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ScienceDirect
Optik 118 (2007) 1–4
Optik Optics
www.elsevier.de/ijleo

# The freeze of intrapulse Raman scattering effect of ultrashort solitons in optical fiber

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Received 15 September 2005; accepted 12 January 2006

## Abstract

The effect of intrapulse Raman scattering (IRS) for the propagation of the femtosecond solitons in an optical fiber is investigated. To factually simulate its influence, a combination of 27 Lorentian lines to fit experimental Raman gain profile is adopted. By using nonlinear Schrödinger equation and finite-difference time domain method, the propagations of femtosecond fundamental solitons in an optical fiber are numerically calculated. When the initial power is suitably enhanced, it is found that the pulse shape is almost the same as initial pulse and the delay Raman response only makes small pulse shift. In other words, when ultrashort soliton is considered, the IRS effect is similarly frozen under the enhanced initial power.


***Keywords:*** Raman effect; Raman gain; Self-frequency shift; Femtosecond soliton; Ultrashort pulse

## 1. Introduction

For the silica fiber, the effect of intrapulse Raman scattering (IRS) plays a most important role among the higher-order nonlinear effects when the nonlinear pulse is of less than the picosecond duration [1,2]. Its effect observed experimentally gives rise to a continuous down-shift of soliton, called the soliton self-frequency shift (SSFS) [3]. It was successfully explained by the use of the delay nature of the Raman response [4]. By the use of adiabatic perturbation theory, it was shown that the SSFS is inversely proportional to soliton duration [5]. Recently, an interesting research topic that the propagation pulse duration is down to or less than the optical phonon oscillation periods in fibers was investigated [6–9]. When the pulse durations are shorter than the optical phonon oscillation period and with respect to the influence of the field-induced change in the vibrational level populations, the self-frequency shift can be suppressed by the coherent saturation of the Raman response [9]. For the small soliton duration ($\approx 20$ fs for silica glass), they predicted the effect of self-suppression of the continuous downshift of the soliton carrier frequency.

For the propagation of ultrashort optical pulses whose width is as short as a few optical cycles or less, the validity of the generalized nonlinear Schrödinger equation described by the slowly varying envelope approximation becomes questionable [10]. By the use of finite-difference methods, the full-vector nonlinear Maxwell's equations have been solved by direct integration [11,12], but this is very time consuming. An accurate wave equation beyond the slowly varying envelope approximation for femtosecond soliton was derived by the iterative method [13]. Comparing three results simulated by the generalized nonlinear Schrödinger equation, the accurate wave

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0030-4026/$ - see front matter 
doi:10.1016/j.ijleo.2006.01.003

equation, and the full Maxwell's equations, it was shown that the generalized nonlinear Schrödinger equation well describes the propagation of the pulse even containing a single optical cycle.

In this paper, we will research the delay Raman response effect for the propagation of the femtosecond solitons in an optical fiber. We take 27 Lorentzian lines to fit experimental Raman gain profile and study numerically its effect. The saturation effect is assumed ignorable because a fraction of the broadband spectrum lays on the phonon transition spectrum. The IRS effect is similarly frozen under unsaturation condition. It is clarified by using nonlinear Schrödinger equation and full Maxwell's equations for the propagations of femtosecond fundamental solitons in an optical fiber. When the initial power is suitably enhanced, we find that the pulse shape is almost the same as initial pulse and the delay Raman response effect only makes small pulse shift.

## 2. Theory model

We have derived the normalized nonlinear Schrödinger equation for a femtosecond pulse propagating in a single-mode fiber with a third-order nonlinearity beyond the slow varying envelope approximation [13]. Introducing the normalized variables $\xi = z/L_D$, $\tau = (t-\beta_1 z)/T_0$, and $u = N_P A/\sqrt{P_0}$, the normalized equation can be written as

$$\begin{aligned}\frac{\partial}{\partial\xi}u = {} & \frac{\mathrm{i}}{2}\frac{\partial^2 u}{\partial\tau^2} + \beta\frac{\partial^3 u}{\partial\tau^3} + \frac{\mathrm{i}\beta_4}{24|\beta_2|T_0^2}\frac{\partial^4 u}{\partial\tau^4} + \mathrm{i}\overline{N}u \\ & - \frac{1}{\omega_0 T_0}\frac{\partial}{\partial\tau}\overline{N}u - \frac{\mathrm{i}\beta_2}{\beta_0 T_0^2}\left\{\frac{1}{2}\frac{\partial^2\overline{N}u}{\partial\tau^2}\right. \\ & + \left[(1-\alpha)\left|\frac{\partial u}{\partial\tau}\right|^2 + \alpha\int_{-\infty}^{\tau}\mathrm{d}\tau' f(\tau-\tau')\left|\frac{\partial u}{\partial\tau'}\right|^2\right] \\ & \left. + \frac{\partial\overline{N}}{\partial\tau}\frac{\partial u}{\partial\tau} + \frac{1}{2}\overline{N}^2 u\right\}, \qquad (1)\end{aligned}$$

where $u$ is normalized electric field, $N_P$ is the order of soliton, $A$ is electric field, $P_0$ is peak power of the soliton, $\beta_i$ is $i$th expansion dispersion, and $T_0$ is pulse duration. In Eq. (1), the normalized parameters are defined: $\beta \equiv \beta_3/(6|\beta_2|T_0)$, dispersion length $L_D = T_0^2/\beta_2$, and $\overline{N} \equiv \overline{N}(z,t) = N_P^2 N(z,t)/P_0$. Here we consider the effect of IRS, and take [1]

$$\begin{aligned}N(z,t) = {} & (1-\alpha)|A(z,t)|^2 \\ & + \alpha\int_{-\infty}^{t}\mathrm{d}t' f(t-t')|A(z,t')|^2, \qquad (2)\end{aligned}$$

where $f(t)$ is the delayed Raman response function [3], and $\alpha = 0.18$ parameterizes the relative strengths of Kerr and Raman interactions [1,2]. The nonlinear loss is produced because a part of pulse energy is absorbed by silica molecules. In this paper, $f(t)$ is obtained by modeling the Raman gain by 27 Lorentzian lines, centered on the different optical phonon frequencies and

$$f(t) = \sum^{27}\frac{\tau_{1i}^2 + \tau_{2i}^2}{\tau_{1i}\tau_{2i}^2}\exp(-t/\tau_{2i})\sin(t/\tau_{1i}), \qquad (3)$$

where the parameters are determined by fitting the imaginary parts of its spectrum to actual Raman gain of fused silica.

It is shown that those more higher-order nonlinear terms in Eq. (1), the coefficients of which are proportional to the second-order dispersion parameter, are much smaller than shock term in a silica-based weakly guiding single-mode fiber [13]. That is, Eq. (1) can approximate to

$$\begin{aligned}\frac{\partial}{\partial\xi}u = {} & \frac{\mathrm{i}}{2}\frac{\partial^2 u}{\partial\tau^2} + \beta\frac{\partial^3 u}{\partial\tau^3} + \frac{\mathrm{i}\beta_4}{24|\beta_2|T_0^2}\frac{\partial^4 u}{\partial\tau^4} \\ & + \mathrm{i}\overline{N}u - \frac{1}{\omega_0 T_0}\frac{\partial}{\partial\tau}\overline{N}u, \qquad (4)\end{aligned}$$

for the propagation of the pulse containing a few optical cycles, even a single optical cycle. So, we use Eq. (4) to describe the IRS effect of the propagation of the ultrashort optical pulse.

To numerically investigate the pulse propagation by using full Maxwell's equations in the fiber, we simply consider one-dimensional electric and magnetic fields, $E_z$ and $H_y$, respectively, propagating along $x$ direction. The Maxwell's equations are written as [10,11]

$$\begin{aligned}&\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial x}, \\ &\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial t}, \qquad (5) \\ &D_z = \varepsilon_0\varepsilon_r E_z + P_z,\end{aligned}$$

where $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity coefficients in free space, $\varepsilon_r$ is the relative permittivity, $D_z$ is the electric field displacement, and $P_z$ is the electric polarization.

$P_z$ consists of linear and nonlinear parts:

$$P_z = P_z^{\mathrm{L}} + P_z^{\mathrm{NL}},$$

where the linear polarization $P_z^{\mathrm{L}}$ and the nonlinear polarization $P_z^{\mathrm{NL}}$ are related to the electric field $E_z(x,t)$ by the following relations:

$$P_z^{\mathrm{L}}(x,t) = \varepsilon_0\int_{-\infty}^{\infty}\chi^{(1)}(t-\bar{t})E_z(x,t)\,\mathrm{d}\bar{t}, \qquad (6)$$

$$\begin{aligned}P_z^{\mathrm{NL}}(x,t) = {} & \varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\chi^{(3)}(t-\bar{t}_1, t-\bar{t}_2, t-\bar{t}_3)E_z(x,\bar{t}_1) \\ & \times E_z(x,\bar{t}_2)E_z(x,\bar{t}_3)\,\mathrm{d}\bar{t}_1\,\mathrm{d}\bar{t}_2\,\mathrm{d}\bar{t}_3, \qquad (7)\end{aligned}$$

where

$$\chi^{(1)}(t) = \frac{\omega_r^2(\varepsilon_s-\varepsilon_\infty)}{\sqrt{\omega_r^2-\delta^2/4}}\exp(-\delta t/2)\sin\left(\sqrt{\omega_r^2-\frac{\delta^2}{4}}t\right)$$

is the first-order susceptibility function, $\chi^{(3)}(t)$ is the third-order susceptibility, $\omega_r$ is dipole resonant frequency, $\varepsilon_\infty$ is the relative material permittivity at infinite frequency, and $\delta$ is damping constant. Considering the effect of IRS, Eq. (7) is rewritten as

$$P_z^{\mathrm{NL}}(x,t) = \varepsilon_0 \chi^{(3)} E_z(x,t) \int_{-\infty}^{\infty} g(t-\bar{t}) E_z^2(x,\bar{t})\,\mathrm{d}\bar{t}, \qquad (8)$$

where $\chi^{(3)}$ is the nonlinear coefficient and $g(t)$ is the response function,

$$g(t) = \alpha\delta(t) + (1-\alpha) f(t). \qquad (9)$$

Here $f(t)$ is the delayed Raman response function, $\delta(t)$ is the delta function, and $\alpha = 0.18$ parameterizes the relative strengths of Kerr and Raman interactions.

## 3. Numerical result and discussion

We consider the effect of IRS for the propagation of the femtosecond fundamental solitons in single-mode fiber. To solve Eq. (4) by the split-step Fourier method with the initial condition $u(\xi = 0, \tau) = \mathrm{sec\,h}(\tau)$, we take the parameters to be: the second-order dispersion $\beta_2 = -20\,\mathrm{fs}^2/\mathrm{mm}$, third-order dispersion $\beta_3 = 0$, and fourth-order dispersion $\beta_4 = 0$. Fig. 1 shows the pulse deviation, $\Delta = \sqrt{\sum_i (u_i - u_i')^2}$, versus different pulse durations at a distance of four soliton periods ($4L_\mathrm{d}$). Here $u_i$ and $u_i'$ are pulses with and without the delay Raman response, respectively, in $i$th time domain resolution point. The saturation effect is assumed ignorable because a small fraction of the broadband spectrum lays on the phonon transition spectrum. One can see that the deviation $\Delta$ is rapidly reduced when pulse duration is less than 10 fs under unsaturation condition. For pulse duration 40 fs case, the deviation is about $1.13T_0(\approx 45\,\mathrm{fs})$ at $4L_\mathrm{d}$ and the pulse shape is shown in Fig. 2. The solid line presents the pulse shape with the delay Raman response. The dotted line presents the pulse shape without the delay Raman response. The tailing edge of pulse shape is outspread by delay Raman response. The change of pulse shape is obvious. And, the peak powers are smaller than initial peak power. For pulse duration 2.5 fs case in Fig. 1, the deviation is only $0.367T_0(\approx 0.92\,\mathrm{fs})$ at $4L_\mathrm{d}$. Fig. 3 shows the pulse shape of pulse duration 2.5 fs at a distance of $4L_\mathrm{d}$ with and without the delay Raman response. One can see that, after propagation $4L_\mathrm{d}$, the change of pulse shape is small and the major change of pulse shape is pulse power. The phenomenon can be explained that the SSFS is slight and negligible and a part of pulse energy is absorbed by silica molecules. To compensate the nonlinear loss and maintain the pulse shape, the power of the initial pulse need to be enhanced to support the enough Kerr effect for balancing the group-velocity dispersion effect when IRS is present. When the delay Raman response effect is

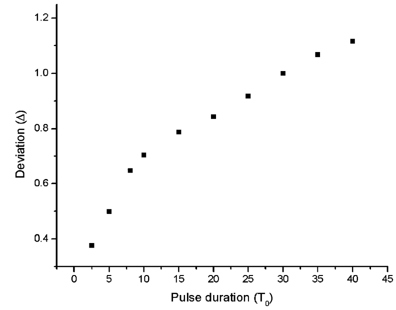


**Fig. 1.** The deviation $\Delta$ versus pulse duration at $4L_\mathrm{d}$ with the delay Raman response.

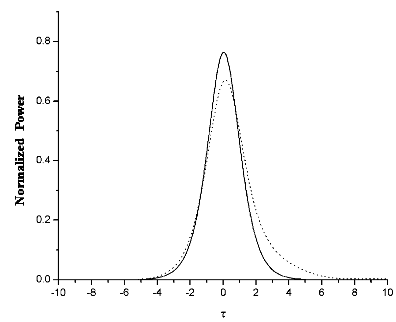


**Fig. 2.** Pulse shapes of pulse duration 40 fs fundamental soliton at $4L_\mathrm{d}$. The solid line is simulated with the delay Raman response. The dot line is simulated without the delay Raman response.

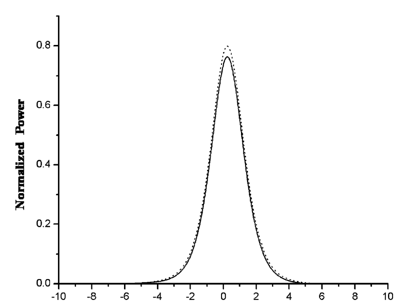


**Fig. 3.** Pulse shapes of pulse duration 2.5 fs fundamental soliton at $4L_\mathrm{d}$. The solid line is simulated with the delay Raman response. The dot line is simulated without the delay Raman response.

small, the enhanced power $P' = |A(z,t)|^2/(1-\alpha)$ is taken. In Fig. 4, we show the pulse shapes of pulse duration 2.5 fs enhanced power soliton after propagation $4L_\mathrm{d}$. Here we take $P' = 1.2195|A(z,t)|^2$. It is seen that the pulse shape is almost the same as initial pulse and delay Raman response only makes small pulse shift.

In addition, we use finite-difference time domain (FD-TD) method to solve full Maxwell's equations and verify the propagation simulated by using Eq. (4) for 2.5 fs fundamental soliton, the pulse duration is about 1.4 fs and nearly one optical cycle. In order to adapt the algorithm of FD-TD, we assume the fiber has dispersion relation: $\beta_2 = -24.57\,\mathrm{fs}^2/\mathrm{mm}$, $\beta_3 = 61.97\,\mathrm{fs}^3/\mathrm{mm}$, and

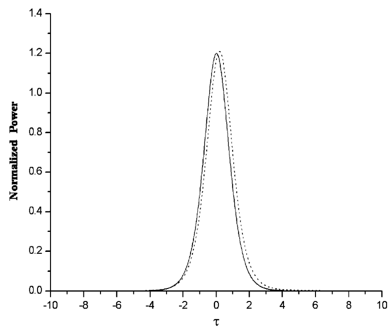


**Fig. 4.** Pulse shape of pulse duration 2.5 fs soliton with enhanced power at $4L_d$ (solid line). The pulse shape of initial pulse is shown by dotted line.

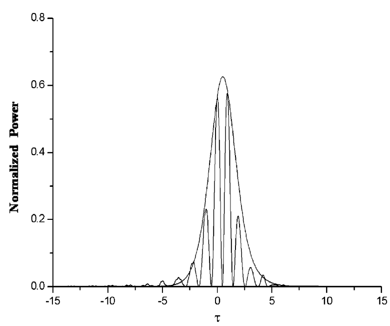


**Fig. 5.** The carrier and envelope formation of pulse duration 1.4 fs soliton at $4L_d$. The carrier is simulated by Eq. (5). The envelope formation is simulated by Eq. (4).

$\beta_4 = -209.64\,\text{fs}^4/\text{mm}$. Fig. 5 shows the pulse shape at $4L_d$, the envelope is simulated by using Eq. (4) and the carrier wave is simulated by Eq. (5). It can be seen that two numerical results are like. That is, the validity of Eq. (4) is not questionable. More further, we simulate such propagations with and without the delay Raman response by FD-TD method for the original power and the enhanced power $P' = 1.2195|A(z,t)|^2$. It is found that the differences of the numerical results with and without the delay Raman response are very small, especially, for the enhanced power case. It is because, comparing with the other high-order effects, the delay Raman response effect is very slight and can be negligible.

## 4. Conclusion

We investigate the effect of IRS for the propagation of the femtosecond solitons in an optical fiber. To factually simulate its influence, a combination of 27 Lorentzian lines to fit experimental Raman gain profile is adopted. The "freeze" of the IRS effect is clarified by using nonlinear Schrödinger equation and FD-TD method under unsaturation condition. It is found that delay response is rapidly reduced when pulse duration is smaller than 10 fs. For the propagation of the fundamental soliton with pulse duration 2.5 fs, it is found that the SSFS is slight and the change of pulse power is very obvious. When the initial power is enhanced, $P' = 1.2195|A(z,t)|^2$, for the same propagation, the pulse shape is almost the same as initial pulse and the delay Raman response only makes small pulse shift. That is, the delay Raman response effect can be negligible when the soliton is down to 2.5 fs or less than that. In other words, for ultrashort soliton, the effect of IRS is similarly frozen under the enhanced initial power.

## Acknowledgment

This work is partially supported by National Science Council, Republic of China, under Contract NSC 93-2212-E-194-015.

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