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Tool replacement for production with a low fraction of defectives

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In manufacturing industry, the tool replacement cost is, in many cases, a significant portion of the production cost. Early tool replacement increases the production cost. Overdue tool replacement, however, results in poor production quality. Accordingly, improving production quality while maintaining a low production cost is essential. The index C_{pk} is regarded as a yield-based index. For a fixed C_{pk} value, the production yield and fraction of defectives can be calculated. In this paper, we present an analytical approach using C_{pk} to determine the optimal tool replacement time. An accurate process capability must be calculated, particularly when the data contain assignable cause variation. Tool wear is a dominant and inseparable component in many machining processes (a systematic assignable cause), and ordinary capability measures become inaccurate because process data are contaminated by the assignable cause variation. Considering process capability changes dynamically, an estimator of C_{pk} is investigated. The closed form of the exact sampling distribution is derived. An effective tool management procedure for determining the optimal tool replacement time is presented for processes with a low fraction of defectives. For illustrative purposes, an application example involving tool wear is presented.

Keywords: Assignable cause; Critical value; Ordinary least square estimate; Process capability index; Tool replacement; Tool wear

1. Introduction

Tool wear control is an important component in many manufacturing factories in order to produce quality products. One of the most important aspects of tool management is the tool replacement policy. With ongoing manufacturing activities, the tool will eventually wear down. While such wear is unavoidable, it must be controlled in order to maintain product quality and efficient tool utilization. The process capability index is now a common language for quantifying process performance, conveying critical information regarding the suitability of a manufacturing process for the required quality standards. Production yield is one of the commonly used criteria for measuring process capability. In practice, a minimal capability requirement would be preset by the customers/engineers. If the prescribed minimum capability fails to be met due to severe tool wear, one would conclude

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that the process is incapable and a tool replacement activity must be initiated. Process capability analysis is applied to determine the optimal tool replacement time. The proposed approach is useful, particularly for processes with a low fraction of defectives requiring low production cost and stringent quality standards.

Process capability analysis has become an important integrated part in applications of statistical process control to the continuous improvement of quality and productivity. The relationship between the actual process performance and the specification limits or tolerance may be quantified using appropriate process capability indices (PCIs). The use of PCIs in industry did not begin in the United States until the early 1980s. Soon after, this explosion of use was expanded into various industries, such as automotive, semiconductor and IC assembly manufacturing, to determine production quality in order to meet stringent customers' specifications. These indices quantify process performance by taking into consideration the process location, process variation, and manufacturing specifications, which reflect process consistency, process accuracy, process yield, and process loss. Four basic capability indices have been defined (Kane 1986, Chan *et al.* 1988, Pearn *et al.* 1992):

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL and LSL are the upper and lower specification limits, T is the target value, and μ and σ are the process mean and the standard deviation of the characteristic, respectively. In the literature, several authors have promoted the use of various process capability indices and have examined their associated properties with different degrees of completeness. Examples include Kushler and Hurley (1992), Rodriguez (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Vännman (1995) Bothe (1997), Spiring (1997), Kotz and Lovelace (1998), Palmer and Tsui (1999), Pearn and Shu (2003), Vännman and Hubele (2003), and references therein. Kotz and Johnson (2002) presented a compact survey for the development of process capability indices with interpretations and comments on some 170 publications appearing during 1992–2000. Spiring *et al.* (2003) consolidated the research findings in the field of process capability for the period 1990–2002.

The assessment of process capability appears to be easy and straightforward to apply. However, some assumptions must be made before applying the capability indices. Those assumptions include (a) the process under investigation must be free from any special or assignable causes (i.e., in control), (b) the process characteristics must follow normal distributions, and (c) the observed values of the quality characteristics must be statistically independent. If the process is out of control in the early stages, it will be unreliable and meaningless to estimate the process capability. Porter and Oakland (1991) pointed out that the two conditions making process capability assessment difficult are: ensuring stability of the mean and standard deviation, and absence of special causes. It is also assumed that observations are statistically independent, which is not always the case in reality. Processes with an uncontrollable, but acceptable, trend, however, are common in practice. Tool wear is an example of a common assignable cause inducing autocorrelation that is physically unavoidable. There are various techniques in current practice used to assess process capability in the presence of an assignable cause. Some approaches attempt to remove the variability associated with the systematic cause. For example, Montgomery (1985) suggested fitting a first-order autoregressive model to the correlated data. Yang and Hancock (1990) recommended that, in computing the C_p index, the unbiased estimator of σ can be obtained as $\sigma/(1-\rho)^{1/2}$, where ρ is defined as the average correction factor. Time series modelling to trended data is also suggested by Alwan and Roberts (1988), who recommend using residuals in monitoring the process. Furthermore, others make the general assumption of linear degradation of the tool. For instance, the procedure suggested by Long and De Coste (1988) removes the linearity by regressing on the means of the subgroups and then determines the process capability. Quesenberry (1988) suggested using regression techniques to handle the tool wear over an interval of tool life assuming that the tool wear rate is known or a good estimate is available.

The existing approaches assume a static process capability over a cycle. Considering that process capability changes dynamically within a cycle, as well as from cycle to cycle, we could circumvent some of the problems encountered. Spiring (1991) considered an application of assessing the process capability index C_{pm} in the presence of a systematic assignable cause that results in a numerical measure of the actual process capability. Although the yield-based index C_{pk} has been widely used in manufacturing industry, existing research has never considered situations with systematic assignable causes. In this paper, we consider the index C_{pk}. A modified estimator of C_{pk} is proposed and an explicit form of the sampling distribution is derived for situations where a systematic assignable cause occurs. In addition, we develop a procedure for choosing the time point for tool replacement. Practitioners can use the proposed procedure to determine whether their process meets the preset capability requirement, and make reliable decisions regarding the optimal tool replacement time.

2. The tool wear problem

It is known that process capability can be assessed only when the process data are statistically independent. The issues of correlation among the samples and its effect on the control chart limits have been discussed by many authors (Vasilopoulos and Stamboulis 1978, Burr 1979). However, the effect of correlation on estimating process capability has been neglected. There are some situations where the assignable causes are systematic, such as tool wear, in which the effects can be decomposed before capability is estimated (often referred to as a constant or consistent process drift). Other examples include accumulation of contaminants and temperature change drift, which must be removed before the natural variability can be analysed for process control purposes (Kotz and Lovelace 1998).

When systematic assignable causes are present and tolerated, the overall variation of the process (σ^2) is composed of variation due to random causes (σ_r^2) and variation due to assignable causes (σ_a^2). That is, $\sigma^2 = \sigma_r^2 + \sigma_a^2$. The capability measures fail to decouple the portion of the overall variation, in the presence of tool wear, contributed by the assignable causes. Consequently, any estimates of process capability will confound the true capability calculations. In order to obtain accurate measures of process capability, any variation due to assignable cause must be removed. Spiring (1989, 1991) viewed this as a dynamic process that is constantly changing as the process tools age, etc. In the dynamic model, the capability of the process will vary, possibly in a predictable fashion. Spiring has devised a modification of the C_{pm} index for this dynamic process under the influence of systematic assignable causes. In the scenario, the goal is to maintain some minimum level of capability at all times to ensure production quality. As a result, the capability will be cyclical in nature, its period defined by the frequency of process/tolling adjustments. Even when assignable cause variation is not systematic, as is the case with tool or die wear, there is a need to be able to deal with random fluctuations of the process mean over time. Typically, deviations from the target value are due to easily determined assignable causes, such as shift-to-shift changes, differences in raw material batches, environmental factors, etc. Wallgren (1996) has also studied the properties and implications of C_{pm} when consecutive measurements are observations of dependent variables resulting from a Markov process in discrete time. This occurs when consecutive measurements from a process are serially corrected. He developed an augmentation of C_{pm}, C_{pmr}, for this situation, based on the first-order autoregressive model (AR(1)).

The most general case discussed assumes only a reasonable predictable recurring pattern with known upper and lower specification limits, target value and the existence of a tool wear problem. Figure 1 illustrates a general relationship that may occur when a tool wear problem exists. The process specifications (i.e. USL, LSL and T), the starting, stopping, tool replacement times (i.e. t_0, t_1, t_2, t_3), and the process output have been included in figure 1. Tool wear is depicted in a nonlinear, increasing fashion, but could be any reasonably consistent recurring pattern (including linear degradation). The change times may represent chronological time, but are more likely to represent production qualities. The process illustrated in figure 1 depicts a systematic tool wear problem with nonlinear cycles over time/production. Similar to assessing the variation in any process, all sources of variation must be examined when considering tool wear. The methodology presented here is

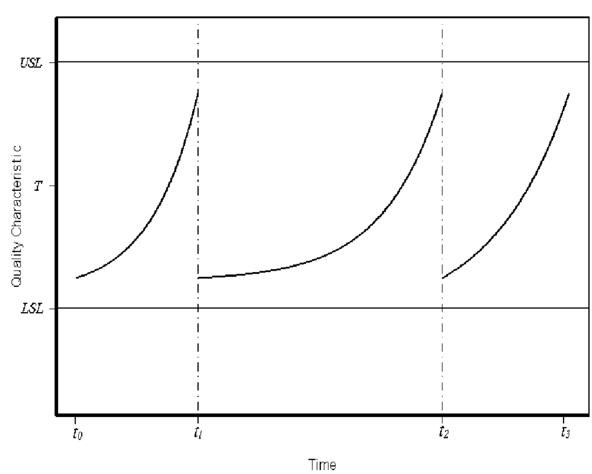

Figure 1. An example of the tool wear problem.

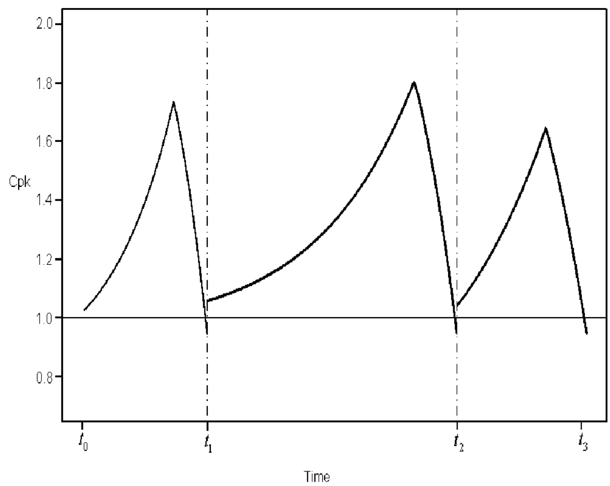

Figure 2. Plot of the changing capability of a process exhibiting tool wear.

only reactionary in its approach to dealing with the problem. To become pro-active in the area of tool wear, steps should be taken to eliminate variation due to an assignable cause. In a process exhibiting a tool wear problem, the traditional measure of the process capability index C_{pk} is affected by the tool wear slope (see figure 2). Thus, such a measure is invalid since it fails to acknowledge that portions of the overall variation are due to assignable causes.

3. Statistical properties of the estimated C_{pk} under tool wear

In this section, we introduce a modification of the C_{pk} index for the dynamic process under tool wear conditions. Subsequently, we obtain an explicit form of the exact cumulative distribution function for the dynamic estimator to measure the fraction of defectives. The cumulative distribution function can be expressed in terms of a mixture of the chi-square distribution and the normal distribution. We then obtain the rth moment, and the first two moments (mean and variance) of the estimated C_{pk} for the dynamic process.

3.1 *Estimation of C_{pk}*

Considering the process capability varies dynamically, tool replacement must not be overdue so that some minimum capability can be maintained. We propose the following modification of C_{pk} for dynamic processes under the condition of systematic assignable cause (tool wear):

$$C_{pk} = \frac{\min\{USL - \mu_t, \mu_t - LSL\}}{3\sigma_{rt}}, \tag{1}$$

where USL and LSL denote the upper and lower specification limit, respectively, μ_t represents the mean and σ_{rt} the variation (due to random causes only) of the process at time period t. Utilizing the identity $\min\{a, b\} = (a + b)/2 - |a - b|/2$, the index C_{pk}

defined in (1) can alternatively be rewritten as

$$C_{pk} = \frac{d - |\mu_t - M|}{3\sigma_{rt}}, \tag{2}$$

where $d=(USL-LSL)/2$ is half of the length of the specification interval, and $M=(LSL-USL)/2$ is the mid-point between the lower and upper specification limit. Monitoring a process's capability will require determination of the value of C_{pk} or a suitable estimate at various times t over each cycle in the lifetime of the tool. Assuming the effect of tool deterioration to be linear over the sampling window only, estimates of C_{pk} are possible that will in fact be free from any contribution of the assignable cause. Hence, the proposed estimator of the process capability can be obtained by replacing μ_t and σ_{rt} by the estimators $\bar{X}_t$ and $[(n-2)MSE_t/(n-1)]^{1/2}$, respectively. Then we have

$$\hat{C}_{pk} = \frac{\min\{USL - \bar{X}_t, \bar{X}_t - LSL\}}{3\hat{\sigma}_{rt}} = \frac{d - |\bar{X}_t - M|}{3\sqrt{[(n-2)MSE_t]/(n-1)}}. \tag{3}$$

The variation $\hat{\sigma}_{rt}$ can be calculated by considering sequentially selected points (i.e. $t_{a1}, t_{a2}, t_{a3}, \ldots, t_{an}$) instead of the sample variance. MSE_t is the mean square error associated with the regression equation $\hat{X}_{a_i} = \hat{\alpha}_a + \hat{\beta}_a t_{a_i}$, where t_{ai} is the sequence number of the sampling unit and $\hat{\beta}_a$ denotes the linear change of tool wear given a unit change in time/production:

$$MSE_t = \frac{\sum_{i=1}^{n} (X_{t_{a_i}} - \hat{X}_{t_{a_i}})^2}{n-2}. \tag{4}$$

3.2 *Sampling distribution*

For convenience in deriving the cumulative distribution function of $\hat{C}_{pk}$, the following notation is introduced:

1. $K=(n-2)MSE/\sigma^2$, which is distributed as χ^2_{n-2};
2. $Z' = \sqrt{n}(\bar{X} - M)/\sigma$, which is distributed as $N(\xi\sqrt{n},\ 1)$ with $\xi = (\mu - M)/\sigma$; and
3. $H = |Z'|$, which is distributed as a folded-normal distribution with probability density function $f_H(h) = \phi(h + \xi\sqrt{n}) + \phi(h - \xi\sqrt{n})$ for $h \geq 0$, where $\phi(\cdot)$ is the probability density function of the standard normal distribution.

For $x > 0$, the cumulative distribution function of $\hat{C}_{pk}$ can be derived as

$$\begin{aligned} F_{\hat{C}_{pk}}(x) &= P(\hat{C}_{pk} \leq x) = P\left(\frac{\sqrt{n-1}(b\sqrt{n} - H)}{3\sqrt{nK}} \leq x\right) \\ &= 1 - P\left(\sqrt{nK} < \frac{\sqrt{n-1}(b\sqrt{n} - H)}{3x}\right) \\ &= 1 - \int_0^\infty P\left(\sqrt{nK} < \frac{\sqrt{n-1}(b\sqrt{n} - H)}{3x}\middle| H = h\right) f_H(h)\mathrm{d}h \\ &= 1 - \int_0^\infty P\left(\sqrt{nK} < \frac{\sqrt{n-1}(b\sqrt{n} - h)}{3x}\right) f_H(h)\mathrm{d}h, \end{aligned}$$

where $b = d/\sigma$. Since K is distributed as χ^2_{n-2}, we have

$$P\left(\sqrt{nK} < \frac{\sqrt{n-1}(b\sqrt{n}-h)}{3x}\right) = 0 \quad \text{for } h > b\sqrt{n} \quad \text{and} \quad x > 0.$$

Therefore,

$$\begin{aligned} F_{\hat{C}_{pk}}(x) &= 1 - \int_0^{b\sqrt{n}} P\left(\sqrt{nK} < \frac{\sqrt{n-1}(b\sqrt{n}-h)}{3x}\right) f_H(h)\mathrm{d}h \\ &= 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-h)^2}{9nx^2}\right) f_H(h)\mathrm{d}h \quad \text{for } x > 0, \end{aligned} \tag{5}$$

where $G(\cdot)$ is the cumulative distribution function of χ^2_{n-2}. Substituting $f_H(h)$ into (5) leads to the result

$$F_{\hat{C}_{pk}}(x) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9nx^2}\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]\mathrm{d}t \quad \text{for } x > 0. \tag{6}$$

Using ordinary least square (OLS) estimates of α_a and β_a and assuming the sampling scheme to be sequential, the computational formula for equation (3) can be expressed alternatively as

$$\hat{C}_{pk} = \frac{d - \left|\bar{X}_{t_a} - M\right|}{3\left[\frac{\sum_{i=1}^n X^2_{t_{ai}}}{n-1} - \frac{2n(2n+1)}{(n-1)^2}\bar{X}^2_{t_a} - \frac{12\left(\sum_{i=1}^n (iX_{t_{ai}})^2\right)}{n(n^2-1)(n-1)} + \frac{12\bar{X}_{t_a}\sum_{i=1}^n (iX_{t_{ai}})}{(n-1)^2}\right]^{1/2}}, \tag{7}$$

where n denotes the subgroup sample size, and $X_{t_{ai}}$ represents the ith value of the quality characteristic in sampling period t_a. The proposed sampling scheme is similar to those schemes used in monitoring a process for control charting procedures. The general format would be to gather k subgroups of size n from each cycle (e.g., the period from t_0 to t_1 in figure 1) over the lifetime of the tool. The value of k is unique to each process, and may change from cycle to cycle within a process. On the other hand, for sample size less than five (i.e. $n < 5$) one must be cautious, and larger samples (e.g. $n < 30$) may also pose a problem. The optimal sample size for assessing process capability in the presence of a systematic assignable cause varies for each process considered (see Spiring (1991) for more details).

3.3 *The rth moment*

Under the assumption of normality, we note that MSE is distributed as $(n-2)^{-1}\sigma^2$ times a chi-square variable with $n-2$ degrees of freedom, symbolically $MSE \sim (n-2)^{-1}\sigma^2\chi^2_{n-2}$, and hence the estimator $\hat{C}_{pk}$ for the dynamic process can be rewritten as

$$\hat{C}_{pk} = \frac{d - \left|\bar{X} - M\right|}{3\sqrt{[(n-2)MSE]/(n-1)}} = \left(\frac{d}{\sigma} - \frac{1}{\sqrt{n}}\frac{\sqrt{n}\left|\bar{X} - M\right|}{\sigma}\right) \Bigg/ \frac{3\chi_{n-2}}{\sqrt{n-1}}. \tag{8}$$

The statistic $\sqrt{n}|\bar{X}-M|/\sigma$ has a folded normal distribution (see Leone *et al.* (1961) for more details about the distribution). Thus, we have

$$E\left(\frac{\sqrt{n}|\bar{X}-M|}{\sigma}\right)=\sqrt{\frac{2}{\pi}}\exp\left\{-\frac{n(\mu-M)^2}{2\sigma^2}\right\}+\frac{\sqrt{n}|\mu-M|}{\sigma}\left\{1-2\Phi\left(-\frac{\sqrt{n}|\mu-M|}{\sigma}\right)\right\}, \tag{9}$$

where $\Phi(\mu)=(2\pi)^{-1/2}\int_{-\infty}^{u}\exp(-t^2/2)\,\mathrm{d}t$ and

$$E\left\{\left(\frac{\sqrt{n}|\bar{X}-M|}{\sigma}\right)^2\right\}=1+\frac{n(\mu-M)^2}{\sigma^2}. \tag{10}$$

The distribution of $\hat{C}_{pk}$ depends on the parameters d/σ and $\sqrt{n}|\bar{X}-M|$. The rth moment about 0 of $\hat{C}_{pk}$ is

$$E(\hat{C}_{pk})^r=\left(\frac{\sqrt{n-1}}{3}\right)^r E(x_{n-2}^{-r})\sum_{j=1}^{r}(-1)^j\left(\frac{d}{\sigma}\right)^{r-j}\left(\frac{1}{\sqrt{n}}\right)^j E\left\{\left(\frac{\sqrt{n}|\bar{X}-M|}{\sigma}\right)^j\right\}. \tag{11}$$

In particular, the first two moments as well as the mean and the variance of $\hat{C}_{pk}$ can be obtained as

$$E(\hat{C}_{pk})=\frac{1}{3}\sqrt{\frac{n-1}{2}}\frac{\Gamma[(n-3)/2]}{\Gamma[(n-2)/2]}\left[\frac{d}{\sigma}-\sqrt{\frac{2}{\pi n}}\mathrm{e}^{-\lambda/2}-\sqrt{\frac{\lambda}{n}}\{1-2\Phi(-\sqrt{\lambda})\}\right],$$

$$\mathrm{Var}(\hat{C}_{pk})=\frac{n-1}{9(n-4)}\left\{\left(\frac{d}{\sigma}\right)^2-2\left(\frac{d}{\sigma}\right)\left[\sqrt{\frac{2}{\pi n}}\mathrm{e}^{-\lambda/2}+\sqrt{\frac{\lambda}{n}}\left(1-2\Phi\left(-\sqrt{\lambda}\right)\right)\right]+\frac{\lambda+1}{n}\right\}-[E(\hat{C}_{pk})]^2,$$

where $\lambda=n(\mu-M)^2/\sigma^2$ and $\Gamma(u)=\int_0^\infty t^{u-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$ is a gamma function.

4. Procedure for optimal tool replacement

Under the normality assumption, we have proved that the cumulative distribution function of $\hat{C}_{pk}$ for a dynamic process can be expressed as a mixture of the chi-square and the normal distributions. Using the index C_{pk}, engineers can access the process performance and monitor the tool replacement. Therefore, to test whether a given process is capable, the statistical hypotheses testing can be considered as

$$H_0: \quad C_{pk}\le C \quad \text{(process is not capable)},$$
$$H_1: \quad C_{pk}> C \quad \text{(process is capable)}.$$

The process is called capable if it produces a low fraction of defectives. Otherwise, the process is not capable. We define the test $\phi^*(x)$, the decision making rule, as the following: $\phi^*(x)=1$ if $\hat{C}_{pk}>c_0$, and $\phi^*(x)=0$ otherwise. Thus, the test ϕ^* rejects the null hypothesis H_0 ($C_{pk}\le C$) if $\hat{C}_{pk}>c_\alpha$, with type I error $\alpha(c_\alpha)=\alpha$, the chance of incorrectly concluding an incapable process ($C_{pk}\le C$) as capable ($C_{pk}\le C$). Based on the CDF of $\hat{C}_{pk}$ expressed in (6), given values of the capability requirement C (i.e. the expected product's fraction of defectives), the parameter ξ, the sample size n, and risk α, the critical value c_α can be obtained by solving the equation $P(\hat{C}_{pk} \geq c_\alpha | C_{pk} = C) = \alpha$ using available numerical integration methods. That is,

$$\int_0^{(3C+|\xi|)\sqrt{n}} G\left(\frac{(n-1)((3C+|\xi|)\sqrt{n}-t)^2}{9nc_\alpha^2}\right)[\phi(t+\xi\sqrt{n})+\phi(t-\xi\sqrt{n})]\,\mathrm{d}t = \alpha. \qquad (12)$$

It should be noted, particularly, that equation (12) is an even function of ξ. Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ we may obtain the same critical value c_α. However, from expression (12) the distribution characteristic parameter $\xi = (\mu - M)/\sigma$ is usually unknown, and has to be estimated in real applications, naturally by substituting μ and σ by the sample mean $\bar{X}$ and the sample variance $S^2 = \Sigma_{i=1}^n (X_i - \bar{X})^2/(n-1)$. To eliminate the need to estimate the parameter ξ, we examine the behaviour of the critical values c_α with respect to the parameter $\xi = 0(0.05)3.00$. We further perform extensive calculations to obtain the critical values c_α for $\xi = 0(0.05)3.00$, $n = 5(5)50$, $C_{pk} = 0.5(0.5)2.0$ with risk $\alpha = 0.05$. Note that the parameter values we investigated, $\xi = 0(0.05)3.00$, cover a sufficiently wide range of applications with process capability analysis. The results indicate that (i) the critical value c_α is increasing in ξ, and is decreasing in n, and (ii) the critical value c_α attains its maximum at $\xi = 1.00$ in all cases with accuracy up to 10^{-3}. Hence, for practical purposes we may solve equation (12) with $\xi = 1.00$ to obtain the required critical values c_α for given C_{pk}, n, and α, without having to further estimate the parameter ξ. Thus, the risk α can be ensured, and the decisions made based on such an approach are indeed more reliable. Figures 3(*a*)–(*d*) plot the curves of the critical value c_α versus the parameter $\xi = 0(0.05)3.00$, $n = 5$, 10, 20, 30 with type I error $\alpha = 0.05$, for levels of $C_{pk} = 0.50$, 1.00, 1.50 and 2.00, respectively.

For the convenience of the user applying our proposed procedure, we tabulate the critical values of $\hat{C}_{pk}$ for various values of $\alpha = 0.01$ and 0.05 with $n = 5(5)30$ in table 1 for commonly recommended minimum capability requirements $C = 1.00$, 1.33, 1.67 and 2.00. For example, if $C = 1.00$ is the minimum capability requirement, then for $\alpha = 0.05$, with sample size $n = 15$, we find $c_\alpha = 1.517$ from table 1. That is, as the estimated process capability drops below the critical value of $\hat{C}_{pk}$, the practitioner should stop the process and reset the tool because there is evidence that the process is nearing the end of its ability to produce a satisfactory product. If the value of $\hat{C}_{pk}$ is greater than the critical value, then the process is considered capable and should be allowed to continue.

5. An application example

To illustrate how the proposed procedure is applied to actual data collected from a factory, we consider the following example taken from a company which manufactures commercial automotive vehicles. A particular type of connecting rod for a diesel engine is investigated. The capability analysis focused on the key characteristic, big end diameter, which is finished by a fine boring process. The upper and lower manufacturing specification limits are set to $USL = 48.6$ mm and $LSL = 47.6$ mm, respectively. If the characteristic data do not fall within the tolerance (LSL, USL), the connecting rod component is considered to be non-conforming/defective, and will not be used to make the diesel engine of that particular model.

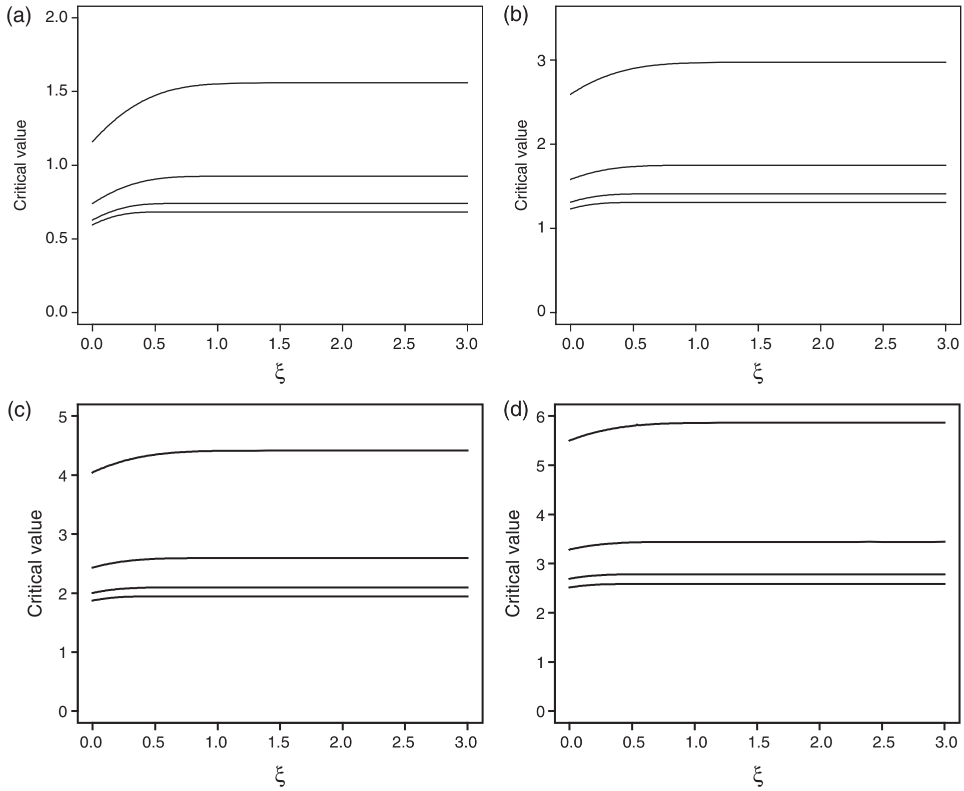

Figure 3. (*a*) Plots of c_α versus ξ for $C_{pk}=0.5$, $\alpha=0.05$, and $n=5$, 10, 20, 30 (from top to bottom). (*b*) Plots of c_α versus ξ for $C_{pk}=1.0$, $\alpha=0.05$, and $n=5$, 10, 20, 30 (from top to bottom). (*c*) Plots of c_α versus ξ for $C_{pk}=1.5$, $\alpha=0.05$, and $n=5$, 10, 20, 30 (from top to bottom). (*d*) Plots of c_α versus ξ for $C_{pk}=2.0$, $\alpha=0.05$, and $n=5$, 10, 20, 30 (from top to bottom).

Table 1. Critical value c_α for dynamic processes with various parameters.

	$C_{pk}=1.00$		$C_{pk}=1.33$		$C_{pk}=1.67$		$C_{pk}=2.00$	
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.01$	$\alpha=0.05$
5	5.206	2.967	6.867	3.918	8.591	4.903	10.269	5.862
10	2.266	1.750	2.980	2.305	3.720	2.881	4.441	3.442
15	1.829	1.517	2.404	2.000	3.002	2.500	3.584	2.987
20	1.644	1.412	2.163	1.863	2.701	2.329	3.226	2.783
25	1.539	1.350	2.026	1.782	2.532	2.229	3.023	2.664
30	1.471	1.309	1.937	1.728	2.420	2.162	2.891	2.584

When the product exits the process, the diameter is measured and recorded. The collected data exhibiting tool wear consist of 100 observations arranged in ten subgroups of size ten each, and is displayed in table 2. Figures 4 plots the individual values in the data series. It can be seen that the observations starting from a higher value (close to the upper limit) gradually decrease to the lower limit due to tool wear.

Table 2. The collected 10 subgroups of size ten (units: mm).

i	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
1	48.550	48.524	48.510	48.510	48.484	48.477	48.471	48.464	48.464	48.451
2	48.444	48.431	48.424	48.391	48.424	48.391	48.398	48.391	48.424	48.404
3	48.424	48.424	48.391	48.391	48.391	48.391	48.424	48.358	48.358	48.325
4	48.305	48.299	48.259	48.292	48.246	48.272	48.292	48.226	48.252	48.226
5	48.226	48.199	48.206	48.226	48.193	48.226	48.160	48.173	48.179	48.160
6	48.160	48.126	48.126	48.093	48.133	48.160	48.093	48.093	48.093	47.961
7	47.994	48.040	48.027	48.060	48.067	48.027	48.027	47.961	47.928	47.895
8	47.895	47.835	47.842	47.829	47.829	47.895	47.835	47.815	47.809	47.809
9	47.822	47.809	47.809	47.796	47.829	47.802	47.829	47.829	47.763	47.729
10	47.763	47.729	47.763	47.696	47.696	47.729	47.696	47.650	47.690	47.696

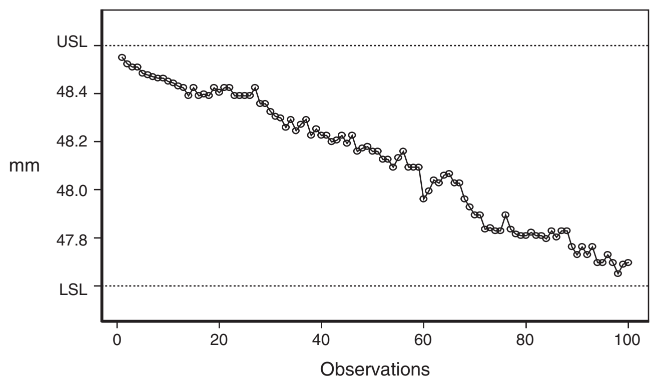

Figure 4. Plot of the original data.

The trend appears to be linearly decreasing. Also, the value of the diameter of each component is influenced by the amount of tool wear at that instant, which is likely to be dependent on the condition of the tool when the previous component was processed. Now, the goal is to maintain some minimum level of capability at all times and to monitor/manage this process under the influence of tool wear. The lower confidence bound is not only essential to assure production yield, but can also be used in capability testing for decision making. In fact, Boyles (1991) showed that the yield is $\geq 2\Phi\ (3C_{pk})–1$, or the fraction of non-conformities is $\leq 2\Phi(–3C_{pk})$. Table 3 displays various values of $C_{pk} = 1.00(0.10)2.00$, and the corresponding maximum possible non-conformities (in ppm). For example, if a process has capability with $C_{pk} \geq 1$, then the production yield would be at least 99.73%. Similarly, to achieve a defect rate of less than 0.544 ppm, a C_{pk} level of 1.67 is needed. At a C_{pk} level of 2.0, the likelihood of a defective part drops to 2 parts per billion (ppb). Suppose the capability requirement for this particular model of diesel engine was defined as 'Capable' if $C_{pk} > 1.00$. When the measure of process capability approaches the minimum acceptable level, the process should be stopped and the tool should be replaced. Thus, applying the proposed capability measure for a dynamic process, practitioners can monitor the process by calculating C_{pk}. The proposed procedure

Table 3. C_{pk} values versus the corresponding non-conformities.

C_{pk}	ppm
1.00	2699.796
1.10	966.848
1.20	318.217
1.30	96.193
1.33	66.073
1.40	26.691
1.50	6.795
1.60	1.587
1.67	0.544
1.70	0.340
1.80	0.067
1.90	0.012
2.00	0.002

Table 4. Estimated C_{pk} for the dynamic process at each time period.

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
$\hat{C}_{pk}$	2.444	2.827	2.699	4.681	6.736	3.664	2.817	2.486	1.872	0.856

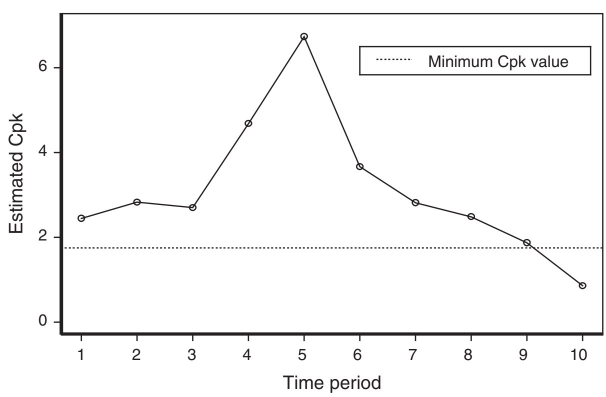

Figure 5. Capability plot for a dynamic process.

for a process involving tool wear is similar to those used in monitoring a process with a control chart.

In this case we find that the critical value of $\hat{C}_{pk}$ is 1.75 by checking table 1 under $\alpha = 0.05$, sample size $n = 10$ and minimum capability requirement $C = 1.00$. When the estimated process capability drops below the critical value of $\hat{C}_{pk}$, the practitioner should stop the process and reset the tool because there is evidence that the process is nearing the end of its ability to produce an acceptable product. For values of $\hat{C}_{pk}$ greater than 1.75 the process is considered capable and is allowed to continue. The calculated $\hat{C}_{pk}$ for a dynamic process at each time period based on the data in table 2 is summarized in table 4. Figure 5 plots the measure of process capability $\hat{C}_{pk}$ for a dynamic process at each time period over a single cycle of the process. It is observed that the estimated $\hat{C}_{pk}$ attains a maximum at time period t_5 and then drops below the line of critical values 1.75 at time period t_{10}. Therefore, based on these results we would suggest that the process should be stopped and the tool should be replaced at time period t_{10} to avoid the production of unacceptable components. As shown in table 4, the $\hat{C}_{pk}$ value is 1.872 at time period t_9; it then drops to near the critical value of 1.75. We recommend that the practitioner sample the characteristic data more frequently after time period t_9. Consequently, the $\hat{C}_{pk}$ value would decrease more smoothly for a more accurate determination of the optimal time for tool replacement in order to maintain production quality.

6. Conclusions

The process capability index C_{pk} is an effective tool for process performance analysis and quality assurance. The index C_{pk} provides a measure of process yield. Given a fixed C_{pk} value, the production yield and fraction of defectives can be calculated. Process capability can be calculated accurately if the data contain no assignable cause variation. Tool wear, however, is a dominant and irremovable component in many machining processes and is a systematic assignable cause. The ordinary measures of process capability are inaccurate because the process data are contaminated by the assignable cause variation. Therefore, developing an effective method to determine the optimal time for tool replacement is essential due to the high product quality requirement and low production cost considerations. For processes with tool wear, an estimator of C_{pk} is investigated. The closed form of the exact sampling distribution is also derived. We have shown that, under the assumption of normality, its sampling distribution is a mixture of the chi-square and the normal distributions. We have implemented the derived results to develop a tool management procedure for assessing process capability at each time period over a single cycle of the process. Critical values for various capability requirements and sample sizes are provided. Considering that the process capability changes dynamically, a procedure using a control chart is developed to monitor the process and determine the optimal tool replacement time to maintain production quality.

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