Phase geographical map for determining the material type of a right-angle prism

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A phase geographical map for determining a right-angle prism is presented. The proposed method is based on total-internal-reflection effects and chromatic dispersion. Under the total-internal-reflection condition, the phase difference between the *S* and *P* polarizations, as a function of the wavelength and refractive index, can be extracted and measured using heterodyne interferometry. Various wavelengths correspond to various refractive index values. The proposed map is convenient in ensuring the prism material using a specific *V* number. The method has the following merits: high stability, ease of operation, and rapid measurement.

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1. Introduction

It is frequently confusing for a depot or laboratory to distinguish between various material types of prisms. Many materials look similar, but these different prism materials present different chromatic dispersion values. Because the refractive index n is dependent on the wavelength (λ) (or color) of light, i.e., $n = n(\lambda)$, dispersion can be measured by determining the relation between the refractive index n and the wavelength λ. Many techniques have been proposed for measuring the refractive index, including the reflectance method,[1–3] Abbe refractometers,[4] the critical angle,[5,6] Brewster's angle,[7,8] the pseudo-Brewster's angle,[9,10] prism couplers,[11–13] total internal reflection (TIR),[14] ellipsometry,[15–20] interference,[21–26] holograms,[27] and the moiré method.[28] However, most of these methods are related to light intensity or interference fringe variation measurements. The proposed method determines a specific V number using the phase method with a simplified optical system for easier operation. Since 1997, Chiu *et al.*[29] and Su *et al.*[30] have proposed a phase method based on common-path heterodyne interferometry for measuring the refractive indices of liquids or prisms. We used this approach with three-wavelength sources to measure the chromatic dispersion and determine the type of prism material in this paper.

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2. Principle

A. Phase Shifts Due to Total-Internal-Reflection Effects

From the paper proposed by Chiu *et al.*,[29] a ray of light in air is incident at θ_i on one surface of a right-angle prism with a refractive index n, as shown in Fig. 1. The incident angle at the prism–air boundary is then θ_1 as

$$\theta_1 = 45° + \sin^{-1}(\sin\theta_i/n). \tag{1}$$

Under TIR conditions, according to Frensel's equation,[31] we can express the phase difference of the S polarization relative to the P polarization as

$$\begin{aligned}\phi &= \delta_s - \delta_p \\ &= 2\tan^{-1}\{[\sin^2\theta_1 - (1/n)^2]^{0.5}/(\tan\theta_1 \sin\theta_1)\},\end{aligned} \tag{2}$$

where δ_s and δ_p are the phase shifts in the S and P polarizations, respectively. To more conveniently and quickly align the optical system, let $\theta_i = 0°$ be the

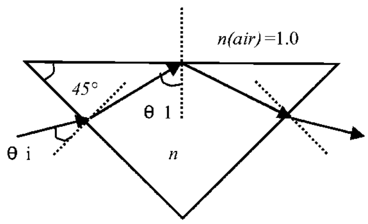

Fig. 1. Total internal reflection inside a right-angle prism.

normal incidence. Equation (2) can then be rewritten simply as

$$\phi = 2 \tan^{-1}\left[(1 - 2/n^2)^{0.5}\right]. \quad (3)$$

Because the refractive index (n) is a function of the wavelength (λ), the phase difference ϕ is also relative to the wavelength.

B. Chromatic Dispersion

From the old Schott dispersion equation[32] we have

$$n^2(\lambda) = A_0 + A_1\lambda^2 + A_2\lambda^{-2} + A_3\lambda^{-4} + A_4\lambda^{-6} + A_5\lambda^{-8}, \quad (4)$$

where the constants ($A_0, A_1, A_2, \ldots$) are derived for each individual material by substituting the known index and wavelength values and solving the resulting simultaneous equations for the constants.

To determine the material of an unknown prism, we measure the phase difference instead of the light intensity using heterodyne technology and three commercial lasers, such as a Nd:YAG laser (λ = 532 nm), an Ar^+ ion laser (λ = 488 nm), and a Ne–He laser (λ = 632.8 nm) instead of a d line (0.5876 μm), F line (0.4861 μm), and C line (0.6563 μm), respectively. To demonstrate the feasibility of the proposed method, we define a specific V number as

$$V' = \frac{\phi_{532}}{\phi_{488} - \phi_{632.8}}, \quad (5)$$

where ϕ_{532}, ϕ_{488}, and $\phi_{632.8}$ are the phase differences at the following wavelengths: 532, 488, and 632.8 nm, respectively. Equations (3)–(5) are used to plot a new map called the phase geographical map. The result is shown in Fig. 2, i.e., the phase difference ϕ_{532} versus the value of V'. Therefore we can use the map to rapidly determine the right-angle prism material type.

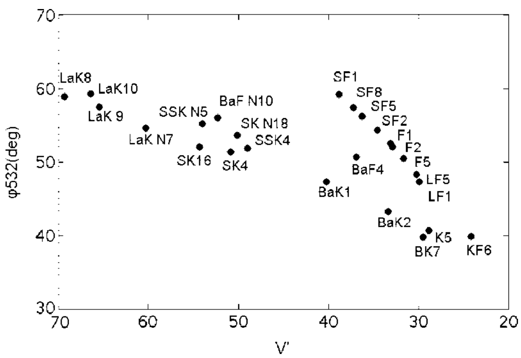

Fig. 2. ϕ_{532} versus V'.

C. Scheme of the System

The scheme for the proposed system is shown in Fig. 3. The lasers are mounted on a linear stage driven by a motor. The lasers are displaced and take turns focusing their beams incident into the system. All beams are 45° polarized, pass through an electro-optic modulator (EOM), and are normal incident into the test prism. From the configuration used in the method of Chiu *et al.*,[29] the light passes through an ANr analyzer, detected by the photodetector D_r, and transferred into an electrical signal such as

$$I_r = |E_r|^2 = 1/2[1 + \cos(\omega t - \phi_r)], \quad (6)$$

where ϕ_r is the phase difference between the S and P polarizations for the light reflected from the beam splitter (BS) and ω is the angular frequency of the electro-optic modulator.[33] On the other hand, the light is transmitted through the BS, enters the prism, and passes through another analyzer ANt, and its intensity on the photodetector D_t is

$$I_t = 1/2[1 + \cos(\omega t - \phi)], \quad (7)$$

where ϕ is the phase difference between the S and P polarizations due to TIR effects and I_t is the test signal. From Eqs. (6) and (7), we can use a phasemeter or a lock-in amplifier to measure the phase difference between I_t and I_r as

$$\phi' = -\phi + \phi_r \quad (8)$$

and can obtain the phase difference ϕ when the initial phase ϕ_r is known.

3. Experimental Results

The experimental setup is shown in Fig. 3. From the results of Chiu *et al.*,[29] the phase difference varies obviously if θ_1 is in the range from 42° to 45°. For normal dispersion, the shorter the wavelength, the larger the phase difference. For easier alignment, we

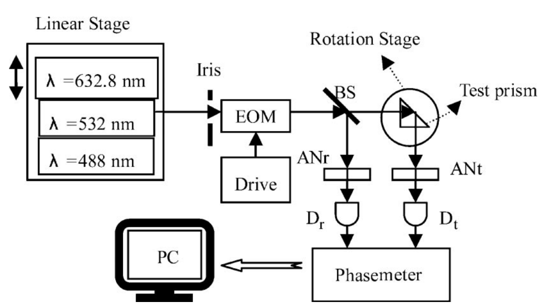

Fig. 3. Experimental system configuration.

Table 1. Experimental Results Compared with Theoretical Data

Parameters	Theoretical Data				Experimental Data			
	BK7	F2	BaF N 10	S-TI H53	BK7	F2	BaF N 10	S-TI H53
ϕ_{532}	39.75	52.01	56.02	65.49	39.76	52.02	56.01	65.49
$\phi_{632.8}$	39.47	51.69	55.80	65.19	39.41	51.63	55.78	65.25
ϕ_{488}	40.70	53.14	56.78	66.47	40.68	53.12	56.77	66.60
V'	32.32	35.87	57.16	51.16	31.31	34.91	56.58	48.51

can measure the phase differences by choosing $\theta_i = 0°$, which is the normal incidence. The minimum measurable refractive index is equal to 1.414.

According to Eqs. (3)–(8), we record these phase differences and calculate the value of V' immediately using a personal computer. These experimental results are shown in Table 1. From these results, the experimental results coincide with the theoretical values.

4. Discussion

From Eq. (3), we can obtain a plot of ϕ versus n at $\theta_1 = 45°$, as shown in Fig. 4. Obviously, the phase difference is proportional to the refractive index. We can define the phase sensitivity using the differential function $d\phi/dn$. If the phase resolution of the system is known, the resolution of the refractive index is achieved using the equation $\Delta n = \Delta\phi(d\phi/dn)^{-1}$, where $\Delta\phi$ presents the phase resolution. Because the

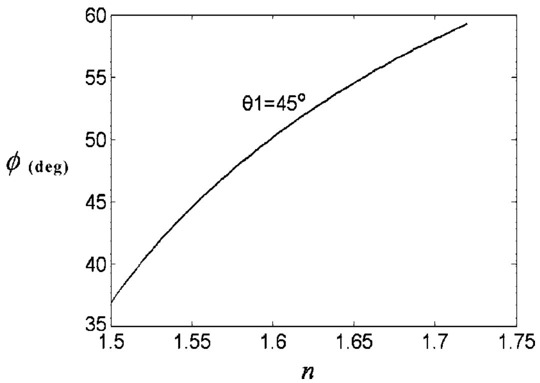

Fig. 4. ϕ versus n for $\theta_1 = 45°$.

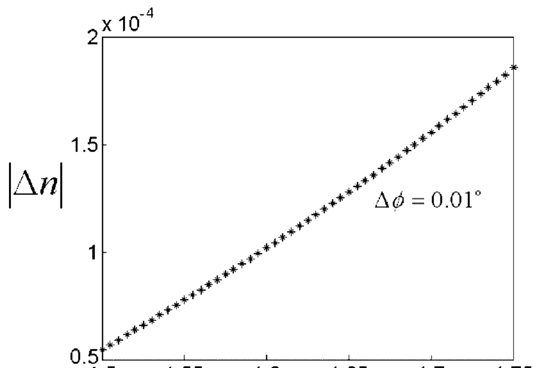

Fig. 5. Error of refractive index versus n for $\Delta\phi = 0.01°$.

S and P polarizations are in a common-path configuration, the two beams disturbed by air or mechanical vibrations cancel each other. The interferences are, therefore, very stable and the phase error is only about the value of $\pm 0.01°$. To simulate the refractive index errors due to the phase error, the simulation curve is plotted as shown in Fig. 5. The errors are below 1.8×10^{-4} in the range of $n = 1.5$–1.75. It is clear that the error increases as n increases.

To consider the errors of V' for $\Delta\phi = \pm 0.01°$, from Eq. (5), $\Delta V'$ is equal to ± 0.5. Using the map, one kind of prism can be distinguished from another.

5. Conclusion

This paper proposed a new method based on TIR effects and common-path heterodyne interferometry for determining a kind of unknown prism. If the phase difference ϕ_{532} and the value of V' are achieved, using the phase geographical map, the kind of prism is known. The method has the following merits: easier alignment and operation, high stability, and rapid measurement.

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