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Control of the spin Hall current in two dimensional electronic gas

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The intrinsic spin Hall conductivity is obtained for a two-dimensional electronic gas (2DEG) in the presence of strain, Rashba coupling, and an external in-plane applied magnetic field. The conduction electrons of [001] oriented quantum well are used to model the 2DEG. The spin current value is dependent on the stress applied in direction [111]. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2234742]

The spin current in the spin Hall effect means a flow of spin angular momentum perpendicular to an applied electrical field with a spin accumulation of opposite magnetization at each edge of the sample and no net charge current. The spin Hall conductivity (SHC) has both *intrinsic* (absence of scattering) and *extrinsic* (induced by scattering) contributions. The intrinsic spin Hall effect was recently predicted as being generated by the splitting of conduction or valence bands, which is induced by intrinsic spin-orbit interactions.[1–3] The spin Hall effect has been detected in experiments dealing with strain effect,[4] and there are reports which place the spin Hall current in the intrinsic regime.[5–8] It seems that a general agreement exists regarding the vanishing of the dc SHC of two-dimensional electronic gas (2DEG) with **k**-linear Rashba coupling and parabolic dispersion in the absence of magnetic field and the presence of scattering, even in the limit of weak disorder.[9,10] On the other hand, in clean samples (where the transport scattering rate τ^{-1} is small compared to the spin-orbit splitting Δ expressed in time units) modeled by a Rashba (**k**-linear) spin-orbit coupling, one finds an intrinsic dc SHC value of $e/8\pi$, independent of details of the impurity scattering, for ac electric field of frequency ω in the range $\tau^{-1}<\omega<\Delta$, in the usual case where both spin-orbit split bands are occupied.[7,10,11] Avoiding speculation that the above intrinsic SHC value would generally be obtained for all **k**-linear coupling systems, we calculate in this work the intrinsic SHC of a specific **k**-linear coupling system, namely, a 2DEG in the presence of strain, Rashba spin coupling, and in-plane magnetic field. Though a more realistic model would involve the presence of impurities and consideration of finite-size effect and of electron-electron interaction, clean samples are considered as experiencing prominently an intrinsic spin Hall effect[7,8] and thus, this theoretical work is motivated from experimental point of view. The 2DEG is obtained using a simple quantum-well (QW) model.[12] We consider for discussion the case of [001]-oriented QW and stress applied in [111] direction.

Firstly, the model of 2DEG is described. Secondly, the intrinsic SHC is calculated with numerical data matching InAs semiconductor. Thirdly, conclusions regarding the spin current are drawn.

For the electron Hamiltonian in the conduction band, we consider the following Hamiltonian:

$$H=\frac{\mathbf{P}^2}{2m_c}+V(z)+H_R+H_P+H_B, \tag{1}$$

where the first two terms describe the orbital motion of the electron confined in the z direction (perpendicular on the xy QW plane) by the $V(z)$ potential. In Eq. (1) e is the elementary charge, m_c is the electron effective mass, $\mathbf{P}=\mathbf{p}+e\mathbf{A}$, with $\mathbf{p}=-i\hbar(\partial/\partial x,\partial/\partial y,\partial/\partial z,)$ the canonical electron momentum, $\mathbf{A}=zB(\sin\theta,-\cos\theta,0)$ is the vector potential, B is the amplitude of magnetic field $\mathbf{B}$, θ is the angle between $\mathbf{B}$ and x, and x is the direction of the electric field. $H_R=\hbar\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_R/2$, with $\boldsymbol{\Omega}_R=\alpha_R(\mathbf{P}\times\mathbf{n})/\hbar$, is the Rashba spin-orbit coupling, α_R is the Rashba coupling factor, $\mathbf{n}$ is the unit vector of the z direction, and σ_i with $i=x,y,z$ are the Pauli matrices in the z representation. $H_P=\hbar\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_P/2$, with $\boldsymbol{\Omega}_{Px}=\lfloor a(\varepsilon_{xy}P_x-\varepsilon_{xz}P_z)+b\lfloor P_x(\varepsilon_{yy}-\varepsilon_{zz})\rfloor\rfloor/\hbar$, is the bulk Pikus interaction (responsible for the strain effect) written for [001]-oriented QW, a and b are constants which determine the magnitude of the splitting,[13] and ε_{ij} is the ij component of the strain tensor. Strain applied in directions [001] and [110] does not yield a σ_z component of the strain Hamiltonian (an effective magnetic field in the z direction). On the other hand, strain applied in [011], [101], or [111] direction induces such a component. The strain Hamiltonian for [011] or [101] direction, even for the simpler case $\mathbf{B}=0$, introduces an additional parameter, e.g., $e_{xxzz}=b(\varepsilon_{xx}-\varepsilon_{zz})/2$ for [011] direction of the applied stress (for this case the Hamiltonian reads $H_P=-e_{xxzz}p_y\sigma_y-e_{yz}p_y\sigma_z$). In the case of [111] direction of the applied stress the only one necessary strain-dependent parameter e_{xy} facilitates discussion and interpretation of possible experimental results too. This case of [111] direction of applied stress, where $\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}$ and $\varepsilon_{xy}=\varepsilon_{yz}=\varepsilon_{xz}$ (Ref. 14), is considered by this work. $H_B=\beta_x\sigma_x+\beta_y\sigma_y$ with $\beta_x=2^{-1}\mu_B gB\cos\theta$, $\beta_y=2^{-1}\mu_B gB\sin\theta$ is the Zeeman term, g is the effective g factor, and $\mu_B=e\hbar/(2m_0)$ is the Bohr magneton. For a less computational effort we consider a semiparabolic confinement potential shape in the z direction, namely, $V(z)=m_c\omega_E^2z^2/2$ for $z\geqslant 0$ and $V(z)=\infty$ for $z<0$; the Schrödinger equation for the orbital motion in the z direction yields wave functions of harmonic oscillator, confined by $V(z)$, having a displaced origin of the z axis.[15] Noticing that momenta p_x and p_y are constants of the motion for H given

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by Eq. (1), the *effective* Hamiltonian $\bar{H}$ is obtained by a quantum average over the wave function of the ground state of the orbital motion in the z direction $\varphi_1(z)$ and of the free motion in the xy plane. The effective Hamiltonian reads

$$\bar{H}=\hbar^2(k_x^2+k_y^2)/2m_c+3\hbar\omega_0/2+\xi_x\sigma_x+\xi_y\sigma_y+\xi_z\sigma_z,$$

where $\xi_x=\gamma_x+\alpha_R\hbar(1+r_f)k_y$, $\xi_y=\gamma_y-\alpha_R\hbar(1+r_f)k_x$, $\xi_z=\alpha_R r_f[\hbar(k_x-k_y)+(\alpha_s-\alpha_c)\bar{z}]$ with $\gamma_x=\beta_x+\alpha_R(1+r_f)\alpha_c\bar{z}$, $\gamma_y=\beta_y-\alpha_R(1+r_f)\alpha_s\bar{z}$, $\alpha_c=eB\cos\theta$, $\alpha_s=eB\sin\theta$, $r_f\equiv\alpha\varepsilon_{xy}/2\alpha_R$, and $\omega_0^2=\omega_E^2+\omega_c^2$, with $\omega_c=eB/m_c$. The magnitude of r_f may be modified by adjusting the values of stress. The electron spin precession around the momentum-dependent effective magnetic field is a useful tool for a mental visualization of the spin Hall effect. In our discussion the effective magnetic field in the z direction changes the explicit form of Bloch equations (as proposed by Ref. 2) used to find the SHC. A supplementary tilt of the precession axes (absent in the case of strain applied in [001] or [110]) has an impact on the value of spin Hall current.

The present analysis of the spin Hall effect of electrons in QW structures is based on the Kubo formalism for a spatially homogenous electric field.[16] Next, we provide an expression of the spin Hall conductivity for the Hamiltonian $\bar{H}$ in the *narrow* QW limit, $\bar{z}\to 0$. The narrow QW limit case can capture the physics of the problem[17] and the amount of algebra necessary to solve this case is moderate. The corresponding Hamiltonian, which models the present problem, reads

$$\bar{H}=\frac{\hbar^2(k_x^2+k_y^2)}{2m_c}+\frac{3\hbar\omega_0}{2}+[\beta_x+\alpha_R\hbar(1+r_f)k_y]\sigma_x+[\beta_y-\alpha_R\hbar(1+r_f)k_x]\sigma_y+\alpha_R r_f\hbar(k_x-k_y)\sigma_z. \quad (2)$$

For zero temperature and noninteracting conduction-band electrons the spin Hall conductivity is given by

$$\sigma_{xy}^{Sz}(\omega)=\frac{e\hbar}{A}\sum_{\mathbf{k},\mu\neq\mu'}(f_{\mu,\mathbf{k}}-f_{\mu',\mathbf{k}})\times\frac{\mathrm{Im}[\langle\mathbf{k},\mu|j_x^{S,z}(t)|\mu',\mathbf{k}\rangle\langle\mathbf{k},\mu'|\mathrm{v}_y|\mu',\mathbf{k}\rangle]}{[E_\mu(\mathbf{k})-E_{\mu'}(\mathbf{k})][E_{\mu'}(\mathbf{k})-E_\mu(\mathbf{k})-\hbar\omega-\mathrm{i}\eta]}. \quad (3)$$

where $f_{\mu,\mathbf{k}}$ is the $T=0$ K Fermi distribution function for energy $E_\mu(\mathbf{k})$ at wave vector $\mathbf{k}$ in a dispersion surface labeled by $\mu=\pm$, and A the xy area. The velocity operators are given by $\mathrm{v}=i[\langle H\rangle,\mathbf{r}]/\hbar$, where $\mathbf{r}$ is the position operator and $\langle\cdots\rangle$ means a quantum average over $\varphi_1(z)$. The spin current operator for the spin moment polarized along the z direction and flowing in the y direction when an electric field is applied in the x direction is given by the generally accepted expression $j_{xy}^{S,z}=4^{-1}(\sigma_z v_x+\mathrm{v}_x\sigma_z)$. The eigenvalues $E_\pm(\mathbf{k})$ and eigenvectors $|\mathbf{k},\pm\rangle$ of Hamiltonian $\bar{H}$ are as follows:

$$E_\pm(\mathbf{k})=\frac{\hbar^2k^2}{2m_c}+\frac{3\hbar\omega_0}{2}\pm\sqrt{\Lambda_1+\Lambda_2}, \quad (4a)$$

where $\Lambda_1=[\alpha_R\hbar(1+r_f)]^2(k_x-k_y)^2$, $\Lambda_2=[\alpha_R\hbar(1+r_f)]^2k^2+4^{-1}(\mu_B gB)^2+\alpha_R(1+r_f)\mu_B gB(k_y\cos\theta-k_x\sin\theta)\geqslant 0$, and

$$|\mathbf{k},\pm\rangle=\frac{1}{\sqrt{1+\rho_\pm^2}}\begin{bmatrix}\rho_\pm\exp(i\delta_\pm)\\1\end{bmatrix}, \quad (4b)$$

where $\rho_\pm=|\alpha_R\hbar(1+r_f)(k_x-k_y)\mp\sqrt{\Lambda_1+\Lambda_2}/\sqrt{\Lambda_2}|$ and $\sin\delta_\pm=-[\alpha_R\hbar(1+r_f)(k_x-k_y)\mp\sqrt{\Lambda_1+\Lambda_2}][\alpha_R\hbar(1+r_f)k_x-\beta_x]/\sqrt{\Lambda_2}\rho_\pm$.

A general analytical expression of the spin Hall conductivity may be obtained in the framework of the above 2DEG model. The presence of stress in [111] direction makes the two surfaces $E_\pm(\mathbf{k})$ crossing only for particular orientations of the magnetic field, namely, $\theta=3\pi/4$ or $\theta=-\pi/4$ at $\mathbf{k}_0=(k_{0x},k_{0y})$. These magnetic field orientations are obtained by imposing $\Lambda_1(\mathbf{k}_0)=\Lambda_2(\mathbf{k}_0)=0$ in Eq. (4a).[18] For the following discussion, we will consider this crossing surface case. With the translation of the origin defined by $\mathbf{k}=\mathbf{K}+\mathbf{k}_0$, the crossing of the surfaces $E_\pm(\mathbf{K})$ in the $\mathbf{K}$ frame by the Fermi energy E_F yields contours which are found with

$$K_{F\pm}^\pm=\frac{-A_{F\pm}\pm\sqrt{A_{F\pm}^2-4B_F}}{2}, \quad (5)$$

where $A_{F\pm}=\sqrt{2}k_0(\cos\alpha+\sin\alpha)\pm 2\hbar^{-1}m_c\alpha_R(1+r_f)\sqrt{r_f^2(1+r_f)^{-2}(1-\sin 2\alpha)+1}$, $B_F=-2\hbar^{-2}m_e(E_F-3\hbar\omega_0/2)+k_0^2$, and α is the polar angle in the $\mathbf{K}$ frame and $k_0=|\mathbf{k}_0|=\mu_B gB/[2\hbar\alpha_R(1+r_f)]$. As $K_{F\pm}^\pm$ must be positive real, condition $A_{F\pm}^2-4B_F\geqslant 0$ must be fulfilled. For the simplest case, $B_F<0\Leftrightarrow E_F>\hbar^2k_0^2/2m_c$, with Eq. (5) one finds the integral contour defined by $K\in[\min K_{F\pm}^+,\max K_{F\pm}^+]$. The intrinsic dc SHC obtained with Eq. (3), within the limits, $\eta\to 0$, and then $\omega\to 0$ is

$$\sigma_{xy}^{Sz}=\frac{e\hbar^3\alpha_R(1+r_f)}{Am_c}\times\sum_{\mathbf{k}}(f_{+\mathbf{k}}-f_{-\mathbf{k}})\frac{k_x[\rho_+(1+\rho_-^2)\sin\delta_+-\rho_-(1+\rho_+^2)\sin\delta_-]}{[E_+(\mathbf{k})-E_-(\mathbf{k})]^2(1+\rho_+^2)(1+\rho_-^2)}. \quad (6)$$

The integral form of Eq. (6) in the $\mathbf{K}$ variable for the crossing surfaces case reads

$$\sigma_{xy}^{Sz}=\frac{e\hbar}{16\pi^2m_c\alpha_R(1+r_f)}\times\int_0^{2\pi}d\alpha\int_{\min K_{F\pm}^+}^{\max K_{F\pm}^+}\frac{dK}{K}\frac{\cos\alpha(k_{0x}+K\cos\alpha)}{r_g^2(1-\sin 2\alpha)+1}\times\left[\frac{r_g(\sin\alpha-\cos\alpha)-\sqrt{r_g^2(1-\sin 2\alpha)+1}}{1+\rho_-^2}-\frac{r_g(\sin\alpha-\cos\alpha)+\sqrt{r_g^2(1-\sin 2\alpha)+1}}{1+\rho_+^2}\right], \quad (7)$$

where $r_g\equiv r_f(1+r_f)^{-1}$. In the limits $B\to 0$ and $r_f\to 0$, the model Hamiltonian from Eq. (2) describes the 2DEG with $\mathbf{k}$-linear Rashba coupling and parabolic dispersion, and the expression of intrinsic dc SHC from Ref. 15 is recovered.

The plane of Fermi energy, which determines the integration contour in Eq. (7), is chosen as being situated above the crossing point of the branches (in this case integration is independent of E_F, see Ref. 19). An energy description of the problem can be found in Ref. 15. Figure 1 shows the intrinsic dc SHC obtained by integration of Eq. (7). We find that the intrinsic dc SHC is practically independent of the in-plane magnetic field for usual values of the magnetic field, $B\in[0,10]$ T. As shown in Fig. 1 the intrinsic dc SHC may be

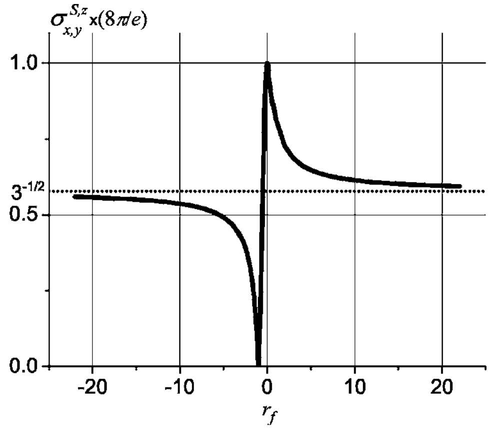

FIG. 1. Variation of intrinsic dc SHC with the parameter r_f. The Fermi energy $E_F=2.7\hbar^2k_0^2/(2m_c)+3h\omega_0^2/2$ is situated above the crossing point of the two branches $E_{\pm}(\mathbf{K})$. The following numerical values are used: $B=5$ T, $g=15$, $m_c=0.024m_0$ (m_0 is the electron mass), $\alpha_R=91\,156$ m/s, $r_f=0.5$, $B=10$ T, and $\theta=3\pi/4$.

changed by applying stress in direction [111]. Thus, the spin current ranges between 0, for $\sigma_{xy}^{Sz}=0$, and maximum value, for $\sigma_{xy}^{Sz}=e/8\pi$. From experimental point of view, an important conclusion is that by variation of r_f (induced by variation of stress) in the interval $[-1,0]$ the spin current is modified from zero to the maximum value in clean QW samples. On the other hand, for values of r_f out of the interval $[-1,0]$ the variation of the intrinsic dc SHC decreases with the applied stress. Consequently, the accuracy of controlling the spin Hall current also decreases with the applied stress. The model described by the Hamiltonian of Eq. (2) predicts [by an analytical integration of Eq. (7)] that $\sigma_{xy}^{Sz}(r_f\to\pm\infty)=e/(\sqrt{3}8\pi)$.

Interesting is the fact that the anisotropy itself of the dispersion branches is not *sufficient* to induce a strain-dependent spin Hall current. In Ref. 20, for a similar model excepting stress presence, but considering scattering effect, increasing electron density (corresponding to increasing Fermi energy) yields a less pronounced variation of the dc SHC (intrinsic plus extrinsic component) with the in-plane magnetic field. This is not in contradiction with the independency of the intrinsic dc SHC of **k**-linear Hamiltonian without effective magnetic field in the z direction, when the Fermi energy level is situated above the crossing point.[15,19] The z component of the effective magnetic is *necessary* to obtain a strain-dependent value of intrinsic SHC. On the other hand, a remarkable *analogy* between the effect of strain on SHC of 2DEG and strain-induced spin relaxation of the electrons of conduction band for the bulk case may be observed: a stress applied in [001] direction has an effect on neither intrinsic dc SHC nor spin relaxation time (as calculated in Ref. 13), but the stress applied in direction [111] affects both quantities. As the two phenomena, the spin Hall effect and spin relaxation are considered for QW and bulk, respectively, this analogy does not hold for the directions [011], [101], and [110]. The three directions are equivalent for the bulk treatment of spin relaxation, but not for the QW case involved by our discussion on the spin Hall effect. Only directions [011] and [101] induce an effective magnetic field in the z direction and consequently can generate a strain-dependent intrinsic dc SHC.

In conclusion, for a 2DEG, in the presence of Rashba coupling and the absence of Dresselhaus coupling, we predict that the intrinsic dc SHC is dependent on both the in-plane magnetic field and applied stress. For [001] direction of QW generating the 2DEG, the intrinsic dc SHC changes between 0 and the universal constant, $\sigma_{xy}^{S,z}=e/8\pi$ as a function of the magnitude of stress applied in [111] direction, when the in-plane magnetic field is oriented at angles $\theta=3\pi/4$ and $-\pi/4$ from the dc electric field direction. Consequently, the magnitude of the intrinsic spin Hall current may be controlled by applying stress in direction [111].

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