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Optics Communications 268 (2006) 23–26
OPTICS COMMUNICATIONS
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A method for measuring two-dimensional refractive index distribution with the total internal reflection of p-polarized light and the phase-shifting interferometry

Zhi-Cheng Jian, Po-Jen Hsieh, Hung-Chih Hsieh, Huei-Wen Chen, Der-Chin Su *

Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 Ta-Hsueh Road, Hsin-Chu 300, Taiwan, ROC

Received 19 January 2006; received in revised form 20 June 2006; accepted 5 July 2006

Abstract

Based on the total internal reflection of p-polarized light and the phase-shifting interferometry, an alternative method for measuring the two-dimensional refractive index distribution of a material is presented. The p-polarized light is incident on the boundary between a right-angle prism and a tested material. When the total internal reflection occurs at the boundary, and the p-polarized light has a phase variation. It depends on the refractive index of the tested material. Firstly, the two-dimensional phase variation distribution of the p-polarized light at the boundary is measured by the four-step phase shifting interferometric technique. Then, substituting the data into the special equations derived from Fresnel equations, the two-dimensional refractive index distribution of the tested material can be obtained.

Keywords: Total internal reflection; Phase-shifting interferometry; Fresnel equations; Two-dimensional refractive index distribution

1. Introduction

Refractive index is an important characteristic constant of an optical material, and it also determines the transparency of the material. Some techniques such as the reflectance method [1], the critical angle method [2,3], the ellipsometry [4], the polarization analysis method [5] and the heterodyne interferometry [6] have been proposed for measuring the refractive index. Although they have good measurement results, they are often used to evaluate the refractive index at one point of a material [7,8]. To overcome this drawback, an alternative method for measuring the two-dimensional refractive index distribution of a tested material is presented in this paper, based on the total internal reflection and the phase-shifting interferometry [9–12]. Firstly, special equations to estimate the phase variation of the p-polarized light under the total internal reflection are derived based on Fresnel equations [13]. Next, the four-step interferometric technique [14] is used to measure the two-dimensional phase variation distribution of the p-polarized light at the boundary between a right-angle prism and a tested material under the total internal reflection. Finally, the measured data are substituted into special equations derived previously, and the two-dimensional refractive index distribution of a tested material can be estimated. To show the validity of this method, a mixed liquid of oils and water was tested. It has several merits such as easy operation, rapid measurement, and a simple optical setup, etc.,

2. Principle

2.1. Phase variation of p-polarized light under total internal reflection

A ray of p-polarized light in air is incident at θ_t on the one side surface of a right-angle prism with refractive index

* Corresponding author. Tel.: +886 3 573 1951; fax: +886 3 571 6631.
E-mail address: t7503@faculty.nctu.edu.tw (D.-C. Su).

0030-4018/$ - see front matter
doi:10.1016/j.optcom.2006.07.009

n_1, as shown in Fig. 1. The light ray is refracted into the prism and it propagates toward the base surface of the prism. At the base surface of the prism, there is a boundary between the prism and the tested material TM of refractive index $n_2(x,y)$ where $n_1 > n_2$. If θ_i is larger than the critical angle, the light is totally reflected at the boundary. According to Fresnel equations, the reflection coefficients of p-polarized light can be expresses as

$$r_p = \frac{n^2\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}} = |r_p|\cdot\exp(i\phi_p), \tag{1a}$$

where $n = \left(\frac{n_2}{n_1}\right)$, $\phi_p(x,y)$ is the phase-variation and it is given as

$$\phi_p = -2\cdot\tan^{-1}\left(\frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cdot\cos\theta_i}\right). \tag{1b}$$

In addition, Eq. (1b) can also be rewritten as

$$n = \sqrt{\frac{-1+\sqrt{1+4\left(\tan\frac{\phi_p}{2}\cdot\cos\theta_i\cdot\sin\theta_i\right)^2}}{2\left(\tan\frac{\phi_p}{2}\cdot\cos\theta_i\right)^2}}. \tag{2}$$

Hence, it is obvious from Eq. (2) that $n_2(x,y)$ can be calculated with the measurement of $\phi_p(x,y)$ under the experimental conditions in which θ_i and n_1 are specified.

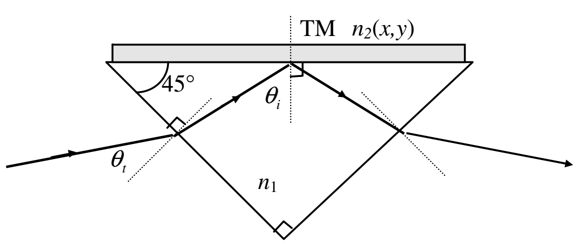

Fig. 1. The total internal reflection at the boundary between a prism and a tested medium.

2.2. Phase variation measurements with phase-shifting interferometry

The schematic diagram of this method is shown in Fig. 2. For convenience, the $+z$-axis is chosen to be along the light propagation direction and the y-axis is along the direction perpendicular to the paper plane. A light beam coming from a laser light source passes through a polarizer P and a beam-expander BE, and it becomes the collimating light. The collimating light enters a modified Twyman–Green interferometer. If the transmission axis of P is located at 0° with respect to the x-axis, then the light becomes the p-polarized light. In the interferometer, the collimating light is incident on a beam-splitter BS and divided into two parts: the transmitted light and the reflected light. The reflected light is normally reflected by a mirror M_1 driven by a piezo-transducer PZT and passes through BS. Then it enters a CCD camera. Here, it acts as the reference light E_r. On the other hand, the transmitted light is reflected by the mirrors M_2 and M_3, and enters a right-angle prism. After it is totally reflected at the boundary between the prism and the tested material TM, it propagates out of the prism. Then, it is normally reflected by a mirror M_4 and comes back along the original path. It is reflected by BS and also enters the CCD camera. Here, it acts as the test light E_t. Therefore, the interference intensity measured by the CCD can be written as

$$\begin{aligned} I(x,y) &= |E_t + E_r|^2 = \left|a_t\cdot e^{i\phi_t} + a_r\cdot e^{i\phi_r}\right|^2 \\ &= a_r^2 + a_t^2 + 2\cdot a_r\cdot a_t\cdot\cos(\phi_t - \phi_r) \\ &= a_r^2 + a_t^2 + 2\cdot a_r\cdot a_t\cdot\cos(2\cdot\phi_p + \psi) \\ &= A(x,y) + B(x,y)\cdot\cos(\phi(x,y)), \end{aligned} \tag{3}$$

where a_i and ϕ_i (i = t or r) represent the amplitude and the phase of E_i, ϕ_p is the phase variation of the p-polarized light under the total internal reflection in the prism, and ψ is the sum of the phase difference owing to the optical path difference between two interfering lights and extra

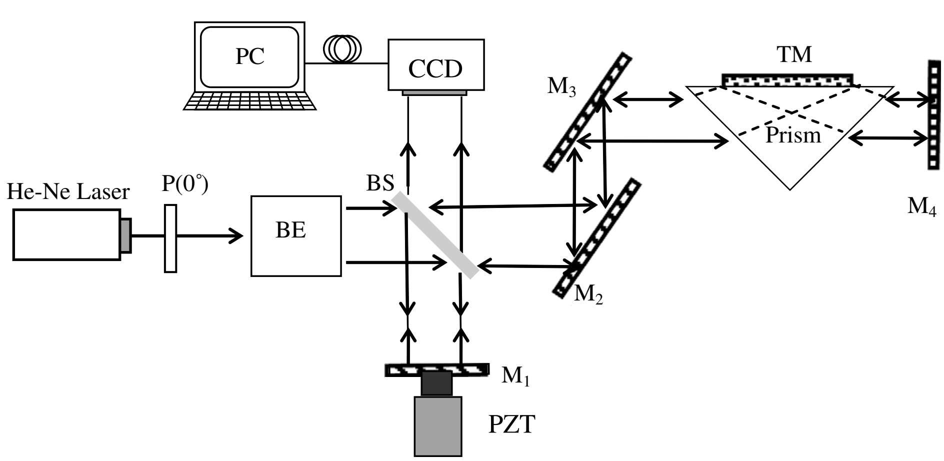

Fig. 2. Schematic diagram for measuring the two-dimensional refractive index distribution of a material. P, polarizer; BE, beam expander; BS, beam splitter; M, mirror; TM, tested material; PC, personal computer.

phase shifts produced by the reflections at BS and mirrors. Then, the phase-shifting interferometric technique is applied to measure the two-dimensional phase distribution $\phi(x,y)$. The CCD takes four interferograms as the PZT moves M_1 to change the phase of the reference light. An extra phase $\frac{\pi}{2}$ is added between two successive interferograms. So the intensities of these four interferograms can be written as

$$I_1(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y)), \tag{4a}$$

$$I_2(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y) + \pi/2), \tag{4b}$$

$$I_3(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y) + \pi), \tag{4c}$$

and

$$I_4(x,y) = A(x,y) + B(x,y)\cos\left(\phi(x,y) + \frac{3\pi}{2}\right). \tag{4d}$$

By solving the simultaneous equations, we get

$$\phi(x,y) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right). \tag{5}$$

From Eq. (5) follows that Eq. (3), we have

$$\phi_p(x,y) = \frac{1}{2}(\phi(x,y) - \psi). \tag{6}$$

In the second measurement let the base surface of the prism free without any tested material. We obtain

$$\phi' = 2 \cdot \phi_a + \psi, \tag{7}$$

where the phase variation ϕ_a can be calculated with the refractive index n_1 of the prism and $n_2 = 1$. Substituting ϕ_a and ϕ' into Eq. (7), the data of ψ can be calculated. Then substituting the data of ψ into Eq. (6), $\phi_p(x,y)$ can be estimated. Finally, the two-dimensional refractive index distribution $n_2(x,y)$ of a tested material can be calculated by using Eq. (1b).

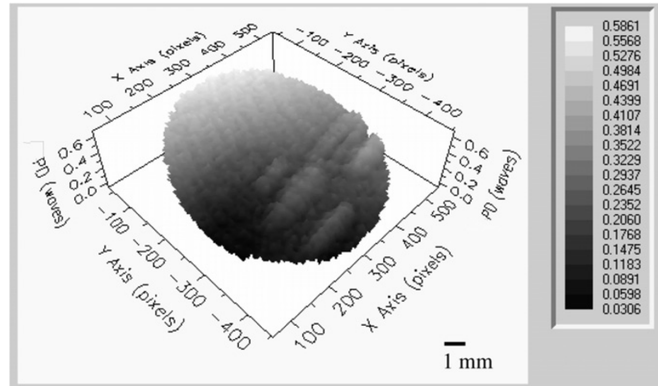

Fig. 3. The two-dimensional phase variation distribution $\phi_p(x,y)$ of the tested material.

$$\Delta n_2 = \frac{1}{n_1}\left|\frac{\partial n_2}{\partial \phi_p}\right| \times \Delta\phi_p = \frac{1}{n_1}\left|\frac{\csc\left(\frac{\phi_p}{2}\right)\left[-2\sec\left(\frac{\phi_p}{2}\right)\cdot\sin^2\theta_i + \cot\left(\frac{\phi_p}{2}\right)\cdot\csc\left(\frac{\phi_p}{2}\right)\cdot\sec^2\theta_i\cdot\left(-1+\sqrt{1+4C}\right)\right]}{2\sqrt{2+8C}\sqrt{\cot^2\left(\frac{\phi_p}{2}\right)\cdot\sec^2\theta_i\left(-1+\sqrt{1+4C}\right)}}\right| \times \Delta\phi_p, \tag{8a}$$

3. Experiments and results

In order to show the feasibility of this method, we tested a mixed liquid of ricinus oil, olive oil, baby oil, and water. Their refractive indices are 1.513, 1.474, 1.463, and 1.33, respectively. A He–Ne laser with a 632.8 nm wavelength, a right-angle prism made of SF8 glass with refractive index $n_1 = 1.68894$ [15], the PZT (PZ-91, Burleigh Instruments, Inc.) with motion sensitivity 0.002 μm/V, and the CCD camera (TM-545, PULNiX Inc.) with 510×492 pixels and 8-bit gray levels were used in this test. It is necessary to choose a suitable incident angle that is larger than the critical angles of all tested materials. In our experiment, the maximum refractive index of the tested material is 1.513, and its critical angle is 63.615°. For our convenience, we choose the condition $\theta_i = 63.62°$. Besides, the incident light intensity and the dynamic range of the CCD camera are so adjusted that they can match with each other. Consequently, the output of the CCD camera is linearly proportional to the light intensity.

The interferograms were sent to a personal computer PC and they were analyzed with the software IntelliWave™ (Engineering Synthesis Design Inc.). The results were depicted as shown in Figs. 3 and 4 by using the software Matlab (MathWorks Inc.). They are the two-dimensional phase variation distribution $\phi_p(x,y)$ and the associated two-dimensional refractive index distribution $n_2(x,y)$ of the tested material, respectively.

4. Discussion

From Eq. (2) we get

where

$$C = \cos^2\theta_i \cdot \sin^2\theta_i \cdot \tan\left(\frac{\phi_p}{2}\right), \tag{8b}$$

Δn_2 and $\Delta\phi_p$ are the errors in n_2 and ϕ_p, respectively. The error $\Delta\phi_p$ may be influenced by the phase-resolution of a phase-shifting interferometry and the polarization-mixing error [16–18]. The gray levels of the minima and the maxima of the interferograms are 0 and 255, respectively, as the phase-shifting interferometry is fully utilized. The tested light is totally reflected twice at the boundary between the prism and the tested material, the theoretical resolution of this method is about 180°/(256) (≅0.703°). In our experiments, the extinction ratio of the polarizer (Newport Inc.)

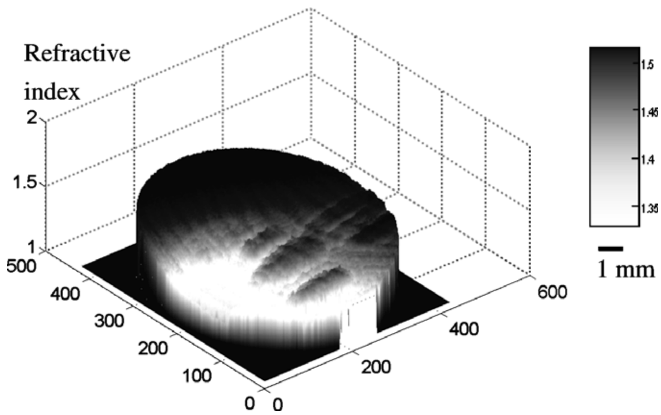

Fig. 4. The two-dimensional refractive index distribution $n_2(x, y)$ of the tested material.

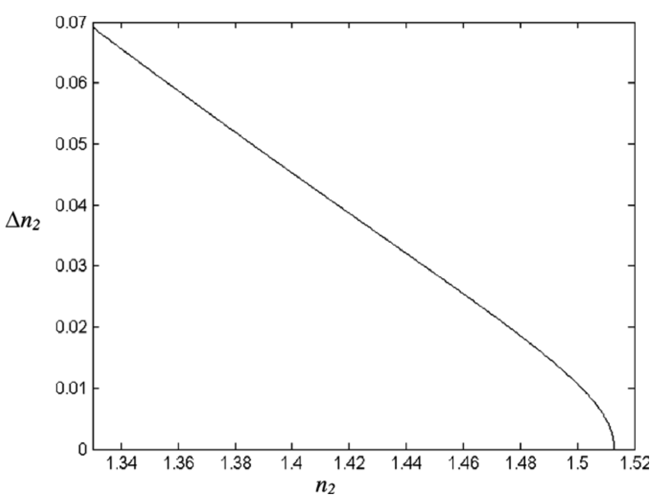

Fig. 5. Relation curve of Δn_2 versus n_2.

is 1×10^{-3}. So the polarization-mixing error is about 0.028°. Hence, the total error of $\Delta\phi_p$ is 0.74°. Substituting the experimental conditions $\theta_i = 63.62°$, $n_1 = 1.68894$, and $\Delta\phi_p = 0.74°$ into Eq. (8) we can obtain the relation curves of Δn_2 versus n_2 as shown in Fig. 5. It is obvious from Fig. 5 that Δn_2 is related to n_2. The measurement error Δn_2 can be decreased to 3.6×10^{-5} when n_2 is near 1.513.

Although the s-polarized light has also a phase variation, it is smaller than that of the p-polarized light. Hence, we choose to measure the phase variation of the p-polarized light in our method to enhance the measurement resolution. The measurable range of this method is limited by the refractive index of the prism. To expand the measurable range, it is better to use a prism with a high refractive index.

5. Conclusion

An alternative method for measuring the two-dimensional refractive index distribution of a material is proposed. First, the phase variation distribution of the p-polarized of the reflected light under the total internal reflection in a prism, whose base is contacted with the tested material, can be measured accurately with the phase-shifting interferometry. Then it is substituted into special equations derived from Fresnel equations, and the two dimensional refractive index distribution of the tested material can be estimated. Its validity has been demonstrated. It has some merits such as simple optical setup, easy operation and rapid measurement.

Acknowledgement

This study was supported in part by the National Science Council, Taiwan, ROC, under Contract NSC 94-2215-E-009-002.

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