System Identification of Partially Restrained Composite Plates Using Measured Natural Frequencies

C. R. Lee[1] and T. Y. Kam[2]

Abstract: A nondestructive evaluation technique established on the basis of a global minimization method is presented for the system identification of laminated composite plates partially restrained by elastic edge supports. Six natural frequencies extracted from the vibration data of the flexibly restrained plate are used to identify the system parameters of the plate. In the identification process, the trial system parameters are used in the Rayleigh–Ritz method to predict the theoretical natural frequencies of the plate, an error function is established to measure the sum of the differences between the experimental and theoretical predictions of the natural frequencies, and the global minimization method is used to search for the best estimates of the parameters by making the error function a global minimum. The accuracy and efficiency of the proposed technique in identifying the parameters of several flexibly supported plates made of different composite materials are studied via both theoretical and experimental approaches. The excellent results obtained in this study have validated the applicability of the proposed technique.

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Introduction

Due to their high strength/stiffness to weight ratios, the fiber reinforced composite plates have been widely used in different industries to fabricate high performance structures. In general, these structures are composed of different kinds of structural members such as plates, shells, and beams, which are interconnected using different joining methods. One popular way to analyze the mechanical behaviors of such structural members is to consider the member being supported at the boundary by equivalent elastic restraints. On the other hand, the attainment of the actual behavioral predictions of such structural members depends on the correctnesses of the elastic constants of the members as well as those of the elastic restraints at the member boundaries. Nevertheless, the determination of the properties of a member and its elastic restraints is usually a difficult problem to be tackled. Furthermore, composite structures fabricated by different methods or curing processes may possess different mechanical properties (Lubin 1982; Schwartz 1983) and because of this the material properties determined from the standard specimens tested in laboratory in general may deviate from those of the laminated composite components manufactured in the factory. For tackling these long existing difficulties, the determinations of realistic system parameters and/or material constants of composite structures have thus become an important topic of research in recent years. For instance, Deobald and Gibson (1988) used a Rayleigh–Ritz/modal analysis technique to determine the elastic constants of composite plates with different boundary conditions. Castagnède et al. (1990) determined the elastic constants of thick composite plates via a quantitative ultrasonic approach. Fallstrom and Jonsson (1991) determined the material constants of anisotropic plates from the frequencies and mode shapes measured by a real-time television-holography system. Nielsen and Toftegaard (1998) used the ultrasonic measurement approach to obtain the elastic constants of fiber-reinforced polymer composites under the influence of absorbed moisture. Several researchers (Berman and Nagy 1983; Kam and Lee 1994; Kam and Liu 1998) developed methods to identify structural stiffness matrices or the element bending stiffnesses of beam structures using measured natural frequencies and mode shapes. Wang and Kam (2000) developed a nondestructive evaluation method in which strains and/or displacements obtained from static testing of laminated composite plates clamped at the edges are used to identify the elastic constants of the plates. Recently, a number of researchers (Moussu and Nivoit 1993; Qian et al. 1997; Sol et al. 1997) have used experimental natural frequencies to identify the elastic constants of laminated composite plates with free boundary conditions. For instance, Moussu and Nivoit (1993) used the method of superposition to determine the elastic constants of free orthotropic plates from measured natural frequencies. Sol et al. (1997) used the method of Bayesian estimation to study the identification of elastic constants from the experimental natural frequencies of free rectangular orthotropic plates. It is noted that all the structures used in the aforementioned system identification studies had regular boundary conditions such as free, clamped, simply supported ends/edges. As for the determination of mechanical properties of plates with elastic restraints, not to mention laminated composite plates partially supported by elastic edge restraints, it seems not much work has been devoted to this area which, however, needs to be explored in depth.

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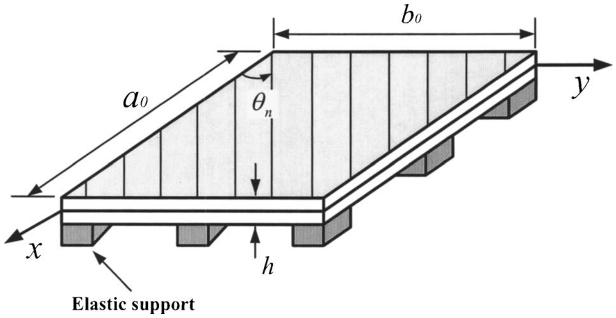

Fig. 1. Flexibly and partially supported laminated composite plate

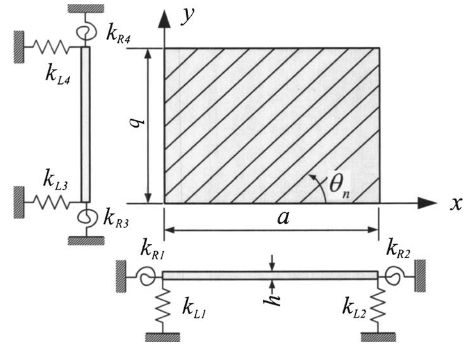

Fig. 2. Model of elastically restrained laminated plate

In this paper, the system parameters identification of laminated composite plates partially supported by edge elastic restraints using measured natural frequencies is studied via both theoretical and experimental approaches. The Rayleigh–Ritz method is used to predict the theoretical natural frequencies of the flexibly and partially supported laminated composite plates using trial system parameters. Vibrating testing of the flexibly supported plates is performed to measure the natural frequencies of the plates. An error function is established to measure the sum of the differences between the experimental and theoretical natural frequencies of the flexibly supported plates. The identification of the system parameters of the flexibly supported plates is then formulated as a constrained minimization problem in which the system parameters are determined by making the error function a global minimum. A normalization technique is used to normalize the trial system parameters in such a way that the search direction can help improve the convergence rate of the solution. In the theoretical study, the system parameters of a number of elastically restrained plates made of different laminated composite materials are identified to illustrate the capability and efficiency of the proposed method. In the experimental investigation, several flexibly and partially supported laminated composite plates are subjected to impulse vibration testing. Measured natural frequencies of the flexibly supported plates are used in the identification of the plate parameters to demonstrate the applications and accuracy of the proposed method.

Plate Vibration Analysis

Consider a thin rectangular symmetrically laminated composite plate of length a_0, width b_0, and constant thickness h. The plate composed of a finite number of orthotropic layers of the same material properties and thickness is partially supported by flexible restraints at the plate periphery as shown in Fig. 1. The x and y coordinates of the plate are taken in the midplane of the plate. The edge flexible supports of the elastically restrained plate can be modeled by longitudinal and torsional springs as shown in Fig. 2. It is noted that the effective length and width of the plate adopted in the vibration analysis are a and b, respectively. For free vibration, the plate vertical displacement $w(x,y,t)$ is assumed to be of the form

$$w(x,y,t) = W(x,y)\sin \omega t \tag{1}$$

where $W(x,y)$=deflection function; and ω=angular frequency. According to the classical lamination theory, the maximum strain energy U_P and maximum kinetic energy T of the plate with the neglect of rotary inertia effect are expressed, respectively, as (Hung et al. 1993)

$$U_P = \frac{1}{2}\int_0^b \int_0^a \left[D_{11}\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right) + D_{22}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 + 4D_{16}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right) + 4D_{26}\left(\frac{\partial^2 W}{\partial y^2}\right)\left(\frac{\partial^2 W}{\partial x \partial y}\right)\right] dx\, dy \tag{2}$$

and

$$T = \frac{1}{2}\rho h \omega^2 \int_0^b \int_0^a W^2 dx\, dy \tag{3}$$

where D_{ij}=bending stiffness coefficients; and ρ=material density. The bending stiffness coefficients are given by

$$D_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij}^{(m)} z^2\, dz \quad i,j = 1,2,6 \tag{4}$$

The transformed lamina stiffness coefficients $\bar{Q}_{ij}^{(m)}$ depending on the material properties and fiber orientation of the mth layer are expressed as

$$\bar{Q}_{11} = Q_{11}C^4 + 2(Q_{12} + 2Q_{66})C^2S^2 + Q_{22}S^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})C^2S^2 + Q_{12}(C^4 + S^4)$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})C^3S + (Q_{12} - Q_{22} + 2Q_{66})CS^3$$

$$\bar{Q}_{22} = Q_{11}S^4 + 2(Q_{12} + 2Q_{66})C^2S^2 + Q_{22}C^4$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})CS^3 + (Q_{12} - Q_{22} + 2Q_{66})C^3S$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})C^2S^2 + Q_{66}(C^4 + S^4)$$

$$\bar{Q}_{12} = \bar{Q}_{21}, \quad \bar{Q}_{16} = \bar{Q}_{61}, \quad \bar{Q}_{26} = \bar{Q}_{62} \tag{5}$$

with

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}; \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}; \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}; \quad C = \cos \theta_m; \quad S = \sin \theta_m \tag{6}$$

where E_1, E_2=Young's moduli in the fiber and transverse directions, respectively; ν_{ij}=Poisson's ratio for transverse strain in the j direction when stressed in the i direction; G_{12}=shear modulus in the 1–2 plane; and θ_m=lamina fiber angle of the mth layer.

For the plate partially restrained by elastic edge supports, additional strain energy stored in the boundary springs exists. Herein, without loss of generality it is assumed that the plate is supported on the edges by $4n$ elastic restraints of which the n elastic restraints on each edge have same constant translational and rotational spring constants. The maximum total strain energy of the flexible restraints is thus written as

$$U_B=\sum_{j=1}^{n}\left\{\frac{K_{L1}}{2}\left(\int_{y_{j-1}}^{y_j}W^2\,dy\right)_{x=0}+\frac{K_{L2}}{2}\left(\int_{y_{j-1}}^{y_j}W^2\,dy\right)_{x=a}\right.$$
$$+\frac{K_{L3}}{2}\left(\int_{x_{j-1}}^{x_j}W^2\,dy\right)_{y=0}+\frac{K_{L4}}{2}\left(\int_{x_{j-1}}^{x_j}W^2\,dy\right)_{y=b}$$
$$+\frac{K_{R1}}{2}\left[\int_{y_{j-1}}^{y_j}\left(\frac{\partial W}{\partial x}\right)^2dy\right]_{x=0}+\frac{K_{R2}}{2}\left[\int_{y_{j-1}}^{y_j}\left(\frac{\partial W}{\partial x}\right)^2dy\right]_{x=a}$$
$$\left.+\frac{K_{R3}}{2}\left[\int_{x_{j-1}}^{x_j}\left(\frac{\partial W}{\partial y}\right)^2dy\right]_{y=0}+\frac{K_{R4}}{2}\left[\int_{x_{j-1}}^{x_j}\left(\frac{\partial W}{\partial y}\right)^2dy\right]_{y=b}\right\}\quad(7)$$

where K_{Li} and K_{Ri} $(i=1,\ldots,4)$=spring constants per unit length of the longitudinal and torsional springs on the ith edge, respectively; and x_j, y_j $(j=1,\ldots,n)$=location coordinates of the ends of the edge supports. The integrals in the above brackets are evaluated at the $4n$ edge supports of the plate. The total strain energy of the flexibly supported plate is then written as

$$U=U_P+U_B\quad(8)$$

Based on the Rayleigh–Ritz method, the deflection function can be expressed in the following nondimensional form

$$W(\xi,\eta)=\sum_{i=1}^{I}\sum_{j=1}^{J}C_{ij}\phi_i(\xi)\varphi_j(\eta)\quad(9)$$

where C_{ij}=undetermined displacement coefficients; $\phi_i(\xi)$, $\varphi_j(\eta)$=orthogonal functions; and ξ,η=normalized coordinates. Herein, the Legendre's orthogonal polynomials are used in Eq. (9) to denote ϕ_i and φ_j. For instance, $\phi_i(\xi)$ is written as

$$\phi_1(\xi)=1$$

$$\phi_2(\xi)=\xi$$

and for $n\geqslant 3$

$$\phi_n(\xi)=[(2n-3)\xi\times\phi_{n-1}(\xi)-(n-2)\times\phi_{n-2}(\xi)]/(n-1)\quad(10)$$

where $\xi=(2x/a)-1$ with $-1\leqslant\xi\leqslant 1$; and $\eta=(2y/b)-1$ with $-1\leqslant\eta\leqslant 1$. It is noted that the above orthogonal polynomials $\phi_i(\xi)$ satisfy the orthogonality condition

$$\int_{-1}^{1}\phi_n(\xi)\phi_m(\xi)d\xi=\begin{cases}0 & \text{if } n\neq m\\ \dfrac{2}{2n-1} & \text{if } n=m\end{cases}\quad(11)$$

Extremization of the functional Π which is defined as $\Pi=U-T$ with respect to the displacement coefficients C_{ij} leads to the following eigenvalue problem:

$$([\mathbf{K}]-\omega^2[\mathbf{M}])\{\mathbf{C}\}=0\quad(12a)$$

with

$$\mathbf{K}=\mathbf{K}_P+\mathbf{K}_B\quad(12b)$$

where $\{\mathbf{C}\}$=displacement coefficients vector; $\mathbf{K}$=structure stiffness matrix; $\mathbf{K}_P$=portion of structure stiffness matrix contributed by plate stiffness; and $\mathbf{K}_B$=portion of structure stiffness matrix contributed by the stiffnesses of edge restraints. The elements of $\mathbf{K}_P$, $\mathbf{K}_B$, and $\mathbf{M}$ are expressed, respectively, as

$$[K_P]_{mnij}=16[D_{11}E_{mi}^{22}F_{nj}^{00}/a^4+D_{12}(E_{mi}^{02}F_{nj}^{20}+E_{mi}^{20}F_{nj}^{02})/(a^2b^2)$$
$$+D_{22}E_{mi}^{00}F_{nj}^{22}/b^4+2D_{16}(E_{mi}^{21}F_{nj}^{01}+E_{mi}^{12}F_{nj}^{10})/(a^3b)$$
$$+2D_{26}(E_{mi}^{01}F_{nj}^{21}+E_{mi}^{10}F_{nj}^{12})/(ab^3)+4D_{66}E_{mi}^{11}F_{nj}^{11}/(a^2b^2)]\quad(13)$$

$$[K_B]_{mnij}=2\sum_{p=1}^{P}\{[K_{L1}J_{nj}^{p}\phi_m(-1)\phi_i(-1)+K_{L2}J_{nj}^{p}\phi_m(1)\phi_i(1)]/a$$
$$+[K_{L3}I_{mi}^{p}\varphi_n(-1)\varphi_j(-1)+K_{L4}I_{mi}^{p}\varphi_n(1)\varphi_j(1)]/b\}$$
$$+8\sum_{p=1}^{P}\{[K_{R1}J_{nj}^{p}\phi_m'(-1)\phi_i'(-1)+K_{R2}J_{nj}^{p}\phi_m'(1)\phi_i'(1)]/a^3$$
$$+[K_{R3}I_{mi}^{p}\varphi_n'(-1)\varphi_j'(-1)+K_{R4}I_{mi}^{p}\varphi_n'(1)\varphi_j'(1)]/b^3\}\quad(14)$$

and

$$[M]_{mnij}=\rho hE_{mi}^{00}F_{nj}^{00}$$

$$m,i=1,2,3,\ldots M,I;\quad n,j=1,2,3,\ldots N,J;\quad p=1,2,3,\ldots P\quad(15)$$

with

$$E_{mi}^{rs}=\int_{-1}^{1}\left[\frac{d^r\phi_m(\xi)}{d\xi^r}\frac{d^s\phi_i(\xi)}{d\xi^s}\right]d\xi;$$

$$F_{nj}^{rs}=\int_{-1}^{1}\left[\frac{d^r\varphi_n(\eta)}{d\eta^r}\frac{d^s\varphi_j(\eta)}{d\eta^s}\right]d\eta;\quad r,s=0,1,2\quad(16a)$$

$$I_{mi}^{p}=\int_{\xi_{p-1}}^{\xi_p}[\phi_m(\xi)\phi_i(\xi)]d\xi;\quad J_{nj}^{p}=\int_{\eta_{p-1}}^{\eta_p}[\varphi_j(\eta)\varphi_n(\eta)]d\eta\quad(16b)$$

where ξ_p and η_p=normalized location coordinates of the ends of the edge supports. The above Rayleigh–Ritz method can be used to predict the natural frequencies of laminated composite plates partially supported by elastic restraints at the plate edges. The theoretically predicted natural frequencies will then be used for the parameters identification of the flexibly supported laminated composite plates as will be described in the following section.

System Identification

In this study, without loss of generality the elastic restraints under consideration are assumed to be made of strip-type pads with cross-sectional area $b_e\times h_e$ and Young's modulus E_e as shown in Fig. 3(a). The spring constants of the pad-type supports are determined via the mechanics of materials approach. It is assumed that

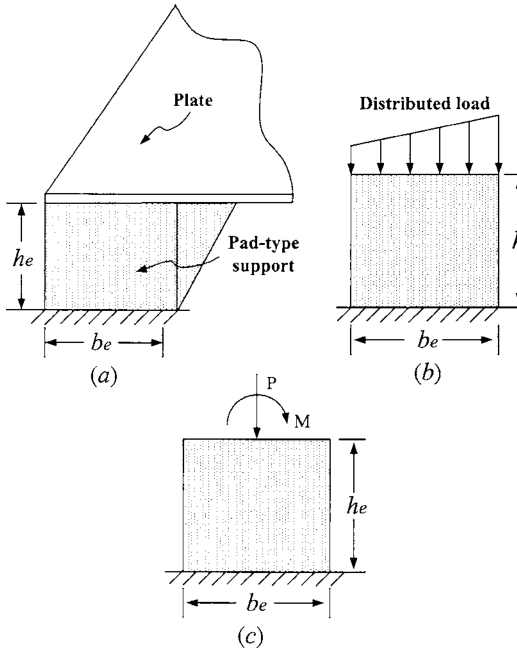

Fig. 3. Geometry and loading condition of pad-type support

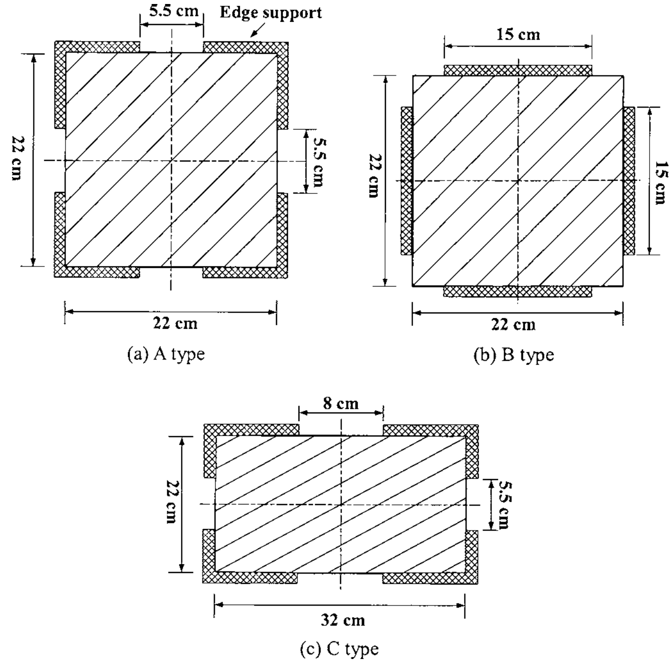

Fig. 4. Supporting condition of elastically restrained laminated composite plates

the plate edges are perfectly bound to the top surfaces of the pads and the load per unit length acting on the top surface of the pad varies linearly across the width of the pad as show in Fig. 3(b). The distributed load per unit length on each pad can be replaced by two components, i.e., moment M and force P as shown in Fig. 3(c). It is further assumed that the plane surface of the pad remains plane after deformation. The vertical displacement δ induced by P alone is obtained as

$$\delta = \frac{Ph_e}{E_e b_e} \tag{17}$$

Then the translational spring constant per unit length of the pad-type support derived from the above equation is

$$K_L = \frac{E_e b_e}{h_e} \tag{18}$$

Similarly, the rotation θ induced by M alone is obtained as

$$\theta = \frac{M h_e}{\frac{1}{12} b_e^3 E_e} \tag{19}$$

Then the rotational spring constant per unit length of the pad-type support derived from the above equation is

$$K_R = \frac{E_e b_e^3}{12 h_e} \tag{20}$$

It is noted that for a plate with length a_0 and width b_0 restrained by pad-like edge supports, the effective length a and width b of the plate become $a_0-(b_e/2)$ and $b_0-(b_e/2)$, respectively. Herein, the problem of identification of system parameters (material and spring constants) of laminated composite plates partially restrained by strip-type pads at the plate edges is formulated as a minimization problem. In mathematical form it is stated as

Minimize

$$e(\mathbf{x}) = (\boldsymbol{\omega}^*)^t(\boldsymbol{\omega}^*)$$

Subject to

$$x_i^L \leqslant x_i \leqslant x_i^U \quad i = 1-5 \tag{21}$$

where $\mathbf{x}=[E_1, E_2, G_{12}, \nu_{12}, E_e]$=vector containing the design variables which presently are material constants of the plate and the Young's modulus of the elastic restraints; $\boldsymbol{\omega}^*=n\times 1$ vector containing the differences between the measured and predicted values of the natural frequencies; $e(\mathbf{x})$=error function measuring the sum of differences between the predicted and measured data; and x_i^L, x_i^U=lower and upper bounds of the material constants. The elements in $\boldsymbol{\omega}^*$ are expressed as

$$\omega_i^* = \frac{\omega_{pi} - \omega_{mi}}{\omega_{mi}} \quad i = 1-6 \tag{22}$$

where ω_{pi}, ω_{mi}=predicted and measured values of the natural frequencies, respectively. In general, the use of any conventional minimization technique to solve the identification problem of Eq. (21) may encounter great difficulty in obtaining the global minimum. Herein, a multistart global minimization method together with an appropriate normalization technique for normalizing the design variables is adopted to solve the above system identification problem. In the proposed method, the above problem of Eq. (21) is first converted into an unconstrained minimization problem by creating the following general augmented Lagrangian (Vanderplaats 1984)

$$\bar{\boldsymbol{\Psi}}(\tilde{\mathbf{x}}, \boldsymbol{\mu}, \boldsymbol{\eta}, r_p) = e(\tilde{\mathbf{x}}) + \sum_{j=1}^{5} (\mu_j z_j + r_p z_j^2 + \eta_j \phi_j + r_p \phi_j^2) \tag{23}$$

with

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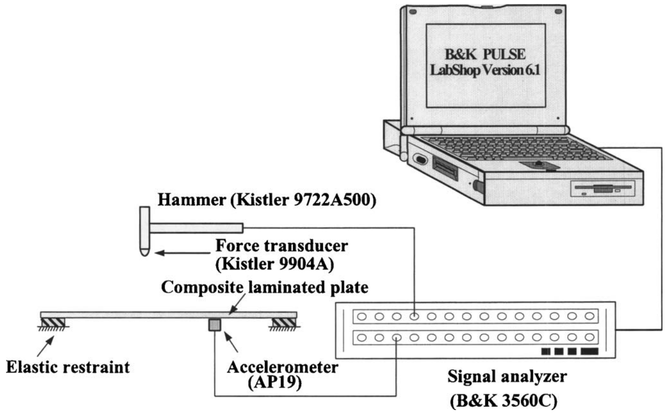

Fig. 5. Experimental setup for impulse vibration testing

$$z_j = \max\left[g_j(\tilde{x}_j), \frac{-\mu_j}{2r_p}\right]$$

$$g_j(\tilde{x}_j) = \tilde{x}_j - \tilde{x}_j^U \leqslant 0$$

$$\phi_j = \max\left[H_j(\tilde{x}_j), \frac{-\eta_j}{2r_p}\right] \tag{24}$$

$$H_j(\tilde{x}_j) = \tilde{x}_j^L - \tilde{x}_j \leqslant 0 \quad j = 1-5$$

where μ_j, η_j, γ_p=multipliers; and max[*,*] takes on the maximum value of the numbers in the bracket. The modified design variables $\tilde{\mathbf{x}}$ are defined as

$$\tilde{\mathbf{x}} = \left(\frac{E_1}{\alpha_1}, \frac{E_2}{\alpha_2}, \frac{G_{12}}{\alpha_3}, \nu_{12}, \frac{E_e}{\alpha_4}\right) \tag{25}$$

where α_i=normalization factor. It is noted that the choice of proper values of the normalization factors can produce appropriate search directions during the minimization process and thus help expedite the convergence of the solution. A detailed study has shown that the suitable values of $\tilde{x}_i$ $(i=1,\ldots,5)$ are greater than 0 and less than 10. Furthermore, it is worth pointing out that the original design variables are used in the Rayleigh–Ritz method to compute the theoretical natural frequencies for determining the error function in Eq. (23). The update formulas for the multipliers μ_j, η_j, and γ_p are

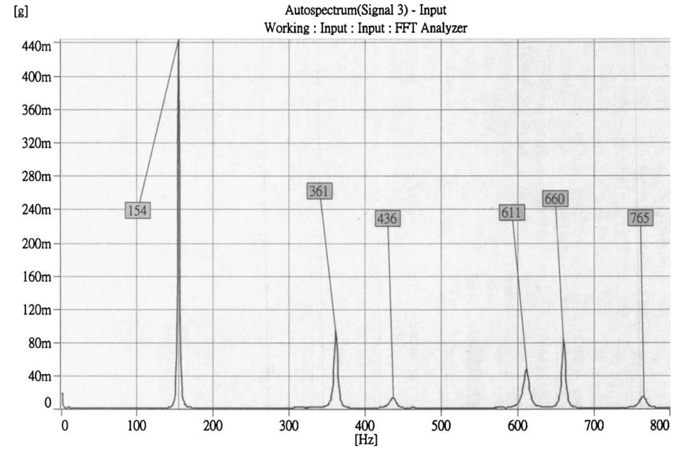

Fig. 6. Frequency response spectrum of $[0°/90°/0°]_{2S}$ plate with A-type supporting condition

$$\mu_j^{n+1} = \mu_j^n + 2r_p^n z_j^n$$

$$\eta_j^{n+1} = \eta_j^n + 2r_p^n \phi_j^n \quad j = 1-5$$

$$r_p^{n+1} = \begin{cases} \gamma_0 r_p^n & \text{if } r_p^{n+1} < r_p^{\max} \\ r_p^{\max} & \text{if } r_p^{n+1} \geqslant r_p^{\max} \end{cases} \tag{26}$$

where the superscript n denotes iteration number; γ_0=constant; and $r_p^{\max}$=maximum value of r_p. The parameters $\mu_j^0, \eta_j^0, r_p^0, \gamma_0, r_p^{\max}$ are chosen as

$$\mu_j^0 = 1.0 \quad \eta_j^0 = 1.0 \quad r_p^0 = 0.4$$

$$\gamma_0 = 2.5 \quad r_p^{\max} = 100 \tag{27}$$

The constrained minimization problem of Eq. (21) has thus become the solution of the following unconstrained optimization problem:

Minimize

$$\bar{\boldsymbol{\Psi}}(\tilde{\mathbf{x}}, \boldsymbol{\mu}, \boldsymbol{\eta}, r_p) \tag{28}$$

The above unconstrained optimization problem is to be solved using a multistart global optimization algorithm. In the adopted optimization algorithm, the objective function is treated as the potential energy of a traveling particle and the search trajectories

Table 1. Experimental Natural Frequencies of Composite Plates

			Natural frequency number											
			1		2		3		4		5		6	
Layup	Geometry	Support condition	Mean	COV (%)	Mean	COV (%)	Mean	COV (%)	Mean	COV (%)	Mean	COV (%)	Mean	COV (%)
$[0°/90°/0°]_{2S}$	Square	A	154	0.3	361	0.47	436	0.36	611	0.14	660	0.21	765	0.16
		B	161	0.0	373	0.25	476	0.14	578	0.34	702	0.16	720	0.63
$[45°/-45°/45°]_{2S}$	Square	A	177	0.45	378	0.44	435	0.42	596	0.23	655	0.37	818	0.07
		B	178	0.24	404	0.21	451	0.27	668	0.23	716	0.29	787	0.16
$[0°]_{12}$	Square	A	152	0.44	241	0.3	386	0.3	416	0.54	601	0.25	616	0.36
		B	158	0.34	244	0.32	405	0.0	535	0.2	561	0.14	600	0.21
$[0°/90°/0°]_{2S}$	Rectangle	C	110	0.0	245	0.22	327	0.16	435	0.18	467	0.28	606	0.15
$[45°/-45°/45°]_{2S}$	Rectangle	C	253	0.39	347	0.15	394	0.23	514	0.41	578	0.27	253	0.22

Note: COV=Coefficient of variation.

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Table 2. Actual Natural Frequencies of Graphite/Exoxy $[0°/90°/0°]_{2S}$ Plate Supported by Elastic Pads (A Type) with Different Rigidities

Young's modulus of elastic pad (MPa)	Spring constant		Natural frequency number					
	K_L (kN/m^2)	K_R (N)	1	2	3	4	5	6
1.0	2.38095	4.96032	151.3163	340.3899	391.3919	575.7812	592.2373	653.2147
15.0	35.7143	74.40476	170.972	394.3665	499.4967	663.1584	786.6243	990.3991

for locating the global minimum are derived from the equation of motion of the particle in a conservative force field (Snyman and Fatti 1987). The design variables, i.e., the plate elastic constants and Young's modulus of the elastic restraints, which make the potential energy of the particle, i.e., the objective function, the global minimum constitutes the solution to the problem. In the minimization process, a series of starting points for the design variables of Eq. (25) are selected at random from the region of interest. The lowest local minimum along the search trajectory initiated from each starting point is determined and recorded. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved. The multistart optimization procedure is terminated when a target probability, typically 0.99, has been exceeded.

Experimental Investigation

In the experimental study, a number of laminated composite plates with different layups were fabricated using T300/2500 graphite/epoxy prepreg tapes produced by Torayca Co., Japan. The dimensions ($a_0 \times b_0 \times h$) of the laminated composite plates were either $22 \times 22 \times 0.15$ cm^3 or $32 \times 22 \times 0.15$ cm^3. The average and coefficient of variation (COV) of the mass density of the plates were 1,548 kg/m^3 and 1.22%, respectively. The elastic constants of the graphite/epoxy composite material were first determined experimentally using the standard specimens in accordance with the revelant ASTM specifications (ASTM 1990). In the material testing, each elastic constant was determined using three specimens. The means and COVs of the elastic constants obtained from the tests are as follows:

$$E_1 = 146.503 \text{ GPa } (0.72\%), \quad E_2 = 9.223 \text{ GPa } (1.19\%)$$

$$G_{12} = 6.836 \text{ GPa } (3.16\%), \quad \nu_{12} = 0.306 \ (0.19\%) \qquad (29)$$

The values in the parentheses in the above equation denote the COVs of the elastic constants of the composite material. Strip-type pads made of foam plastic materials were used to support the laminated composite plates. The elastic constant E_e of the flexible pads was also determined following the standard ASTM tensile testing procedure. The mean and COV of E_e are 2.028 MPa and 2.3%, respectively. The cross-sectional dimensions of the pads were b_e=0.5 cm and h_e=2.1 mm.

Table 3. Identified System Parameters of Graphite/Epoxy $[0°/90°/0°]_{2S}$ Plate Supported by Elastic Pads (A Type) with E_e=1.0 MPa Using Actual Natural Frequencies.

Starting point number	Stage	System parameter					Number of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	
1	Initial	194.538	0.849	16.519	0.21825	4.433	12
	Final	146.5032	9.2231	6.836	0.306	1	
2	Initial	224.703	17.862	0.225	0.41863	13.719	10
	Final	87.7715	3.3359	3.0379	0.4682	19.7024	
3	Initial	234.44	22.888	16.971	0.19005	4.258	12
	Final	146.503	9.2232	6.836	0.306	1.0	
4	Initial	251.766	22.601	7.614	0.10768	14.206	11
	Final	146.503	9.223	6.836	0.306	1.0	
5	Initial	165.37	17.196	3.731	0.09233	10.058	7
	Final	87.7696	3.3364	3.0385	0.4682	19.7025	
6	Initial	163.239	35.29	16.643	0.01602	7.718	11
	Final	146.5037	9.2218	6.8358	0.3061	1.0	
7	Initial	62.819	20.423	7.079	0.26991	12.009	7
	Final	87.7599	3.3384	3.0404	0.4682	19.7024	
8	Initial	52.512	17.035	17.208	0.02261	17.514	9
	Final	87.7607	3.3382	3.0403	0.4682	19.7024	
9	Initial	196.172	25.595	11.252	0.01915	13.345	10
	Final	146.5032	9.2231	6.836	0.306	1.0	
Global minimum		146.503 (0%)	9.223 (0%)	6.836 (0%)	0.306 (0%)	1.0 (0%)	Probability 0.9923

Note: Values in the parentheses denote percentage difference between identified and actual data.

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Table 4. Identified System Parameters of Graphite/Epoxy $[0°/90°/0°]_{2S}$ Plate Supported by Elastic Pads (A Type) with E_e=15.0 MPa Using Actual Natural Frequencies

Starting point number	Stage	System parameter E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	Number of iterations
1	Initial	194.551	21.895	16.967	0.13555	7.055	12
	Final	146.5032	9.2231	6.8361	0.306	15.0	
2	Initial	175.037	25.389	14.534	0.27722	8.494	12
	Final	146.5034	9.2231	6.8364	0.3059	15.0	
3	Initial	48.043	25.994	3.448	0.25919	8.535	13
	Final	146.503	9.223	6.836	0.306	15.0	
4	Initial	299.286	9.985	8.23	0.37452	19.554	10
	Final	146.503	9.223	6.836	0.306	15.0	
Global minimum		146.503 (0%)	9.223 (0%)	6.836 (0%)	0.306 (0%)	15.0 (0%)	0.9921

Note: Values in the parentheses denote percentage difference between identified and actual data.

The laminated composite plates partially supported by the flexible strip-type pads at the plate edges were subjected to impulse vibration testing. The supporting conditions of the composite plates are shown in Fig. 4 in which A- and B-type supporting conditions are for the square $(22\times 22\times 0.15\ \text{cm}^3)$ composite plates while the C-type supporting condition is for the rectangular $(32\times 22\times 0.15\ \text{cm}^3)$ composite plates. In the vibration testing of the flexibly supported composite plate, a hand held impulse hammer (Kistler 9722A500, Kistler Instrument, United States) was used to excite the composite plate at different points on the plate, a force transducer (Kistler 9904A, Kistler Instrument, United States) attached to the hammer's head to measure the input forces, an accelerometer of mass 0.14g (AP19, APTechnology, The Netherlands) to pick up the vibration response data at different locations on the plate, and a data acquisition and analysis system (B&K 3560C and B&K Pulse Labshop, Version 6.1) to process the vibration data from which the natural frequencies of the composite plates were extracted. The experimental setup for the impulse vibration testing of the plates is shown in Fig. 5. It is noted that each flexibly supported composite plate was then tested 15 times. Each time when the plate was tested a set of vibration data was produced for constructing the frequency response spectrum of the plate from which the first six natural frequencies of the plate were extracted. Without loss of generality, it was assumed that the small modal damping ratios with values less than 2% had negligible effects on the first six natural frequencies of the plates. Therefore, in determining the natural frequencies from the frequency response spectra, the frequencies associated with the peak responses were treated as the natural frequencies of the plates. For illustration purposes, Fig. 6 shows a typical frequency response spectrum of the square $[0°/90°/0°]_{2S}$ plate with A-type supporting conditions. It is noted that the first six natural frequencies of the plate can be easily identified from the frequency response spectrum as shown in the figure. The means and COVs of the first six measured natural frequencies of the elastically restrained laminated composite plates with different layups and supporting conditions determined from the impulse vibration testing of the plates are listed in Table 1. It is noted that the COVs of the natural frequencies obtained from the tests are less than 0.45%. The means of the measured natural frequencies of the plates are to be used in the following identification of the plate parameters.

Results and Discussion

Before proceeding to the system identification of the elastically restrained laminated composite plates that have been tested, a number of numerical examples are first given to illustrate the capability of the proposed method in identifying the elastic constants of plates made of different laminated composite materials and partially supported by elastic pads with different rigidities. The first example is the system identification of the square $[0°/90°/0°]_{2S}$ plate with A-type supporting condition shown in Fig. 4. The $[0°/90°/0°]_{2S}$ plate is made of graphite/epoxy composite material with the actual elastic constants just as the means given in Eq. (29). The actual natural frequencies of the $[0°/90°/0°]_{2S}$ plate supported by elastic pads with different Young's moduli are computed and tabulated in Table 2. The actual natural frequencies of the plate are then treated as "measured" natural frequencies in the system parameters identification process. The upper and lower bounds of the system parameters are chosen based on experience

$$0\leqslant E_1\leqslant 400\ \text{GPa};\quad 0\leqslant E_2\leqslant 40\ \text{GPa};\quad 0\leqslant G_{12}\leqslant 20\ \text{GPa}$$

$$0\leqslant \nu_{12}\leqslant 0.5;\quad 0\leqslant E_e\leqslant 20\ \text{MPa} \tag{30}$$

The modified design variables of Eq. (25) are obtained via the use of the following normalization factors:

Table 5. Actual Natural Frequencies of Glass/Epoxy $[0°/90°/0°]_{2S}$ Plate Supported by Elastic Pads (A Type) with Different Rigidities

Young's modulus of elastic pad (MPa)	Spring constant K_L (kN/m^2)	K_R (N)	Natural frequency number 1	2	3	4	5	6
1.0	2.38095	4.96032	87.91181	208.9391	235.5438	344.3321	384.733	428.8373
15.0	35.7143	74.40476	101.7888	236.4288	272.623	382.7989	466.449	557.0408

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Table 6. Identified System Parameters of Glass/Epoxy $[0°/90°/0°]_{2S}$ Plate Supported by Elastic Pads (A Type) with Different Rigidities Using Actual Natural Frequencies

Case	Actual E_e (MPa)	Number of starting points	Average number of iteration	System parameter E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)
1	1.0	7	8	43.5(0%)	11.5(0%)	3.45(0%)	0.27(0%)	1.0(0%)
2	15.0	8	10	43.5(0%)	11.5(0%)	3.45(0%)	0.27(0%)	15.0(0%)

Note: Values in the parentheses denote percentage difference between identified and actual data.

$$\alpha_1 = 100 \quad (31a)$$

and

$$\alpha_i = 10 \quad i = 2-4 \quad (31b)$$

In view of Eqs. (30) and (31), the feasible intervals of the modified design variables are thus obtained as [0, 4], [0, 4], [0, 2], [0, 0.5], and [0, 2] for E_1, E_2, G_{12}, ν_{12}, and E_e, respectively. Since the modified design variables are less than 10, the search for the solution can be expedited. For the case where the elastic pads have E_e=1.0 MPa, nine starting points are randomly generated and for each starting point around 7–12 iterations are required to locate the lowest local minimum. The starting points, the lowest local minima for the starting points, numbers of iterations required to obtain the lowest local minima, and the global minimum are listed in Table 3. It is noted that the actual system parameters have been identified with probability greater than 0.99. Similarly, for the case where the elastic pads have E_e=15 MPa, the results are listed in Table 4. Again it is noted that the actual system parameters have also been identified in an efficient and effective way where only four starting points have been randomly generated and around 12 iterations required to locate the lowest local minima for the starting points. Next consider the system identification of the square $[0°/90°/0°]_{2S}$ plate with same supporting condition but made of glass/epoxy composite material of which the properties are as follows:

$$E_1 = 43.5 \text{ GPa}, \quad E_2 = 11.5 \text{ GPa}, \quad G_{12} = 3.45 \text{ GPa}, \quad \nu_{12} = 0.27, \quad \rho = 2{,}000 \text{ kg/m}^3 \quad (32)$$

The actual natural frequencies of the glass/epoxy $[0°/90°/0°]_{2S}$ plate supported by the elastic pads with different Young's moduli are computed and listed in Table 5. The actual natural frequencies treated as the "measured" ones together with the bounds in Eq. (30) and normalization factors in Eqs. (31) are used in the present method to identify the system parameters of the glass/epoxy plate supported by the elastic pads with different rigidities. The estimates of the system parameters together with the numbers of starting points and the average numbers of iterations required to obtain the global minima for the cases under consideration are listed in Table 6. It is noted that the present method is also able to produce the actual system parameters in an efficient and effective way for the flexibly supported glass/epoxy $[0°/90°/0°]_{2S}$ plate.

Now the present method is applied to the system identification of the elastically restrained laminated composite plates which have been tested. The measured natural frequencies of the $[0°/90°/0°]_{2S}$ plate with A-type supporting conditions in Table 1 are used as an example to illustrate the system identification process of the present method. Table 7 lists the randomly generated starting points, the lowest local minima obtained for the starting points, the numbers of iterations required for getting the lowest local minima, and the global minimum. It is noted that only six starting points are needed to obtain the global minimum with

Table 7. System Identification of Square $[0°/90°/0°]_{2S}$ Plate with A-Type Supporting Condition Using Experimental Natural Frequencies

Starting point number	Stage	System parameter E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)	Number of iterations
1	Initial	44.645	34.586	4.881	0.42941	5.985	8
	Final	141.164	8.982	6.982	0.30002	2.211	
2	Initial	160.933	0.683	17.640	0.37422	3.853	8
	Final	141.162	8.982	6.983	0.30001	2.211	
3	Initial	238.425	0.880	19.186	0.03901	4.472	11
	Final	141.158	8.983	6.984	0.30001	2.211	
4	Initial	55.035	16.731	17.782	0.03920	16.305	10
	Final	109.693	5.521	3.529	0.30015	18.005	
5	Initial	51.864	7.121	0.005	0.17907	6.870	10
	Final	141.167	8.981	6.982	0.30001	2.211	
6	Initial	213.219	36.831	9.558	0.33293	1.689	5
	Final	141.171	8.980	6.981	0.30002	2.211	
Global minimum		141.167 (−3.64%)	8.981 (−2.62%)	6.982 (2.14%)	0.30001 (−1.96%)	2.211 (9.02%)	Probability 0.9959

Note: Values in the parentheses denote percentage difference between identified and actual data.

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Table 8. Identified System Parameters of Plates Subjected to Impulse Vibration Testing

		Identified system parameter				
Layup	Support condition	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	E_e (MPa)
$[0°/90°/0°]_{2S}$	B	141.896	9.041	7.045	0.30035	1.984
		(−3.14%)	(−1.97%)	(3.06%)	(−1.85%)	(−2.17%)
$[45°/-45°/45°]_{2S}$	A	148.425	8.505	6.506	0.30002	1.992
		(1.31%)	(−7.78%)	(−4.83%)	(−1.95%)	(−1.78%)
	B	144.040	9.853	6.994	0.29998	2.021
		(−1.68%)	(6.83%)	(2.31%)	(−1.97%)	(−0.35%)
$[0°]_{12}$	A	148.983	9.817	7.005	0.30015	1.965
		(1.69%)	(6.44%)	(2.47%)	(−1.91%)	(−3.11%)
	B	143.770	9.447	7.499	0.30421	1.901
		(−1.87%)	(2.43%)	(9.70%)	(−0.58%)	(−6.26%)
$[0°/90°/0°]_{2S}$	C	142.187	9.750	6.355	0.29751	2.144
		(−2.95%)	(5.71%)	(−7.04%)	(−2.77%)	(5.72%)
$[45°/-45°/45°]_{2S}$	C	143.608	9.326	6.199	0.29996	1.996
		(−1.98%)	(1.12%)	(−9.32%)	(−1.97%)	(−1.58%)

Note: Values in parentheses denote percentage difference between predicted and measured data.

probability exceeding 0.99, the numbers of iterations for getting the lowest local minima are less than or equal to 11, and the percentage differences between the actual and identified system parameters are less than or equal to 9.02%. In a similar way, the system parameters of the other flexibly supported laminated composite plates that have been tested can also be identified using the measured natural frequencies listed in Table 1. The identified system parameters and their associated percentage errors for the tested plates are listed in Table 8. Again, it is noted that very good estimates of the system parameters with percentage differences less than or equal to 9.7% have also been obtained for the plates. It is also noted that for all the tested plates, better estimates of E_1 and ν_{12} of which the percentage differences are less than or equal to 3.64% can be obtained. It is worth pointing out that the differences between the identified and actual system parameters of the plates that have been tested in this study may be due to improper modeling of the flexibly supported plates and existence of noise in the measurement data. Regarding the modeling of the elastic edge pads, the mechanics of materials approach adopted to calculate the spring constants of the edge supports might be oversimplified in simulating the actual behaviors of the edge supports. Therefore, a more sophisticated method such as the finite element method may be needed to model the elastic edge supports if more accurate results are desired. On the other hand, the effects of measurement noise on the identified system parameters and the sources of the noise should be studied in detail so that the measurement noise incurred in the vibration testing of the plates can be suppressed or the detrimental effects of the measurement noise can be minimized or even eliminated.

Conclusions

A nondestructive evaluation method for system identification of laminated composite plates partially supported by elastic pad-like restraints using six experimentally measured natural frequencies of the plates have been presented. The system identification process in the proposed method has included the utilization of the Rayleigh–Ritz method for predicting the theoretical natural frequencies of the elastically restrained laminated composite plates using trial values of the system parameters, the construction of the error function that measures the sum of the differences between the experimental and theoretical predictions of the system natural frequencies, the use of a multistart global minimization method to identify the elastic constants by making the error function a global minimum, and a design variables normalization technique for expediting the convergence of the search of the global minimum. Both numerical and experimental investigations have been conducted to demonstrate the capability, effectiveness, accuracy, and applications of the proposed method. In the numerical study, it has been shown that the proposed method can identify the actual system parameters of partially and elastically restrained plates made of different laminated composite materials in an efficient and effective way. In the experimental investigation, several flexibly restrained laminated composite plates with different layups and supporting conditions have been subjected to impulse testing and the first six natural frequencies of the plates have been measured. The uses of the measured natural frequencies in the proposed method have also produced very good estimates of the system parameters for the laminated composite plates with percentage differences less than or equal to 3.64% for E_1, 7.78% for E_2, 9.70% for G_{12}, 2.77% for ν_{12}, and 9.02% for E_e.

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References

ASTM. (1990). *Standards and literature references for composite materials*, 2nd Ed., West Conshohocken, Pa.

Berman, A., and Nagy, E. J. (1983). "Improvement of a large analytical model using test data." *AIAA J.*, 21, 1168–1173.

Castagnède, B., Jenkins, J. T., Sachse, W., and Baste, S. (1990). "Optimal determination of the elastic constants of composite materials from ultrasonic wave-speed measurements." *J. Appl. Phys.*, 67,

2753–2761.
Deobald, L. R., and Gibson, R. F. (1988). "Determination of elastic constants of orthotropic plates by a modal analysis/Rayleigh-Ritz technique." *J. Sound Vib.*, 124(2), 269–283.
Fallstrom, K. E., and Jonsson, M. (1991). "A nondestructive method to determine material properties in anisotropic plate." *Polym. Compos.*, 12(5), 293–305.
Hung, K. C., Lim, M. K., and Liew, K. M. (1993). "Boundary beam characteristics orthonormal polynomials in energy approach for vibration of symmetric laminates. II: Elastically restrained boundaries." *Compos. Struct.*, 26, 185–209.
Kam, T. Y., and Lee, T. Y. (1994). "Crack size identification using an expanded mode method." *Int. J. Solids Struct.*, 31, 925–940.
Kam, T. Y., and Liu, C. K. (1998). "Stiffness identification of laminated composite shafts." *Int. J. Mech. Sci.*, 40(9), 927–936.
Lubin, G. (1982). *Handbook of composites*, van Nostrand Reinhold, London.
Moussu, F., and Nivoit, M. (1993). "Determination of elastic constants of orthotropic plates by a modal analysis/method of superposition." *J. Sound Vib.*, 165(1), 149–163.
Nielsen, S. A., and Toftegaard, H. (1998). "Ultrasonic measurement of elastic constants in fiber-reinforced polymer composites under influence of absorbed moisture." *Int. J. Ultrasonic*, 38, 242–246.
Qian, G. L., Hoa, S. V., and Xiao, X. (1997). "A vibration method for measuring mechanical properties of composite, theory and experiment." *Compos. Struct.*, 39(1–2), 31–38.
Schwartz, M. M. (1983). *Composite materials handbook*, McGraw-Hill, New York.
Snyman, J. A., and Fatti, L. P. (1987). "A multi-start global minimization algorithm with dynamic search trajectories." *J. Optim. Theory Appl.*, 54(1), 121–141.
Sol, H., Hua, J., Visscher, J., Vantomme, J., and Wilde, W. P. (1997). "A mixed numerical/experimental technique for the nondestructive identification of the stiffness properties of fibre reinforced composite materials." *NDT & E Int.*, 30(2), 85–91.
Vanderplaats, G. N. (1984). *Numerical optimization techniques for engineering design with applications*, McGraw–Hill, New York.
Wang, W. T., and Kam, T. Y. (2000). "Material characterization of laminated composite plates via static testing." *Compos. Struct.*, 50(4), 347–352.

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