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A soft computing method for multi-criteria decision making with dependence and feedback

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Abstract

In this paper, the decision making problems with the dependence and the feedback effects are considered. Although the analytic network/hierarchy process (ANP/AHP) has been proposed to deal with the problems above, several problems make the method impractical. In this paper, we proposed the fuzzy decision maps (FDM), which incorporates the eigenvalue method, the fuzzy cognitive maps (FCM), and the weighting equation, to overcome the problem of preferential independent and the shortcomings of the ANP. In addition, two numerical examples are used to demonstrate the proposed method. On the basis of the numerical results, we can conclude that the proposed method can soundly deal with the decision making problems with the dependence and the feedback effects.

Keywords: Decision making; Analytic network/hierarchy process (ANP/AHP); Fuzzy decision maps (FDM); Fuzzy cognitive maps (FCM); Preferential independent

1. Introduction

Multi-criteria decision making (MCDM) involves determining the optimal alternative among multiple, conflicting, and interactive criteria [1]. Many methods, which are based on multiple attribute utility theory (MAUT), have been proposed (e.g. the weighted sum and the weighted product methods) to deal with the MCDM problems. The concept of MAUT is to aggregate all criteria to a specific unidimension which is called utility function to evaluate alternatives. Although many papers have been proposed to discuss the aggregation operator of MAUT [2], the main problem of MAUT is the assumption of preferential independence [3,4].

On the assumption of preferential independence, it can be seen that the dependence and the feedback effects cannot be considered. However, the real-life situation usually emerges the dependence and the feedback effects simultaneously while making decisions. The analytic network process (ANP) was proposed in [5,6] to overcome the problem of dependence and feedback among criteria or alternatives. The ANP is the general

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form of the analytic hierarchy process (AHP) [7], which has been used for multi-criteria decision making (MCDM), to release the restriction of hierarchical structure, and has been applied to project selection [8,9], product planning, strategic decision [10,11], and optimal scheduling [12].

The advantages of the ANP are that it is not only appropriate for both quantitative and qualitative data types, but it also can overcome the problem of interdependence and feedback among criteria. Although the ANP have been widely used in various applications, two main problems should be highlighted as follows. The first is the problem of comparison. In the ANP, the decision maker is asked to answer the question like "How much importance does a criterion have compared to another criterion with respect to our interests or preferences?" However, sometimes the questions are hard even for the expert to answer the question above due to some questions are anti-intuitive. We will highlight the problem again in Section 3. Furthermore, the key for the ANP is to determine the relationship structure among features in advance [9]. The different structure results in the different priorities. However, it is usually hard for the decision maker to give the true relationship structure by considering many criteria.

In this paper, we proposed the fuzzy decision maps (FDM), which incorporates the eigenvalue method, the fuzzy cognitive maps (FCM) [13,14], and the weighting equation, to overcome the problem of preferential independent and the shortcomings of the ANP. Not only dependence effects but also feedback effects can be considered to derive the best alternative. Besides, two numerical examples are used to demonstrate the proposed method and compared with the ANP. On the basis of the numerical results, we can conclude that FDM can provide another method to deal with the structural MCDM problem.

The rest of this paper is organized as follows. In Section 2, we describe the contents of the analytic network process. Fuzzy decision maps are proposed in Section 3. Two numerical examples, which are used here to demonstrate the proposed method, are in Section 4. Discussions are presented in Section 5 and conclusions are in the last section.

2. The analytic network process

Since the ANP/AHP has been proposed by Saaty, it has been widely used to deal with the dependence and the feedback decision making. The method of the ANP can be described as follows. The first phase of the ANP is to compare the criteria in whole system to form the supermatrix. This is done through pairwise comparisons by asking "How much importance does a criterion have compared to another criterion with respect to our interests or preferences?" The relative importance value can be determined using a scale of 1–9 to represent equal importance to extreme importance [5,7]. The general form of the supermatrix can be described as follows:

$$
\begin{array}{cc}
 & \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \\ e_{11}\cdots e_{1n_1} & e_{21}\cdots e_{2n_2} & \cdots & e_{m1}\cdots e_{mn_n} \end{array} \\
\boldsymbol{W} = \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \vdots \\ C_m & \begin{array}{c} e_{m1} \\ e_{m2} \\ \vdots \\ e_{mn_m} \end{array} \end{array} &
\begin{bmatrix} \boldsymbol{W}_{11} & \boldsymbol{W}_{12} & \cdots & \boldsymbol{W}_{1m} \\ \boldsymbol{W}_{21} & \boldsymbol{W}_{22} & \cdots & \boldsymbol{W}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{W}_{m1} & \boldsymbol{W}_{m2} & \cdots & \boldsymbol{W}_{mm} \end{bmatrix}
\end{array}
$$

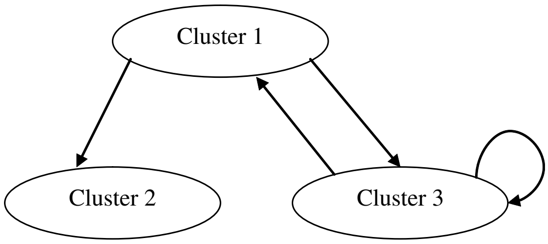

Fig. 1. The structure of the case 1.

where C_m denotes the mth cluster, e_{mn} denotes the nth element in mth cluster, and $\boldsymbol{W}_{ij}$ is the principal eigenvector of the influence of the elements compared in the jth cluster to the ith cluster. In addition, if the jth cluster has no influence to the ith cluster, then $\boldsymbol{W}_{ij} = 0$.

Therefore, the form of the supermatrix depends much on the variety of the structure. There are several structures which were proposed by Saaty including hierarchy, holarchy, suparchy, and intarchy to demonstrate how the structure affects the supermatrix. Here, two simple cases, which both have three clusters, are used to display how to form the supermatrix based on the structures.

The supermatrix can be formed as the following matrix:

$$\boldsymbol{W} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} 0 & 0 & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & 0 & W_{33} \end{bmatrix} \end{array}$$

In Fig. 2, the second case is more complex than the first case.

Then, the supermatrix of the second case can be expressed as

$$\boldsymbol{W} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & 0 \end{bmatrix} \end{array}$$

After forming the supermatrix, the weighted supermatrix is derived by transforming all columns sum to unity exactly. This step is much similar to the concept of Markov chain for ensuring the sum of these probabilities of all states equals to 1. Next, we raise the weighted supermatrix to limiting powers such as Eq. (1) to get the global priority vector or called weights.

$$\lim_{k \to \infty} \boldsymbol{W}^k \tag{1}$$

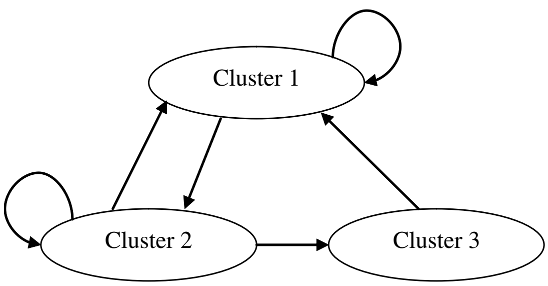

Fig. 2. The structure of the case 2.

In addition, if the supermatrix has the effect of cyclicity, the limiting supermatrix is not the only one. There are two or more limiting supermatrices in this situation, and the Cesaro sum would be calculated to get the priority. The Cesaro sum is formulated as

$$\lim_{k\to\infty}\left(\frac{1}{N}\right)\sum_{j=1}^{N}\boldsymbol{W}_j^k \tag{2}$$

to calculate the average effect of the limiting supermatrix (i.e. the average priority weights) where $\boldsymbol{W}_j$ denotes the jth limiting supermatrix. Otherwise, the supermatrix would be raised to large powers to get the priority weights. The detailed discussion of the mathematical processes of the ANP can refer to [5,15].

In order to describe the concrete procedures of the ANP, a simple example of system development is demonstrated to derive the priority of each criterion. As we know, the key to develop a successful system depending on the match of human and technology factors. Assume the human factor can be measured by the criteria of business culture (C), end-user demand (E), and management (M). On the other hand, the technology factor can be measured by the criteria of employee ability (A), process (P) and resource (R). In addition, human and technology factors are affected with each other as like as shown in Fig. 3.

The first step of the ANP is to compare the importance between each criterion. For example, the first matrix below is to ask the question "For the criterion of employee ability, how much the importance does one of the human criteria than another." The other matrices can easily be formed with the same procedures. The next step is to calculate the influence (i.e. calculate the principal eigenvector) of the elements (criteria) in each component (matrix) using the eigenvalue method.

Ability	Culture	End-user	Management	Eigenvector	Normalization
Culture	1	3	4	0.634	0.634
End-user	1/3	1	1	0.192	0.192
Management	1/4	1	1	0.174	0.174
Process	Culture	End-user	Management	Eigenvector	Normalization
Culture	1	1	1/2	0.250	0.250
End-user	1	1	1/2	0.250	0.250
Management	2	2	1	0.500	0.500
Resource	Culture	End-user	Management	Eigenvector	Normalization
Culture	1	2	1	0.400	0.400
End-user	1/2	1	1/2	0.200	0.200
Management	1	2	1	0.400	0.400
Culture	Ability	Process	Resource	Eigenvector	Normalization
Ability	1	5	3	0.637	0.637
Process	1/5	1	1/3	0.105	0.105
Resource	1/3	3	1	0.258	0.258
End-user	Ability	Process	Resource	Eigenvector	Normalization
Ability	1	5	2	0.582	0.582
Process	1/5	1	1/3	0.109	0.109
Resource	1/2	3	1	0.309	0.309
Management	Ability	Process	Resource	Eigenvector	Normalization
Ability	1	1/5	1/3	0.136	0.136
Process	5	1	3	0.654	0.654
Resource	3	1/3	1	0.210	0.210

Now, we can form the supermatrix based on the eigenvectors above and the structure in Fig. 3. Since the human factor can affect the technology factor, and vise versa, the supermatrix is formed as follows:

e_{11} ← **C**; $w_{1_2 2_3}$ → (E, R) entry 0.200; $\boldsymbol{W}_{22}$ → lower-right block

		C	**E**	**M**	**A**	**P**	**R**
Cluster 1	**C**	0	0	0	0.634	0.250	0.400
	E	0	0	0	0.192	0.250	0.200
	M	0	0	0	0.174	0.500	0.400
Cluster 2	**A**	0.637	0.582	0.136	0	0	0
	P	0.105	0.109	0.654	0	0	0
	R	0.258	0.309	0.210	0	0	0

Then, the weighted supermatrix is obtained by ensuring all columns sum to unity exactly.

	C	**E**	**M**	**A**	**P**	**R**
C	0	0	0	0.634	0.250	0.400
E	0	0	0	0.192	0.250	0.200
M	0	0	0	0.174	0.500	0.400
A	0.637	0.582	0.136	0	0	0
P	0.105	0.109	0.654	0	0	0
R	0.258	0.309	0.210	0	0	0

Last, by calculating the limiting power of the weighted supermatrix, the limiting supermatrix is obtained as follows:

	C	**E**	**M**	**A**	**P**	**R**
C	0	0	0	0.464	0.464	0.464
E	0	0	0	0.210	0.210	0.210
M	0	0	0	0.324	0.324	0.324
A	0.463	0.463	0.463	0	0	0
P	0.284	0.284	0.284	0	0	0
R	0.253	0.253	0.253	0	0	0

(When k is even)

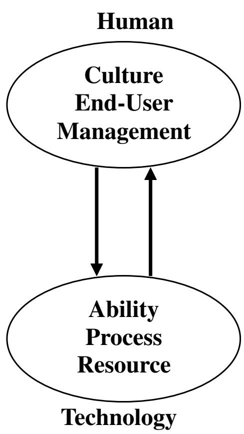

Fig. 3. The structure of the system development.

and

	C	**E**	**M**	**A**	**P**	**R**
C	0.464	0.464	0.464	0	0	0
E	0.210	0.210	0.210	0	0	0
M	0.324	0.324	0.324	0	0	0
A	0	0	0	0.463	0.463	0.463
P	0	0	0	0.284	0.284	0.284
R	0	0	0	0.253	0.253	0.253

(When k is odd)

As we see, the supermatrix has the effect of cyclicity, and the Cesaro sum (i.e. add the two matrices and dividing by two) is used here to obtain the final priorities as follows:

	C	**E**	**M**	**A**	**P**	**R**
C	0.233	0.233	0.233	0.233	0.233	0.233
E	0.105	0.105	0.105	0.105	0.105	0.105
M	0.162	0.162	0.162	0.162	0.162	0.162
A	0.231	0.231	0.231	0.231	0.231	0.231
P	0.142	0.142	0.142	0.142	0.142	0.142
R	0.127	0.127	0.127	0.127	0.127	0.127

In this example, the criterion of culture has the highest priority (0.233) in system development and the criterion of end-user has the least priority (0.105).

In order to show the effect of the different structure in the ANP, the other structure, which has the feedback effect on human factors, is considered as shown in Fig. 4.

There are two methods to deal with the self-feedback effect. The first method simply place 1 in diagonal elements and the other method performs a pairwise comparison of the criteria on each criterion. In this example, we use the first method. With the same steps above, the unweighted supermatrix, the weighted supmatrix, and the limiting supermatrix can be obtained as follows, respectively:

	C	**E**	**M**	**A**	**P**	**R**
C	1	0	0	0.634	0.250	0.400
E	0	1	0	0.192	0.250	0.200
M	0	0	1	0.174	0.500	0.400
A	0.637	0.582	0.136	0	0	0
P	0.105	0.109	0.654	0	0	0
R	0.258	0.309	0.210	0	0	0

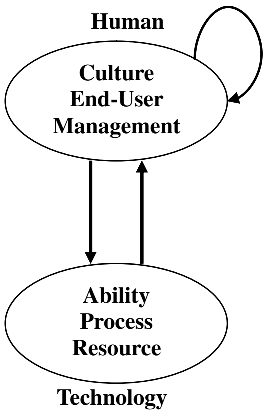

Fig. 4. The structure of system development with feedback effects.

$$\begin{array}{c} \\ \mathbf{C} \\ \mathbf{E} \\ \mathbf{M} \\ \mathbf{A} \\ \mathbf{P} \\ \mathbf{R} \end{array}\begin{array}{c} \begin{array}{cccccc} \mathbf{C} & \mathbf{E} & \mathbf{M} & \mathbf{A} & \mathbf{P} & \mathbf{R} \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0 & 0.634 & 0.250 & 0.400 \\ 0 & 0.5 & 0 & 0.192 & 0.250 & 0.200 \\ 0 & 0 & 0.5 & 0.174 & 0.500 & 0.400 \\ 0.319 & 0.291 & 0.068 & 0 & 0 & 0 \\ 0.053 & 0.055 & 0.327 & 0 & 0 & 0 \\ 0.129 & 0.155 & 0.105 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ \mathbf{C} \\ \mathbf{E} \\ \mathbf{M} \\ \mathbf{A} \\ \mathbf{P} \\ \mathbf{R} \end{array}\begin{array}{c} \begin{array}{cccccc} \mathbf{C} & \mathbf{E} & \mathbf{M} & \mathbf{A} & \mathbf{P} & \mathbf{R} \end{array} \\ \begin{bmatrix} 0.310 & 0.310 & 0.310 & 0.310 & 0.310 & 0.310 \\ 0.140 & 0.140 & 0.140 & 0.140 & 0.140 & 0.140 \\ 0.216 & 0.216 & 0.216 & 0.216 & 0.216 & 0.216 \\ 0.154 & 0.154 & 0.154 & 0.154 & 0.154 & 0.154 \\ 0.095 & 0.095 & 0.095 & 0.095 & 0.095 & 0.095 \\ 0.084 & 0.084 & 0.084 & 0.084 & 0.084 & 0.084 \end{bmatrix} \end{array}$$

Since the effect of cyclicity does not exist in this example, the final priorities are directly obtained by limiting the power to converge. Although the criterion of culture also has the highest priority, the priority changes from 0.233 to 0.310. On the other hand, the least priority is resource (0.084) instead of end-user. Compare with the priorities of the two examples, the structures play the key to both the effects and the results. In addition, it should be highlighted that when we raise the weighted matrix to limiting power, the weighted matrix should always be the stochastic matrix.

From the contents of the ANP above, it is clear that the key for the ANP is to determine the network structure among all features in advance [9] and answer the questions precisely. However, sometimes both of them are hard for the decision maker to give. Next, we propose the fuzzy decision map method to overcome the problems of the ANP for dealing with the MCDM problems with dependence and feedback in Section 3.

3. Fuzzy decision maps

In order to deal with the problem of dependence and feedback among criteria, we first depict the FCM as shown in Fig. 5 to illustrate the situation of decision making. In Fig. 5, e_{ij} denotes the interaction effect from

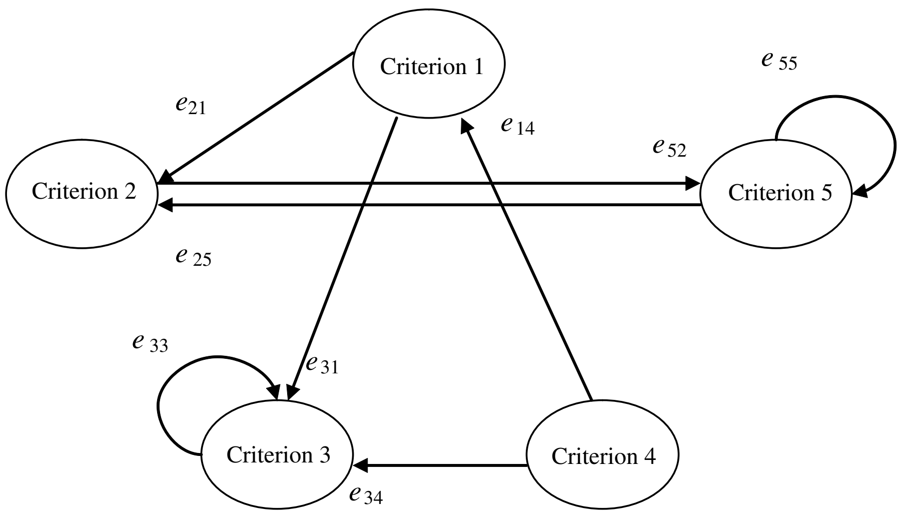

Fig. 5. The problem of a decision map.

the jth criterion to the ith criterion, and e_{ii} indicates the compound effect of the ith criterion. As we know, due to the problem of compound and interaction effects, it is hard for decision makers to make a good decision using the simple weighted method.

A way to overcome the problems above is to obtain the information of influences among criteria and then to derive the finial weights by considering the influences among criteria. However, since these criteria may have loop or feedback relationships, it is hard to derive the influences among criteria. Next, we first employ the FCM to derive the influence among criteria and then obtain the finial weights by using the weighted formulation.

FCM, which was first proposed by Koska [14,15], extends the original cognitive maps [16] by incorporating fuzzy measures to provide a flexible and realistic method for extracting the fuzzy relationships among objects in a complex systems. Recently, FCM have been widely employed in the applications of political decision-making, business management, industrial analysis, and system control [17–19], except for the area of MCDM. The concepts of FCM can be described as follows.

Given a 4-tuple $(N, \boldsymbol{E}, \boldsymbol{C}, f)$ where $N = \{N_1, N_2, \ldots, N_n\}$ denotes the set of n objects, $\boldsymbol{E}$ denotes the connection matrix which is composed of the weights between objects, $\boldsymbol{C}$ is the state matrix, where $\boldsymbol{C}^{(0)}$ is the initial matrix and $\boldsymbol{C}^{(t)}$ is the state matrix at certain iteration t, and f is a threshold function, which indicates the weighting relationship between $\boldsymbol{C}^{(t)}$ and $\boldsymbol{C}^{(t+1)}$. Several formulas have been used as threshold functions such as

$$f(x) = \begin{cases} 1 & \text{if } x \geqslant 1 \\ 0 & \text{if } x < 1 \end{cases}, \quad \text{(Hard line function)}$$

$$f(x) = \tanh(x) = (1 - e^{-x})/(1 + e^{-x}), \quad \text{(Hyperbolic-tangent function)}$$

and

$$f(x) = 1/(1 + e^{-x}). \quad \text{(Logistic function)}$$

The influence of the specific criterion to other criteria can be calculated using the following updating equation:

$$\boldsymbol{C}^{(t+1)} = f(\boldsymbol{C}^{(t)}\boldsymbol{E}), \quad \boldsymbol{C}^{(0)} = \boldsymbol{I}_{n\times n}, \tag{3}$$

where $\boldsymbol{I}_{n\times n}$ denotes the identity matrix.

The vector-matrix multiplication operation to derive successive FCM states is iterated until it converges to a fixed point situation or a limit state cycle. The state vector remains unchanged for successive iterations is called a fixed point situation and the sequence of the state vector keeps repeating indefinitely is called a limit state cycle.

Now, we can summarize the proposed method to derive the priorities of criteria as follows:

Step 1. Compare the importance among criteria to derive the local weight vector using the eigenvalue approach;
Step 2. Depict the fuzzy cognitive map to indict the influence among criteria by the expert;
Step 3. Calculate Eq. (3) for obtaining the steady-state matrix;
Step 4. Derive the global weight vector. In order to derive the global weights, we should first normalize the local weight vector (z) and the steady-state matrix ($\boldsymbol{C}^*$) as follows:

$$z_n = \frac{1}{\lambda} z, \tag{4}$$

and

$$\boldsymbol{C}_n^* = \frac{1}{\gamma} \boldsymbol{C}^*, \tag{5}$$

where λ is the largest element of z and γ is the largest row sum of $\boldsymbol{C}^*$. Then, we can obtain the global weight vector by using the following weighting equation:

$$\boldsymbol{w} = z_n + \boldsymbol{C}_n^* z_n. \tag{6}$$

Next we used two numerical examples to demonstrate the proposed method in Section 4.

4. Numerical examples

In this section, two numerical examples are employed to demonstrate the proposed method and compared the results with the ANP. The first example is the multi-criteria decision problem about purchasing cars. The second example is modified by Fig. 1 to consider the more complicated decision problem. Note that in this paper we use two threshold functions including the pure-linear and the hyperbolic-tangent functions to indicate the relationships among criteria.

Example 1. Consider a decision maker try to purchase a car according to the following four criteria including Price (P), Durability (D), Robustness (R), and Repair Cost (C). For choosing the best alternative, we should derive the weights of each criterion and calculate the weighted scores of each car. In order to derive the local weights of each criterion, we first compare the importance among criteria using the following matrix:

	Price	Durability	Robustness	Repair Cost	Local Weights
Price	1	1/3	1/5	1/3	0.1370
Durability	3	1	1/3	1/2	0.2999
Robustness	5	3	1	2	0.8218
Repair Cost	3	2	1/2	1	0.4647

From the matrix above, we can calculate the local weights by using the eigenvalue method. Next, since a criterion may have interaction effects with other criteria, we then depict the FCM to indict the influence among criteria as shown in Fig. 6.

On the basis of Fig. 6, we can formulate the connection matrix as follow:

$$\boldsymbol{E} = \begin{array}{c} \\ P \\ D \\ R \\ C \end{array} \begin{array}{c} \begin{array}{cccc} P & D & R & C \end{array} \\ \begin{bmatrix} 0 & 0.40 & 0.30 & 0 \\ 0.20 & 0 & 0.25 & 0.15 \\ 0.15 & 0.15 & 0 & 0.35 \\ 0.2 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Next, we can obtain the two steady-state matrices by calculating Eq. (3) to the convergent state using the pure-linear and the hyperbolic-tangent functions as follows:

$f(x) = x$	Price	Durability	Robustness	Repair Cost
Price	0.2465	0.5763	0.5180	0.2678
Durability	0.3691	0.2096	0.4131	0.3260
Robustness	0.3296	0.3082	0.1759	0.4578
Repair Cost	0.2493	0.1153	0.1036	0.0536
$f(x) = \tanh(x)$				
Price	0.0402	0.2181	0.1813	0.0480
Durability	0.1243	0.0359	0.1471	0.1031
Robustness	0.1050	0.0978	0.0280	0.1851
Repair Cost	0.1037	0.0221	0.0183	0.0049

After obtaining the influences among criteria, we can employ Eq. (6) to obtain the global weights. In addition, on the information of importance among criteria, we can calculate the global weights using the ANP. The comparison of the ANP and the proposed method can be presented as shown in Table 1.

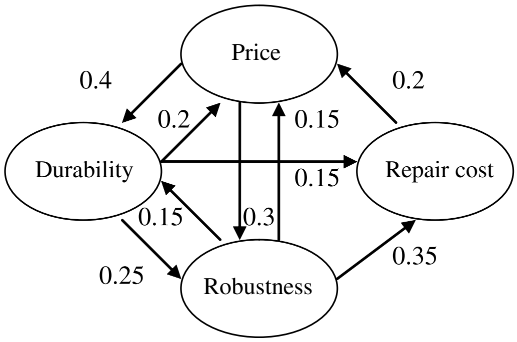

Fig. 6. An fuzzy cognitive map for Example 1.

Table 1
The comparison of the ANP and the proposed method

Weights	Price	Durability	Robustness	Repair Cost
Original weights	0.0795	0.1740	0.4768	0.2696
ANP	0.2852	0.1766	0.3166	0.2216
FCM ($f(x) = x$)	0.2165	0.1965	0.3993	0.1877
FCM ($f(x) = \tanh(x)$)	0.2102	0.2332	0.3765	0.1801

It should be highlighted that although we employ the ANP to derive the global weights, a main problem should be highlighted to display the shortcoming of the ANP as follows. In order to form the supermatrix, we should ask the question like "For the criterion of Price, how much the importance does Durability than Robustness?" or "For the criterion of Robustness, how much the importance does Price than Repair Cost?" The questions above usually are strange and hard even for the expert to answer. We will deeply discuss the problem above in Section 5.

Example 2. In this example, five criteria are used to select the best alternative. In order to derive the local weights, we first compare the importance between the criteria and then employ the eigenvalue method to obtain the eigenvector.

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Local Weights
Criterion 1	1	3	1	5	1/3	0.4716
Criterion 2	1/3	1	1/3	3	3	0.4437
Criterion 3	1	3	1	5	1/3	0.4716
Criterion 4	1/5	1/3	1/5	1	1/2	0.1154
Criterion 5	3	1/3	3	2	1	0.5874

Next, suppose the relationships among criteria above can be depicted using the FCM as shown in Fig. 7:

From Fig. 7, it can be seen that the problem above contains the compound and the interaction effects simultaneously. Next, we present the proposed method to determine the best alternative as follows.

First, on the basis of Fig. 7, we can formulate the connection matrix as follow:

$$\boldsymbol{E} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 0.35 & 0.35 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0.15 & 0 & 0 \\ 0.50 & 0 & 0.45 & 0 & 0 \\ 0 & 0.45 & 0 & 0 & 0.25 \end{bmatrix} \end{array}$$

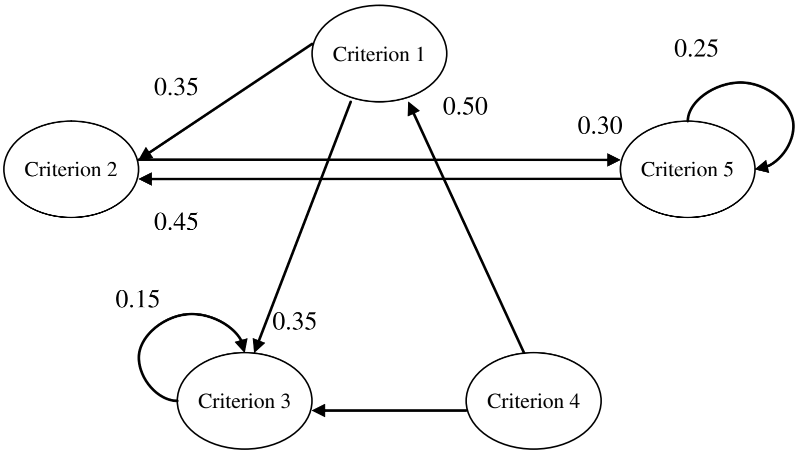

Fig. 7. A fuzzy cognitive map for Example 2.

Table 2
The global weights using pure-linear and hyperbolic-tangent functions

Weights	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5
Original weights	0.2257	0.2123	0.2257	0.0552	0.2811
FCM ($f(x) = x$)	0.2165	0.1909	0.1447	0.1622	0.2857
FCM ($f(x) = \tanh(x)$)	0.2238	0.1842	0.1516	0.1654	0.2751

Next, by using the pure-linear and the hyperbolic-tangent functions, we can calculate the steady-state matrices as follows:

$f(x) = x$	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5
Criterion 1	0	0.4268	0.4118	0	0.1707
Criterion 2	0	0.2195	0	0	0.4878
Criterion 3	0	0	0.1765	0	0
Criterion 4	0.5000	0.2134	0.7353	0	0.0853
Criterion 5	0	0.7317	0	0	0.6260
$f(x) = \tanh(x)$					
Criterion 1	0	0.1800	0.1868	0	0.0308
Criterion 2	0	0.0396	0	0	0.1761
Criterion 3	0	0	0.0809	0	0
Criterion 4	0.2449	0.0445	0.2812	0	0.0076
Criterion 5	0	0.2605	0	0	0.1850

Finally, using Eq. (6), we can obtain the global weights as shown in Table 2.

Next, we provide the depth discussions according to the results of the numerical examples above in Section 5.

5. Discussions

Structural MCDM problems involve determining the best alternatives by considering the dependence and the feedback effects among criteria. In order to deal with the problems above, the crucial point is to derive the global weights by considering the dependence and the feedback effects. Although the ANP/AHP has been proposed to handle this problems, a more easy and convenient approach is limited.

In this paper, the fuzzy decision maps, which combine the eigenvalue method, the fuzzy cognitive maps, and the weighting equation, are proposed to deal with the structural MCDM problems. The local weights are first derived using the eigenvalue method. Then, the FCM is verified to indicate the influence between criteria. Next, the steady-state matrix is derived using the updating equation. Finally, the global weights are obtained using the weighting equation.

From the process of the numerical examples above, we can describe two main shortcomings of the ANP as follows. First, since some questions are hard even for the expert to compare the importance among criteria, the final solution is doubtable. Second, the result of the ANP is highly dependent on the network structure. However, the true network structure is hard to identify due to the interaction effect between two criteria may be caused by another criterion.

In contrast, the advantages of the proposed methods can be summarized as follows. First, we can employ the different threshold functions to indicate the various kinds of relationship among criteria. Second, instead of asking the weary questions like the ANP, all we have to do is to judge the degree of influences between criteria. Third, both the compound and the interaction effects can easily be solved using the proposed method. Fourth, only the direct influences should be identified. The indirect influences can be generated by the steady-state matrix. In addition, we can use the direct and the indirect influences for other applications.

6. Conclusions

The MCDM problems with dependence and feedback effects are hard for the decision maker to make a good decision. Although the ANP have been widely used to deal with this problem, some shortcomings should be overcome for proving the satisfaction solution. In this paper, the FDM method is proposed to deal with the structural MCDM problems. Without answering the troublesome questions and verifying the true structure, only the influence between criteria should be given using the proposed method. On the basis of the numerical results, we can conclude that the proposed method can soundly deal with the structural MCDM problems with dependence and feedback effects.

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