5 or else go to step 3 or step 4.
Step 8. Go to step 2 to smooth next node until the end of the smoothing.
Step 9. Check the convergence of the whole mesh smoothing procedures. If it is not convergence, go to step 2 or else end off the smoothing procedures.

3.2. Design parameters

In the MGA mesh smoothing procedures, the design parameters were chosen to represent the nodes position. In order to avoid degenerate elements, the search space was restricted within a triangular area for each adjacent element in both triangular and quadrilateral mesh smoothing. To represent a node lay on a triangle, we choose two parameters r_1 and r_2, which relate to the barycentric coordinates as the following: Consider a triangle T (Fig. 1) with vertices V_1 and V_3 in barycentric coordinates u, v and w, such that any point on the triangle can be expressed as

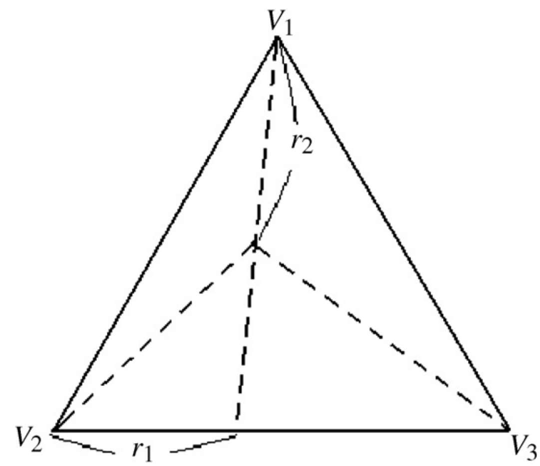

Fig. 1. Design parameters definition on a triangle.

$$V = uV_1 + vV_2 + wV_3, \quad u + v + w = 1 \tag{1}$$

Now, with $0 < r_i < 1$, $i = 1, 2$, the barycentric coordinates can be given as

$$\begin{aligned} u &= r_1(1 - r_2) + (1 - r_1)(1 - r_2) = 1 - r_2 \\ v &= r_2(1 - r_1) \\ w &= r_1 r_2 \end{aligned} \tag{2}$$

Substituting (2) into (1), we get

$$V = (1 - r_2)V_1 + r_2(1 - r_1)V_2 + r_1 r_2 V_3 \tag{3}$$

As shown in Fig. 1, the vertices V_1, V_2 and V_3 are collinear when $r_2 = 1$. To avoid that, we let $0 < r_2 < 0.5$. After the position of point V is obtained, the exact position is calculated by mapping it to the original surface based on its corresponding triangular patch and the local reconstructed surface function (Section 2.2).

3.3. Fitness function

In this study, the fitness function is a criterion to judge the mesh quality and defined with the mesh quality measurement index. For the triangular mesh quality measurement, the common used quality index is

$$\alpha = 2\sqrt{3}\frac{\|AB \times AC\|}{\|AB\|^2 + \|BC\|^2 + \|AC\|^2} \tag{4}$$

where A, B, C are the vertices of a triangle. According to this mesh quality measurement, the fitness function for the triangular mesh smoothing F_t is defined as

$$F_t = \sum_{i=1}^{n} \alpha_i \tag{5}$$

where n is the number of adjacent triangular elements.

For the quadrilateral mesh quality measurement, Knupp [37] presented an algebraic mesh quality metrics from the Jacobian matrix. The definition of quadrilateral shape quality metric is as follows: for a plane quadrilateral mesh, let the coordinates of the four nodes be (x_k, y_k), $k = 0, 1, 2, 3$. The Jacobian matrices, A_k, one at each node of the quadrilateral:

$$A_k = \begin{pmatrix} x_{k+1} - x_k & x_{k+3} - x_k \\ y_{k+1} - y_k & y_{k+3} - y_k \end{pmatrix} \tag{6}$$

where the indices $k + 1$ and $k + 3$ are taken modulo four, for example, if $k = 1$ then $k + 3$ becomes 0. Four metric tensors are obtained by the combinations $A_k^{\mathrm{T}} A_k$. Let $\lambda_{ij}^k, i, j = 1, 2$, be the ijth component of the kth metric tensor. Geometrically, at the kth node, λ_{11}^k is the square of the length of the side connecting nodes k and $k + 1$, λ_{22}^k is the square of the length of the side connecting nodes k and $k + 3$. Let θ_k be the angle between the two sides joined at the kth node, the quadrilateral shape quality metric can be expressed as

$$\beta = \frac{8}{\sum_{k=0}^{3} (1 + r_k^2)/(r_k \sin\theta_k)} \tag{7}$$

where $r = \sqrt{\lambda_{22}/\lambda_{11}}$ is the length ratio of kth node. This concept can be extended to measure quadrilateral surface mesh quality directly. Then the fitness function for the quadrilateral mesh smoothing F_q is defined as

$$F_q = \sum_{i=1}^{n} \beta_i \tag{8}$$

where n is the number of adjacent quadrilateral elements.

4. Results and discussion

The proposed approach is tested in several unorganized point datasets. A simple geometry of saddle shape with prescribed surface function is used to validate the procedure and to compare the performance of our MGA approach and the conjugate gradient method (Table 2). It is then applied to complicated geometries, such as wing-fuselage, which is often used in preliminary aircraft design, and biological dataset of shapes of a foot and a rat, which are constructed from contours by a three-dimensional laser scanning and from a sequence of segmented bio-images, respectively. The geometrical reconstruction of bio-images is a crucial pre-processing for physical modeling in biology and biomechanics. So, the applications are used to demonstrate not only the effectiveness but also the practical use of our approach. In order to show the capability of our MGA mesh smoothing approach, all of the smoothed results are obtained by smoothing the primary meshes directly without any additional mesh treatment. The mesh quality is measured according to Section 3.3 that the triangular mesh quality index is α and the quadrilateral mesh quality index is β. The results are summarized in Tables 3 and 4 for triangular and quadrilateral surface meshes, respectively. Significant enhancement is found by our approach. Furthermore, in Table 3, the worst quadrilateral mesh quality is set as 0.0001, which represents that one of the interior angle is greater than 179° and the codes are run in Linux PC with a dual core AMD 2 GHz CPU and 3 GHz RAM.

The first example is a basic mathematical function of saddle shape (Fig. 2), which allows us to scrutinize the performance of the approach. The surface function is denoted as the following:

$$z = x^2/10 - y^2/4 \tag{9}$$

The saddle shape surface, unlike the usual well-behaved elliptic shape, has negative curvature, which is appealing to be used as a test case for the surface smoothing [38]. The triangular surface mesh is generated based on

Table 3
Mesh quality improvement for the triangular surface meshes

Object	Number of node	Number of element	Worst quality	Mean quality	CPU time (s)
Wing-fuselage	1419	2432	0.2535 0.2535	0.8483 0.9046	3.0500
Foot	4039	8000	0.3803 0.3469	0.8275 0.9192	23.8200
Rat	25,670	51,354	0.2225 0.2460	0.8963 0.9453	150.0509

Table 4
Mesh quality improvement for the quadrilateral surface meshes

Object	Number of node	Number of element	Worst quality	Mean quality	CPU time (s)
Wing-fuselage	1405	1200	0.0001 0.4215	0.6982 0.8580	4.1000
Foot	4010	3971	0.0001 0.4111	0.7049 0.8632	19.6400
Rat	25,374	25,383	0.0001 0.2553	0.7568 0.8899	88.8099

Table 2
The comparison between conjugate gradient method and MGA method using the saddle geometry

Method	Object	Number of node	Number of element	Worst quality	Mean quality	CPU time (s)
Conjugate gradient MGA	Triangular saddle mesh	601	1104	0.6834 0.4591	0.9562 0.9509	0.8000 3.8199
Conjugate gradient MGA	Quadrilateral saddle mesh	597	548	0.4099 0.5696	0.8736 0.9001	0.8400 2.3199

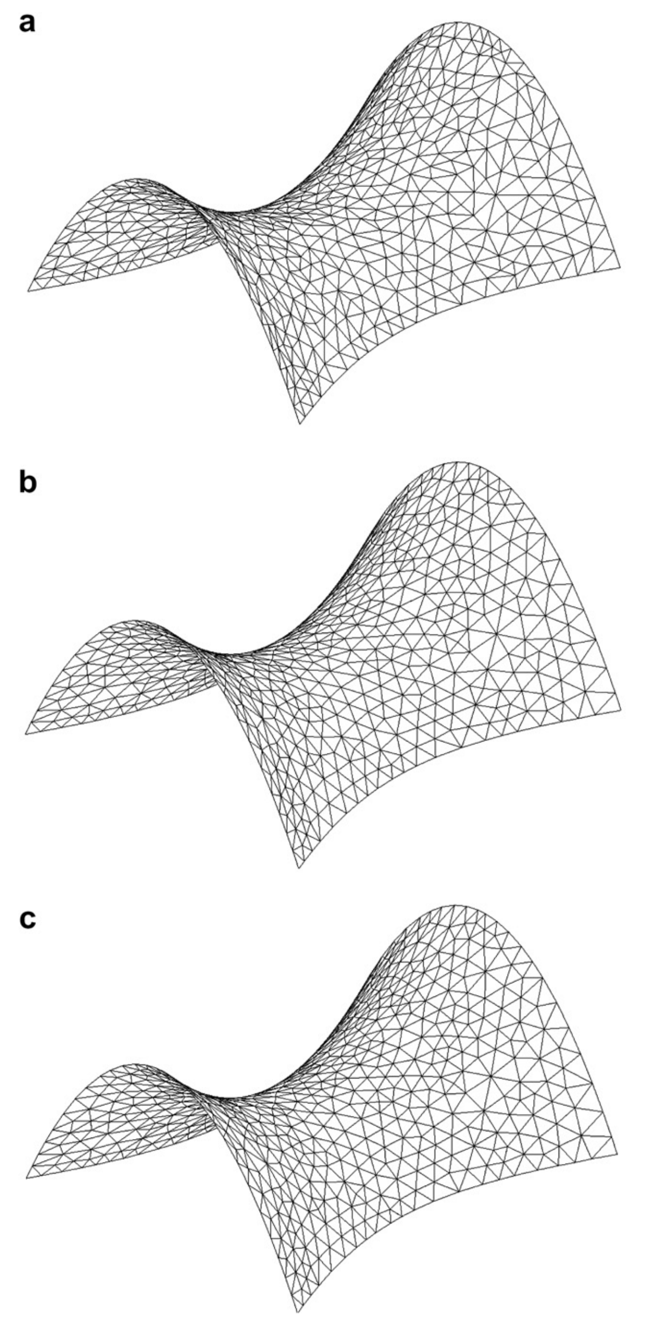

Fig. 2. Triangular surface mesh of saddle: (a) original surface mesh; (b) surface mesh after MGA smoothing; and (c) surface mesh after conjugate gradient smoothing.

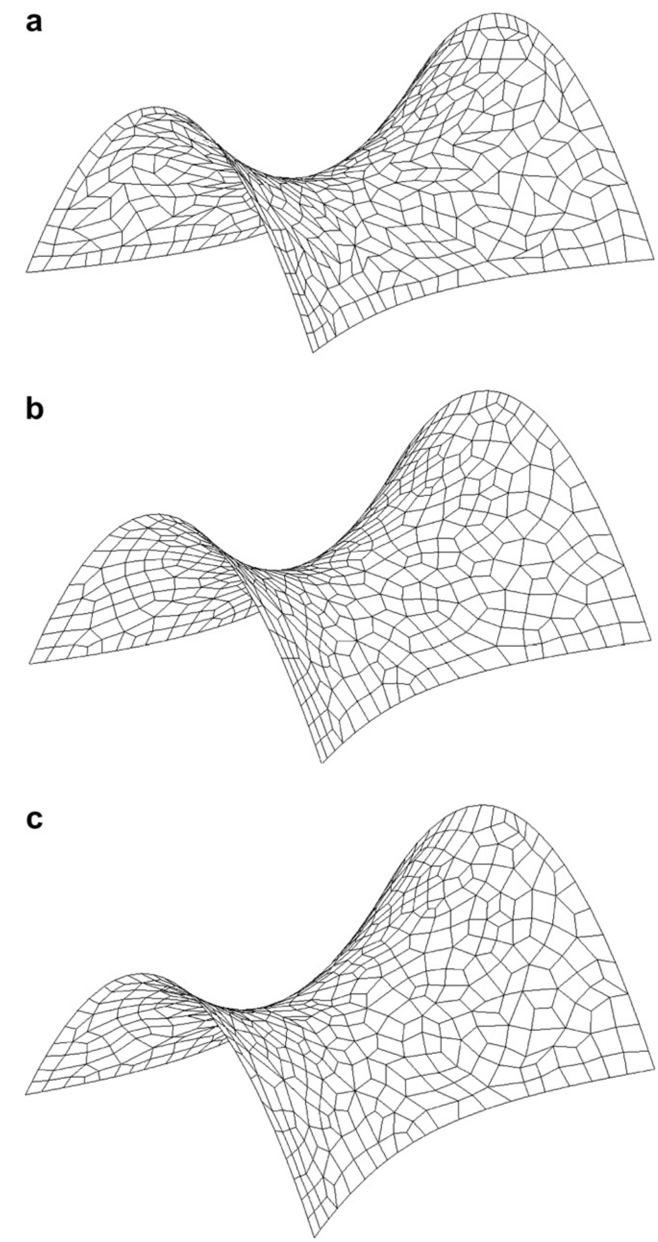

Fig. 3. Quadrilateral surface mesh of saddle: (a) original surface mesh; (b) surface mesh after MGA smoothing; and (c) surface mesh after conjugate gradient smoothing.

Delaunay triangulation and the quadrilateral surface mesh converted from the triangular mesh (Section 2.1). The performance of MGA can be observed by comparing with that of the conjugate gradient method in the saddle shape. Table 2 shows that the improvement of the mean quality of triangular mesh reaches to 0.9562 and 0.9509 by both approaches, but the CPU time of our MGA approach is approximate five times of the conjugate gradient method. This is one drawback of GA, but can be easily tackled by parallelism and there are several parallel GA [19] developed with great success. The approach is similarly employed to the quadrilateral mesh. Our quadrilateral meshes are generated using a popular conversion scheme or fission scheme [26–30]. The conversion scheme essentially merges two neighboring triangles to form a new quadrilateral. This method may introduce ill-posed quadrilaterals, and needs further treatments [24,25] for practical use. Table 2 shows that the improvement of mean quality of quadrilateral mesh reaches to 0.8736 and 0.9001 by employing the conjugate gradient method and our MGA approach, respectively. It is clear that our performance as a global method achievement for mesh quality improvement is better than the conjugate gradient method. The enhancements of the mesh quality can be also visually observed from Figs. 2 and 3 for triangular and quadrilateral surface mesh, respectively.

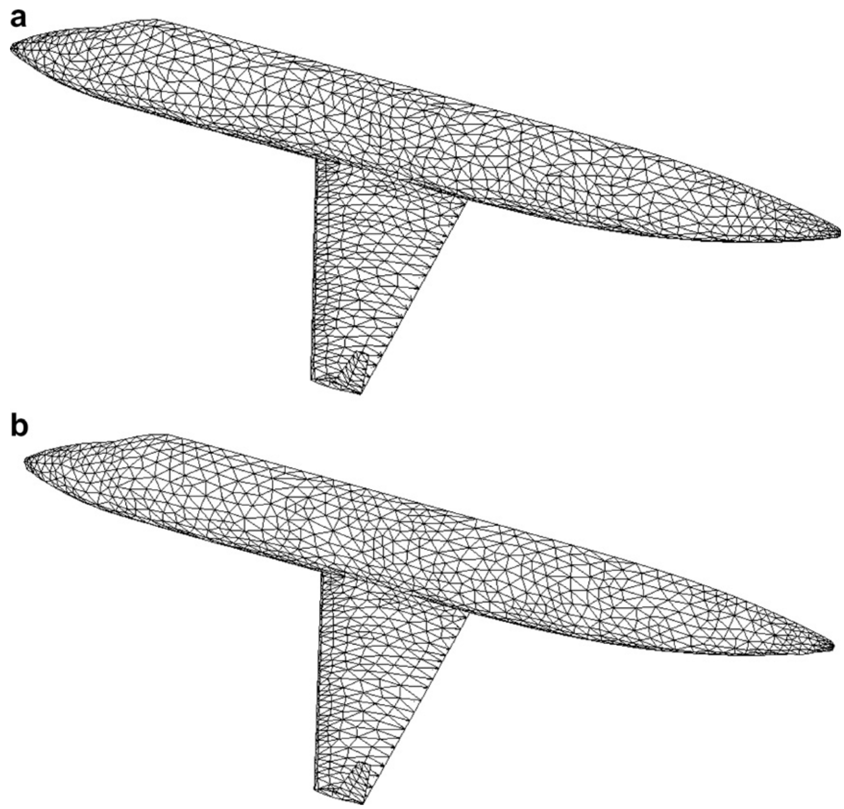

Fig. 4. Triangular surface mesh of wing-fuselage: (a) original surface mesh and (b) surface mesh after smoothing.

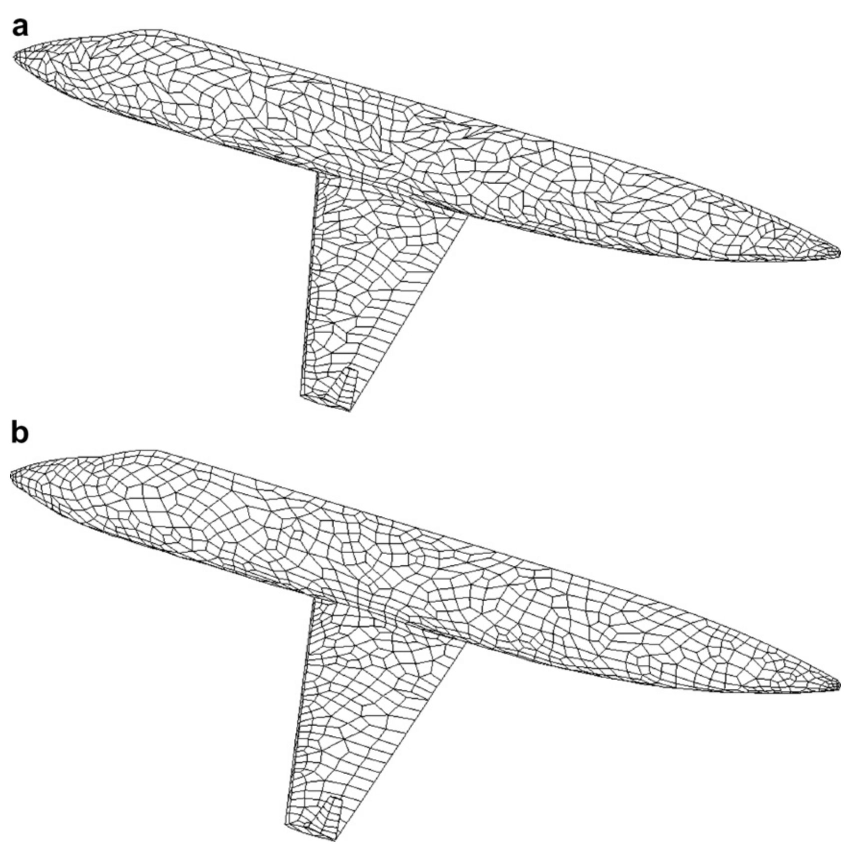

Fig. 5. Quadrilateral surface mesh of wing-fuselage: (a) original surface mesh and (b) surface mesh after smoothing.

The following example is a complicated geometry of wing-fuselage configuration of NCKU-ILD-101 [39]. Such kind of geometrical model is often used in CFD simulation for aircraft preliminary design. The usual practice for generating a surface mesh for such a geometrical model is based on a pre-defined patch interpolation. The current data are prepared from a point datasets that is well generated by usual mesh generation methods of Delaunay triangulation. The point datasets are deliberately reduced from 9962 points to 1314 points, but the feature of the geometry is carefully preserved by using AMIRA [22]. The reduction makes mesh so coarse that it is difficult to maintain quality for simulation. However, it is well suited to test the performance of the current MGA procedure. The points at the edges and joints have to be marked and fixed to avoid singularity during the smoothing. The results are shown in Figs. 4 and 5 for triangular and quadrilateral meshes, respectively. The improvement is significant, which can be seen from the quality indices measured in Tables 3 and 4. The mean quality index of triangular mesh is enhanced from 0.8483 to 0.9046. It is well known that quadrilateral mesh is less stiff than triangular mesh and allows more degrees of freedom to move the mesh to change the shape of elements. As a result, it needs more

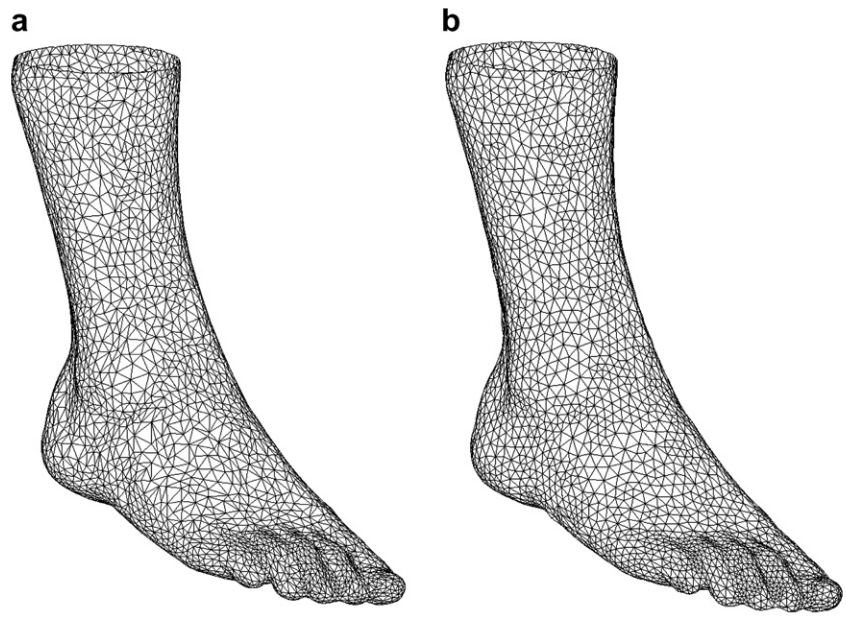

Fig. 6. Triangular surface mesh of foot: (a) original surface mesh and (b) surface mesh after smoothing.

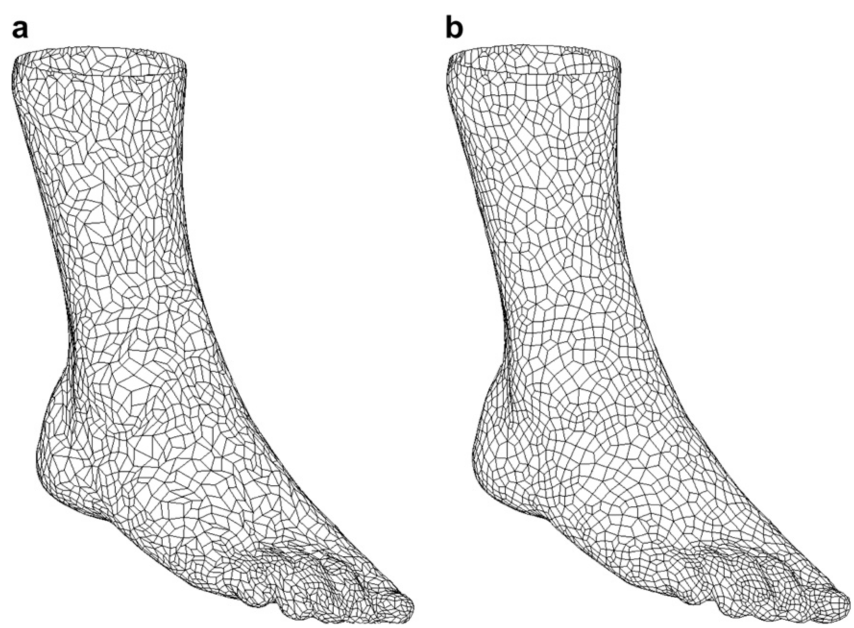

Fig. 7. Quadrilateral surface mesh of foot: (a) original surface mesh and (b) surface mesh after smoothing.

smoothing treatments after generating the surface mesh. Our results show significant improvement in terms of worst and average property measured within elements. The mean and worst qualities are enhanced from 0.6982 to 0.8580 and 0.0001 to 0.4215, respectively.

Due to the advance of image processing, geometry reconstruction and mesh generation, nowadays larger and more complicated geometry models can be reconstructed from a sequence of bio-medical images data set. The need for such reconstruction is immense and it has become a common practice for bio-medical and bioengineering study. By scrutinizing the reconstruction procedure for such geometry models, it can be found that the surface geometry is usually defined by a set of unorganized points, or at least by a sequence of un-associated contours, that is identified by pattern recognition methods on each image of interest. The surface triangulation is not straightforward. It is even more challenging for surface mesh quality enhancement in that the re-arrangement of the point distribution needs higher order interpolation methods for accuracy. Two test cases are given to demonstrate the capability of the proposed methods for tackling these challenging issues.

The first example is the foot shape model of Polhemus [40]. The original model was created by using FastSCAN with the FastRBF Extensions. The surface data points are measured by the laser scanner FastSCAN, and then the data points are reconstructed to form the geometrical model by the software FastRBF Extensions. It is a very convenient and efficient way to reconstruct a geometrical model, especially when the surface function is not

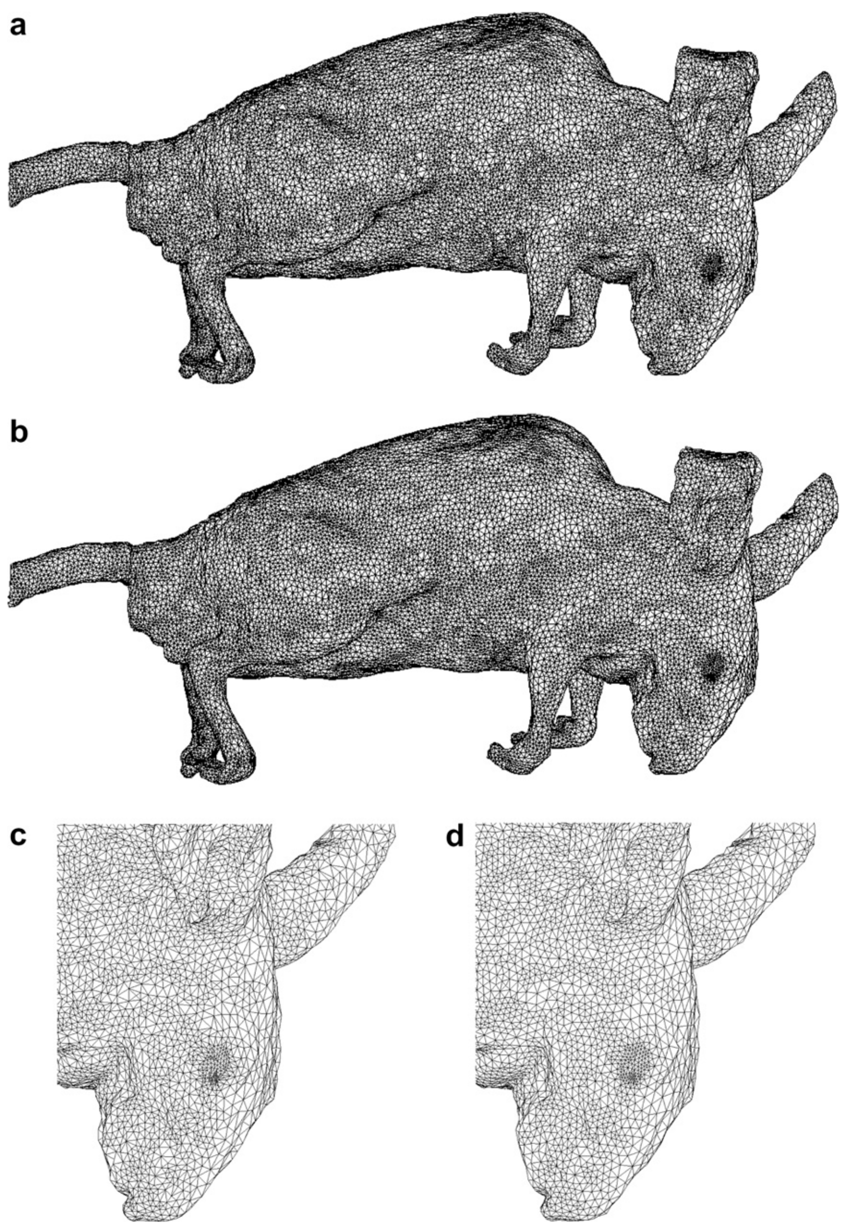

Fig. 8. Triangular surface mesh of rat: (a) original surface mesh; (b) surface mesh after smoothing; (c) original surface mesh (enlarged); and (d) surface mesh after smoothing (enlarged).

pre-defined, by using a laser scanner system. In this study, the original foot model contains 25,845 nodes. Similarly, the original data points are reduced to 4039 and it contains 8000 triangles on the surface. Our MGA approach constructs a high order interpolation and iteratively optimizes the point distribution in terms of local evolution. The triangular mesh quality was enhanced as expected from 0.8275 to 0.9192 (Fig. 6). The original surface mesh obtained is actually well defined. However, the current approach can still give further enhancement of the mesh quality. For systematical comparison, the quadrilateral mesh is generated and smoothed in a similar fashion. The results of obvious improvement are as expected that the surface quadrilaterals are more regular after the MGA approach (Fig. 7). This also can be readily observed from the mean quality, which is improved from 0.7049 to 0.8632 (Table 4).

The last case is a rat shape that is reconstructed from a sequence of image slices of histological sections from Ryutaro Himeno of RIKEN [41]. The original reconstructed model contains skin, bone, organs and so on, and it can help people to observe the whole model of rat virtually without dissect a real rat. This approach can be used to reconstruct more models of biology. In this study, we extract the skin model and reduce it by using AMIRA. As shown in Fig. 8, this model contains 25,670 nodes and 51,354 elements. The reconstructed shape is rough and irregular and the primary surface mesh contains many poor triangles elements. Some of the nodes connect to three

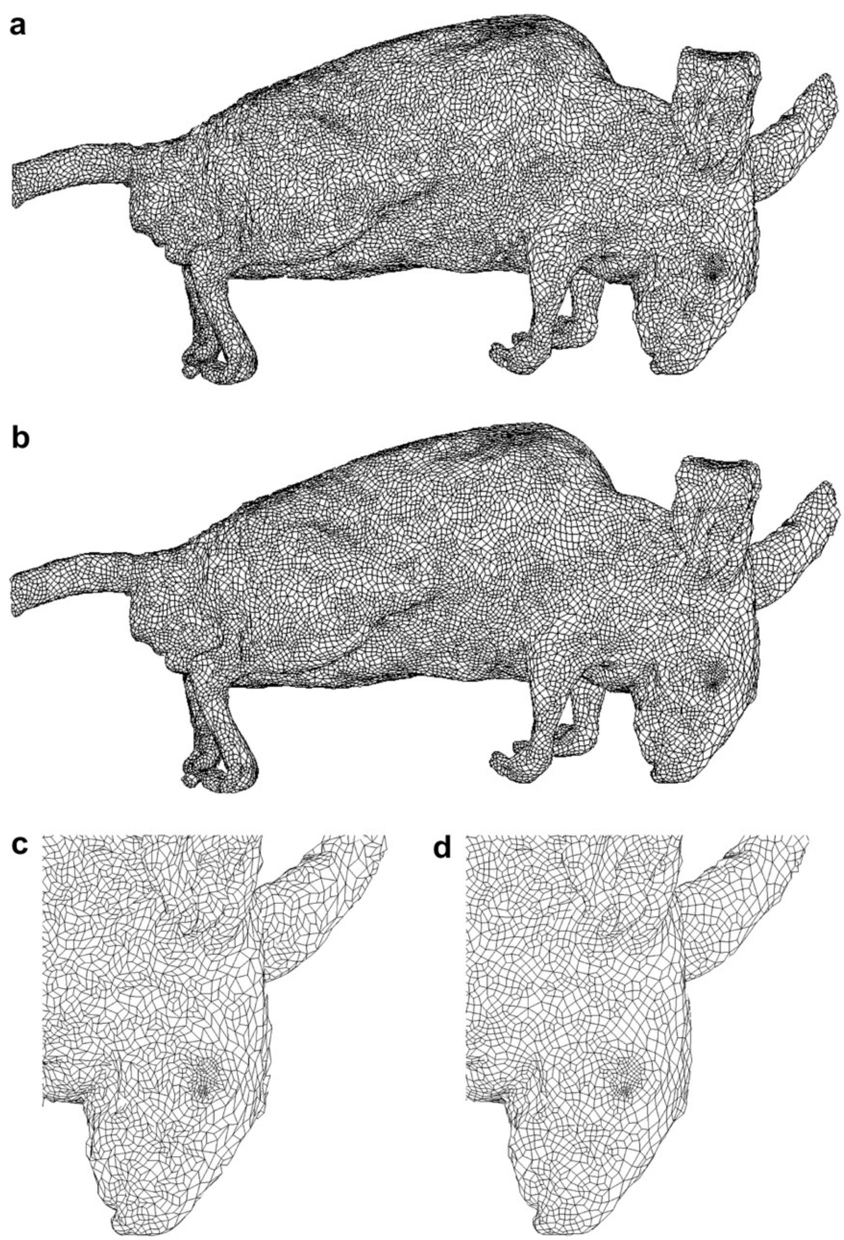

Fig. 9. Quadrilateral surface mesh of rat: (a) original surface mesh; (b) surface mesh after smoothing; (c) original surface mesh (enlarged); and (d) surface mesh after smoothing (enlarged).

elements and others connect to more than nine elements. The poor connectivity is treated first by the mesh structure modification operators [23–25]. To avoid interpolation error, the singular points, which locate at the boundary or contain large curvature variation, need be identified and fixed before the surface mesh smoothing. The MGA procedure is then used to re-construct surface interpolation function and to smooth the mesh by optimizing the point distribution. The result is shown in Fig. 8b and the mean mesh quality improved from 0.8963 to 0.9453. The similar improvement for quadrilateral mesh can be found in Fig. 9 and Table 4.

5. Conclusions

In this study, we present an evolutionary procedure, which integrates the micro genetic algorithm and a C^1 surface function interpolation method, applied to surface meshing on a set of unorganized points. Original surface meshes are generated based on Delaunay triangulation or reduced from the original models. Quadrilateral meshes are generated via the conversion of the triangular meshes and elementarily modified by the mesh structure modification operators. A C^1 interpolation function is constructed using the primary local elements to ensure the surface geometrical feature is preserved. The meshes are then smoothed using the micro-genetic algorithm on the reconstructed high order surface elements. The procedure is then tested in complicated and large-scale point datasets. The results show that the procedure successfully achieves better surface meshing with mesh quality significantly enhanced for different kinds of practical applications.

Generally, the generation of surface mesh is based on a pre-defined surface function, either in parametric patches or in algebraic form. In this study, the given data of model is just a set of unorganized points. In this situation, the major problem of surface mesh generation is of the mesh smoothing. As mention above, the given points are first triangulated to form a triangular mesh and/or converted them into quadrilateral mesh, and then surface mesh smoothing procedure is applied to improve the mesh quality. A MGA mesh smoothing procedure is adopted in this study, which allows us to avoid the calculation of search direction and step size and to enable a global search for optimum. Furthermore, a C^1 surface function interpolation method is integrated into the MGA mesh smoothing procedure to ensure the geometrical accuracy of models during the surface mesh smoothing.

The extensions of the proposed method are to enhance the efficiency of MGA and the accuracy of the reconstructed surface functions. The MGA as one of the GA methods is by nature well suited to parallelization. Therefore, our further enhancement will be in performance issues of parallelism. The accuracy of the reconstructed local interpolation functions on surfaces determines the baseline of the accuracy of the entire model. Higher order interpolation methods can be further developed to serve the purpose.

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