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ORIGINAL ARTICLE

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Bayesian approach for measuring EEPROM process capability based on the one-sided indices C_{PU} and C_{PL}

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Abstract The purpose of process capability analysis is to provide numerical measures on whether a process is capable of reproducing items meeting the manufacturing specifications. Capability analyses have received considerable recent research attention and increased usage in process assessments and purchasing decisions. Most existing research works on capability analysis focus on estimating and testing process capability based on the traditional distribution frequency approach. In this paper, we propose a Bayesian approach based on the indices C_{PU} and C_{PL} to measure EEPROM process capability, in which the specifications are one-sided rather than two-sided. We obtain the credible intervals of C_{PU} and C_{PL} and develop a Bayesian procedure for capability testing. The posterior probability p, for which the process under investigation is capable, is derived. The credible interval is a Bayesian analog of the classical lower confidence interval. A process satisfies the manufacturing capability requirements if all the points in the credible interval are greater than the pre-specified capability level w. To make this Bayesian procedure practical for in-plant applications, a real example of an EEPROM manufacturing process is investigated, demonstrating how the Bayesian procedure can be applied to actual data collected in the factories.

Keywords Bayesian approach · Credible interval · Process capability indices · Posterior probability

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1 Introduction

In recent years, numerous process capability indices (PCIs), including C_p, C_a, C_{PU}, C_{PL}, C_{pk}, C_{pm}, and C_{pmk}, have received substantial research attention in quality assurance as well as statistical literatures. These indices have been popularly used in the manufacturing industry, providing measures on whether a process is capable of reproducing items meeting the quality requirements preset by the product designer. The use of PCIs in industry began in the United States in the early 1980s. Soon after, this explosion of use expanded into various industries, including automotive, semiconductor, and IC manufacturing industries, to measure product qualities meeting the manufacturing specification. Examples include [6, 10, 12, 21] and many others. These capability indices are convenient tools to the manufacturer for monitoring process quality, and a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. These indices are defined in the following (see [8, 3, 15]):

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_a = 1 - \frac{|\mu - m|}{d}$$

$$C_{PU} = \frac{USL - \mu}{3\sigma}, \quad C_{PL} = \frac{\mu - LSL}{3\sigma},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation (overall process variation), m=(USL+

LSL)/2, d=(USL-LSL)/2, and T is the target value. While C_p, C_a, C_{pk}, C_{pm}, and C_{pmk} are appropriate measures for normal processes with two-sided manufacturing specifications (which require both USL and LSL), C_{PU} and C_{PL} have been designed particularly for processes with one-sided manufacturing specifications (which require only USL or LSL, but not both). In practice, the process mean and process standard deviation are usually unknown, but they can be estimated from a sample of n measurements $\{x_1, x_2,..., x_n\}$. The most common estimates of μ and σ are:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,\ s = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2\right]^{1/2}.$$

The current practice of measuring production quality by evaluating the point estimates of the capability indices have been criticized, since there is no sampling error assessment of these estimates. The sample estimate of the index calculated from sample data has never been accurate. Therefore, the decisions made in concluding the capability measures directly from the sample estimate are unreliable. Existing methods for measuring production quality have focused on the traditional distribution frequency approaches. However, the sampling distributions are usually complicated, and this makes establishing the exact confidence interval difficult. An alternative approach for measuring process capability is to use the Bayesian method, by specifying a prior distribution for the parameter of interest, to obtain the posterior distribution of the parameter. Then, one makes inference to the parameter using its posterior distribution given the sample observations. It is not difficult to obtain the posterior distribution when a prior distribution is given, even in the case where the form of the posterior distribution is complicated, as one could always use numerical methods or Monte Carlo methods [7] to obtain an approximate point estimate or interval estimate. This is the advantage of the Bayesian approach over the traditional distribution frequency approach. In this paper, we propose a Bayesian approach using the capability indices C_{PU} and C_{PL}, for measuring EEPROM manufacturing quality where the specifications are one-sided rather than two-sided. We obtain the credible interval of C_{PU} and C_{PL}, and propose, accordingly, a Bayesian procedure for capability testing. The posterior probability p, for which the process under investigation is capable, is derived in Sect. 4. In Sect. 5, to make this Bayesian procedure practical for in-plant applications, we tabulate the minimum values of C*(p), for which the posterior probability p reaches various desirable confidence levels. Based on the test, we also develop a simple but practical step-by-step procedure. Practitioners can use the proposed procedure to determine whether their manufacturing processes are capable of reproducing product items satisfying the preset manufacturing quality.

2 EEPROM process capability requirement

An electrically erasable programmable read-only memory (EEPROM) chip is a user-modifiable read-only memory chip that can be erased and reprogrammed (written onto) repeatedly through the application of a higher electrical voltage. It is usually used in portable phones, PHS phones, compact portable terminals, consumer products (such as cordless phones and audio systems), industrial equipment, including measuring instruments and PLCs, OA products, such as printers and scanners, in-house telephone switches, and other communications equipment. The product investigated here is a 128-bit EEPROM organized as a 16×8 configuration with a 2-wire serial interface, as depicted in Fig. 1. The low-voltage design permits operations down to 1.8 volts, which maintains a maximum standby current of only 1 μA, with a typical active current of only 500 μA. This EEPROM is available in 8-pin PDIP, 8-pin SOIC (150 mil), 8-pin TSSOP, and 5-pin SOT-23 packages. This EEPROM supports a bi-directional 2-wire bus and data transmission protocol. A device that sends data onto the bus is defined as a transmitter, and a device receiving data is defined as a receiver.

2.1 Process capability requirement for OLC

The output leakage current (OLC) is an essential product quality characteristic which has a significant impact on the product quality. For the OLC of a particular model of EEPROM, the upper specification limit, USL, is set to 5 μA. For processes with one-sided specifications, some minimum capability requirements have been recommended

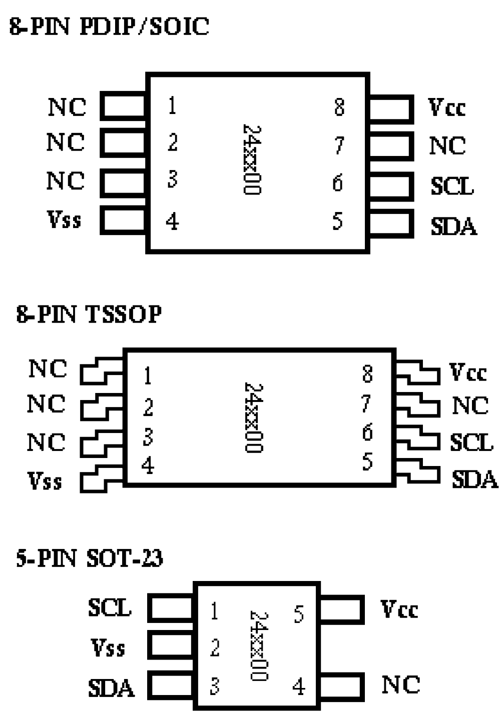

Fig. 1 A 128-bit EEPROM chip

Table 1 Some minimum capability requirements based on C_{PU} for new and special processes

C_{PU} value	Production process types
1.25	Existing processes
1.45	New processes or existing processes regarding safety, strength, or critical parameters
1.60	New processes on safety, strength, or critical parameters

[11], as specific process types must run under some designated quality conditions. Those recommendations are summarized in Table 1. For existing manufacturing processes, the capability must be no less than 1.25, and for new manufacturing processes, the capability must be no less than 1.45. For existing manufacturing processes, regarding safety, strength, or critical parameters, the capability must be no less than 1.45, and for new manufacturing processes, regarding safety, strength, or critical parameters, the capability must be no less than 1.60.

For normally distributed processes with one-sided specification limit USL, the process yield P(X< USL) is:

$$P\left(\frac{X-\mu}{3\sigma}<\frac{USL-\mu}{3\sigma}\right)=P\left(\frac{1}{3}Z<C_{PU}\right)$$
$$=P(Z<3C_{PU})=\Phi(3C_{PU}),$$

where Z is the standard normal distribution N(0, 1). Therefore, the corresponding non-conforming units in parts per million (NCPPM) for a well-controlled normally distributed process can be calculated, exactly, as $NCPPM = 10^6\times[1-\Phi(3C_{PU})]$. Consequently, the production yield for the usual existing processes should target no more than 88 PPM, noting that NCPPM≤200 PPM is the common standard used in most microelectronic industries for products with two-sided specifications. The production yield for newly set up processes on safety, strength, or with critical parameters, however, should target no more than 0.8 PPM, a more stringent requirement set for possible mean shift or variation change.

3 Sample calculations of C_{PU} and C_{PL}

To estimate the indices C_{PU} and C_{PL}, we consider the natural estimators, which are defined as the following:

$$\widehat{C}_{PU}=\frac{USL-\overline{x}}{3s},\ \widehat{C}_{PL}=\frac{\overline{x}-LSL}{3s},$$

where $\overline{x}=\sum_{i=1}^{n}x_i/n$ and $s=\left[(n-1)^{-1}\sum_{i=1}^{n}(x_i-\overline{x})^2\right]^{1/2}$ are conventional estimators of μ and σ, which may be obtained from a process that is demonstrably stable (in control). Chou and Owen [5] investigated the natural estimators $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$, and showed that, under the normality assumption, the estimators $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$ are distributed as $ct_{n-1}(\delta)$, where $c=(3\sqrt{n})^{-1}$ and $t_{n-1}(\delta)$ is a non-central t distribution with n−1 degrees of freedom and non-centrality parameter $\delta=3\sqrt{n}C_{PU}$ and $\delta=3\sqrt{n}C_{PL}$, respectively. Both estimators are biased, but Pearn et al. [15] showed that, by adding the well-known bias correction factor $b_{f}=(2/f)^{1/2}\Gamma(f/2)/\Gamma[(f-1)/2]$ to $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$, then the unbiased estimators $b_{n-1}\widehat{C}_{PU}$ and $b_{n-1}\widehat{C}_{PL}$ can be obtained, which have been denoted as $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$. That is, $E(\widetilde{C}_{PU})=C_{PU}$ and $E(\widetilde{C}_{PL})=C_{PL}$. Since $b_f<1$ $(n>2)$, then $Var(\widetilde{C}_{PU})<Var(\widehat{C}_{PU})$ and $Var(\widetilde{C}_{PL})<Var(\widehat{C}_{PL})$. Further, since both estimators depend only on the sufficient and complete statistics $(\overline{x},\ s^2)$, $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$ are uniformly minimum variance unbiased estimators (UMVUE) of C_{PU} and C_{PL}, respectively.

Based on the two UMVUEs, $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$, Pearn and Chen [14] implemented the statistical theory of hypotheses testing, and developed a simple but practical procedure accompanied with convenient tabulated critical values for engineers/practitioners to use in decision making for their factory applications. Formulas for computing the power of the corresponding test are also obtained based on the following probability density function, putting $C_{PU}=Y$ and $\delta=3\sqrt{n}C_{PU}$, then:

$$f(y)=\frac{3\sqrt{n/(n-1)}\times 2^{-n/2}}{b_{n-1}\sqrt{\pi}\Gamma[(n-1)/2]}$$
$$\times\int_0^\infty t^{(n-2)/2}\exp\left\{-\frac{1}{2}\left[t+\left(\frac{3y\sqrt{nt}}{b_{n-1}\sqrt{n-1}}-\delta\right)^2\right]\right\}dt,$$

where $b_{n-1}=\left(\frac{2}{n-1}\right)^{1/2}\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{n-2}{2}\right)^{-1}$.

Lin and Pearn [9] developed efficient SAS computer programs to calculate the critical values and p values needed based on the cumulative distribution function for the capability testing. An illustrative application of capability testing to the voltage level translator was also given. Pearn and Shu [16, 19] further developed an efficient algorithm with the Matlab computer program to find the lower confidence bounds conveying critical information regarding the minimal true process capability. However, their investigations are all based on traditional distribution frequency approaches.

4 A Bayesian approach for capability testing

Cheng and Spiring [4] proposed a Bayesian procedure for assessing process capability index C_p. Shiau et al. [20] applied a similar Bayesian approach to index C_{pm} and index C_{pk}, but under the restriction that the process mean μ equals the midpoint of the two specification limits, m (a rather impractical assumption for most factory applications, since, in this case, C_{pk} reduces to C_p). In the following, we consider a Bayesian procedure for the one-sided capability indices C_{PU} and C_{PL}, derive a Bayesian interval estimate for C_{PU} and C_{PL}, and propose, accordingly, a Bayesian procedure for testing process capability. A 100*p*% credible interval is the Bayesian analog of the classical 100*p*% confidence interval, where *p* is the confidence level for the interval. The credible interval covers 100*p*% of the posterior distribution of the parameter [1]. Assuming that the measures $\mathbf{x}=\{x_1, x_2,..., x_n\}$ are a random sample taken from independent and identically distributed (i.i.d.) $N(\mu, \sigma^2)$, a normal distribution with mean μ and variance σ^2. Then, the likelihood function for μ and σ is:

$$L(\mu,\ \sigma|\mathbf{x}) = \left(2\pi\sigma^2\right)^{-n/2} \times \exp\left\{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right\}.$$

The first step for the Bayesian approach is to find an appropriate prior. If prior information about the parameters is available, it should be incorporated in the prior density. When there is little or no prior information, we want a prior with minimal influence on the inference. One of the most widely used non-informative priors is the so-called reference prior, which is a non-informative prior that maximizes the difference between the information (entropy) on the parameter provided by the prior and by the posterior. In other words, the reference prior allows the prior to provide as little information as possibly about the parameter (see [2] for more details). Therefore, in this paper, we adopt the following non-informative reference prior:

$$\pi(\mu,\sigma) = 1/\sigma, \quad 0 < \sigma < \infty.$$

The posterior probability density function (PDF), $f(\mu, \sigma|\mathbf{x})$ of (μ, σ) may be expressed as the following:

$$f(\mu,\sigma|\mathbf{x}) \propto L(\mu,\sigma|\mathbf{x}) \times \pi(\mu,\sigma) \propto \sigma^{-(n+1)} \times \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right),$$

Since:

$$\int_0^\infty \int_{-\infty}^{\infty} \sigma^{-(n+1)} \times \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right) d\mu\, d\sigma = \int_0^\infty \sigma^{-(n+1)} \exp\left(-\frac{1}{\beta\sigma^2}\right) \times \left[\int_{-\infty}^{\infty} \exp\left(-\frac{n(\mu-\bar{x})^2}{2\sigma^2}\right) d\mu\right] d\sigma = \sqrt{\frac{\pi}{2n}}\,\Gamma(\alpha)\beta^\alpha.$$

And in order to satisfy the integration property, that the probability over the PDF is 1, so that:

$$f(\mu,\sigma|\mathbf{x}) = \frac{2\sqrt{n}}{\sqrt{2\pi}\Gamma(\alpha)\beta^\alpha}\sigma^{-(n+1)} \times \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right) \tag{1}$$

where $\alpha = (n-1)/2$, $\beta = \left[\sum_{i=1}^{n}(x_i-\bar{x})^2/2\right]^{-1} = \left[(n-1)s^2/2\right]^{-1}$. As mentioned before, it is natural to consider the quantity $\Pr\{\text{process is capable} \mid \mathbf{x}\}$ in the Bayesian approach. Since the indices C_{PU} and C_{PL} are our major concern in this paper, so we are interested in finding the posterior probability $p=\Pr\{C_{PU}>w \mid \mathbf{x}\}$ or $\Pr\{C_{PL}>w \mid \mathbf{x}\}$ for some fixed positive number *w*. Therefore, given a pre-specified capability level $w>0$, the posterior probability based on index C_{PU} that a process is capable is given as:

$$
\begin{aligned}
p &= \Pr\{C_{PU} > w|\mathbf{x}\} = \Pr\left\{\frac{USL-\mu}{3\sigma} > w|\mathbf{x}\right\} = \Pr\{\mu + 3\sigma w < USL|\mathbf{x}\} \\
&= \int_0^\infty \int_{-\infty}^{USL-3\sigma w} \frac{2\sqrt{n}}{\sqrt{2\pi}\Gamma(\alpha)\beta^\alpha}\sigma^{-(n+1)} \times \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right) d\mu\, d\sigma \\
&= \int_0^\infty \frac{2\sqrt{n}}{\sqrt{2\pi}\Gamma(\alpha)\beta^\alpha}\sigma^{-(n+1)} \times \exp\left(-\frac{1}{\beta\sigma^2}\right)\int_{-\infty}^{USL-3\sigma w} \exp\left(-\frac{n(\mu-\bar{x})^2}{2\sigma^2}\right) d\mu\, d\sigma \\
&= \int_0^\infty \frac{2\sigma^{-n}}{\Gamma(\alpha)\beta^\alpha}\exp\left(-\frac{1}{\beta\sigma^2}\right) \times \Phi\left(\frac{USL-3\sigma w-\bar{x}}{\sigma/\sqrt{n}}\right) d\sigma \\
&= \int_0^\infty \frac{2\sigma^{-n}}{\Gamma(\alpha)\beta^\alpha}\exp\left(-\frac{1}{\beta\sigma^2}\right) \times \Phi\left(3\sqrt{n}\left(\widehat{C}_{PU} \times \frac{s}{\sigma} - w\right)\right) d\sigma
\end{aligned}
$$

By changing variable, let $y=\beta\sigma^2$. Then, $dy=2\beta\sigma d\sigma$ and $\frac{s}{\sigma}=\sqrt{\frac{2}{(n-1)y}}$. Therefore, the posterior probability p may be rewritten as:

$$
\begin{aligned}
p &= \Pr\{C_{PU} > w|\mathbf{x}\} \\
&= \int_0^\infty \frac{1}{\Gamma(\alpha)y^{\alpha+1}}\exp\left(-\frac{1}{y}\right) \\
&\quad \times \Phi\left(3\sqrt{n}\left(\frac{\widetilde{C}_{PU}}{b_{n-1}} \times \sqrt{\frac{2}{(n-1)y}} - w\right)\right) dy
\end{aligned} \tag{2}
$$

where $\alpha = (n-1)2$, $\beta = \left[\sum_{i=1}^{n}(x_i-\bar{x})^2/2\right]^{-1} = [(n-1)s^2/2]^{-1}$, $-\infty < \mu < \infty$, $0 < \sigma < \infty$, $b_{n-1} = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ and $\mathbf{\Phi}$ is the cumulative distribution function of the standard normal distribution.

On the other hand, the posterior probability based on the index C_{PL} that a process is capable is given as:

$$
\begin{aligned}
p &= \Pr\{C_{PL} > w|\mathbf{x}\} = \Pr\left\{\frac{\mu-LSL}{3\sigma} > w\Big|\mathbf{x}\right\} = \Pr\{\mu - 3\sigma w > LSL|\mathbf{x}\} \\
&= \int_0^\infty \int_{LSL+3\sigma w}^{\infty} \frac{2\sqrt{n}}{\sqrt{2\pi}\Gamma(\alpha)\beta^\alpha}\sigma^{-(n+1)} \times \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right) d\mu\, d\sigma \\
&= \int_0^\infty \frac{2\sigma^{-n}}{\Gamma(\alpha)\beta^\alpha}\exp\left(-\frac{1}{\beta\sigma^2}\right) \times \left[1-\Phi\left(\frac{LSL+3\sigma w-\bar{x}}{\sigma/\sqrt{n}}\right)\right] d\sigma \\
&= \int_0^\infty \frac{2\sigma^{-n}}{\Gamma(\alpha)\beta^\alpha}\exp\left(-\frac{1}{\beta\sigma^2}\right) \times \Phi\left(3\sqrt{n}\left(\widehat{C}_{PL} \times \frac{s}{\sigma} - w\right)\right) d\sigma \\
&= \int_0^\infty \frac{1}{\Gamma(\alpha)y^{\alpha+1}}\exp\left(-\frac{1}{y}\right) \times \Phi\left(3\sqrt{n}\left(\frac{\widetilde{C}_{PL}}{b_{n-1}} \times \sqrt{\frac{2}{(n-1)y}} - w\right)\right) dy
\end{aligned} \tag{3}
$$

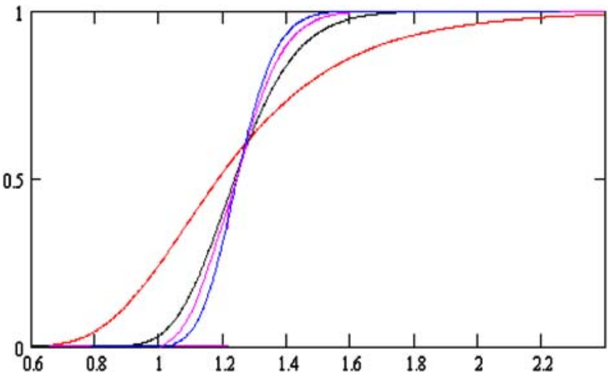

Fig. 2 Probability p versus C*(p) for n=10(30)100, w=1.25

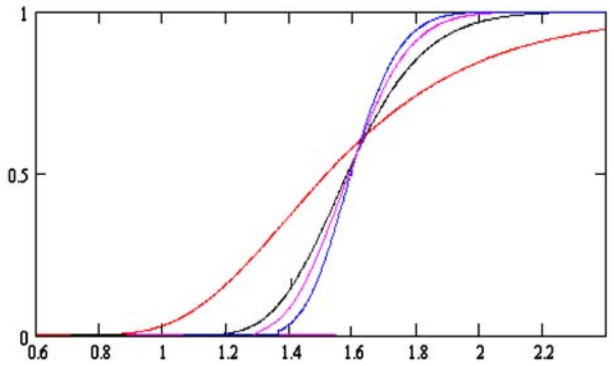

Fig. 4 Probability p versus C*(p) for n=10(30)100, w=1.60

5 Procedure for testing process capability

For the convenience of presentation, we let C_I be either C_{PU} or C_{PL}, $\widehat{C}_I$ denote either $\widehat{C}_{PU}$ or $\widehat{C}_{PL}$, and $\widetilde{C}_I$ denote either $\widetilde{C}_{PU}$ or $\widetilde{C}_{PL}$, then, from Eqs. 2 and 3, the posterior probability based on the one-sided indices C_{PU} and C_{PL} can be rewritten as:

$$p = \Pr\ \{\text{the process is capable}|\mathbf{x}\} = \Pr\{C_I > w|\mathbf{x}\} = \int_0^\infty \frac{1}{\Gamma(\alpha)y^{\alpha+1}}\exp\left(-\frac{1}{y}\right) \times \Phi\left(3\sqrt{n}\left(\frac{\widetilde{C}_I}{b_{n-1}} \times \sqrt{\frac{2}{(n-1)y}} - w\right)\right)dy \tag{4}$$

where $\alpha = (n-1)/2$, $b_{n-1} = [2(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$, and Φ is the cumulative distribution function of the standard normal distribution. Note that the posterior probability p depends on n, w, and $\widetilde{C}_I$. From Eq. 4, by noticing that there is a one-to-one correspondence between p and C* when n and w are given, and by the fact that $\widetilde{C}_I$ can be calculated from the process data, we find that the minimum value of C*(p) required to ensure that the posterior probability p reaches a certain desirable level can be useful in assessing process capability. Thus, the value w, denoted by $L_{C_I}(p)$, satisfies:

$$p = \Pr\{C_I > w|\mathbf{x}\} = \Pr\{C_I > L_{C_I}(p)|\mathbf{x}\} \tag{5}$$

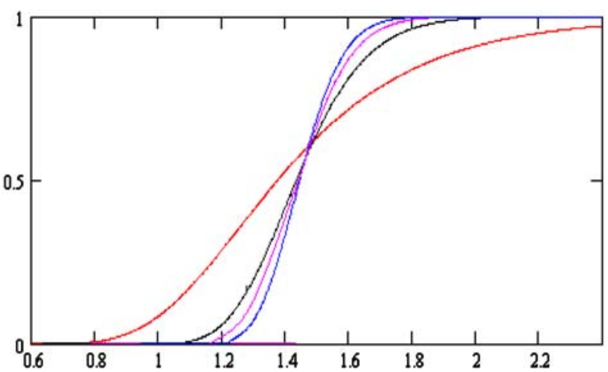

Fig. 3 Probability p versus C*(p) for n=10(30)100, w=1.45

Table 2a Critical values C*(p) for w=1.25, n=10(10)300, and p=0.99, 0.975, and 0.95

n	p=0.99	p=0.975	p=0.95
10	2.420	2.124	1.910
20	1.927	1.780	1.667
30	1.763	1.659	1.576
40	1.677	1.593	1.526
50	1.622	1.551	1.493
60	1.584	1.521	1.470
70	1.555	1.498	1.452
80	1.532	1.480	1.438
90	1.513	1.465	1.426
100	1.498	1.453	1.416
110	1.485	1.443	1.408
120	1.474	1.434	1.401
130	1.464	1.426	1.394
140	1.455	1.419	1.389
150	1.448	1.413	1.384
160	1.441	1.407	1.379
170	1.434	1.402	1.375
180	1.429	1.398	1.372
190	1.424	1.393	1.368
200	1.419	1.389	1.365
210	1.414	1.386	1.362
220	1.410	1.383	1.359
230	1.406	1.379	1.357
240	1.403	1.377	1.355
250	1.400	1.374	1.352
260	1.396	1.371	1.350
270	1.393	1.369	1.348
280	1.391	1.367	1.346
290	1.388	1.365	1.345
300	1.386	1.362	1.343

A $100p\%$ credible interval for C_I is $[L_{C_I}(p), \infty]$, where p is a number between 0 and 1, say, 0.95 for a 95% confidence interval, which means that the posterior probability that the credible interval contains C_I is p. Figures 2, 3, and 4 plot the probability p versus $C^*(p)$ from Eq. 5 for n=10(30)100 with w=1.25, 1.45, 1.60, respectively. From these figures, we can see that the larger the sample size, the steeper the curve. That is, the larger is the sample size, the smaller the critical value $C^*(p)$.

In our Bayesian approach, we say that the process is capable in a Bayesian sense if all the points in this credible interval are greater than a pre-specified value of w, say 1.00 or 1.25. When this happens, we have $p=\Pr\{C_I>w \mid \mathbf{x}\}$. In other words, to see if a process is capable (with capability level w and confidence level p), we only need to check if $\tilde{C}_I > C^*(p)$. Throughout this paper, it is assumed that the process measurements are independent and identically distributed (i.i.d.) from a normal distribution, and the process is under statistical control. We remark that the estimation of these capability indices is meaningful only when the process is under statistical control. Table 2a,b,c tabulates the values of $C^*(p)$ for w=1.25, 1.45, and 1.60, n=10(10)300, for p=0.95, 0.975, and 0.99, respectively. For example, if w=1.25 is the minimum capability requirement, then for p=0.95, n=50, $C^*(p)$=1.493. Thus, the value of $\tilde{C}_I$ calculated from sample data must satisfy $\tilde{C}_I \geq 1.493$ to conclude that C_{PU} (or C_{PL}) ≥1.25 (process is capable).

As a result, to judge if a given process meets the capability requirements, we first determine the pre-specified value w, the capability requirement, and the α-risk or the confidence level p for the interval. Checking the appropriate table or solving Eq. 4 (the program is available from the authors), we may obtain the critical value $C^*(p)$ based on given values of p, sample size n, and, next, to calculate $\tilde{C}_I$ from samples. If the estimated value $\tilde{C}_I$ is greater than the critical value $C^*(p)$, then we may conclude that the process meets the capability requirements ($C_I > w$). Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirements. In this case, we would believe that $C_I \leq w$. In the following section, we present a simple step-by-step procedure for

Table 2b Critical values $C^*(p)$ for w=1.45, n=10(10)300, and p=0.99, 0.975, and 0.95

n	p=0.99	p=0.975	p=0.95
10	2.793	2.452	2.206
20	2.225	2.057	1.927
30	2.038	1.918	1.823
40	1.939	1.843	1.766
50	1.876	1.794	1.728
60	1.832	1.760	1.701
70	1.798	1.734	1.681
80	1.772	1.713	1.664
90	1.751	1.696	1.651
100	1.734	1.682	1.640
110	1.719	1.670	1.630
120	1.706	1.660	1.622
130	1.695	1.651	1.615
140	1.685	1.634	1.609
150	1.676	1.636	1.603
160	1.668	1.630	1.598
170	1.661	1.624	1.593
180	1.654	1.619	1.589
190	1.648	1.614	1.585
200	1.643	1.609	1.581
210	1.638	1.605	1.578
220	1.633	1.601	1.575
230	1.629	1.598	1.572
240	1.625	1.595	1.569
250	1.621	1.591	1.567
260	1.617	1.589	1.565
270	1.614	1.586	1.562
280	1.611	1.583	1.560
290	1.608	1.581	1.558
300	1.605	1.578	1.556

Table 2c Critical values $C^*(p)$ for w=1.60, n=10(10)300, and p=0.99, 0.975, and 0.95

n	p=0.99	p=0.975	p=0.95
10	3.074	2.700	2.430
20	2.450	2.265	1.122
30	2.244	2.112	2.009
40	2.135	2.030	1.946
50	2.066	1.977	1.904
60	2.018	1.939	1.875
70	1.981	1.910	1.852
80	1.953	1.888	1.835
90	1.930	1.870	1.820
100	1.910	1.854	1.808
110	1.894	1.841	1.798
120	1.880	1.830	1.789
130	1.868	1.820	1.781
140	1.857	1.811	1.774
150	1.847	1.804	1.767
160	1.839	1.797	1.762
170	1.831	1.790	1.757
180	1.824	1.785	1.752
190	1.817	1.779	1.748
200	1.811	1.774	1.744
210	1.805	1.770	1.740
220	1.800	1.766	1.737
230	1.796	1.762	1.734
240	1.791	1.758	1.731
250	1.787	1.755	1.728
260	1.783	1.752	1.725
270	1.779	1.749	1.723
280	1.776	1.746	1.721
290	1.773	1.743	1.718
300	1.769	1.741	1.716

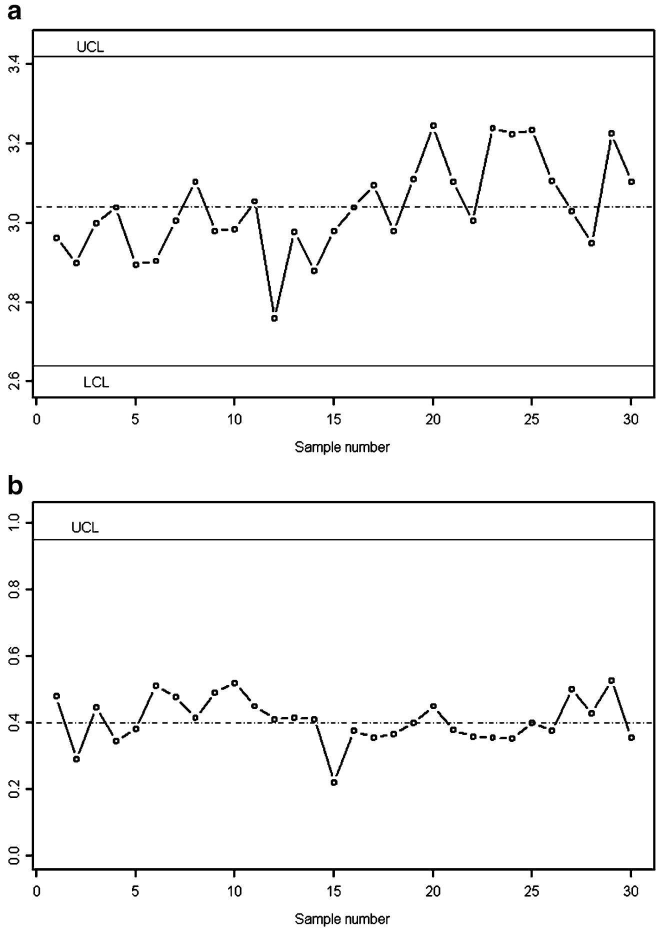

Fig. 5 **a** $\overline{X}$ control chart of the process. **b** S control chart of the process

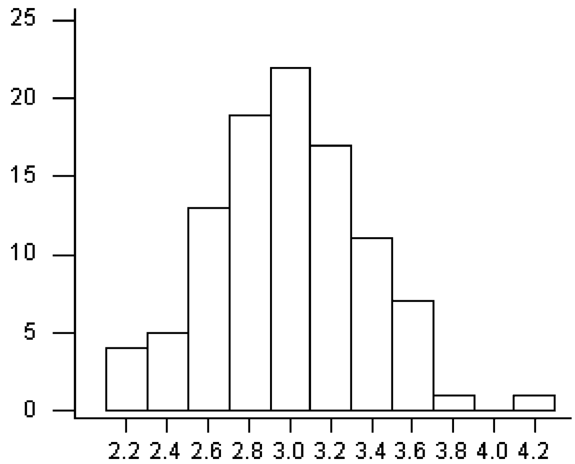

Fig. 6 Histogram

testing the process capability. The practitioners can apply the procedure to their in-plant applications to obtain reliable decisions.

5.1 Test procedure

Step 1

Decide the definition of "capable" (w, normally set to 1.00, 1.25, 1.45, or 1.60), and the confidence level p for the interval (normally set to 0.99, 0.975, or 0.95). The chance of the true C_{PU} or C_{PL} lying in this interval is p.

Step 2

Calculate the value of b_{n-1} and the estimator $\widehat{C}_I$ from the collected sample, then $\widetilde{C}_I = b_{n-1} \times \widetilde{C}_I$ can be obtained.

Step 3

Check the appropriate table to find the critical value $C^*(p)$ based on given values of p, sample size n, and w.

Step 4

Conclude that the process is capable ($C_I > w$) if $\widetilde{C}_I$ value is greater than the critical value $C^*(p)$. Otherwise, we do not have enough information to conclude that the process is capable.

6 Measuring EEPROM process capability

We consider the following example taken from a company located on the Hsinchu Science-Based Industrial Park (HSIP) in Taiwan, which designs and manufactures standard flash memory EEPROM and mixed-signal products, such as PLL, ADC DAC, and many others. The manufacturing specifications for a 128-bit EEPROM chip has an upper specification limit USL=5μA for the output leakage current (OLC), as mentioned before. If the OLC is

Table 3 A total of 100 observations

2.74	2.25	2.98	3.14	3.08	2.85	3.21	2.51	3.19	2.75
3.68	3.23	2.90	3.05	2.58	3.31	2.52	3.16	2.62	2.95
2.85	2.80	3.03	3.05	2.54	2.44	2.82	3.01	2.93	3.39
2.47	3.08	2.40	3.22	2.77	3.05	4.15	2.59	3.28	3.56
2.75	3.38	3.49	2.54	2.28	2.93	3.54	3.49	3.09	3.17
3.17	2.66	3.35	2.77	2.68	3.15	3.23	2.77	2.30	2.17
3.35	2.76	2.20	2.75	3.58	2.70	2.78	2.99	3.63	3.44
2.91	2.67	3.56	2.73	2.90	2.41	3.20	3.86	3.02	3.39
3.26	3.60	2.89	3.18	3.03	2.60	2.70	3.25	3.32	2.67
2.61	3.09	3.07	2.89	3.49	3.14	2.96	2.87	2.97	3.26

```
  1    2 1
  5    2 2223
 15    2 4444555555
 34    2 6666666777777777777
(17)   2 88888889999999999
 49    3 00000000000011111111
 29    3 2222222223333333
 13    3 44445555
  5    3 666
  2    3 8
  1    4 1
```

Fig. 7 Stem-and-leaf plot

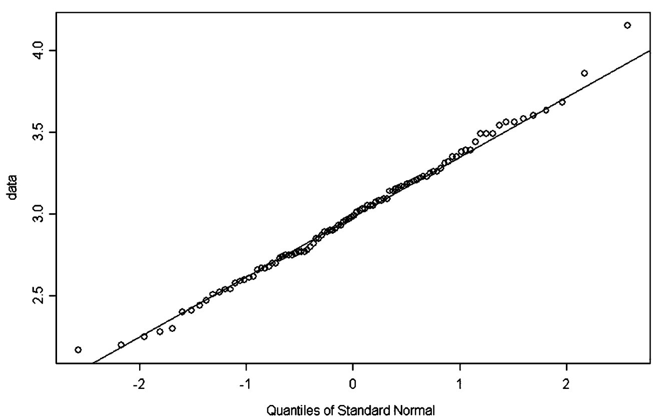

Fig. 9 Normal probability plot

greater than 5 μA, then the EEPROM chip is considered to be a nonconforming product, and will not be used to make the EEPROM chip of that particular model. The capability requirement for this particular model of EEPROM chip was defined as "capable" if C_{PU}>1.45.

And, as mentioned before, in order to make the estimation of these capability indices meaningful, it is necessary to check whether the manufacturing process is under statistical control. We use the $\overline{X}$ and S charts for retrospectively testing whether the process is in control. A partial historical data, including a group of 30 samples of size 10, and the corresponding $\overline{X}-S$ charts are displayed in Fig. 5a,b. The $\overline{X}-S$ control charts show that all the sample points are within the control limits without any special pattern, and the process is justified to be in well control. Therefore, we could consider the process to be stable, so we could proceed with the capability measure.

A total of 100 observations collected from a stable process in the factory are displayed in Table 3. Figure 6 displays the histogram of the sample data. Figure 7 displays the stem-and-leaf plot of the sample data. Figure 8 displays the box-whisker plot of these data, and the normal probability plot of the 100 sample data is plotted in Fig. 9. The data analysis results justify that the process is fairly close to the normal distribution.

From Figs. 6, 7, 8, and 9, and the Shapiro-Wilk test for normality confirming this with p-value>0.1, it is evident to take into consideration that the data collected from the factory are normal distributed. The sample mean $\overline{X}=2.987$ and the sample standard deviation s=0.382 are first calculated. For n=100, we calculate the value of the estimator $\widehat{C}_{PU}=\left(USL-\overline{X}\right)/3s=1.757$ and b_{n-1}=0.992 based on the sample size of n. We assume that the α-risk is 0.05, the critical value is found to be $C^*(p)$=1.640 from Table 3, based on w=1.45, p=0.95, and n=100. Since $\widetilde{C}_{PU}=b_{n-1}\times\widehat{C}_{PU}=1.743$ is larger than the critical value $C^*(p)$=1.640 in this case, we, therefore, conclude that, with a 95% level of confidence, the 128-bit EEPROM chip manufacturing process satisfies the requirement "C_{PU}>1.45." Thus, at least 99.99863% of the produced EEPROM chips conformed to the manufacturing specifications, with a fraction of nonconformities at 13.614 PPM, which is considered a satisfactory figure and reliable in terms of product quality (originally set by the product designers or the manufacturing engineers).

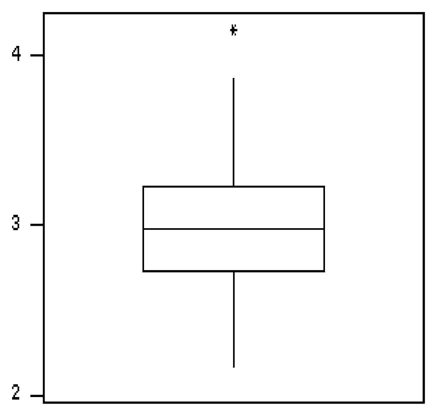

Fig. 8 Box-whisker plot

7 Conclusion

Process capability indices have been proposed in the manufacturing industry to provide numerical measures on process capability, which are effective tools for quality assurance. Usual practices in measuring production quality have focused on the traditional distribution frequency approach. In this paper, we considered estimating and testing the one-sided capability indices C_{PU} and C_{PL} using a Bayesian approach. We obtained the credible intervals of C_{PU} and C_{PL}, and proposed, accordingly, a Bayesian procedure for capability testing. The posterior probability p for which the process under investigation is capable is derived. The credible interval, a Bayesian analog of the classical lower confidence interval, is obtained. To make this Bayesian procedure practical for in-plant applications, we tabulate the minimum values of $C^*(p)$ for which the posterior probability p reaches various desirable confidence levels. A factory example of the manufacture of EEPROM chips was investigated, showing how the Bayesian procedure can be applied to in-plant applications.

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