Expert Systems with Applications 31 (2006) 525–530
Expert Systems with Applications
www.elsevier.com/locate/eswa

Interval multidimensional scaling for group decision using rough set concept

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Abstract

Multidimensional scaling (MDS) is a statistical tool for constructing a low-dimension configuration to represent the relationships among objects. In order to extend the conventional MDS analysis to consider the situation of uncertainty under group decision making, in this paper the interval-valued data is considered to represent the dissimilarity matrix in MDS and the rough sets concept is used for dealing with the problems of group decision making and uncertainty simultaneously. In addition, two numerical examples are used to demonstrate the proposed method in both the situation of individual differences scaling and the conventional MDS analysis with the interval-valued data, respectively. On the basis of the results, we can conclude that the proposed method is more suitable for the real-world problems.

Keywords: Multidimensional scaling (MDS); Individual differences scaling; Uncertainty; Interval-valued data; Rough sets

1. Introduction

The goal of multidimensional scaling (MDS) is to represent the relationships among objects by constructing a configuration of n points in low dimension from pairwise comparisons of similarities/dissimilarities among a set of n objects. The dissimilarities can be calculated by Euclidean distance or other weighted distance such as Manhattan or maximum value distances. Generally, MDS can be categorized into metric MDS and non-metric MDS according to whether the dissimilarity values are quantitative or qualitative. More detailed discussions about MDS can refer to Mead (1992) which reviews the development of MDS.

Although MDS has been successfully used in various areas such as psychophysics, sensor analysis, and marketing, the issues of group decision making and uncertainty should be considered for more applications. The first problem, which also called individual differences scaling, involves assessing the dissimilarities by more than one person. It is clear that the dissimilarity matrix may be inconsistent in this situation. The other problem is uncertainty, which is caused by human subjective interpretation or incomplete information. In this paper, we propose a method, which can deal with the problems of group decision making and uncertainty in the MDS analysis simultaneously using the concept of rough sets.

Several algorithms such as EMD (McGee, 1968), CEMD (McGee, 1968), INDSCAL (Carroll & Chang, 1970), and ACOVS (Jöreskog, 1970) have been proposed to handle the problem of group decision making in MDS. These methods are generally based on the monotonic transformation of the observed dissimilarities or weighting the subject space to obtain a compromise solution. However, due to the limitations of human judgment and incomplete information, it is hard even for experts to quantify the dissimilarity value of certain pairs of objects. In this situation, interval-valued data are more suitable to represent human imprecision. Although this idea of interval-valued data for MDS has been developed in Denceux and Masson (2000, 2002), it should be highlighted that their method only deal with the problem of group decision making or human imprecision. However, in this paper, we deal with the problems of group decision making and human imprecision simultaneously. We will use a numerical example to discuss the problem of their method in Section 2.

In this paper, the degree of uncertainty can be represented as two cycles. The possibility of the actual output falls into the internal cycle, which is also called the lower approximation, is higher than the external cycle, which is also called the upper approximation based on the opinions of experts. However, we should also concern both cycles especially when the actual output falls into the area of the external cycle may cause substantial loss or earnings.

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0957-4174/$ - see front matter
doi:10.1016/j.eswa.2005.09.060

In addition, a reduced model, which is suitable for a single decision maker with interval-valued data, is derived for extending the proposed method. Two numerical examples are used to demonstrate the proposed method. From the numerical results, we can conclude that the proposed method can well handle the problems of group decision making and human imprecision simultaneously in the MDS analysis. More information can be obtained using the proposed method by the decision maker.

The rest of this paper is organized as follows. In Section 2, we describe the problem of individual differences scaling with interval-valued data. The approximation space, which is derived using the concept of rough sets, is given in Section 3. In Section 4, we will present the proposed method. Two numerical examples are used in Section 5 to show the proposed method. Discussions are presented in Section 6 and conclusions are given in Section 7.

2. The problem of interval-valued data in individual differences scaling

Individual differences scaling is used to handle the problem, which more than one person evaluate the dissimilarity matrix in the MDS analysis. This method requires one dissimilarity matrix for each subject and all subjects are assumed to have the same underlying configuration for each object. A simple method for dealing with this problem is to average the pairwise dissimilarities and to form a single dissimilarity matrix. However, this method has been critical for obtaining the poor results (Ashby, Maddox, & Lee, 1994) and it also prevents analyzing the differences among participants. In order to overcome the above problems, many algorithms mentioned in Section 1 have been proposed to take individual differences into account.

Although these algorithms have been widely used in various fields, these methods do not consider the problem of human imprecision or incomplete information. The concept of the interval-valued data has been incorporated into the MDS analysis in Denceux and Masson (2000, 2002) for considering human subjects, but it works only in the situation of one person to assess dissimilarity matrix under the situation of uncertainty rather than group decision making. We can use a numerical example to highlight the problem of Denoeux and Masson's method when considering the situations of individual differencing scaling and uncertainty simultaneously.

Assume there are four objects in the MDS analysis and two experts are asked to assess the dissimilarity matrix using the interval-valued data as shown in Tables 1 and 2, respectively.

Table 1
The interval-valued dissimilarity matrix given by Expert 1

Δ_1	O_1	O_2	O_3	O_4
O_{11}	0			
O_{12}	(1,8)	0		
O_{13}	(1,7)	(1,8)	0	
O_{14}	(1,6)	(1,7)	(1,8)	0

Table 2
The interval-valued dissimilarity matrix given by Expert 2

Δ_2	O_1	O_2	O_3	O_4
O_{21}	0			
O_{22}	(2,10)	0		
O_{23}	(3,10)	(2,10)	0	
O_{24}	(4,10)	(3,10)	(2,10)	0

Table 3
The final interval-valued dissimilarity matrix

Δ	O_1	O_2	O_3	O_4
O_1	0			
O_2	(1,10)	0		
O_3	(1,10)	(1,10)	0	
O_4	(1,10)	(1,10)	(1,10)	0

Based on Tables 1 and 2, we can determine the final interval-valued dissimilarity matrix by Eq. (1)

$$\Delta = [\delta^-_{jk}, \delta^+_{jk}] \quad \forall\, i \neq j; \qquad \delta^-_{jk} = \min_i [\delta_{ijk}],$$
$$\delta^+_{jk} = \max_i [\delta_{ijk}] \tag{1}$$

where Δ is the interval-valued dissimilarity matrix and δ_{ijk} denotes the dissimilarity value in the jth row and the kth column with the ith expert. The final interval-valued dissimilarity matrix can be represented as shown in Table 3.

On the basis of Table 3, we can see that the dissimilarity matrix seems irrational and is too wide to provide any useful information for the decision maker. Although this example is the extreme situation in the real world, we can understand that Denoeux and Masson's method is not suitable for the situations of individual differences scaling and uncertainty simultaneously. Next, in order to derive the proposed method, we will first describe the concept of the approximation space using rough sets theory in Section 3.

3. Approximation space with rough sets concept

Rough sets, which were proposed by Pawlak (1982), are mathematical algorithms to deal with the problem of vagueness or uncertainty. Rough sets have been used in the area of multicriteria decision analysis (Greco, Matarazzo, & Slowinski, 2001; Pawlak & Slowinski, 2001), variable reduction (Beynon, 2001), knowledge acquisition (Grzymala-Busse, 1988; Pawlak, 1997), etc. to solve the uncertainty problem in the real word applications. One main advantage of rough sets is that rough sets do not need any pre-assumption or preliminary information about data such as the degree of membership function (Grzymala-Busse, 1988; Pawlak, Grzymala-Busse, Slowinski, & Ziarko, 1995). Recently, rough sets and fuzzy sets theory are used together to complement each other (Chakrabarty, Biswas, & Nanda, 2000; Mordeson, 2001; Radzikowska & Kerre, 2002) rather than compete it (Dubois & Prade, 1991). More detailed discussions about the processes of rough sets theory can be referred to (Walczak & Massart, 1999).

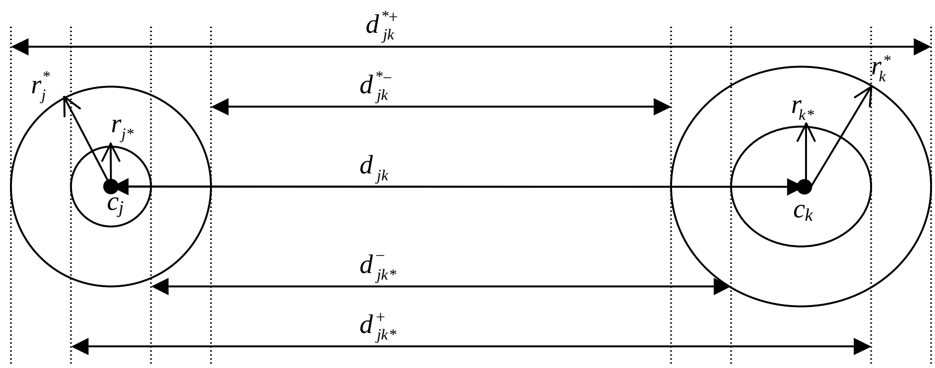

Fig. 1. Concepts of the lower and upper radii between two objects.

The original concept of the approximation space in rough sets can be described as follows. Given an approximation space

$$\text{apr} = (U, A) \tag{2}$$

where U is the universe which is a finite and non-empty set, and A is the set of the attributes. On the basis of the approximation space, we can define the lower and upper approximations of a set as follows.

Let X be a subset of U and $B \subset A$. Then, the lower approximation of X in A can be represented as

$$\underline{\text{apr}}(A) = \{x | x \in U, \text{Ind}(B) \subset X\} \tag{3}$$

and the upper approximation of X in A can be represented as

$$\overline{\text{apr}}(A) = \{x | x \in U, \text{Ind}(B) \cap X \neq \phi\} \tag{4}$$

where

$$\text{Ind}(B) = \{(x_i, x_j) \in U \times U, f(x_i, a) = f(x_j, a) \quad \forall\, a \in B\}. \tag{5}$$

Note that Eq. (3) represents the least composed set in A contained by X which is called the best upper approximation of X in A and Eq. (4) represents the greatest composed set in A contained by X which is called the best lower approximation. After constructing the upper and lower approximations, the boundary can be represented as

$$\text{BN}(A) = \overline{\text{apr}}(A) - \underline{\text{apr}}(A) \tag{6}$$

On the concepts of the approximation spaces, we can derive the proposed method for dealing with the problems of individual differences scaling and uncertainty in the MDS analysis.

4. Individual differences scaling with rough sets concept

From the rough sets concept, we can define the lower and upper dissimilarity matrices to describe the situation of individual differences scaling with the interval-valued data using the follow equations

$$\underline{apr}(\Delta_{jk}) = \bigcap_{i=1}^{n} \delta_{ijk} = (\delta_{jk*}^-, \delta_{jk}^{*-}) \quad \forall\, j \neq k \tag{7}$$

and

$$\overline{apr}(\Delta_{jk}) = \bigcup_{i=1}^{n} \delta_{ijk} = (\delta_{jk*}^+, \delta_{jk}^{*+}) \quad \forall\, j \neq k \tag{8}$$

where Δ_{jk} is the interval-valued dissimilarity of the jth row and the kth column, δ_{ijk} denotes the interval-valued dissimilarity of the jth row and the kth column with the ith individual. In addition, $\delta_{jk^*}^-$, δ_{jk}^{*-} denote the left and right values of the lower dissimilarity and $\delta_{jk^*}^+$, δ_{jk}^{*+} denote the left and right values of the upper dissimilarity. Then, the boundary can be determined as the following equation:

$$\text{BN}(\Delta_{jk}) = \overline{\text{apr}}(\Delta_{jk}) - \underline{\text{apr}}(\Delta_{jk}) \tag{9}$$

The boundary of the dissimilarity matrix can be interpreted as the uncertain degree of the pure upper approximation. Next, we depict two cycles as shown in Fig. 1 to describe the concepts of the proposed method. In this figure, the concept of uncertainty can be represented by extending the point to the cycle and the radius r indicates the degree of uncertainty. For example, the radius r_{j^*} indicates the uncertain degree of the lower approximation in the jth object and the radius r_j^* denotes the uncertain degree of the upper approximation in the jth object. Obviously, the uncertain degrees of the upper approximation are larger than the lower approximation.

In addition, on the basis of Fig. 1 we can describe the relationships among objects using the following equations

$$d_{jk} = d_{jk^*}^+ - (r_{j^*} + r_{k^*}) \tag{10}$$

$$d_{jk} = d_{jk^*}^- + (r_{j^*} + r_{k^*}) \tag{11}$$

$$d_{jk} = d_{jk}^{*+} - (r_j^* + r_k^*) \tag{12}$$

$$d_{jk} = d_{jk}^{*-} + (r_j^* + r_k^*) \tag{13}$$

where d_{jk} denotes the dissimilarity value between the centers c_j and c_k, $d_{jk^*}^+$ and $d_{jk^*}^-$ denote the maximum and minimum dissimilarities between the lower approximations r_{j^*} and r_{k^*}, respectively, and the maximum and minimum dissimilarities between the upper approximations r_j^* and r_k^* can be denoted as d_{jk}^{*+} and d_{jk}^{*-}, respectively.

From rough sets concept, we can summarize that the internal cycle (i.e. the lower approximation) indicates the intersection of the experts' agreements and the external cycle (i.e. the upper approximation) indicates the union of the experts' agreements. Since the internal and external cycles are the different degrees of uncertainty, we should minimize the sum of uncertainty to obtain the optimal radii. That is, we can transform the above problems into the following mathematical programming model:

$$\min \sum_{j=1}^{n} (r_j^* + r_{j^*}) \tag{14}$$

$$\text{s.t.} \quad d_{jk}^{*-} \le \delta_{jk}^{*-} \quad \forall j \ne k \tag{15}$$

$$d_{jk}^{*+} \ge \delta_{jk}^{*+} \quad \forall j \ne k \tag{16}$$

$$d_{jk^*}^{-} \le \delta_{jk}^{-} \quad \forall j \ne k \tag{17}$$

$$d_{jk^*}^{+} \ge \delta_{jk^*}^{+} \quad \forall j \ne k \tag{18}$$

$$d_{jk}^{*-} \ge d_{jk^*}^{-} \ge 0 \quad \forall j \ne k \tag{19}$$

$$r_{j^*}, r_j^* \ge 0 \quad \forall j = 1, \ldots, n \tag{20}$$

By substituting Eqs. (15)–(18) by Eqs. (10)–(13), we can derive the optimum lower and upper radii using the following linear programming model:

$$\min \sum_{j=1}^{n} (r_j^* + r_{j^*}) \tag{21}$$

$$\text{s.t.} \quad r_j^* + r_k^* \ge \frac{1}{2}(\delta_{jk}^{*+} - \delta_{jk}^{*-}) \quad \forall j \ne k \tag{22}$$

$$r_{j^*} + r_{k^*} \ge \frac{1}{2}(\delta_{jk^*}^{+} - \delta_{jk^*}^{-}) \quad \forall j \ne k \tag{23}$$

$$r_j^* \ge r_{j^*} \ge 0 \quad \forall j = 1, \ldots, n \tag{24}$$

In addition, we can also reduce the proposed method by defining the pessimistic index (PI) for considering the situation of a single decision maker. The pessimistic index can be presented by the following equation

$$r_{j^*} = \eta r_j^*, \quad 0 \le \eta \le 1 \tag{25}$$

where η is the pessimistic index which can be assigned by the expert. When $\eta = 0$, then $r_{j^*} = c_j$ indicates the most optimistic situation, i.e. the lower approximation is equal to zero. On the other hand, when $\eta = 1$, then $r_{j^*} = r_j^*$ indicates the most pessimistic situation. The problem of finding the optimal upper and lower radii can be derived by solving the following linear programming model:

$$\min \sum_{j=1}^{n} (r_j^* + r_{j^*}) \tag{26}$$

$$\text{s.t.} \quad r_j^* + r_k^* \ge \frac{1}{2}(\delta_{jk}^{*+} - \delta_{jk}^{*-}) \quad \forall j \ne k \tag{27}$$

$$r_{j^*} = \eta r_j^*, \quad 0 \le \eta \le 1, \tag{28}$$

$$r_j^* \ge 0 \quad \forall j = 1, \ldots, n \tag{29}$$

Note that it is clear that the possibility model (Denceux & Masson, 2002) is one special case (i.e. $\eta = 0$) of the proposed method. Next, we used two numerical examples to demonstrate the two situations of the proposed method.

5. Numerical examples

In this paper, in order to provide the convenient results for decision making, the configuration map is only represented using two dimensions. Next, we first use a numerical example to demonstrate how the interval-valued data can be used in the situation of individual differences scaling using the rough sets concept.

Example 1. Assume there are three experts to assess the dissimilarity matrix using the interval-valued data and the dissimilarity matrices can be represented in Tables 4–6, respectively.

By using Eqs. (7) and (8), we can construct the lower and upper dissimilarity matrices in Tables 7 and 8, respectively.

Table 4
The interval-valued dissimilarity matrix given by Expert 1 in Example 1

Δ_1	A_1	A_2	A_3	A_4	A_5
A_{11}	0				
A_{12}	(3,7)	0			
A_{13}	(7,9)	(6,8)	0		
A_{14}	(2,3)	(4,6)	(6,8)	0	
A_{15}	(5,8)	(9,10)	(5,7)	(4,7)	0

Table 5
The interval-valued dissimilarity matrix given by Expert 2 in Example 1

Δ_2	A_1	A_2	A_3	A_4	A_5
A_{21}	0				
A_{22}	(2,6)	0			
A_{23}	(6,9)	(5,8)	0		
A_{24}	(1,4)	(5,7)	(7,8)	0	
A_{25}	(4,7)	(8,10)	(4,6)	(5,6)	0

Table 6
The interval-valued dissimilarity matrix given by Expert 3 in Example 1

Δ_3	A_1	A_2	A_3	A_4	A_5
A_{31}	0				
A_{32}	(4,6)	0			
A_{33}	(5,8)	(6,9)	0		
A_{34}	(2,4)	(3,6)	(6,8)	0	
A_{35}	(5,7)	(9,10)	(4,7)	(4,5)	0

Table 7
The lower dissimilarity matrix in Example 1

Δ_*	A_1	A_2	A_3	A_4	A_5
A_1	0				
A_2	(4,6)	0			
A_3	(7,8)	(6,8)	0		
A_4	(2,3)	(5,6)	(7,8)	0	
A_5	(5,7)	(9,10)	(5,6)	(5,5)	0

Table 8
The upper dissimilarity matrix in Example 1

Δ^*	A_1	A_2	A_3	A_4	A_5
A_1	0				
A_2	(2,7)	0			
A_3	(5,9)	(5,9)	0		
A_4	(1,4)	(3,7)	(6,8)	0	
A_5	(4,8)	(8,10)	(4,7)	(4,7)	0

The lower dissimilarity matrix is the intersection of all dissimilarity matrices and the upper dissimilarity matrix is the union of all dissimilarity matrices.

From the lower and upper approximation matrices, we can calculate every cycle's lower and upper radii by solving Eqs. (21)–(24) and the configuration can be depicted as shown in Fig. 2.

Example 2. Next, we use another numerical example to demonstrate how to apply the proposed method for the situation of only one decision maker. This example has six objects and the dissimilarity value is given using the interval-valued data as shown in Table 9. The last row represents the degrees of pessimistic index.

On the basis of Eqs. (26)–(29), we can derive the upper and lower radii by solving the linear programming model and the corresponding 2D configuration can be depict as shown in Fig. 3.

As shown in Fig. 3, the configuration map also shows the internal and external cycles to represent the degrees of uncertainty even with a single expert. Next, we describe the discussions according to our two numerical examples in Section 6.

6. Discussions

In Section 5, we demonstrate two numerical examples to show the applications of individual differences scaling using the interval-valued data using the rough sets concept. The main concept is that the degrees of uncertainty can be represented using the concept of the cycle. When the cycle is bigger, the degrees of uncertainty or vagueness are increasing. However, the degree of uncertainty is different when the method of group decision making is adopted. In this paper, the internal cycle,

Table 9
The interval-valued dissimilarity matrix given in Example 2

Δ	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0					
A_2	(4,6)	0				
A_3	(5,8)	(1,3)	0			
A_4	(1,3)	(4,7)	(4,5)	0		
A_5	(1,3)	(4,6)	(3,5)	(5,7)	0	
A_6	(5,8)	(1,3)	(1,2)	(2,4)	(3,6)	0
PI	0.3	0.5	0.5	0.2	0.1	0.7

which is also called the common ground, denotes the all evaluators' intersection. In contrast, the external cycle denotes the all the evaluators' union. Using those two cycles, we can incorporate the different degrees of uncertainty under the situation of group decision making in the MDS analysis.

In addition, we can highlight the differences of each cycle and show various situations of uncertainty as shown in Fig. 2. For example, in the first numerical example, even {A3} and {A4} have the same external cycle, but the internal cycles are different. This situation indicates that they have different degrees of uncertainty, i.e. {A4} is more uncertain than {A3}.

In order to extend the application of the proposed method, try to consider the application of the MDS analysis in clustering. When we cluster data, the closer position is considered to be the same cluster in MDS analysis. However, if we consider the situation of uncertainty among data, it may obtain different results.

We can use an example to show this situation. Assume the 2D configuration can be depicted as shown in Fig. 4.

On the basis of Fig. 4, we can see that although {A3} is close to {A5, A6} rather than {A1, A2}, it is sensible to cluster {A1, A2, A3} to the same cluster by considering the degree of uncertainty. The same situation can be found in {A4, A5, A6}.

In addition, in order to extend the proposed method in the conventional MDS with the interval-valued data, we can use the pessimistic index to represent the acceptable uncertainty. Compared with the proposed method, in the conventional MDS analysis there are only the center value, which is presented in the 2D configuration. We can consider that the conventional MDS is one special case of the proposed method with the crisp data and the pessimistic index is equal to zero. However, since

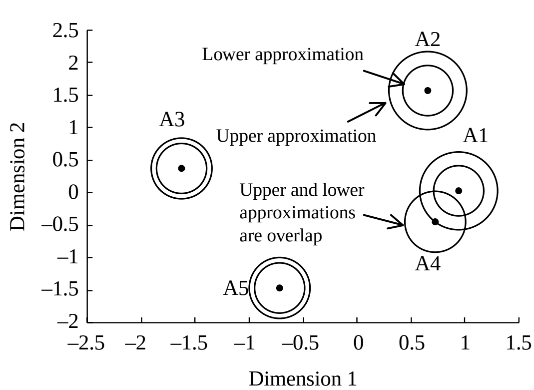

Fig. 2. The 2D configuration with rough sets concept in Example 1.

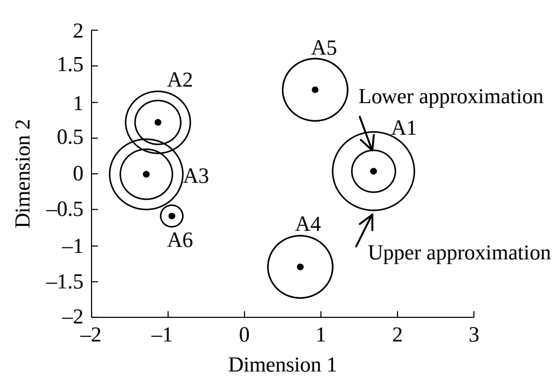

Fig. 3. The 2D configuration with rough sets concept in Example 2.

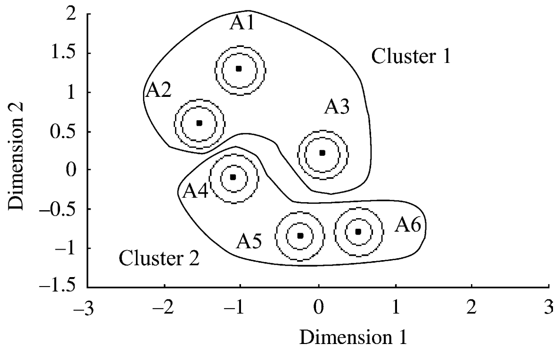

Fig. 4. The application of the proposed concept for clustering.

the problem of uncertainty usually exist in the real word, the proposed method should be more suitable in this situation.

On the basis of the implementation, we can see the differences between the proposed method and the model of (Denceux & Masson, 2000, 2002). First, in the proposed method each center point has two cycles, in which the internal cycle represents the optimistic situation and the external cycle represents the pessimistic situation. However, in Masson and Denceux's possibility model, only one cycle, called the pessimistic situation, is used to represent uncertainty. Second, the proposed method is suitable for the situation of individual differences scaling and uncertainty simultaneously. In addition, from Fig. 4 we know that even if each cycle has the same upper radius, it does not necessarily have the same lower radius, and vice versa.

Our method is not only applicable for individual differences scaling, but is also suitable for the conventional MDS analysis by incorporating the pessimistic index. On the basis of the numerical examples, we can conclude that the proposed method can show more information among objects' relationships. This is useful for decision makers to make critical decisions under the situation of uncertainty.

7. Conclusions

Although uncertainty and vagueness usually exist in the real-world problems, the degree of uncertainty can be reduced when we have some useful information. This information can be obtained from expert's common ground and used for knowledge discovery. In this paper, the conventional individual differences scaling is extended to describe the situation of human subjects or uncertainty using the interval-valued data. The different degrees of uncertainty are divided into two cycles using the rough sets concept. The internal cycle describes the situation in which experts think the actual outputs surely fall and the external cycle is the situation in which experts think the actual outputs possibly fall. In addition, two numerical examples are used to demonstrate the processes of the proposed method. On the basis of the results, we can conclude that the proposed method is suitable for both individual differences scaling and conventional MDS under the situation of uncertainty.

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