

Central fringe identification using a heterodyne interferometric technique and a tunable laser-diode

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Abstract

A novel method combining a heterodyne technique and a tunable laser diode for identifying the central fringe of an interferogram is presented. It has some merits, such as simple optical structure, easier operation, etc. Its feasibility is demonstrated.

1. Introduction

White-light interferometry [1] has been applied to the measurement of displacement, pressure, temperature, strain, and other quantities that can be converted into displacement. When it is applied to these applications, it is necessary to identify the central fringe [2] (zeroth order) of the interferogram. In this paper, a novel method combining a heterodyne technique [3,4] and a tunable laser-diode for identifying the central fringe of an interferogram is presented. Besides identifying the central fringe, this method can be used to quickly judge the longer arm of an interferometer. And its feasibility is demonstrated.

2. Principle

The schematic diagram of this novel method is shown in Fig. 1. The linearly polarized light from a tunable laser diode LD [5] passing through an electro-optic modulator EO [6,7] incides on a beam-splitter BS, and is divided into two parts, the reflected light

and the transmitted light. The reflected light passes through an analyzer AN_r and enters into a photodetector D_r . If the amplitude of the light detected by D_r is E_1 , then the intensity measured by D_r is $I_r = |E_1|^2$. I_r is the reference signal. On the other hand,

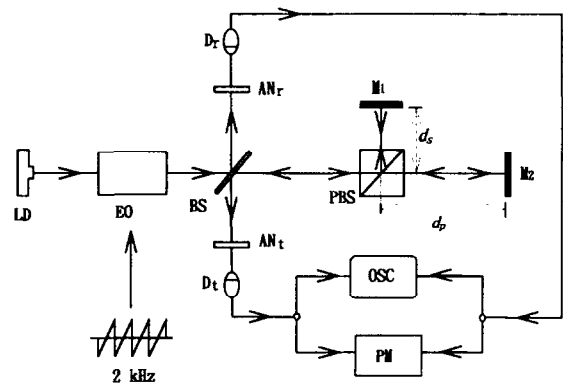


Fig. 1. Schematic diagram of a method of identifying the central fringe of an interferometer. LD, laser diode. EO, electro-optic modulator. BS, beam-splitter. AN, analyzer. D, photodetector. PBS, polarization beam-splitter. M, mirror. OSC, oscilloscope. PM, phase meter.

the transmitted light enters a Michelson interferometer and is divided into two parts by the polarization beam-splitter PBS; the reflected s-polarization light and the transmitted p-polarization light. The former is reflected by a mirror M_1 , then is reflected by PBS and BS again. It passes through an analyzer AN_t and is detected by the photodetector D_t . It acts as the reference light in the Michelson interferometer. The latter is reflected by M_2 and returns along the original path. After being reflected by BS, it passes through AN_t and also enters into D_t . It acts as the test light in the Michelson interferometer. If the amplitudes of the reference light and the test light are E_2 and E_3 , respectively, then D_t measures the interference intensity of E_2 and E_3 , i.e., $I_t = |E_2 + E_3|^2$. I_t is the test signal. Here, the central fringe of the Michelson interferometer will be identified.

2.1. The intensities of the reference signal and the test signal

For convenience, the $+z$ axis is chosen along the propagation direction and the y -axis is along the vertical direction. Let the incident light be linearly polarized at 45° with respect to the x -axis, then its Jones vector [8] can be written as

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (1)$$

If the fast axis of EO is along the x -axis, and an external sawtooth voltage signal with angular frequency ω and amplitude $V_{\lambda/2}$, the half-wave voltage of EO, is applied to EO, then the retardation produced by EO can be expressed as ωt . And if the transmission axis of AN_r is 45° with respect to the x -axis, then we have

$$\begin{aligned} E_1 &= AN_r(45^\circ) EO(\omega t) E_{in} e^{i\phi} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i\phi} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} (e^{i(\omega t/2)} + e^{-i(\omega t/2)}) e^{i\phi}, \end{aligned} \quad (2)$$

where ϕ is the phase due to the optical path. Hence, the intensity of the reference signal is

$$I_r = |E_1|^2 = \frac{1}{2} (1 + \cos \omega t). \quad (3)$$

On the other hand, let the angle between the transmission axis of AN_t and the x -axis be 45° , then we have

$$\begin{aligned} E_2 &= AN_t(45^\circ) R_{PBS} EO(\omega t) E_{in} e^{i\phi_s} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \\ &\quad \times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-ik(2d_s)} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i(\omega t/2 + 2kd_s)}, \end{aligned} \quad (4)$$

and

$$\begin{aligned} E_3 &= AN_t(45^\circ) T_{PBS} EO(\omega t) E_{in} e^{i\phi_p} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix} \\ &\quad \times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-ik(2d_p)} \\ &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{i(\omega t/2 - 2kd_p)}, \end{aligned} \quad (5)$$

where R_{PBS} and T_{PBS} are the transformation matrices of PBS for the reflected light and the transmitted light, respectively; $k = 2\pi/\lambda$ is the wavevector; ϕ_s and ϕ_p are the phases corresponding to the optical path lengths $2d_s$ and $2d_p$ of the reference light and test light, respectively in the Michelson interferometer. Therefore, the intensity of the test signal is given by

$$\begin{aligned} I_t &= |E_2 + E_3|^2 \\ &= \frac{1}{2} \left\{ 1 + \cos \left[\omega t - (4\pi/\lambda)(d_p - d_s) \right] \right\}, \end{aligned} \quad (6)$$

where λ is the wavelength. From Eq. (3) and Eq. (6), it is obvious that both the reference signal and the test signal are sinusoidal with angular frequency ω .

2.2. Identification of central fringe

If the wavelength of the laser-diode has a small variation $\Delta\lambda$ [9], it is obvious that Eq. (3) remains unchanged and Eq. (6) is changed to

$$I_t = \frac{1}{2} \left[1 + \cos \left(\omega t - \frac{4\pi d}{\lambda + \Delta\lambda} \right) \right], \quad (7)$$

where $d = d_p - d_s$. Comparing Eq. (7) and Eq. (6), the phase variation is given as

$$\Delta\phi = \frac{-4\pi d}{\lambda + \Delta\lambda} - \frac{-4\pi d}{\lambda} = \frac{4\pi d\Delta\lambda}{\lambda^2}. \quad (8)$$

Hence Eq. (7) can be rewritten as

$$I'_t = \frac{1}{2} [1 + \cos(\omega t - 4\pi d/\lambda + \Delta\phi)]. \quad (9)$$

For easier observation, the signals detected by photodetectors D_r and D_t are monitored by an oscilloscope. Two sinusoidal signals with the angular frequency ω can be observed. Between these two signals, there is a phase difference $4\pi d/\lambda + \Delta\phi$. The phase shift $\Delta\phi$, being proportional to the optical path difference d , will appear as the wavelength of LD is changed slightly. If the lengths of the two arms of the Michelson interferometer are equivalent, i.e., $d = 0$, then $\Delta\phi$ equals zero despite the wavelength variation. This means that the waveforms of

the test signal I'_t will remain unchanged without any shift even if there is any wavelength variation.

If the wavelength of LD is changed between λ and $\lambda + \Delta\lambda$, then the waveforms of I'_t will shift back and forth relative to the waveforms of I_r , and the shifting range is proportional to the optical path difference d . Either mirror M_1 or M_2 is so adjusted that the shifting range of the waveforms of I'_t gradually decreases. As the shifting range equals zero, that is, the waveforms of I'_t remain unchanged, then the central fringe is identified. For enhancing the resolution, a phase meter is introduced to measure the phase difference between the signals I'_t and I_r . When the phase difference remains unchanged, the photodetector D_t detects the central fringe of the interferogram. Under this condition, the lengths of the two arms of the Michelson interferometer are exactly equivalent.

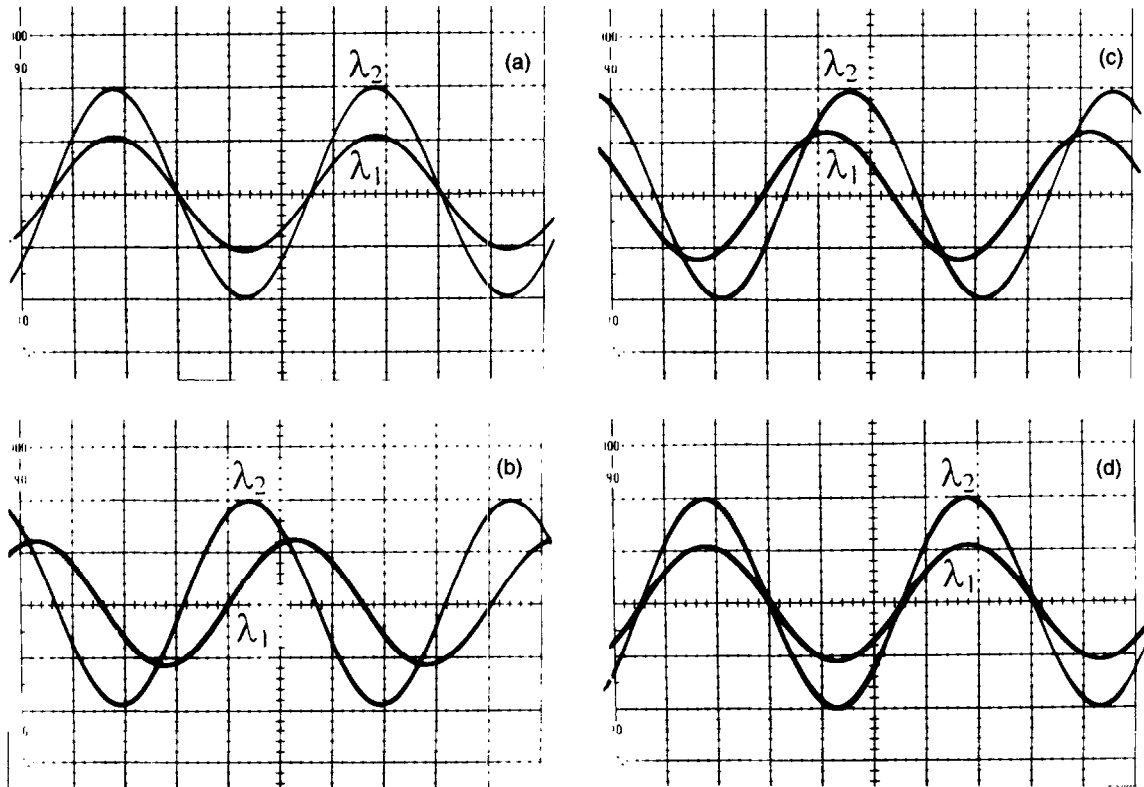


Fig. 2. The variations of waveforms of (a) the reference signal, (b) the test signal for $d = 800 \mu\text{m}$, (c) the test signal for $d = -400 \mu\text{m}$, and (d) the test signal for $d = 0$, as the wavelength of the laser diode is changed from 670.98 nm (labelled λ_1) to 671.02 nm (labelled λ_2).

3. Experiments and results

In order to show the feasibility, a laser diode (HL6720G) manufactured by Hitachi Ltd. and an electro-optic modulator (PC200/2) manufactured by Electro-Optics Developments Ltd. are used. That laser diode has a central wavelength of 671 nm and its wavelength variation is proportional to the injection current with a rate $+0.01$ nm/mA. Its output power is a function of the injection current, too. Its maximum wavelength variation in the same longitudinal mode is 0.04 nm. As for that electro-optic modulator, its half-wave voltage is 220 V and the frequency of the external modulated sawtooth signal is 2 kHz.

Let the wavelength of the laser diode be changed from 670.98 nm to 671.02 nm by controlling the injection current, then the waveforms of the reference signal and the test signal are taken after d.c. bias are filtered out. The results are shown in Fig. 2. The amplitudes of these waveforms are changed because of the injection current variation, the smaller one (labelled λ_1) corresponds to 670.98 nm and the larger one (labelled λ_2) 671.02 nm, respectively, in these figures. Fig. 2a represents the waveforms of the reference signal, and it does not shift. Fig. 2b and Fig. 2c represent the waveforms of the test signals for $d \doteq 800$ μm and $d \doteq -400$ μm respectively. Both of them are shifted, but in different directions; Fig. 2b shifts forward and Fig. 2c backward. The amount of the shifting range in Fig. 2b is almost twice that of Fig. 2c. Fig. 2d represents the case of $d = 0$, that is, the waveforms do not shift and it means the photodetector D_t detects the central fringe of the interferogram.

4. Discussion

From Eq. (4), the resolution is given as [9]

$$|d| = \frac{\Delta\phi}{4\pi} \frac{\lambda^2}{|\Delta\lambda|}. \quad (10)$$

In our experiment, a phase meter with 0.01° angular resolution is used. After substituting these experimental data into Eq. (10), a resolution of 0.2 μm is

obtained. Besides the enhancement of angular resolution of the phase meter, the increment of the tunable wavelength range will improve the resolution of this method.

5. Conclusion

In this paper, a novel method combining a heterodyne technique and a tunable laser diode for identifying the central fringe of an interferogram is presented. It has some merits, such as

- (i) Simple optical structure.
- (ii) Easier operation because the coherent length of a laser diode is much longer than that of a white light source.
- (iii) According to the shifting direction of the test signal, it is very easy to judge which arm of an interferometer is longer.

Moreover, the feasibility is demonstrated and it has 0.2 μm resolution.

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