[13] D. M. Chickering, D. Geiger, and D. Heckerman, "Learning Bayesian networks: Search methods and experimental results," in *Preliminary Papers 5th Int. Workshop on Artificial Intelligence and Statistics,* 1995, pp. 112–128.
[14] D. Heckerman, D. Geiger, and D. M. Chickering, "Learning Bayesian networks: The combination of knowledge and statistical data," in Microsoft, Technical Report MSR-TR-94-09, 1994.
[15] R. R. Bouckaert, "Properties of Bayesian belief networks learning algorithms," in *Proc. 10th Annual Conf. Uncertainty in Artificial Intelligence,* Washington, 1994, pp. 102–109.
[16] P. Larrañaga, R. H. Murga, M. Poza, and C. M. H. Kuijpers, "Structure learning of Bayesian networks by hybrid genetic algorithms," in *Preliminary Papers 5th Int. Workshop on Artificial Intelligence and Statistics,* 1995, pp. 310–316.
[17] R. R. Bouckaert "Optimizing causal orderings for generating DAG's from data," in *Proc. 8th Conf. Uncertainty in Artificial Intelligence,* 1992, pp. 9–16.
[18] M. Singh and M. Valtorta, "An algorithm for the construction of Bayesian network structures from data," in *Proc. 9th Conf. Uncertainty in Artificial Intelligence,* Washington, DC, 1993, pp. 259–265.
[19] W. Lam and F. Bacchus, "Learning Bayesian belief networks. An approach based on the MDL principle," *Computational Intelligence,* vol. 10, no. 4, 1994.
[20] ——, "Using causal information and local measures to learn Bayesian networks," in *Proc. 9th Conf. Uncertainty in Artificial Intelligence,* Washington, DC, 1993, pp. 243–250.
[21] P. Larrañaga, M. Poza, Y. Yurramendi, R. H. Murga, and C. M. H. Kuijpers, "Structure learning of Bayesian networks by genetic algorithms: A performance analysis of control parameters," *IEEE Trans. Pattern Anal. Mach. Intell.,* in press.
[22] G. M. Provan and M. Singh, "Learning Bayesian networks using feature selection," in *Preliminary Papers 5th Int. Workshop on Artificial Intelligence and Statistics,* FL, 1995, pp. 450–456.
[23] L. R. Andersen, J. H. Krebs, and J. D. Andersen, "STENO: An expert system for medical diagnosis based on graphical models and model search," *J. Appl. Stat.,* vol. 18, no. 1, pp. 139–153, 1991.
[24] S. Kreiner, "On tests of conditional independence," Statistical Research Unit, University of Copenhagen, Res. Rep. 89/14, 1989.
[25] R. M. Fung and S. L. Crawford, "Constructor: A system for the induction of probabilistic models," in *Proc. AAAI,* Boston, MA, 1990, pp. 762–769.
[26] S. L. Lauritzen, B. Thiesson, and D. J. Spiegelhalter, "Diagnostic systems created by model selection methods—A case study," in *Preliminary Papers 4th Int. Workshop on Artificial Intelligence and Statistics,* 1993, pp. 93–105.
[27] H. Akaike, "New look at the statistical model identification," *IEEE Trans. Automat. Contr,* vol. 19, pp. 716–722, 1974.
[28] D. Madigan, A. E. Raffery, J. C. York, J. M. Bradshaw, and R. G. Almond, "Strategies for graphical model selection," in *Preliminary Papers 4th Int. Workshop on Artificial Intelligence and Statistics,* 1993, pp. 331–336.
[29] R. Mechling and M. Valtorta, "PaCCIN: A parallel constructor of Markov networks," in *Preliminary Papers 4th Int. Workshop on Artificial Intelligence and Statistics,* 1993, pp. 405–410.
[30] G. M. Provan, "Model selection for diagnosis and treatment using temporal influence diagrams," in *Preliminary Papers 5th Int. Workshop on Artificial Intelligence and Statistics,* 1995, pp. 469–480.
[31] J. H. Holland, *Adaptation in Natural and Artificial Systems.* Ann Arbor, MI: The Univ. of Michigan Press, 1975.
[32] U. K. Chakraborty and D. G. Dastidar, "Using reliability analysis to estimate the number of generations to convergence in genetic algorithms," *Inform. Proc. Lett.,* vol. 46, no. 4, pp. 199–209, 1993.
[33] A. E. Eiben, E. H. L. Aarts, and K. M. van Hee, "Global convergence of genetic algorithms: An infinite Markov chain analysis," *Computing Science Notes,* Eindhoven Univ. of Tech., 1990.
[34] G. Rudolph, "Convergence analysis of canonical genetic algoritms," submitted to *IEEE Trans. Neural Networks.*
[35] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning.* Addison-Wesley, 1989.
[36] L. Davis, Ed., *Handbook of Genetic Algorithms.* New York: Van Nostrand Reinhold, 1991.
[37] P. Larrañaga, C. M. H. Kuijpers, and R. H. Murga, "Evolutionary algorithms for the travelling salesman problem: A review of representations and operators," submitted to *Artif. Intell. Rev.*
[38] D. E. Goldberg and J. R. Lingle, "Alleles, loci and the traveling salesman problem," in *Proc. Int. Conf. Genetic Algorithms and Their Applications,* Pittsburgh, PA, 1985, pp. 154–159.
[39] I. M. Oliver, D. J. Smith, and J. R. C. Holland, "A study of permutation crossover operators on the TSP," in *Proc. 2nd Int. Conf. on Genetic Algorithms and Their Applications,* Cambridge, MA, 1987, pp. 224–230.
[40] L. Davis, "Applying adaptive algorithms to epistatic domains," in *Proc. Int. Joint Conf. on Artificial Intelligence,* Los Angeles, CA, 1985, pp. 162–164.
[41] G. Syswerda, "Schedule optimization using genetic algorithms," in [36], pp. 332–349.
[42] H. Mühlenbein, "Parallel genetic algorithms, population genetics and combinatorial optimization," in *Proc. 3rd Int. Conf. on Genetic Algorithms,* Arlington, VA, 1989, pp. 416–421.
[43] P. Larrañaga, C. M. H. Kuijpers, M. Poza, and R. H. Murga, "Optimal decomposition of Bayesian networks by genetic algorithms," Dept. of Com. Science and Art. Intel., Univ. of the Basque Country, Int. Rep. EHU-KZAA-IKT-3-94, 1994.
[44] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution Programs.* Berlin: Springer-Verlag, 1992.
[45] W. Banzhaf, "The "molecular" traveling salesman," *Biol. Cybern.,* vol. 64, pp. 7–14, 1990.
[46] D. B. Fogel, "A parallel processing approach to a multiple traveling salesman problem using evolutionary programming," in *Proc. 4th Annual Parallel Processing Symp.,* Fullerton, CA, 1990, pp. 318–326.
[47] D. Whitley, "The GENITOR algorithm and selection pressure: Why rank-based allocation of reproductive trials is best," in *Proc. 3rd Int. Conf. on Genetic Algorithms,* Arlington, VA, 1989, pp. 116–121.
[48] K. A. de Jong, "An analysis of the behavior of a class of genetic adaptive systems," Ph.D. Dissertation, Univ. of Michigan, 1975.
[49] E. H. Herskovits, "Computer based probabilistic-network construction," Doctoral Dissertation, Dept. Medical Information Sciences, Stanford University, Stanford, CA, 1991.

A New Method for Evaluating Weapon Systems Using Fuzzy Set Theory

Shyi-Ming Chen

***Abstract*—This paper presents a new method for evaluating weapon systems using fuzzy set theory. The proposed method is more flexible than the one presented in [11] due to the fact that it allows each item of criteria to have a different weight represented by a triangular fuzzy number. Furthermore, because the proposed method does not need to perform complicated entropy weight calculations as described in [11], its execution is much faster than the one shown in [11].**

I. Introduction

In [11], Mon *et al.* have presented a method for evaluating weapon systems using fuzzy Analytic Hierarchy Process (AHP) based on entropy weights [10], where an example is used to illustrate the method. The example is reviewed as follows. Assume that there are three tactical missile systems A, B, and C to be evaluated, where the tactical specification data of the three missile systems and the expert's opinions are listed in Tables I and II (data source [12]) for

Manuscript received November 5, 1994; revised May 28, 1995. This work was supported in part by the National Science Council, Republic of China, under Grant NSC 84-2213-E-009-100.

The author is with the Department of Computer and Information Science, National Chiao Tung University, Hsinchu 30050, Taiwan, R.O.C.

Publisher Item Identifier S 1083-4427(96)03847-7.

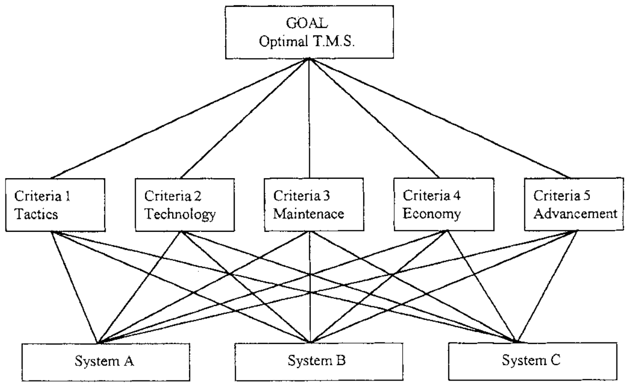

Fig. 1. Structure model for evaluating three tactical missile systems.

TABLE I
CHARACTERISTICS AND EXPERTS' OPINIONS

Items	System A	System B	System C
Operation condition requirement	Higher	General	General
Safety	Good	General	General
Defilade	General	Good	General
Simplicity	General	General	General
Assembility	General	General	Poor
Combat capability	Good	General	General
Material limitation	Higher	General	Higher
Mobility	Poor	Good	General
Modulation	General	Good	General
Standardization	General	General	Good

the decision making process. The structure model presented in [11] for evaluating the three tactical missile systems is shown in Fig. 1.

In Fig. 1, the tactic criteria includes the following items:

1) Effective range.
2) Flight height.
3) Flight velocity.
4) Reliability.
5) Firing accuracy.
6) Destruction rate.
7) Kill radius.

The technology criteria includes the following items:

1) Missile scale.
2) Reaction time.
3) Fire rate.
4) Anti-jam.
5) Combat capability.

The maintenance item includes the following items:

1) Operation condition requirement.
2) Safety.
3) Defilade.
4) Simplicity.
5) Assembility.

The economy criteria includes the following items:

1) System cost.
2) System life.
3) Material limitation.

The advancement criteria includes the following items:

1) Modulization.
2) Mobility.
3) System standardization.

However, some drawbacks exist in the method presented by Mon *et al.* [11].

TABLE II
TACTICAL SPECIFICATION DATA OF THE THREE TACTICAL MISSILE SYSTEMS

Items	System A	System B	System C
Effective range (km)	43	36	38
Flight height (m)	25	20	23
Flight velocity (M. No)	0.72	0.8	0.75
Fire rate (round/min)	0.6	0.6	0.7
Reaction time (min)	1.2	1.5	1.3
Missile scale (cm) ($1 \times d$-span)	$521 \times 35 - 135$	$381 \times 34 - 105$	$445 \times 35 - 120$
Firing accuracy (%)	67	70	63
Destruction rate (%)	84	88	86
Kill radius (m)	15	12	18
Anti-jam (%)	68	75	70
Reliability (%)	80	83	76
System cost (10 000)	800	755	785
System life (year)	7	5	5

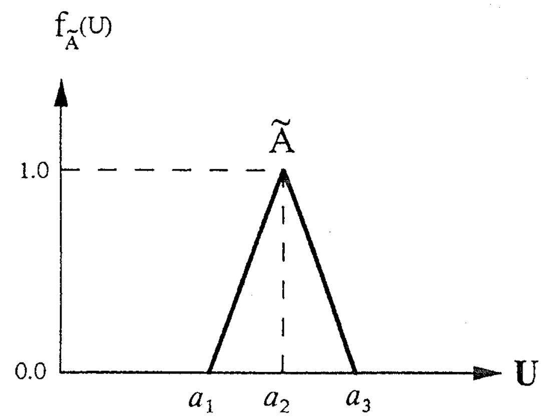

Fig. 2. A triangular fuzzy number.

TABLE III
TRIANGULAR FUZZY NUMBERS AND THEIR CORRESPONDING MEMBERSHIP FUNCTIONS

Triangular fuzzy numbers	Membership functions
$\tilde{1}$	(1, 1, 2)
$\tilde{2}$	(1, 2, 3)
$\tilde{3}$	(2, 3, 4)
$\tilde{4}$	(3, 4, 5)
$\tilde{5}$	(4, 5, 6)
$\tilde{6}$	(5, 6, 7)
$\tilde{7}$	(6, 7, 8)
$\tilde{8}$	(7, 8, 9)
$\tilde{9}$	(8, 9, 9)

1) Their method assumed that each item in each criteria has the same weight. For example, under tactic criteria, the items "reliability" and "flight height" have the same weight, respectively. However, in a real-world application, if we can allow each item of criteria to have a different weight, then there is room for more flexibility.
2) Their method is not efficient enough due to the fact that it must perform complicated entropy weight calculations.

In this paper, we present a new method to overcome the drawbacks of the one presented in [11], where we allow the items shown in Tables I and II to have different weights represented by triangular fuzzy numbers. The proposed method is more flexible than the one presented in [11] due to the fact that it allows each item of criteria to have a different weight. Furthermore, because the proposed method

TABLE IV
FUZZY SCORES OF THE THREE TACTICAL MISSILE SYSTEMS

Item Numbers	Items	System A	System B	System C
1	Operation condition requirement	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
2	Safety	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
3	Defilade	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
4	Simplicity	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$
5	Assembility	$\tilde{2}$	$\tilde{2}$	$\tilde{1}$
6	Combat capability	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
7	Material limitation	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$
8	Mobility	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
9	Modulization	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
10	Standardization	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$
11	Effective range	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
12	Flight height	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
13	Flight velocity	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
14	Fire rate	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$
15	Reaction time	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
16	Missile scale	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
17	Firing accuracy	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
18	Destruction rate	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
19	Kill radius	$\tilde{2}$	$\tilde{1}$	$\tilde{3}$
20	Anti-jam	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
21	Reliability	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
22	System cost	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
23	System life	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$

does not need to perform the complicated entropy weight calculations as described in [11], its execution is much faster than the one presented in [11].

II. BASIC CONCEPTS OF FUZZY SET THEORY

In the following, we briefly review basic concepts of fuzzy set theory from [1]–[9], [13], and [14]. Let U be the universe of discourse, $U = \{u_1, u_2, \cdots, u_n\}$. A fuzzy set $\tilde{A}$ of U is a set of ordered pairs $\{(u_1, f_{\tilde{A}}(u_1)), (u_2, f_{\tilde{A}}(u_n)), \cdots, (u_n, f_{\tilde{A}}(u_n))\}$, where $f_{\tilde{A}}$ is the membership function of $\tilde{A}$, $f_{\tilde{A}}$: $U \rightarrow [0,1]$, and $f_{\tilde{A}}(u_i)$ indicates the grade of membership of u_i in $\tilde{A}$. A fuzzy set $\tilde{A}$ is convex if and only if $\forall\ u_1, u_2$ in U

$$f_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \geq \min(f_{\tilde{A}}(u_1), f_{\tilde{A}}(u_2)) \tag{1}$$

where $\lambda \in [0,1]$. A fuzzy set $\tilde{A}$ is normal if and only if $\exists u_i \in U, f_{\tilde{A}}(u_i) = 1$. A fuzzy number is a fuzzy subset in U which is both convex and normal. The α-cut of the fuzzy number $\tilde{A}$ is denoted by $\tilde{A}_\alpha$, where

$$\tilde{A}_\alpha = \{u_i | f_{\tilde{A}}(u_i) \geq \alpha\} \tag{2}$$

and $\alpha \in [0,1]$.

A triangular fuzzy number $\tilde{A}$ can be parameterized by a triplet (a_1, a_2, a_3) shown in Fig. 2, where the membership function of the triangular fuzzy number $\tilde{A}$ is defined by

$$f_{\tilde{A}}(u) = \begin{cases} 0, & u < a_1 \\ \dfrac{u - a_1}{a_2 - a_1}, & a_1 \leq u \leq a_2 \\ \dfrac{a_3 - u}{a_3 - a_2}, & a_2 \leq u \leq a_3 \\ 0, & u > a_3 \end{cases}. \tag{3}$$

The set of triangular fuzzy numbers we used in this paper and their corresponding membership functions are show in Table III. From Table III, we can see that $\tilde{1}$ is the smallest fuzzy number and $\tilde{9}$ is the largest fuzzy number.

Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers, where

$$\tilde{A} = (a_1, a_2, a_3),$$
$$\tilde{B} = (b_1, b_2, b_3).$$

According to [8] and [9], the fuzzy number arithmetic operations can be summarized as follows:

$$\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \tag{4}$$

$$\tilde{A} \ominus \tilde{B} = (a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \tag{5}$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \tag{6}$$

$$\tilde{A} \oslash \tilde{B} = (a_1, a_2, a_3) \oslash (b_1, b_2, b_3) = (a_1/b_3, a_2/b_2, a_3/b_1). \tag{7}$$

TABLE V
THE WEIGHTS OF ITEMS AND THE FUZZY SCORES OF TACTICAL MISSILE SYSTEMS WITH RESPECT TO THE ITEMS SHOWN IN TABLE IV

Item Numbers	Weights	System A	System B	System C
1	$\tilde{W}_1$	$\tilde{F}_{1A}$	$\tilde{F}_{1B}$	$\tilde{F}_{1C}$
2	$\tilde{W}_2$	$\tilde{F}_{2A}$	$\tilde{F}_{2B}$	$\tilde{F}_{2C}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
23	$\tilde{W}_{23}$	$\tilde{F}_{23A}$	$\tilde{F}_{23B}$	$\tilde{F}_{23C}$

TABLE VI
THE WEIGHTS OF THE ITEMS AND THE FUZZY SCORES OF THE SYSTEMS

Item Numbers	Weights	System A	System B	Systems C
1	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
2	$\tilde{6}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
3	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
4	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$
5	$\tilde{3}$	$\tilde{2}$	$\tilde{2}$	$\tilde{1}$
6	$\tilde{9}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$
7	$\tilde{5}$	$\tilde{2}$	$\tilde{1}$	$\tilde{2}$
8	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
9	$\tilde{5}$	$\tilde{1}$	$\tilde{2}$	$\tilde{1}$
10	$\tilde{3}$	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$
11	$\tilde{7}$	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
12	$\tilde{1}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
13	$\tilde{9}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
14	$\tilde{9}$	$\tilde{1}$	$\tilde{1}$	$\tilde{2}$
15	$\tilde{9}$	$\tilde{3}$	$\tilde{1}$	$\tilde{2}$
16	$\tilde{4}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
17	$\tilde{9}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
18	$\tilde{7}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
19	$\tilde{6}$	$\tilde{2}$	$\tilde{1}$	$\tilde{3}$
20	$\tilde{8}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
21	$\tilde{9}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$
22	$\tilde{8}$	$\tilde{1}$	$\tilde{3}$	$\tilde{2}$
23	$\tilde{8}$	$\tilde{2}$	$\tilde{1}$	$\tilde{1}$

III. A NEW METHODOLOGY TO EVALUATE WEAPON SYSTEMS

In the following, we present a new method to deal with weapon system selection problems. Assume that the decision-maker can

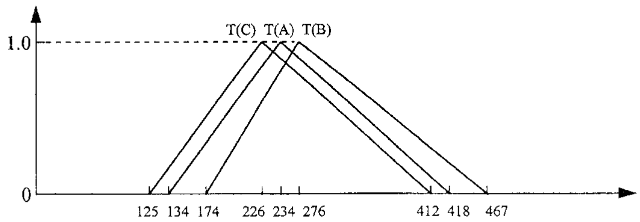

Fig. 3. Membership functions of triangular fuzzy numbers $T(A), T(B)$, and $T(C)$.

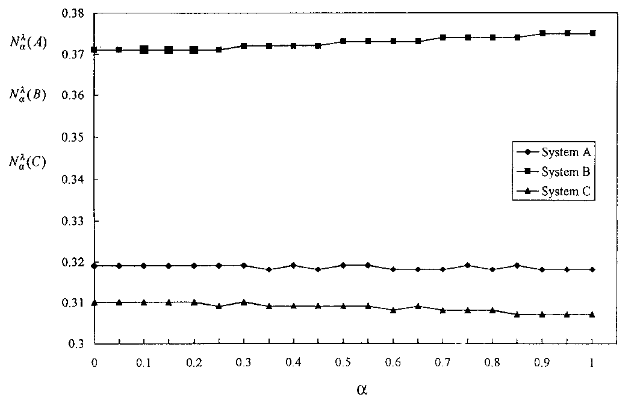

Fig. 4. Values of $N_\alpha^\lambda(A), N_\alpha^\lambda(B)$, and $N_\alpha^\lambda(C)$ for $\lambda = 0.5$ (moderate decision-maker).

assign a fuzzy score (i.e., $\tilde{1}, \tilde{2}, \tilde{3}, \cdots$) to each weapon system with respect to each item shown in Table I and Table II, respectively, where the fuzzy scores are represented by triangular fuzzy numbers shown in Table III. The larger the value of the fuzzy score, the more the suitability of the system with respect to the criteria item. For example, consider the three tacticle missile systems shown in Table I and Table II. We can assign the fuzzy scores to the three weapon systems using triangular fuzzy numbers.

Assume that the weights of the items and the fuzzy scores of the three tactical missile systems with respect to the items presented in Table IV are shown in Table V, where $\tilde{W}_1, \tilde{F}_{1A}, \tilde{F}_{1B}, \tilde{F}_{1C}, \tilde{W}_2, \tilde{F}_{2A}, \tilde{F}_{2B}, \tilde{F}_{2C}, \cdots, \tilde{W}_{23}, \tilde{F}_{23A}, \tilde{F}_{23B}$, and $\tilde{F}_{23C}$ are triangular fuzzy numbers parameterized by triplets as shown in Table III.

Then, the total fuzzy scores of system A, denoted by $T(A)$, can be evaluated as follows:

$$T(A) = \tilde{W}_1 \otimes \tilde{F}_{1A} \oplus W_2 \otimes \tilde{F}_{2A} \oplus \cdots \oplus \tilde{W}_{23} \otimes \tilde{F}_{23A} \quad (8)$$

the total fuzzy scores of system B, denoted by $T(B)$, can be evaluated as follows:

$$T(B) = \tilde{W}_1 \otimes \tilde{F}_{1B} \oplus \tilde{W}_2 \otimes \tilde{F}_{2B} \oplus \cdots \oplus \tilde{W}_{23} \otimes \tilde{F}_{23B} \quad (9)$$

the total fuzzy scores of system C, denoted by $T(C)$, can be evaluated as follows:

$$T(C) = \tilde{W}_1 \otimes \tilde{F}_{1C} \oplus \tilde{W}_2 \otimes \tilde{F}_{2C} \oplus \cdots \oplus \tilde{W}_{23} \otimes \tilde{F}_{23C} \quad (10)$$

where $T(A), T(B)$, and $T(C)$ are triangular fuzzy numbers.

Let the α-cuts of $T(A), T(B)$, and $T(C)$ be denoted by $[a_1^{(\alpha)}, a_2^{(\alpha)}], [b_1^{(\alpha)}, b_2^{(\alpha)}]$, and $[c_1^{(\alpha)}, c_2^{(\alpha)}]$, respectively, where $\alpha \in [0,1]$. Furthermore, the scores $D_\alpha^\lambda(A), D_\alpha^\lambda(B)$, and $D_\alpha^\lambda(C)$ with fixed α and fixed index of optimism $\lambda, \lambda \in [0,1]$, can be evaluated, where the index of optimism λ indicates the degree of optimism of the decision-maker [11]. A smaller λ indicates a higher degree

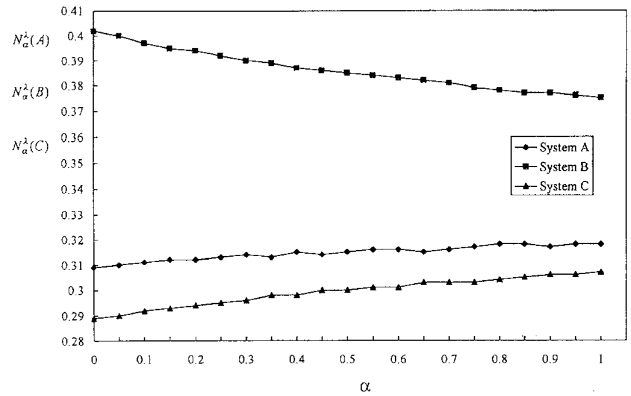

Fig. 5. Values of $N_\alpha^\lambda(A), N_\alpha^\lambda(B)$, and $N_\alpha^\lambda(C)$ for $\lambda = 1$ (pessimistic decision-maker).

of optimism. Let

$$D_\alpha^\lambda(A) = \lambda a_1^{(\alpha)} + (1-\lambda) a_2^{(\alpha)} = p_1 \quad (11)$$

$$D_\alpha^\lambda(B) = \lambda b_1^{(\alpha)} + (1-\lambda) b_2^{(\alpha)} = p_2 \quad (12)$$

$$D_\alpha^\lambda(C) = \lambda c_1^{(\alpha)} + (1-\lambda) c_2^{(\alpha)} = p_3, \quad (13)$$

and let

$$N_\alpha^\lambda(A) = \frac{p_1}{p_1 + p_2 + p_3} \quad (14)$$

$$N_\alpha^\lambda(B) = \frac{p_2}{p_1 + p_2 + p_3} \quad (15)$$

$$N_\alpha^\lambda(C) = \frac{p_3}{p_1 + p_2 + p_3}. \quad (16)$$

The values of $N_\alpha^\lambda(A), N_\alpha^\lambda(B)$, and $N_\alpha^\lambda(C)$ indicate the degree of suitability of the selection with respect to the systems A, B, and C for fixed α and λ, respectively, where $\alpha \in [0,1], \lambda \in [0,1], N_\alpha^\lambda(A) \in [0,1], N_\alpha^\lambda(B) \in [0,1]$, and $N_\alpha^\lambda(C) \in [0,1]$. The larger the value, the more the suitability of the selection of the system.

IV. A Numerical Example

Assume that three tactical missile systems A, B, and C are to be evaluated, where the tactical specification data of the three missile systems and the expert's opinions are listed in Tables I and II, respectively. Assume that the decision-maker can assign different weights to the items shown in Tables I and II, respectively, and assume that the decision-maker can assign fuzzy scores to the systems with respect to the items shown in Tables I and II, respectively, where the weights and the fuzzy scores are represented by triangular fuzzy numbers shown in Table III. Furthermore, assume that the weights of the items and the fuzzy scores of the systems with respect to the items assigned by the decision-maker are shown in Table VI.

Then, based on formulas (8)–(10), we can get

$$\begin{aligned} T(A) = {} & \tilde{5}\otimes\tilde{2}\oplus\tilde{6}\otimes\tilde{2}\oplus\tilde{2}\otimes\tilde{1}\oplus\tilde{3}\otimes\tilde{1}\oplus\tilde{3}\otimes\tilde{2}\oplus\tilde{9}\otimes\tilde{2} \\ & \oplus\tilde{5}\otimes\tilde{2}\oplus\tilde{7}\otimes\tilde{1}\oplus\tilde{5}\otimes\tilde{1}\oplus\tilde{3}\otimes\tilde{1}\oplus\tilde{7}\otimes\tilde{3}\oplus\tilde{1} \\ & \otimes\tilde{1}\oplus\tilde{9}\otimes\tilde{1}\oplus\tilde{9}\otimes\tilde{1}\oplus\tilde{9}\otimes\tilde{3}\oplus\tilde{4}\otimes\tilde{1}\oplus\tilde{9}\otimes\tilde{2} \\ & \oplus\tilde{7}\otimes\tilde{1}\oplus\tilde{6}\otimes\tilde{2}\oplus\tilde{8}\otimes\tilde{1}\oplus\tilde{9}\otimes\tilde{2}\oplus\tilde{8}\otimes\tilde{1}\oplus\tilde{8}\otimes\tilde{2} \\ = {} & (4,5,6)\otimes(1,2,3)\oplus(5,6,7)\otimes(1,2,3) \\ & \oplus(1,2,3)\otimes(1,1,2)\oplus(2,3,4)\otimes(1,1,2) \\ & \oplus(2,3,4)\otimes(1,2,3)\oplus(8,9,9)\otimes(1,2,3) \\ & \oplus(4,5,6)\otimes(1,2,3)\oplus(6,7,8)\otimes(1,1,2) \\ & \oplus(4,5,6)\otimes(1,1,2)\oplus(2,3,4)\otimes(1,1,2) \\ & \oplus(6,7,8)\otimes(2,3,4)\oplus(1,1,2)\otimes(1,1,2) \end{aligned}$$

$$
\begin{aligned}
&\oplus (8,9,9) \otimes (1,1,2) \oplus (8,9,9) \otimes (1,1,2)\\
&\oplus (8,9,9) \otimes (2,3,4) \oplus (3,4,5) \otimes (1,1,2)\\
&\oplus (8,9,9) \otimes (1,2,3) \oplus (6,7,8) \otimes (1,1,2)\\
&\oplus (5,6,7) \otimes (1,2,3) \oplus (7,8,9) \otimes (1,1,2)\\
&\oplus (8,9,9) \otimes (1,2,3) \oplus (7,8,9) \otimes (1,1,2)\\
&\oplus (7,8,9) \otimes (1,2,3) = (134,234,418).
\end{aligned}
$$

$$
\begin{aligned}
T(B) =\ & \tilde{5} \otimes \tilde{1} \oplus \tilde{6} \otimes \tilde{1} \oplus \tilde{2} \otimes \tilde{2} \oplus \tilde{3} \otimes \tilde{1} \oplus \tilde{3} \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{1}\\
& \oplus \tilde{5} \otimes \tilde{1} \oplus \tilde{7} \otimes \tilde{3} \oplus \tilde{5} \otimes \tilde{2} \oplus \tilde{3} \otimes \tilde{1} \oplus \tilde{7} \otimes \tilde{1} \oplus \tilde{1}\\
& \otimes \tilde{3} \oplus \tilde{9} \otimes \tilde{3} \oplus \tilde{9} \otimes \tilde{1} \oplus \tilde{9} \otimes \tilde{1} \oplus \tilde{4} \otimes \tilde{3} \oplus \tilde{9} \otimes \tilde{3}\\
& \oplus \tilde{7} \otimes \tilde{3} \oplus \tilde{6} \otimes \tilde{1} \oplus \tilde{8} \otimes \tilde{3} \oplus \tilde{9} \otimes \tilde{3} \oplus \tilde{8} \otimes \tilde{3} \oplus \tilde{8} \otimes \tilde{1}\\
=\ & (4,5,6) \otimes (1,1,2) \oplus (5,6,7) \otimes (1,1,2)\\
& \oplus (1,2,3) \otimes (1,2,3) \oplus (2,3,4) \otimes (1,1,2)\\
& \oplus (2,3,4) \otimes (1,2,3) \oplus (8,9,9) \otimes (1,1,2)\\
& \oplus (4,5,6) \otimes (1,1,2) \oplus (6,7,8) \otimes (2,3,4)\\
& \oplus (4,5,6) \otimes (1,2,3) \oplus (2,3,4) \otimes (1,1,2)\\
& \oplus (6,7,8) \otimes (1,1,2) \oplus (1,1,2) \otimes (2,3,4)\\
& \oplus (8,9,9) \otimes (2,3,4) \oplus (8,9,9) \otimes (1,1,2)\\
& \oplus (8,9,9) \otimes (1,1,2) \oplus (8,9,9) \otimes (2,3,4)\\
& \oplus (8,9,9) \otimes (2,3,4) \oplus (6,7,8) \otimes (2,3,4)\\
& \oplus (5,6,7) \otimes (1,1,2) \oplus (7,8,9) \otimes (2,3,4)\\
& \oplus (8,9,9) \otimes (2,3,4) \oplus (7,8,9) \otimes (2,3,4)\\
& \oplus (7,8,9) \otimes (1,1,2) = (174,276,467).
\end{aligned}
$$

$$
\begin{aligned}
T(C) =\ & \tilde{5} \otimes \tilde{1} \oplus \tilde{6} \otimes \tilde{1} \oplus \tilde{2} \otimes \tilde{1} \oplus \tilde{3} \otimes \tilde{1} \oplus \tilde{3} \otimes \tilde{1} \oplus \tilde{9} \otimes \tilde{1}\\
& \oplus \tilde{5} \otimes \tilde{2} \oplus \tilde{7} \otimes \tilde{2} \oplus \tilde{5} \otimes \tilde{1} \oplus \tilde{3} \otimes \tilde{2} \oplus \tilde{7} \otimes \tilde{2} \oplus \tilde{1}\\
& \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{2} \oplus \tilde{4} \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{1}\\
& \oplus \tilde{7} \otimes \tilde{2} \oplus \tilde{6} \otimes \tilde{3} \oplus \tilde{8} \otimes \tilde{2} \oplus \tilde{9} \otimes \tilde{1} \oplus \tilde{8} \otimes \tilde{2} \oplus \tilde{8} \otimes \tilde{1}\\
=\ & (4,5,6) \otimes (1,1,2) \oplus (5,6,7) \otimes (1,1,2)\\
& \oplus (1,2,3) \otimes (1,1,2) \oplus (2,3,4) \otimes (1,1,2)\\
& \oplus (2,3,4) \otimes (1,1,2) \oplus (8,9,9) \otimes (1,1,2)\\
& \oplus (4,5,6) \otimes (1,1,2) \oplus (6,7,8) \otimes (1,2,3)\\
& \oplus (4,5,6) \otimes (1,1,2) \oplus (2,3,4) \otimes (1,2,3)\\
& \oplus (6,7,8) \otimes (1,2,3) \oplus (1,1,2) \otimes (1,2,3)\\
& \oplus (8,9,9) \otimes (1,2,3) \oplus (8,9,9) \otimes (1,2,3)\\
& \oplus (8,9,9) \otimes (1,2,3) \oplus (3,4,5) \otimes (1,2,3)\\
& \oplus (8,9,9) \otimes (1,1,2) \oplus (6,7,8) \otimes (1,2,3)\\
& \oplus (5,6,7) \otimes (2,3,4) \oplus (7,8,9) \otimes (1,2,3)\\
& \oplus (8,9,9) \otimes (1,1,2) \oplus (7,8,9) \otimes (1,2,3)\\
& \oplus (7,8,9) \otimes (1,1,2) = (125,226,412).
\end{aligned}
$$

The membership functions of $T(A), T(B)$, and $T(C)$ are shown in Fig. 3, respectively.

Based on formulas (11)–(16), we have used Turbo C^{++} version 3.0 to write a computer program on a PC/AT for calculating the values of $N_\alpha^\lambda(A), N_\alpha^\lambda(B)$, and $N_\alpha^\lambda(C)$ with respect to different values of $\alpha(\alpha = 0, 0.05, 0.1, \cdots, 1)$ and $\lambda(\lambda = 0.5, 1, 0)$ as shown in Figs. 4–6, respectively. From Figs. 4–6, we can see that system B is the best selection for all the degrees of optimism λ, where $\lambda \in [0, 1]$.

V. Conclusions

In this paper, we have presented a new method for evaluating weapon systems to overcome the drawbacks of the one presented in

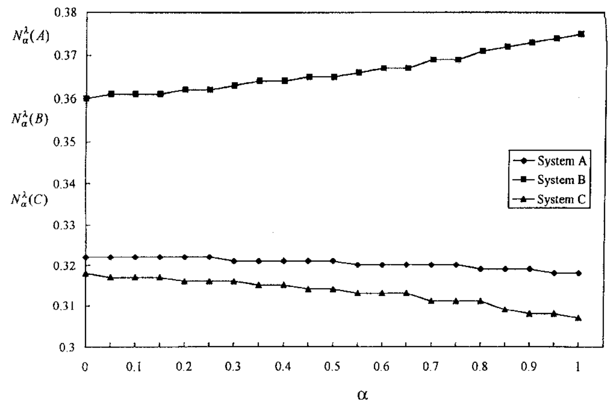

Fig. 6. Values of $N_\alpha^\lambda(A), N_\alpha^\lambda(B)$, and $N_\alpha^\lambda(C)$ for $\lambda = 0$ (optimistic decision-maker).

[11]. Because the proposed method allows the items of criteria to have different weights represented by triangular fuzzy numbers, it is more flexible than the one presented in [11]. Furthermore, because the proposed method does not need to perform the complicated entropy weight calculations as described in [11], its execution is much faster than the one presented in [11].

Acknowledgment

The author would like to thank Dr. D. L. Mon and Dr. C. H. Cheng for their encouragement in this work. The author also would like to thank Mr. Y. J. Horng and Mr. W. T. Jong for their assistance in the computer simulations.

References

[1] S. M. Chen, "A new approach to handling fuzzy decisionmaking problems," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, no. 6. pp. 1012–1016, Nov./Dec. 1988.

[2] ——, "A new approach to inexact reasoning for rule-based systems," *Cybern. Syst.: Int. J.*, vol. 23, no. 6, pp. 561–582, Nov./Dec. 1992.

[3] ——, "A weighted fuzzy reasoning algorithm for medical diagnosis," *Decision Support Syst.*, vol. 11, no. 1, pp. 37–43, Jan. 1994.

[4] ——, "Fuzzy system reliability analysis using fuzzy number arithmetic operations," *Fuzzy Sets Syst.*, vol. 64, no. 1, pp. 31–38, May 1994.

[5] ——, "A new method for handling multicriteria fuzzy decision-making problems," *Cybern. Syst.: Int. J.*, vol. 25, no. 3, pp. 409–420, May/June 1994.

[6] C. H. Cheng and D. L. Mon, "Fuzzy system reliability analysis by interval of confidence," *Fuzzy Sets Syst.*, vol. 56, no. 1, pp. 29–35, May 1993.

[7] A. Kandel, *Fuzzy Mathematical Techniques with Applications.* Reading, MA: Addison-Wesley, 1986.

[8] A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic Theory and Applications.* New York: Van Nostrand Reinhold, 1991.

[9] ——, *Fuzzy Mathematical Models in Engineering and Management Science.* Amsterdam: North-Holland, 1988.

[10] T. Ke, "Target decision by entropy weight and fuzzy," *Syst. Eng. Theory and Practice*, vol. 5, 1992 (in Chinese).

[11] D. L. Mon, C. H. Cheng, and J. C. Lin, "Evaluating weapon system using fuzzy analytic hierarchy process based on entropy weight," *Fuzzy Sets Syst.*, vol. 62, no. 2, pp. 127–134, Mar. 1994.

[12] J. H. Wen, *Guided Missile System Analysis and Design.* Beijing: Beijing Natural Science & Engineering Univ., 1989 (in Chinese).

[13] L. A. Zadeh, "Fuzzy sets," *Inform. Contr.*, vol. 8, pp. 338–353, 1965.

[14] H. J. Zimmermann, *Fuzzy Set Theory and Its Applications.* Norwell, MA: Kluwer, 1991.