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Detail-preserving smoothing with morphology and fuzzy reasoning

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Abstract. *This study investigates a new approach to removing noise that preserves the fine details which are based on combination of mathematical morphology and fuzzy reasoning. We extract some features from an image using mathematical morphology, then input these features, and the image itself, into fuzzy-rule-based systems to produce a more graceful image.*

1 Introduction

Morphological filters,[1,2] such as closings and openings, are able to remove impulsive noise while preserving such geometrical structures as edges and lines. However, some fine details, for instance texture patterns, cannot be preserved even when multiple structuring elements are employed in morphological filtering. In other words, some fine details are frequently treated as noise by morphological filters. In such situations, we encounter the problem of ambiguity or uncertainty between noises and fine details.

Fuzzy set theory[3,4] is a mathematical tool used in situations involving modeling ambiguity or uncertainty. Fuzzy-rule-based systems are tools used to perform reasoning tasks. In this paper, we will use morphological operations to extract fine details or noisy features from an image. Then, these features will be processed using a rule-based system such that a new better quality image can be obtained by approximate reasoning. This approach will be used in performing morphological filtering and image enhancement. The noise removal system can be described by Fig. 1.

2 Mathematical Morphology

Mathematical morphology provides a sharp-based approach to digital image processing. Most morphological operations tend to simplify image data, preserve their essential shape characteristics, and eliminate irrelevancies. Thus, they are useful for feature extraction, shape analysis, and nonlinear filtering. In this section, we will briefly describe the four basic types of morphological operations: dilations, erosions, openings and closings.

Let $f, s: E \rightarrow R^*$, where E is a Euclidean space and R^* is the set of all extended real numbers. Then:

1. The *dilation* of f by s, denoted by $f \oplus s$, is defined by

$$(f \oplus s)(x) = \sup\{f(x-z)+s(z);\ \ z \in E\}. \tag{1}$$

2. The *erosion* of f by s, denoted by $f \ominus s$, is defined by

$$(f \ominus s)(x) = \inf\{f(x+z)-s(z);\ \ z \in E\}. \tag{2}$$

3. The *closing* of f by s, denoted by $f \bullet s$, is defined by

$$f \bullet s = (f \oplus s) \ominus s. \tag{3}$$

4. The *opening* of f by s, denoted by $f \circ s$, is defined by

$$f \circ s = (f \ominus s) \oplus s. \tag{4}$$

Morphological operations possess many algebraic properties. For instance, closings and openings are increasing, translation-invariant, and idempotent. For more details, the readers can refer to Refs. 5 and 6.

3 Approximate Reasoning

In our daily life, we often make inferences whose antecedents and consequences contain fuzzy concepts. Such an inference can not be made adequately using methods that are based on classical two-valued logic. In order to make such an inference, Zadeh[7] suggested an inference rule called the ''compositional rule of inference'' (CRI). Later, a new approach was proposed by Mamdani and Assilian.[8] This new form, fuzzy if-then rules, was based on Zadeh's.[9] Other reasoning methods, such as Tsukamoto's and the TSK fuzzy model, can be found in the literature.[10–18] In this paper, we'll use Tsukamoto's fuzzy models and TSK fuzzy models to remove noise from signal and image.

3.1 *Tsukamoto's Fuzzy Models*

In Tsukamoto's fuzzy models,[12] the consequent part of each fuzzy if-then rule is represented by a linguistic value with a monotonical membership function, the inferred output of each rule is defined as a crisp value induced by the minimum or product of the degree match between the input and antecedent part. The overall output is taken as the

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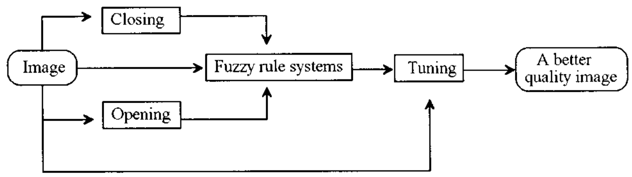

Fig. 1 Noise removal by approximate reasoning structure.

weighted average of each rule's output. Figure 2 illustrates the reasoning mechanism in a two-rule two-input inference system that was proposed by Tsukamoto.

Inference rules:

Rule 1: if x is A_1 and y is B_1 then z is C_1.
Rule 2: if x is A_2 and y is B_2 then z is C_2.

Since each rule infers a crisp output, the process of defuzzification in Tsukamoto's fuzzy model involves a weighted average for each rule's output. This avoids the time- consuming process of defuzzification. However, there is one problem in the consequence part of the linguistic variable's membership function. That is, a closed form of the membership function in the term of the consequence part is required. To solve this problem, we partition this term as ''positive'' and ''negative;'' it is natural to define either open-right or open-left membership, which is monotonically increasing or decreasing, respectively.

3.2 *TSK Fuzzy Models*

The TSK fuzzy model was proposed by Takagi, Sugeno, and Kang.[10,11] A typical fuzzy rule in a TSK fuzzy model can be described as the form:

if x is A and y is B then $z=f(x,y)$,

where A and B are fuzzy sets in the antecedent part, and $z = f(x,y)$ is a crisp function in the consequent part. Usually $f(x,y)$ is a polynomial of the input variables x and y, but it can be any function as long as it can appropriately describe the output system around the fuzzy region specified by the antecedent of the rule. The fuzzy inference system with two-rule, two-input is illustrated in Fig. 3.

4 Noise Removal by Morphology and Fuzzy Reasoning

In this section, we utilize approximate reasoning in morphological filtering to perform removing noise on signals and images. In general, removing noise from signals or images often causes blurring effects. In other words, the process of removing noise often distorts thin lines, edges, or fine details. In this paper, we use a new approach based on mathematical morphology and the fuzzy-rule system to remove noise. We then obtain a more graceful signal or image. This task is based on the fuzzy if-then rules approach. How to define the corresponding fuzzy sets and rules to achieve our goal is an important problem.

4.1 *Basic Idea of Smoothing*

As mentioned before, morphological filters are able to remove impulsive noises and preserve geometrical structures. In particular, morphological closings are frequently used to remove negative impulsive noise, and morphological openings are frequently used to remove positive noise. However, detail might also be removed by morphological filters. Now, we use fuzzy concept to decide whether a pixel in an image is a noisy one or not. To make such a decision, we measure the difference of this pixel and its neighborhoods with the help of morphological operations.

Let f denote a digital image with domain R and grey scale M, and let S be a structuring element. Then, the residue images $f\bullet s - f$ and $f - f \circ s$ implicitly contain the information about fine details and noise. Thus, our basic idea is that if $f\bullet s(p) - f \circ s(p)$ is small, then p is considered to be a point of fine detail. On the other hand, if $f\bullet s(p) - f \circ s(p)$ is large, then p is considered to be a point of either positive or negative noise. We define positive and negative noise in the following:

Definition 1: *Positive noise* at p:

$$\text{if } f\bullet s(p)-f\circ f(p) \text{ is large and}$$

$$f\bullet s(p)-f(p)<f(p)-f\circ s(p).$$

Definition 2: *Negative noise* at p:

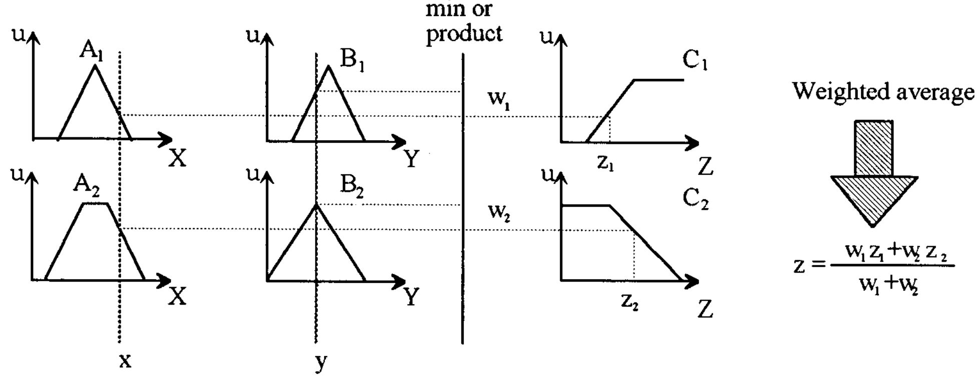

Fig. 2 Tsukamoto's fuzzy inference system.

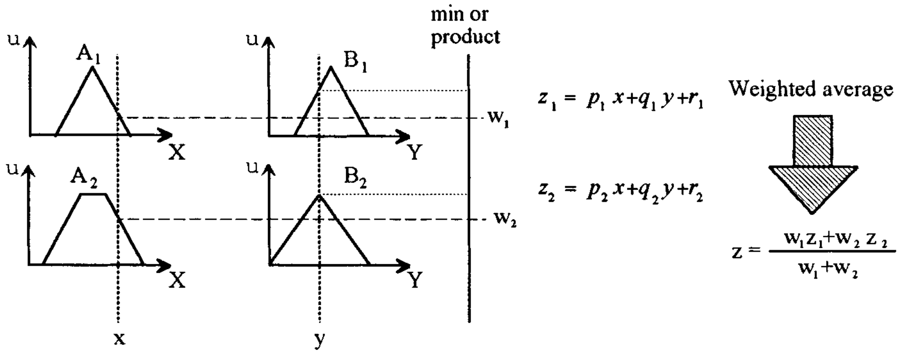

Fig. 3 TSK fuzzy inference model.

$$\text{if } f\bullet s(p)-f\circ f(p) \text{ is large and}$$

$$f\bullet s(p)-f(p)>f(p)-f\circ s(p).$$

Based on the above observations, we will propose a filtering method by using approximate reasoning. Let FINE_DETAIL and NOISE be two fuzzy sets on M. For noise removal, in this subsection, we adopt the TSK fuzzy reasoning model and establish the following rules:

Rule 1: If x is FINE_DETAIL then y is x.

Rule 2: If x is POSITIVE NOISE then y is c_1.

Rule 3: If x is NEGATIVE NOISE then y is c_2.

Here, $c_2 = -c_1$, and c_1 is obtained from another fuzzy inference system, which is described in next subsection. The fuzzy sets FINE_DETAIL and NOISE and their corresponding membership functions are defined as follows: Taking $S(x;0,20,40)$ as FINE_DETAIL and $1 - S(x;20,60,100)$ as NOISE. Their corresponding membership functions are illustrated in Fig. 4, where $S(x;\alpha,\beta,\gamma)$ is the S-function

$$S(x;\alpha,\beta,\gamma)=\begin{cases} 0, & x\leqslant\alpha, \\ 2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right), & \alpha\leqslant x\leqslant\beta, \\ 1-2\left(\dfrac{x-\gamma}{\gamma-\alpha}\right), & \beta\leqslant x\leqslant\gamma, \\ 1, & \gamma\leqslant x. \end{cases} \tag{5}$$

Now, for each pixel p, if we input the value $x_0 = f\bullet s(p) - f\circ s(p)$ into the above system, we will get an output value y_0 after approximate reasoning. Then, the value of the final output image g at pixel p can be obtained by the following equation:

$$\text{if positive noise then } g(p)=f\circ s(p)+y_0$$

$$\text{else } g(p)=f\bullet s(p)-y_0. \tag{6}$$

To expedite our experiment, an artificial noisy test image is created by

$$f_n(x,y)=f(x,y)+n(x,y), \tag{7}$$

where $f(x,y)$ is the original image and $n(x,y)$ represents the added random noise.

In order to show the practical effects on image processing of fuzzy filters, we demonstrate the image smoothing by a typical example of fuzzy filters and we evaluate the filters performance by a simple measure MSE (mean square error) among the original, degraded and processed images. In this paper, we consider that

$$\text{Closing}(f)=f\bullet L_0\wedge f\bullet L_{\pi/4}\wedge f\bullet L_{\pi/2}\wedge f\bullet L3_{\pi/4} \tag{8}$$

and

$$\text{Opening}(f)=f\circ L_0\vee f\circ L_{\pi/4}\vee f\circ L_{\pi/2}\vee f\circ L3_{\pi/4} \tag{9}$$

where the structure elements of L_0, $L_{\pi/4}$, $L_{\pi/2}$ and $L3_{\pi/4}$ are shown in Fig. 5.

4.2 *Fuzzy Interpolation Algorithm*

In this subsection, we propose an image interpolation algorithm based on fuzzy reasoning. The algorithm extracts the local self-similarity properties with a window scanning in an image and uses these properties to interpolate pixels in the current window simultaneously. We can obtain more enhanced and high quality images than those by using bilinear interpolation.

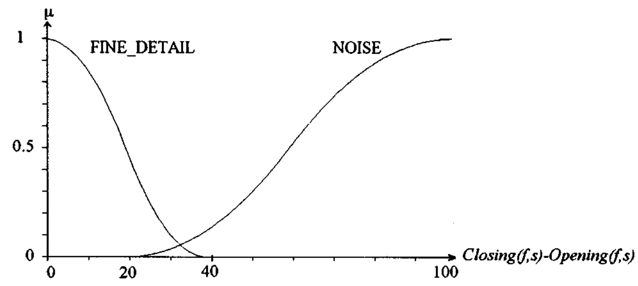

Fig. 4 The membership functions of FINE_DETAIL and NOISE.

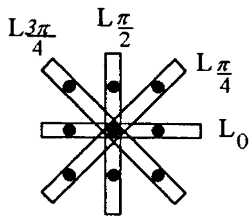

Fig. 5 Directional smoothing structure elements.

Consider an interpolation in the 1-D curve of point (x,y). Its neighbors are l_1, l_2, r_1 and r_2. The relationship of the 1-D signal sequence is described as follows.

The proposed interpolation algorithm at point (x,y) is briefly described in the following:

1. Tsukamoto's fuzzy reasoning model is imitated.
2. Define linguistic variables in the fuzzy-rules system. They are LOW, NMED, PMED and HIGH in the antecedent part and LOW′, NMED′, PMED′ and HIGH′ in the consequent part. We define them as LOW $= 1 - S(x;-100,-50,0)$, NMED(NEGATIVE MED) $= S(x;-80,-40,0)$, PMED(POSITIVE MED) $= 1 - S(x;0,40,80)$ and HIGH$=S(x;0,50,100)$; LOW′$=1-S(x;-60,-30,0)$, NMED′$=S(x;-60,-30,0)$, PMED′$=1-S(x;0,30,60)$ and HIGH$=S(x;0,30,60)$. The S-function was introduced in the previous subsection.
3. The interpolation rules are given in Table 1, where LEFT$=y_{l_1}-y_{l_2}$ and RIGHT$=y_{r_1}-y_{r_2}$, and illustrated in Fig. 6.
4. After defuzzification (weighted average), we can obtain a crisp value c_1, which is the value desired in Section 4.1.

5 Experimental Results and Discussions

In this section, we describe a series of experiments. All of the test images in our experiment have a resolution of 256 by 256 pixels with 8 bits/pixel gray-scale quantization. First, we demonstrate our results using 1-D signals. Then, we perform image noise reduction on an image of ''Lena.''

5.1 *One-Dimensional Signals and Noise Removal*

To see the effect of our proposed method easily, we consider the filtering of a simple 1-D signal in this subsection. This signal is shown in Fig. 7(a) and some results in Fig. 7(b)–(e). For comparison with our method, we consider the conventional median and mean filter.

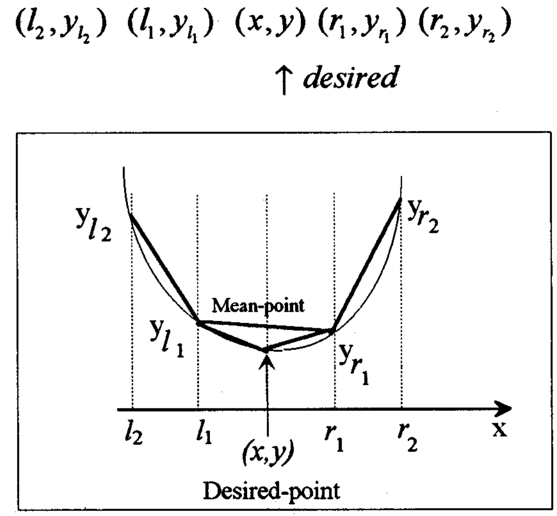

Fig. 6 The fuzzy interpolation method.

5.2 *Image Smoothing*

Computer simulations have been carried out to compare the performance of the proposed filter with the mean filter, the median filter and so on. The original image is corrupted by random noise, with probability $p = 0.01$, 0.02, or 0.05, and is then used for simulations. Table 2 shows the MSE of the results by various filtering. The proposed filter shows good results regardless of the MSE. The visual quality can be observed in Fig. 8(a)–(e), in the case of random noise having probability 0.02. Clearly all the other filters can remove the noise, but that they also blur the details. On the other hand, the proposed filter can remove noise and preserve the details.

Figure 8(a) is the original ''Lena'' image. Figure 8(b) shows the image corrupted with random noise having probability $p = 0.02$. Figure 8(c) shows the resulting noisy image with closing-and-opening average. Figure 8(d) shows the noise image filtered with a 3×3 median filter. It is clear that the fine details in Lena's hair are destroyed by this operation. Finally, Fig. 8(e) shows the result of smoothing the noisy image by our proposed method. The measures of the original and processed images are given in Table 2.

6 Conclusions

In this paper, we have explored a combination of mathematical morphology and fuzzy reasoning for use in noise removal. Morphological operations have some good properties. Furthermore, the gray-scale morphology suggests parallel computer architectures (array and pipelines) for

Table 1 Fuzzy interpolation rules.

LEFT \ RIGHT	LOW	NMED	PMED	HIGH
LOW	LOW′	NMED′	NMED′	PMED′
NMED	NMED′	NMED′	PMED′	PMED′
PMED	NMED′	PMED′	PMED′	PMED′
HIGH	PMED′	PMED′	PMED′	HIGH′

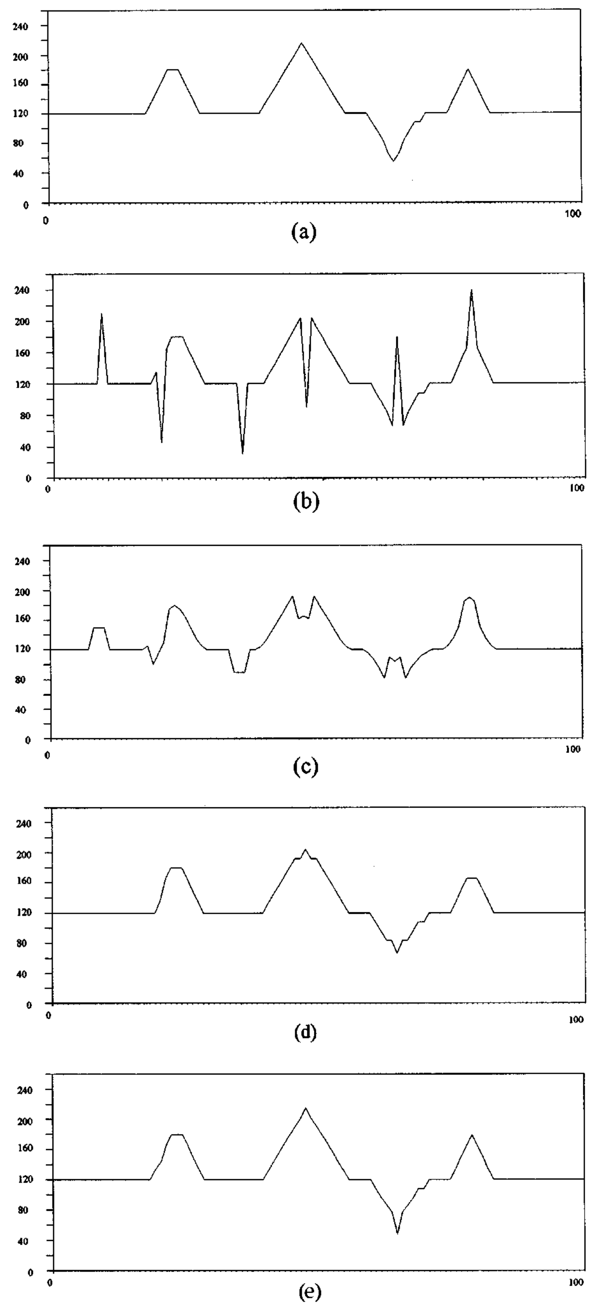

Fig. 7 1-D signals: (a) the original 1-D signal, (b) 1-D signal with noise, (c) the result of a mean filter with mask length 3, (d) the result of a median filter with mask length 3, (e) output of our proposed method.

Table 2 Mean square error for test image with random noise.

	MSE			
Noise Prob.	Unfiltered	(Closing+opening)/2	3×3 med.	Our method
0.01	42.20	17.59	51.13	7.06
0.02	88.10	31.41	50.80	10.28
0.05	229.19	78.51	53.75	24.78

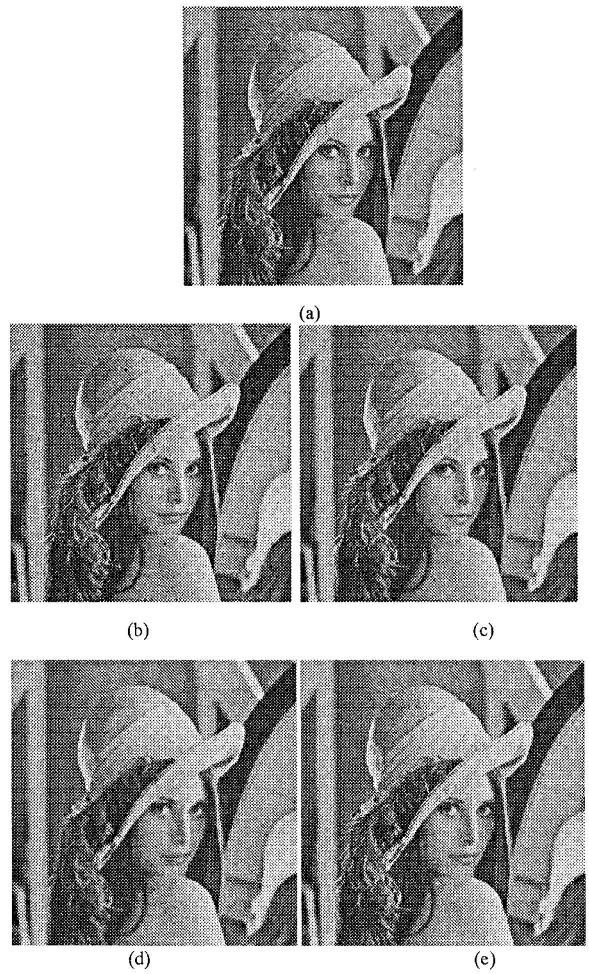

Fig. 8 Smoothing of noise image: (a) original image. (b) random noise, $p = 0.02$, (c) output of closing and opening average, (d) output of the 3 × 3 median, (e) output of fuzzy reasoning.

processing in real time. Here, we not only propose a novel method for image smoothing but provide a new research field of mathematical morphology and image processing. It is also worth noting that the membership function in this paper is extracted from expert's views and experimental result. However, such membership functions do not achieve the optimal results. We can use the popular neural network or genetic algorithm approach to obtain more flexible membership functions or fuzzy rules. Our future research will employ morphological and fuzzy inference models to perform line detection, edge detection, and segmentation in image processing.

Acknowledgments

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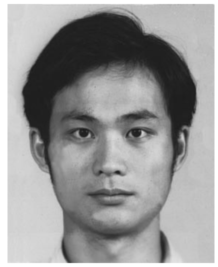
References

1. J. Serra, *Image Analysis and Mathematical Morphology*, Academic Press, New York (1982).
2. J. Serra, *Image Analysis and Mathematical Morphology, Vol. 2: Theoretical Advances*, Academic Press, London (1988).
3. L. A. Zadeh, ''Outline of a new approach to analysis of complex

systems and decision process,'' *IEEE Trans. Systems Man Cybernet.* **SMC-3**, 28–44 (1973).

4. J. C. Bezdek and S. K. Pal, Eds., *Fuzzy Models for Pattern Recognition*, IEEE Press, New York (1992).
5. R. M, Haralick, S. R. Sternberg, and X. Zhuang, ''Image analysis using mathematical morphology,'' *IEEE Trans. Pattern Anal. Machine Intell.* **PAMI-9**(4), 532–550 (1987).
6. Y.-C. Hsueh, ''Mathematical morphology on l-images,'' *Signal Process.* **26**, 221–241 (1992).
7. L. A. Zadeh, Calculus of fuzzy restriction, in *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, L. A. Zadeh *et al.*, Eds., pp. 1–39, Academic Press, New York (1975).
8. E. M. Mamdani and S. Assilian, ''Application of fuzzy algorithms for control of simple dynamic plant,'' *Proc. IEE* **121**, 1585–1588 (1974).
9. L. A. Zadeh, ''Outline of new approach to the analysis of complex systems and decision process,'' *IEEE Trans. Systems Man Cybernet.* **3**, 28–44 (1973).
10. M. Sugeno and G. T. Kang, ''Structure identification of fuzzy model,'' *Fuzzy Sets and Systems* **28**, 15–33 (1988).
11. T. Takagi and M. Sugeno. ''Fuzzy identification of system and its applications to modeling and control,'' *IEEE Trans. Systems Man Cybernet.* **SMC-15**, 116–132 (1988).
12. Y. Tasukamoto, An approach to fuzzy reasoning method, in *Advances in Fuzzy Set Theory and Applications*, M. M. Gupta, R. K. Ragade, and R. R. Yager, Eds., pp. 137–149, North-Holland, Amsterdam (1979).
13. H. Nakanishi, I. B. Turksen, and M. Sugeno, ''A review and comparison of six reasoning methods,'' *Fuzzy Sets and Systems* **57**, 257–294 (1993).
14. I. B. Turksen and Y. Tian, ''Combination of rules or their consequences in fuzzy expert systems,'' *Fuzzy Sets and Systems* **58**, 3–40 (1993).
15. I. B. Turksen and Y. Tian, ''Interval valued fuzzy set representation on multiple antecedents fuzzy s-implications and reasoning,'' *Fuzzy Sets and Systems* **52**, 1–25 (1992).
16. R. R. Yager and L. A. Zadeh, *An Introduction to Fuzzy Logic Applications in Intelligent Systems*, Academic Press, 1992.
17. R. C. Hardie and C. G. Boncelet, ''LUM filters: a class of rank-order-based filters for smoothing and sharpening,'' *IEEE Trans. Signal Process.* **41**(3), 1061–1076 (1993).
18. C.-T. Sun, ''Rule-Base Structure Identification in an Adaptive-Network-Based Fuzzy Inference System,'' *IEEE Trans. Fuzzy Systems* **2**(1), 64–73 (1994).



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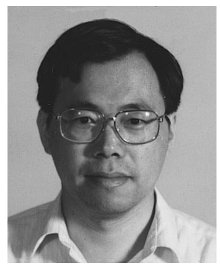

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