三十四年勝利來臨。復員鎭江。有人接收五子登科，我們一介不取。工作起居都在省立圖書館的一間藏書室中，夜晚卽以破書架作牀鋪，仍舊過着艱苦的日子，生活絕未因勝利的果實而有所改善。三十六年機關改制，我被調回南京。三十八年共匪作亂，京滬撤守，我們最後撤出，經上海、廣州而轉到了臺灣。從廣州到臺灣，押運了許多公物，乘的是免費差輪，駛駛停停，在海上就誤了不少日子。兒女在船上染病，無法就醫，病勢日重。到基隆登陸，立卽送進臺大醫院。大兒經醫治獲救，小女抵抗力弱，翌日卽告不治，又使我失去了唯一的千金幼苗。（前生兩女，勝利後相繼夭折。）（女作家徐鍾珮女士在中央副刊發表的『失去的幼苗』卽描述此事，曾編入中學國文教科書。）

在臺灣一恍又是二十多年，眞是當初做夢也不曾想到。終年勞勞碌碌，戰戰兢兢，生活雖日趨安定，心頭却充滿了憂憤。『神州不復，易興陸沉之嘆；中年已往，莫釋哀樂之懷。』往事不堪回首，來日尚有可期。希望早日反攻復國，直搗黃龍。

四十年歷盡滄桑，冷眼旁觀：『眼看他起高樓，眼看他樓塌了。』有人爬得高，跌得重；有人升得快，去得早；有人爭得到，拚了命；有人變了節，遺了臭；這些熟識的、熱衷於名利的朋友們，到頭來無非是夢幻泡影一場空。我們膠柱鼓瑟，死心眼兒，澹泊自甘，潔身自愛，雖然清風兩袖，位卑職小而不足道，却也心安理得，樂在其中。記得校長唐文治先生有云：『人生惟有廉節重，世界須憑氣骨撐。』岳武穆也有詩云：『飲酒讀書四十年，方知頭上有青天。男兒欲到淩煙閣，第一功名不愛錢。』平生皆奉爲座右之銘，時時警惕。因此做人不欺不詐，大處着眼，小處留心，尚能謹言愼行；辦事盡心盡力，不爲求名，不爲求利，只爲工作而生活。

人世本是一個大舞臺，一個機關好似一個戲班子，我們每一個同事都是演員。不論你演的是什麼角色，主角也好，配角也好，在班子裡，在舞臺上總要大家齊心合力，努力表演。不鬆懈，不取巧，才能成功，才能爭得團體的榮譽。我所扮演的雖是配角，且已接近尾聲；但在落幕之前，我仍希望能演得更生動，更精彩，更有意義。

GRAPHIC SOLUTION of $\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$

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IN ELEMENTARY algebra work problems, water-pipe delivery problems, and similar problems are based on the relationship

$$\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$$

The units for the three variables may be of any magnitude, fundamental or derived. The only requirement is that all three variables be in the same units.

For example, if a job can be finished by A alone in 3 days and by B alone in 6 days and can be finished by A and B working together in r days, then r can be found by the above formula, $1/r=1/3+1/6$. Here, the uniform units used are days. However, if we know that the job can be done by A and B working together in 2 days and that the same job can be done by A alone in 3 days, how long does it take B to finish it alone? Here, r and p are 2 and 3 days, respectively; q can be found by solving the equation $1.q=1/2-1/3$.

Similarly, suppose a tank of water can be emptied in 10 minutes by opening pipe A alone, in 6 minutes by opening pipe B alone, and in r minutes by opening pipes A and B together. Then, using the same formula, $1/r=1/10+1/6$. Here the uniform units used are minutes.

When more than two men work or more than two pipes are open, the right-hand member of the equation can be extended, thus:

$$\frac{1}{r}=\frac{1}{p}+\frac{1}{q}+\frac{1}{s}+\cdots+\frac{1}{n}.$$

There are many physical phenomena to which this method applies. When two electrical resistances r_1 and r_2 are connected in parallel, the combined resistance R is given by $1/R=1/r_1+1/r_2$. Here r_1 and r_2 represent resistance in ohms. Again, when two electrical capacitors are connected in series the combined capacitance C is given by $1/C=1/c_1+1/c_2$, where c_1 and c_2 are individual capacitances in farads or microfarads.

In the study of spherical mirrors and lenses, the focal length f of a mirror or a lens can be found using $1/f=1/d_0+1d_i$. Here d_0 is the distance of an object from the vertex of a mirror or from the center of a lens, and d_i is the distance of the image from the vertex or from the center. The units of distance can be centimeters, meters, inches, or feet, so long as they are the same for all the variables in the formula.

A graphic method is available for the approximate solution of the equation $1/r=1/p+1/q$ when any two of the three variables are given. This method is presented here not so much to show another method of solution as to

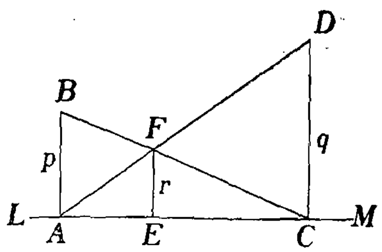

Fig 1

demonstrate the close relationship between algebra and geometry. The graphic method is described and proved as follows.

Two line segments, $\overline{AB}$ and $\overline{CD}$ are drawn perpendicular to $\overleftrightarrow{LM}$, as shown in figure 1. Segments $\overline{AD}$ and $\overline{BC}$ are drawn, intersecting at F, and $\overline{FE}$ is drawn perpendicular to $\overleftrightarrow{LM}$. Then, as will be proved, $1/EF=1/AB+1/CD$, or $1/r=1/p+1/q$.

Proof. In triangle ABC,

$$\frac{p}{r}=\frac{AC}{EC}. \qquad (1)$$

In triangle CDA,

$$\frac{q}{r}=\frac{AC}{AC-EC}, \qquad (2)$$

From (1),

$$EC=AC\cdot\frac{r}{p}.$$

Substituting into (2),

$$\frac{q}{r}=\frac{AC}{AC-AC\cdot\frac{r}{p}},$$

or

$$\frac{q}{r}=\frac{1}{1-\frac{r}{p}}.$$

$$q\left(1-\frac{r}{p}\right)=r.$$

$$q-q\cdot\frac{r}{p}=r.$$

Dividing by qr,

$$\frac{1}{r}-\frac{1}{p}=\frac{1}{p},$$

or

$$\frac{1}{r}=\frac{1}{p}+\frac{1}{q}.$$

It should be noted that the above formula is independent of the distance AC between the line segments $\overline{AB}$ and $\overline{CD}$; therefore these segments may be drawn any distance apart, and the value of r remains the same. This may be seen in figure 2, where the segment p and the four different segments all of length q are drawn at different distances apart, but the vertical segments $r_1, r_2, r_3, \cdots$ are the same length. This property facilitates the solution of more complicated problems, as will be shown later.

Now let us use the graphic method to solve the work problem given at the beginning of this article. Use a convenient unit to represent 1 day. As shown in figure 3, draw two line segments of lengths $AB=3$ and $CD=6$, both perpendicular to $\overleftrightarrow{LM}$. Draw $\overline{AD}$ and $\overline{BC}$, intersecting at F. Draw $\overline{FE}$ perpendicular to $\overline{LM}$. Then $\overline{FE}$ measures 2 units, which represent 2 days, the time required to finish the job when A and B work together.

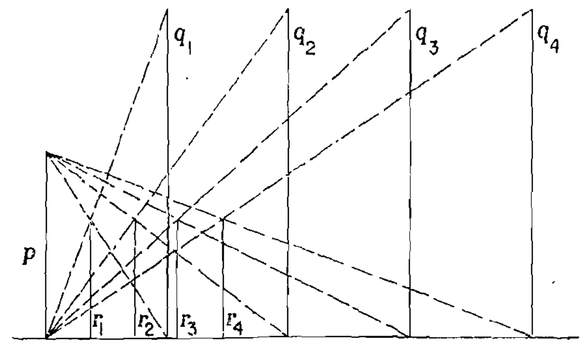

Fig. 2

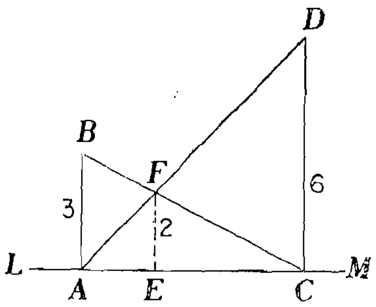

Fig. 3

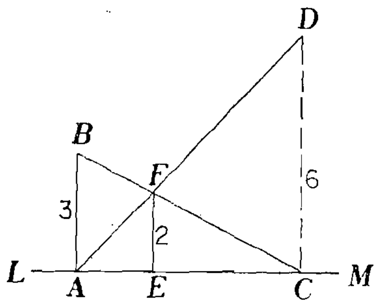

Fig. 4

Again, suppose we know that A working alone can finish the job in 3 days, but A and B working together can finish it in 2 days. How many days does it take B to finish it when working alone? Using the same scale as before, draw $AB = 3$ and $EF = 2$, bothper pendicular to $\overleftrightarrow{LM}$ (see fig. 4). Draw $\overrightarrow{AF}$ and $\overrightarrow{BF}$. $\overrightarrow{BF}$ meets $\overleftrightarrow{LM}$ at C. Erect a perpendicular to $\overleftrightarrow{LM}$ at C, intersecting $\overrightarrow{AF}$ at D. Then $\overline{CD}$ is the required line segment; it measures 6 units, indicating 6 days, the time required by B to finish the job when working alone.

Similarly, water-pipé problems and problems involving resistances-in parallel, inductances in parallel, and capacitances in series can all be worked out by the graphic method. No numerical example is necessary for any one of these problems, because they are all done in the same way as work problems. The graphic method is also helpful when there are more than two men working, more than two resistances in parallel, and so on. Suppose there are four resistances of 20, 60, 85, and 140 ohms in parallel; the total resistance, R, is:

$$\frac{1}{R}=\frac{1}{20}+\frac{1}{60}+\frac{1}{85}+\frac{1}{140}.$$

Figure 5 shows a graphic solution of this problem. Use whatever scale is convenient. First resistances of 20 and 60 are combined, giving e; then 85 and e are combined, giving f; finally 140 and f are combined to obtain g. Depending on scale and accuracy of consrtruction, a value between 11.5 and 12 is obtained (numerical calculation gives 11.7 ohms).

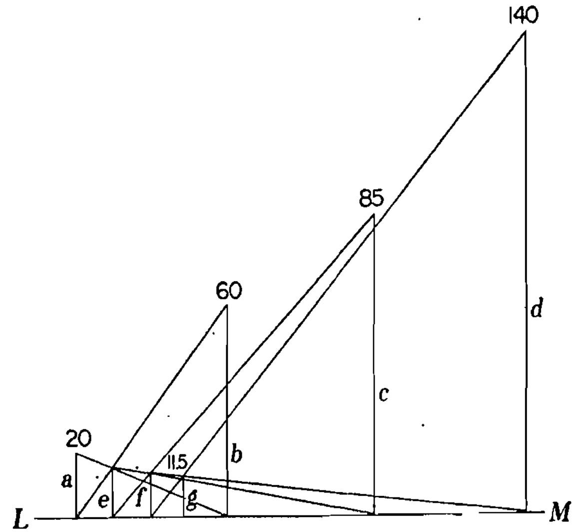

Fig. 5

It may be noted that this problem could have been solved equally well by various other pairings. For example, a might have been paired with b to obtain e, then c with d to obtain (say) f', then f' with e to obtain g. In fact, any possible pairing can be used.

If negative values are introduced, the use of the formula and its graphic solution can be extended to the

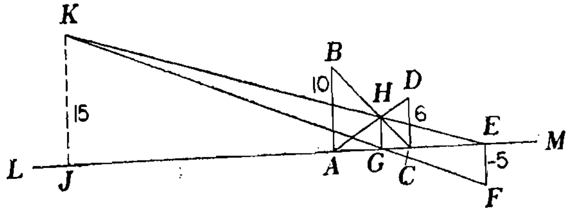

Fig 6

solution of more complicated forms of water-tank problems. Suppose a bathtub can be filled in 10 minutes by the hot-water faucet alone and in 6 minutes by the cold-water faucet alone. When the tub is full, it can be emptied in 5 minutes by the drainpipe. If both faucets and the drainpipe are open, can the tub be filled? If so, how long does it take?

By formula, $1/r=1/0+1/6-1/5$, or $1/r=1/15$, and $r=15$ minutes.

Figure 6 is the graphic solution of the same problem. The construction is selfevident, and no explanation of procedure is necessary. The line JK is the line segment required; its length above the line LM indicates that the tub is filled in 15 minutes, the result as obtained by calculation. (轉載「美國數學教師月刋」)

本文 $\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$ 之圖解法係陳廣沅學長在麻州執教時，閒暇學圃，與隣人園中閒話，偶獲靈感，遣意之作，原文載美國「數學教師」學會之月刋。雖爲三角老定理，而以圖解之，創意可謂新穎別緻，陳老學長眞是寓學理於趣味，樂在其中也。

據本刋探悉，陳學長胃疾已大致康復，現優遊紐州婿家，孟春時節駕車探花訪友，陶性怡神。並答應編者，一俟完全復原，當將一年來侍候妻病，學校教書不能繼續，搬家冒雪遊行，到後家俱不到以及到新地方後，找醫生，找銀行，找保險，考車牌，修車等趣聞經過，爲文撰載友聲。

——編者 註