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Production quality and yield assurance for processes with multiple independent characteristics

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Abstract

Process capability indices have been widely used in the manufacturing industry providing numerical measures on process potential and process performance. Capability measure for processes with single characteristic has been investigated extensively, but is comparatively neglected for processes with multiple characteristics. In real applications, a process often has multiple characteristics with each having different specifications. Singhal [Singhal, S.C., 1990. A new chart for analyzing multiprocess performance. Quality Engineering 2 (4), 397–390] proposed a multi-process performance analysis chart (MPPAC) for analyzing the performance of multi-process product. Using the same technique, several MPPACs have been developed for monitoring processes with multiple independent characteristics. Unfortunately, those MPPACs ignore sampling errors, and consequently the resulting capability measures and groupings are unreliable. In this paper, we propose a reliable approach to convert the estimated index values to the lower confidence bounds, then plot the corresponding lower confidence bounds on the MPPAC. The lower confidence bound not only gives us a clue minimum actual performance which is tightly related to the fraction of non-conforming units, but is also useful in making decisions for capability testing. A case study of a dual-fiber tip process is presented to demonstrate how the proposed approach can be applied to in-plant applications.

Keywords: Bootstrap sampling; Lower confidence bound; MPPAC control chart; Process capability indices

1. Introduction

During the last decade, numerous process capability indices (PCIs), including C_p, C_a, C_{pu}, C_{pl},

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and C_{pk}, have been proposed in the manufacturing industry to provide numerical measures on process performance, which are effective tools for quality/reliability assurance (see Kane, 1986; Chan et al., 1988; Pearn et al., 1992, 1998; Kotz and Lovelace, 1998; Kotz and Johnson, 2002 for more details). These indices are defined as

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma}, \quad C_{pu} = \frac{\mathrm{USL} - \mu}{3\sigma},$$
$$C_{pl} = \frac{\mu - \mathrm{LSL}}{3\sigma}, \quad C_a = 1 - \frac{|\mu - m|}{d},$$
$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\},$$

where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, σ is the process standard deviation, $m = (\mathrm{USL} + \mathrm{LSL})/2$ is the mid-point of the specification interval and $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half the length of the specification interval. For normally distributed processes, C_p, C_a and C_{pk} indices are appropriate measures for processes with two-sided specifications. The index C_p measures the overall process variation relative to the specification tolerance, therefore only reflects process potential (or process precision). The index C_a measures the degrees of process centering, which alerts the user if the process mean deviates from its center. Therefore, the index C_a only reflects process accuracy. The index C_{pk} takes into account process variation as well as process centering, providing process performance in terms of yield (proportion of conformities). Given a fixed value of C_{pk}, the bounds on process yield p can be expressed as $2\Phi(3C_{pk}) - 1 \leqslant p \leqslant \Phi(3C_{pk})$ (Boyles, 1991), where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. For instance, if $C_{pk} = 1.00$, then it guarantees that the yield will be no less than 99.73%, or equivalent to no more than 2700 parts per million (ppm) of non-conformities. On the other hand, the indices C_{pu} and C_{pl} have been designed particularly for processes with one-sided manufacturing specifications, which measure the-smaller-the-better and the-larger-the-better process capabilities, respectively. For normally distributed processes with one-sided specification limit, USL or LSL, the relationship between the one-sided capability indices and the process yield can be calculated as $p_u = P(X < \mathrm{USL}) = \Phi(3C_{pu})$ and $p_l = P(X > \mathrm{LSL}) = \Phi(3C_{pl})$.

In factory applications a product usually has multiple characteristics with each having different specifications, which need to be monitored and controlled hence is a difficult and time-consuming task for factory engineers. A multi-process performance analysis chart (MPPAC) proposed by Singhal (1990), which evaluates the performance of a multi-process product with symmetric bilateral specifications. Singhal (1991) further presented a MPPAC with several well-defined capability zones by using the process capability indices C_p and C_{pk} for grouping the processes in a multiple process environment into different performance categories on a single chart. Using the same technique, several modified control charts have been developed for monitoring processes with single or multiple independent characteristics. Pearn and Chen (1997) proposed a modification to the C_{pk} MPPAC combining the more-advanced process capability indices, C_{pm} or C_{pmk}, to identify the problems causing the processes failing to center around the target. By combining Singhal's MPPAC with asymmetric process capability index C_{pa}, Chen et al. (2001) introduced a process capability analysis chart (PCAC) to evaluate process performance for an entire product composed of multiple characteristics with symmetric and asymmetric specifications. Pearn et al. (2002) introduced a MPPAC to the chip resistors applications based on the incapability index C_{pp}. Chen et al. (2003) also developed a control chart for processes with multiple characteristics based on the generalization of yield index S_{pk} proposed by Boyles (1994). We should note that the process mean μ and the process variance σ^2 are usually unknown in practice. In order to calculate the index value, sample data must be collected and a great degree of uncertainty may be introduced into capability assessments due to sampling errors. However, those existing research works on MPPAC are restricted to assuming the value of μ and σ^2 are known or obtaining quality information from one single sample of each process ignoring sampling errors. The information provided from the existing MPPAC, therefore, is unreliable and misleading resulting in incorrect decisions. In this paper, we propose a reliable approach to obtain the lower confidence bounds and apply it to the modified C_{pk} MPPAC. A real-world application to the dual-fiber tips manufacturing process is presented for illustration.

2. Capability measure for multiple characteristics

2.1. Processes with multiple dependent characteristics

Process capability analysis often entails characterizing or assessing processes or products based on more than one engineering specification or quality characteristic (variable). When these variables are related characteristics, the analysis should be based on a multivariate statistical technique. Chen (1994) and Boyles (1996) and others have presented multivariate capability indices for assessing capability. Wang and Chen (1998–1999) and Wang and Du (2000) proposed multivariate equivalents for C_p, C_{pk}, C_{pm} and C_{pmk} based on the principal component analysis, which transforms numbers of original related measurement variables into a set of uncorrected linear functions. Moreover, a comparison of three recently proposed multivariate methodologies for assessing capability are illustrated and their usefulness is discussed in Wang et al. (2000). However, those indices and methodologies were appropriate for products with either multiple unilateral specifications or multiple bilateral specifications exclusively. For practical applications, most multi-process products are composed of numerous unilateral specifications and bilateral specifications, and customers are satisfied when all quality characteristics of an entire product meet preset specifications. Therefore, neither univariate process capability indices nor multivariate process capability indices can meet the needs for the requirements.

2.2. Processes with multiple independent characteristics

For processes with multiple characteristics, Bothe (1992) considered a simple measure by taking the minimum of the measure of each single characteristic. For example, consider a v characteristics process with v yield measures $P_1, P_2, \ldots,$ and P_v, the overall process yield is measured as $P = \min\{P_1, P_2, \ldots, P_v\}$. We note that this approach does not reflect the real situation accurately. Suppose the process has five characteristics ($v = 5$), with equal characteristic yield measures $P_1 = P_2 = P_3 = P_4 = P_5 = 99.73\%$. Using the approach considered by Bothe (1992), the overall process yield is calculated as $P = \min\{P_1, P_2, P_3, P_4, P_5\} = 99.73\%$ (or 2700 ppm of non-conformities). Assuming that the five characteristics are mutually independent, then the actual overall process yield should be calculated as $P = P_1 \times P_2 \times \cdots \times P_5 = 98.66\%$ (or 134,000 ppm of non-conformities), which is significantly less than that calculated by Bothe (1992). Generally, the quality characteristics of a product can be classified into three types: the-nominal-the-best, the-smaller-the-better and the-larger-the-better types. C_{pk}, C_{pu} and C_{pl} are three indices to evaluate the process capabilities on the MPPAC. For a multi-process product, assume there are n_k processes of the-nominal-the-best type evaluated by $C_{pkj}, j = 1, 2, \ldots, n_k$, n_u the-smaller-the-better processes evaluated by C_{puj}, $j = 1, 2, \ldots, n_u$, and n_l processes of the-larger-the-better type evaluated by C_{plj}, $j = 1, 2, \ldots, n_l$. Thus, as described earlier, the general form of process yield can be calculated for unilateral characteristics as $p_{uj} = \Phi(3C_{puj})$, $j = 1, 2, \ldots, n_u$ or $p_{lj} = \Phi(3C_{plj})$, $j = 1, 2, \ldots, n_l$ and $p_{kj} \geqslant 2\Phi(3C_{pkj}) - 1$, $j = 1, 2, \ldots, n_k$ for bilateral specifications.

Assume the individual process yields are independent, the entire process yield p_T can be calculated as

$$p_T = \prod_{i \in G} \prod_{j=1}^{n_i} p_{ij},$$

where $G = \{k, u, l\}$. Furthermore, utilizing the inequality $\Phi(x) \geqslant 2\Phi(x) - 1$, the above relations of process yield can be rewritten as: $p_{ij} \geqslant 2\Phi(3C_{pij}) - 1$, $i \in \{k, u, l\}$, $j = 1, 2, \ldots, n_i$. Then, the overall process yield p_T can be described as

$$p_T = \prod_{i \in G} \prod_{j=1}^{n_i} p_{ij} \geqslant \prod_{j=1}^{n_i} [2\Phi(3C_{pkj}) - 1] \times \prod_{j=1}^{n_u} \Phi(3C_{puj}) \times \prod_{j=1}^{n_l} \Phi(3C_{plj}) \geqslant \prod_{i \in G} \prod_{j=1}^{n_i} [2\Phi(3C_{pij}) - 1].$$

In general, the overall process yield of a multi-process product is lower than any individual process yield, namely, $p_T \leqslant p_{ij}$. Similarly, when the overall process yield (or entire product capability) is preset to satisfy the required level, the individual process yield (or individual process capability) should exceed the preset standard for the entire product. Based on the above analysis, if each characteristic is mutually independent and normally distributed, the process yield can be evaluated in terms of an integrated process capability index C_T in the following:

$$C_T = \frac{1}{3}\Phi^{-1}\left(\left[\left(\prod_{i\in G}\prod_{j=1}^{n_i}[2\Phi(3C_{pij})-1]+1\right)\Big/2\right]\right).$$

Some minimum capability requirements have been recommended in the manufacturing industry (see Montgomery, 2001), for specific process types, which must run under some more designated stringent quality conditions.

3. A reliable modified MPPAC for capability control

Process capability index measures the ability of the process to reproduce products that meet specifications. However, the fact that process capability indices combine information about closeness to target and process spread, and express the capability of a process by a single number, may in some cases also be held as one of their major drawbacks. If, for instance, the process is found non-capable, the operator is interested in knowing whether this non-capability is caused by the fact that the process output is off target or that the process spread is too large, or if the result is a combination of these two factors. In order to circumvent this shortage of process capability indices, recent research suggests that different graphical methods be used to support the improvement initiative aimed at accomplishing more capable processes (see, e.g., Gabel, 1990; Boyles, 1991; Tang et al., 1997; Deleryd and Vännman, 1999). The modified C_{pk} MPPAC proposed by Pearn and Chen (1997) is shown in Fig. 1, with five capability zones corresponding to the five process conditions for $C_{pk} =$ 1.00, 1.33, 1.67, and 2.00.

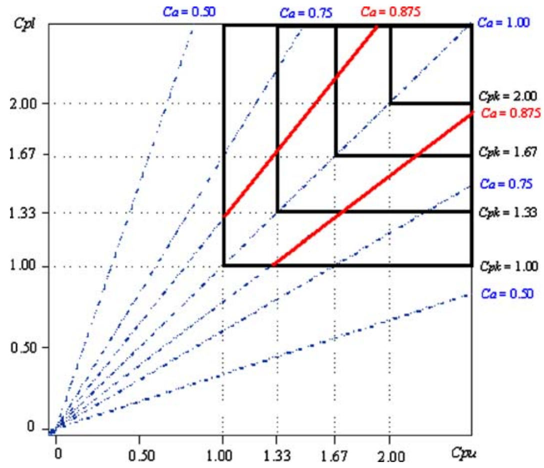

Fig. 1. The modified C_{pk} MPPAC with capability zones for C_{pk} = 1.00, 1.33, 1.67, and 2.00.

In this modified MPPAC, C_{pu} and C_{pl} represent the X-axis and Y-axis, respectively. Whereas C_p is the average of C_{pu} and C_{pl}, namely, $C_p = (C_{pu} + C_{pl})/2$ and C_{pk} is the minimum value of the X- and Y-axes, namely, $C_{pk} = \min\{C_{pu}, C_{pl}\}$. Thus, based on C_{pk} MPPAC, the vertical and horizontal axes of the chart are to evaluate the-larger-the-better and the-smaller-the-better characteristics, respectively. Furthermore, a few subsidiary lines of C_a can be added on MPPAC for precisely controlling the process centering. Off-diagonal subsidiary lines are plotted when C_a are 0.500, 0.750 and 0.875 in Fig. 1. Note that we will assume the preset target value T at the mid-point of the specification (i.e. $m = T$). $C_a < 0.875$ indicates that the process is not accurate; actions to shift the process mean closer to the process target are required. Namely, $C_a \geqslant 0.875$ indicates a process with good accuracy. In general, C_a cannot be too small since a smaller C_a implies the process mean shifts farther away from the process target and results in much process loss. Let $r = |\mu - m|/d$, then the values of μ are $m + r \times d$ and $m - r \times d$ for each C_a. The slope of the corresponding subsidiary line is $(1 + r)/(1 - r)$ when the process mean is greater than the process target, and the slope of the corresponding subsidiary line is $(1 - r)/(1 + r)$ when the process mean is smaller than the process target. Table 1 briefly displays the

Table 1
The values of C_a with the corresponding μ, r, and slope of lines

C_a	μ	r	Slope
1.000	m	0.000	1.000
0.875	$m + 0.125 \times d$	0.125	1.286
	$m - 0.125 \times d$	0.125	0.778
0.750	$m + 0.250 \times d$	0.250	1.667
	$m - 0.250 \times d$	0.250	0.600
0.500	$m + 0.500 \times d$	0.500	3.000
	$m - 0.500 \times d$	0.500	0.333
0.250	$m + 0.750 \times d$	0.750	7.000
	$m - 0.750 \times d$	0.750	0.143
0.000	USL	1.000	∞
	LSL	1.000	0.000

values of C_a with the corresponding μ, r, and slope of the subsidiary lines.

In quality improvement, reduction of variation from the target is as important as increasing the process yield and reduction process spread. A modified C_{pk} MPPAC put the two concepts: closeness to target and small spread in more efficient way than by using a process capability index alone. If the exact values of μ and σ are known, then the modified C_{pk} MPPAC can easily be used. That is, if the corresponding value of (C_{pu}, C_{pl}) is inside the capability region, then the process is defined to be capable, and if the value is outside, the process is defined as non-capable. In practice, though, we never know the true values of capability indices. In the next section we will develop the procedure to construct the lower confidence bounds of indices C_{pk}, C_a, C_{pu}, and C_{pl} for each characteristic type. These lower confidence bounds can be simultaneously plotted on a single chart to check if the process output is off target or that the process spread is too large, or if the result is a combination of these two.

4. Lower confidence bounds for production yield assurance

As noted before, several MPPACs have been developed for monitoring processes with multiple characteristics. In current practice of implementing those charts, practitioners simply plot the estimated index values on the chart then make conclusions on whether processes meet the capability requirement and directions need to be taken for further capability improvement. Such approach is highly unreliable since the estimated index values are random variables and sampling errors are ignored. A reliable approach is to first convert the estimated index values to the lower confidence bounds then plot the corresponding lower confidence bounds on the MPPAC. Using lower confidence bounds, the MPPAC applications become more efficient and the results are not misleading.

4.1. Lower confidence bounds on C_{pk}

Construction of the exact lower confidence bounds on C_{pk} is complicated since the distribution of $\widehat{C}_{pk}$ involves the joint distribution of two non-central t distributed random variables, or alternatively, the joint distribution of the folded-normal and the chi-square random variables, with an unknown process parameter even when the samples are given (Pearn et al., 1992). Numerous methods for obtaining approximate confidence bounds of C_{pk} have been proposed, including Bissell (1990), Chou et al. (1990), Zhang et al. (1990), Porter and Oakland (1991), Kushler and Hurley (1992), Rodriguez (1992), Nagata and Nagahata (1994), Tang et al. (1997) and many others. Under the assumption of normality, Pearn and Lin (2004) obtain an exactly explicit form of the cumulative distribution function of the natural estimator $\widehat{C}_{pk}$ as

$$F_{\widehat{C}_{pk}}(y) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9ny^2}\right) \times \left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right] \mathrm{d}t,$$

for $y > 0$, where $b = d/\sigma$, $\xi = (\mu - m)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution with $n - 1$ degrees of freedom, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution. Hence, given the sample of size n, the confidence level $1 - \alpha$, the estimated value $\widehat{C}_{pk}$ and the parameter ξ, using numerical integration technique with iterations, the $100(1 - \alpha)\%$ lower confidence bounds for C_{pk}, L_C, and $b_L = 3L_C + |\xi|$, can be obtained by solving the following equation,

$$\int_0^{b_L\sqrt{n}} G\left(\frac{(n-1)(b_L\sqrt{n}-t)^2}{9n\widehat{C}_{pk}^2}\right) \times \left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right] \mathrm{d}t = \alpha.$$

4.2. Lower confidence bounds on C_a

On the assumption of normality, Pearn et al. (1998) showed that the natural estimator $\widehat{C}_a = 1 - |\bar{x} - m|/d$ of the process accuracy index C_a, is the maximum likelihood estimator, consistent, asymptotically efficient and $\sqrt{n}(\widehat{C}_a - C_a)$ converges to $N(0, 1/(3C_p^2))$ in distribution. And the statistic $\sqrt{n}|\bar{x} - m|/\sigma$ has a folded normal distribution as defined by Leone et al. (1961). Therefore, owing to $\sqrt{n}(\widehat{C}_a - C_a)$ converging to $N(0, 1/(3C_p^2))$, $3\sqrt{n}\widetilde{C}_p(\widehat{C}_a - C_a)$ converges to $N(0,1)$ in distribution, An approximate $100(1 - \alpha)\%$ confidence interval of C_a can be established as

$$\left[\widehat{C}_a - \frac{z_\alpha/2}{3\sqrt{n}\widetilde{C}_p}, \widehat{C}_a + \frac{z_\alpha/2}{3\sqrt{n}\widetilde{C}_p}\right],$$

where $\widetilde{C}_p = b_{n-1}\widehat{C}_p$, $b_{n-1} = (2/(n-1))^{1/2} \times \Gamma[(n-1)/2]/\Gamma[(n-2)/2]$, and z_α is the upper αth quantile for the standard normal distribution. While a $100(1 - \alpha)\%$ lower confidence bound of C_a, L_{C_a}, can be constructed using only the lower limit as $\widehat{C}_a - z_\alpha/(3\sqrt{n}\widetilde{C}_p)$.

4.3. Lower confidence bounds on C_{pu} and C_{pl}

Chou and Owen (1989) showed that under normality assumption the estimators $\widehat{C}_{pu}$ and $\widehat{C}_{pl}$ are distributed as $(3\sqrt{n})^{-1}t_{n-1}(\delta)$, where $t_{n-1}(\delta)$ is distributed as the non-central t distribution with $n - 1$ degrees of freedom and the non-centrality parameter $\delta = 3\sqrt{n}C_{pu}$ and $\delta = 3\sqrt{n}C_{pl}$, respectively. A $100(1 - \alpha)\%$ lower confidence bound L_C for C_{pu} satisfies $\Pr(C_{pu} \geqslant L_C) = 1 - \alpha$. It can be written as

$$\Pr\left(\frac{\mathrm{USL} - \mu}{3\sigma} \geqslant L_C\right) = \Pr(t_{n-1}(\delta_1) \leqslant t_1) = 1 - \alpha,$$

where $t_1 = 3\widehat{C}_{pu}\sqrt{n}$ and $\delta_1 = 3\sqrt{n}L_C$. Thus, L_C can be obtained by solving the above cumulative distribution function of $t_{n-1}(\delta_1)$. Similarly, a 100 $(1 - \alpha)\%$ lower confidence bound for C_{pl} can be obtained by solving $\Pr(C_{pl} \geqslant L_C) = 1 - \alpha$.

5. Bootstrap confidence bound for overall capability testing

Statistical hypothesis testing used for examining whether the process capability meets the customers' demands, can be stated as follows: H_0: $C_T \leqslant C$ versus H_1: $C_T > C$. The null hypothesis states that the overall process capability is no greater than the minimum capability level C. We conclude that the entire product capability satisfies the required level if the sample statistic $\widehat{C}_T$ is greater than the critical value (or p-value $< \alpha$) or the lower confidence bound of C_T is greater than the capability requirement C. Otherwise, we reverse the conclusion. Unfortunately, the exact sampling distribution of $\widehat{C}_T$ is intractable. Efron (1979, 1982) introduced a non-parametric, computational intensive but effective estimation method called the "Bootstrap", which is a data based simulation technique for statistical inference. Efron and Tibshirani (1986) developed three types of bootstrap confidence interval, including the standard bootstrap

(SB) confidence interval, the percentile bootstrap (PB) confidence interval, and the biased corrected percentile bootstrap (BCPB) confidence interval. Efron and Tibshirani (1986) indicated that a rough minimum of 1000 bootstrap resamples is usually sufficient to compute reasonably accurate confidence interval estimates. We apply these three bootstrap methods to the entire product capability measure C_T to obtain the confidence bounds. In order to obtain more reliable results, $B = 10{,}000$ bootstrap resamples are taken and these 10,000 bootstrap estimates of C_T are calculated and ordered in ascending order. The notations $\widehat{C}_T$ and $\widehat{C}^*_T(i)$ will be used to denote the estimator of C_T and the associated ordered bootstrap estimates. For instance, $\widehat{C}^*_T(1)$ is the smallest of the 10,000 bootstrap estimates of C_T. For each single characteristic, the C_{pu}, C_{pl}, and C_{pk} values can be estimated by their estimators $\widehat{C}_{puj} = (\mathrm{USL}_j - \bar{x}_j)/s_j$, $j = 1, 2, \ldots, n_u$, $\widehat{C}_{plj} = (\bar{x}_j - \mathrm{LSL}_j)/s_j$, $j = 1, 2, \ldots, n_l$, and $\widehat{C}_{pkj} = (d_j - |\bar{x}_j - m_j|)/(3 \times s_j)$, $j = 1, 2, \ldots, n_k$, where $\bar{x}_j$ and s_j are the sample mean and standard deviation of the j-th characteristic. Thus, the bootstrap estimates of C_T are defined as

$$\widehat{C}_T = \frac{1}{3}\Phi^{-1}\left(\left[\left(\prod_{i\in S}\prod_{j=1}^{n_i}\left[2\Phi(3\widehat{C}_{pij}) - 1\right] + 1\right)\Big/2\right]\right).$$

Based on the SB method, the $100(1-\alpha)\%$ lower confidence bound for C_T is $\widehat{C}_T - z_\alpha \times S_{\widehat{C}_T}$, where $S_{\widehat{C}_T}$ is the sample standard deviation of $\widehat{C}_T$. From the ordered collection of $\widehat{C}^*_T(i)$, the α percentage and the $(1-\alpha)$ percentage points are used to obtain $100(1-2\alpha)\%$ PB confidence interval for C_T is $[\widehat{C}^*_T(\alpha B),\ \widehat{C}^*_T((1-\alpha)B)]$. While a $100(1-\alpha)\%$ lower confidence bound can be constructed by using only the lower limit $\widehat{C}^*_T(\alpha B)$. That is, for a 95% lower confidence bound for C_T based on the PB method with $B = 10{,}000$ would be obtained as $\widehat{C}^*_T(500)$. For the BCPB method, it calculates the probability $p_0 = P(\widehat{C}^*_T \leqslant \widehat{C}_T)$ and computes the inverse of the cumulative distribution of a standard normal based on p_0 as $z_0 = \Phi^{-1}(p_0)$, $p_L = \Phi(2z_0 - z_\alpha)$. The $100(1-\alpha)\%$ BCPB lower confidence bound can be obtained as $\widehat{C}^*_T(p_L B)$. Therefore, to determine whether the total product capability is capable or not, the minimum requirement level C and the significant level α-risk are first decided. And if the lower confidence bound of C_T, is greater than the capability requirement C, we conclude that the entire product capability satisfies the required level. Otherwise, we reverse the conclusion.

6. A case study

In the following, we consider a case study to demonstrate how the modified C_{pk} MPPAC and the lower confidence bound can be used in analyzing processes with multiple characteristics. The case we investigate involves a process manufacturing the dual-fiber tips, which is used in making fiber optic cables. For a particular model of the dual-fiber tips, the specifications of characteristics are presented in Table 2, which is taken form a optical communication manufacturing factory located on Science-based Industrial Park in Taiwan, devoted to the optical fiber component module products, such as single-fiber tips for collimators, isolators, switches, WDM, circulators, etc., and dual-fiber tips for WDM, hybrid isolators, compact circulators, etc. The applications of these fiber

Table 2
Specifications of characteristics for the dual-fiber tips

Characteristic	Type	LSL	Target	USL
Capillary diameter	Nominal-the-best	1.795 mm	1.800 mm	1.805 mm
Capillary length	Nominal-the-best	6.00 mm	6.25 mm	6.50 mm
Wedge	Nominal-the-best	7.5°	8°	8.5°
Core diameter	Nominal-the-best	126 μm	127 μm	128 μm
Return loss	Larger-the-better	60 dB	–	–
Polishing direction	Smaller-the-better	–	–	5°

tips are fabricated with high performance optical fiber ends and precise glass capillary.

The key quality characteristics of a dual-fiber tip include (1) capillary diameter, length, wedge and core diameter, which are nominal-the-best specifications, (2) return loss, which is the-larger-the-better specification, (3) polishing direction, which is the-smaller-the-better specification. Customers expect all of the quality characteristics of a dual-fiber tip to meet or exceed expected levels. Fig. 2 shows a sample of the dual-fiber tips. We take a random sample of size 60, for the dual-fiber tips from a stable (under statistical control) process in the factory, and measure the six product quality characteristics, the capillary diameter (I), length (II), wedge (III), core diameter (IV), return loss (V), and polishing direction (VI). For these 60 measurements of each characteristics, under the Shapiro–Wilk test for normality, the result confirms that all the p-value >0.1. That is, it is reasonable to assume that the process data collected from the factory are normally distributed. The calculated sample mean, sample standard deviation, the estimated PCIs, $\widehat{C}_{pu}$, $\widehat{C}_{pl}$, $\widehat{C}_{p}$, $\widehat{C}_{a}$, $\widehat{C}_{pk}$, a 95% lower confidence bound of C_a, and lower confidence bound of C_{pk} (C_{pu} or C_{pl}), L_C, are summarized in Table 3.

The modified C_{pk} MPPAC for the six processes based on the estimated PCI values and the lower confidence bound listed in Table 3, are displayed in Figs. 3 and 4, respectively. Table 4 displays the manufacturing quality and capability groupings for the six dual-fiber tips processes using the estimated values (unreliable) and the lower confidence bounds (reliable) associated with the corresponding non-conformities (NC) expressed in ppm (with asterisks * indicating incorrect groupings). Therefore, from these figures and tables, an

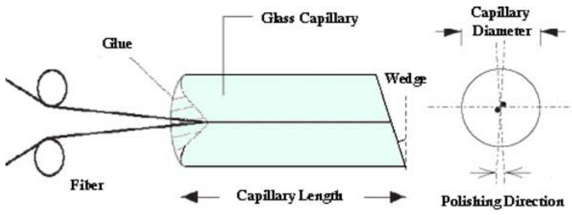

Fig. 2. A sample of the dual-fiber tips.

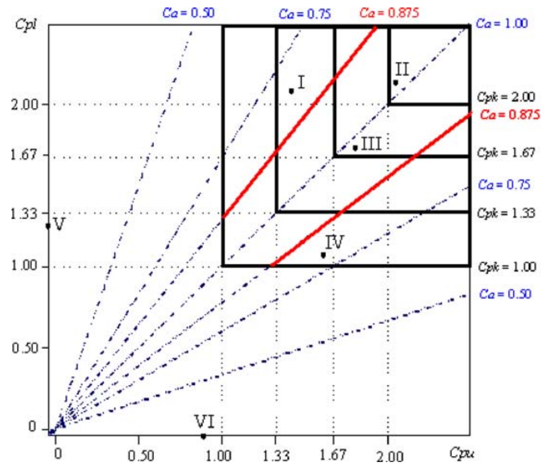

Fig. 3. The modified C_{pk} MPPAC for estimated index.

approach widely used in current industrial applications based on the estimated PCI values only, we note that such MPPAC obviously conveys unreliable information and is misleading, which should be avoided in real applications.

Hence, based on the analysis of this chart as Fig. 4, it provides directions and priority for processes important to mining process defect. We can make some conclusions and recommendations to these six processes in the following:

Table 3
The calculated sample mean, sample standard deviation, the estimated capability indices and lower confidence bound

Code	Characteristic	$\bar{x}$	s	$\widehat{C}_{pu}$	$\widehat{C}_{pl}$	$\widehat{C}_{p}$	$\widehat{C}_{a}$	L_{C_a}	$\widehat{C}_{pk}$	L_C
I	Capillary diameter	1.8009	0.00097	1.412	2.032	1.722	0.820	0.748	1.412	1.184
II	Capillary length	6.255	0.04035	2.024	2.107	2.065	0.980	0.908	2.024	1.706
III	Wedge	7.99°	0.0959	1.773	1.703	1.738	0.980	0.908	1.703	1.433
IV	Core diameter	126.8	0.2458	1.627	1.085	1.356	0.800	0.728	1.085	0.904
V	Return loss	63.6	0.9547	–	1.257	–	–	–	1.257	1.051
VI	Polishing direction	4.2°	0.3027	0.881	–	–	–	–	0.881	0.728

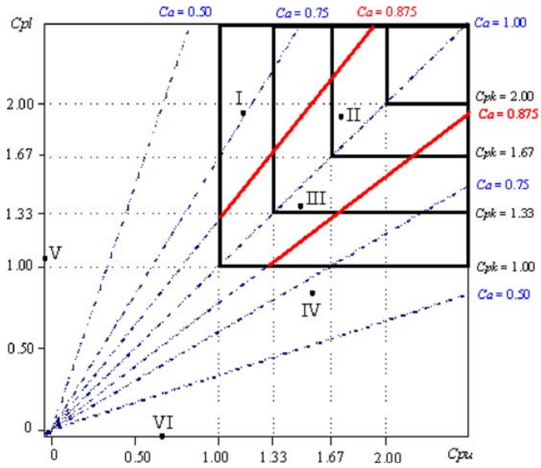

Fig. 4. The modified C_{pk} MPPAC for the LCB.

(a) The plotted points IV and VI are not located within the contour of $C_{pk} = 1.00$. It indicates that the process has a very low capability. For the point IV, since the lower confidence bound of C_a is 0.728, that is, the process of core diameter represents that the process mean is towards the lower specification limit (process mean is smaller than target value), and the poor capabilities are mainly contributed by the significant process departure from target. Thus, both characteristics core diameter and polishing direction are candidates for high-priority quality improvement effort focus. Under the six-sigma program, the quality improvement effort could focus on the reduction of process variability and the decrease of the process mean from the target to improve the process quality.

(b) The plotted points I and V lie within the contours of $1.00 \leqslant C_{pk} < 1.33$. The point I lies inside the area, which is to the left of the 45° target line (slope = 1) represents processes where the process mean is towards the upper specification limit (process mean is greater than the target value). On the other hand, for the point V, the lower confidence bound of C_{pu} is 1.051, the process is capable and the corresponding non-conformities are about 800 ppm. Thus, quality improvement effort for these processes should be first focused on reducing their process departure from the target value T for the process of capillary diameter, then the reduction of the process variance.

(c) Process wedge (III) lies inside the contours of $1.33 \leqslant C_{pk} < 1.67$, the process is "Satisfactory". And the lower confidence bound of C_a is close to the 45° target line ($C_a = 1$). Thus, the quality improvement effort for process wedge could be focused on the reduction of the process variation.

(d) The plotted point II lies inside the contours of $1.67 \leqslant C_{pk} < 2.00$, and the lower confidence bound of C_a is greater than 0.875. The corresponding non-conformities of process are 0.309 ppm only. Thus, stringent control for characteristic capillary length could be reduced since the process is "Excellent".

6.1. Overall process yield analysis

The sample estimates of C_T and three bootstrap lower confidence bounds of C_T for the dual-fiber tips can be calculated from the sample. Table 5

Table 4
Estimated value and lower confidence bound (LCB) of capability indices with their groupings for the six characteristics

Code	Characteristic	Estimated index	NC	Grouping	LCB	NC	Grouping
I	Capillary diameter	1.412	22.75	Satisfactory*	1.184	383.32	Capable
II	Capillary length	2.024	0.0013	Super*	1.706	0.309	Excellent
III	Wedge	1.703	0.324	Excellent*	1.433	17.16	Satisfactory
IV	Core diameter	1.085	1133.9	Capable*	0.904	6687.9	Incapable
V	Return loss	1.257	81.30	Capable	1.051	799.74	Capable
VI	Polishing direction	0.881	4108.8	Incapable	0.728	14481	Incapable

Table 5
Calculations for overall yield index ($\widehat{C}_T$) and three lower confidence bounds based on bootstrap technique and the corresponding non-conformities

Dual-fiber tips	$\widehat{C}_T$	Bootstrap lower confidence bound of C_T		
		SB method	PB method	BCPB method
Index value	0.864	0.756	0.763	0.755
Non-conformities	9526 ppm	23317 ppm	22079 ppm	23452 ppm

displays the manufacturing quality and its corresponding ppm of non-conformities for the dual-fiber tips processes using the estimated $\widehat{C}_T$ values and the lower confidence bounds of C_T based on the three bootstrap methods.

Based on the analysis of Table 5, we could find that the modified product capability obtained using three bootstrap methods are certainly more reliable than the estimated $\widehat{C}_T$ index values, since the sampling errors are considered in the LCB approach. In fact, as the sample estimate $\widehat{C}_T$ may overestimate the true capability (overall process yield), it conveys unreliable and misleading information, which should be avoided in factory applications. The lower confidence bound not only gives us a clue on the minimal actual performance of the process which is tightly related to the fraction of non-conforming units, but is also useful in making decisions for capability testing.

7. Conclusions

Process capability indices establish the relationship between the actual process performance and the manufacturing specifications, which quantify process potential and process performance, are essential to any successful quality improvement activities and quality program implementation. Capability measure for processes with single characteristic has been investigated extensively, but is comparatively neglected for processes with multiple characteristics. In real applications, a process often has multiple characteristics with each having different specifications. MPPAC can be used for evaluating the performance of a multi-process product, sets the priorities among multiple processes for capability improvement and indicates if reducing the variability, or the departure of the process mean should be the focus of improvement. However, existing applications on MPPAC control charts simply look at the estimated indices values and then make a conclusion on which the given process is classified, is highly unreliable and misleading since they didn't considered sampling errors. We proposed a reliable approach to convert the estimated index values into the lower confidence bounds, then plot the corresponding lower confidence bounds on the MPPAC. The lower confidence bound obtained by analytical method gives us reliable performance measure for each single characteristic and overall production yield. Based on the proposed approach, the practitioners can make reliable decisions for capability testing and monitoring the performance of all process characteristics simultaneously.

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