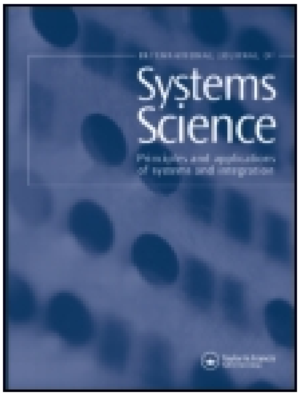

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Control chart techniques have been widely used in the manufacturing industry for controlling and monitoring process performance and are practical tools for quality improvement. When dealing with variable data, one usually employs the $\bar{X}$ chart and R chart (or S chart) to detect the process mean and process variance change. These charts are easy to understand and effectively communicate critical process information without using words and formulae. In this paper, we develop a new multiple-process performance analysis chart (MPPAC), using the process loss index L_e to control the product quality and/or reliability for multiple manufacturing processes. Upper confidence bounds are applied to the L_e MPPAC to ensure the capability groupings are accurate, which is essential to product quality assurance. The L_e MPPAC displays the multiple-process relative inconsistency and process relative off-target degree on one single chart in order to provide simultaneous capability control for multiple processes. We demonstrate the applicability of the proposed L_e MPPAC incorporating the upper confidence bounds by presenting a case study on some liquid-crystal display module manufacturing processes, to evaluate the factory performance.

Keywords: Multiple-process performance analysis chart; Process capability indices; Process loss indices; Upper confidence bound

1. Introduction

Process capability indices (PCIs), including C_p, C_a, C_{pk}, C_{pm} and C_{pmk} (Kane 1986, Chan *et al.* 1988, Pearn *et al.* 1992, 1998), have been widely used in the manufacturing industry to measure whether the product quality meets the preset specifications, particularly, in automated, semiconductor and integrated-circuit (IC) assembly manufacturing industries. Those indices provide the manufacturers the means for monitoring their quality levels. By analysing the PCIs, a production department can improve and enhance a poor process to meet their customers' need. Those indices have been defined as

$$C_p = \frac{\mathrm{USL}-\mathrm{LSL}}{6\sigma}, \quad C_a = 1 - \frac{|\mu - T|}{d}, \tag{1}$$

$$C_{pk} = \min\left\{\frac{\mathrm{USL}-\mu}{3\sigma}, \frac{\mu-\mathrm{LSL}}{3\sigma}\right\},$$

$$C_{pm} = \frac{\mathrm{USL}-\mathrm{LSL}}{6\left(\sigma^2 + (\mu - T)^2\right)^{1/2}}, \tag{2}$$

$$C_{pmk} = \min\left\{\frac{\mathrm{USL}-\mu}{3\left(\sigma^2 + (\mu - T)^2\right)^{1/2}}, \frac{\mu-\mathrm{LSL}}{3\left(\sigma^2 + (\mu - T)^2\right)^{1/2}}\right\}, \tag{3}$$

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where μ is the process mean, σ is the process standard deviation, USL is the upper specification limit, LSL is the lower specification limit, T is the target value and $d = (\text{USL} - \text{LSL})/2$ is the half-length of the specification interval.

The index C_p considers the overall process variability relative to the manufacturing tolerance, which reflects product consistency (process precision). The index C_a measures the degree of process centring (the ability to cluster around the centre), which has been regarded as the process accuracy index. The index C_{pk} takes the process mean into consideration but can fail to distinguish between on-target and off-target processes. The index C_{pk} is a yield-based index which provides lower bounds on process yield. The index C_{pm} takes the proximity of process mean from the target value into account, which is more sensitive to process departure than C_{pk}. The design of C_{pm} is based on the average process loss relative to the manufacturing tolerance, providing an upper bound on the average process loss, which has been alternatively called the Taguchi index. The index C_{pmk} is constructed from combining the modifications to C_p that produced C_{pk} and C_{pm}, which inherits the merits of both indices.

Hsiang and Taguchi (1985) first used the loss function to improve process quality, focusing on reducing the process variation around the target value. Johnson (1992) introduced the relative expected loss index L_e for processes with symmetric tolerances. Tsui (1997) rewrote $L_e = L_{pe} + L_{ot}$ to provide an uncontaminated separation between information concerning process relative inconsistency loss L_{pe} and process relative off-target loss L_{ot}. The index L_e is defined as the ratio of the 'expected quadratic loss' to the 'square of the half-specification-width':

$$L_e = \int_{-\infty}^{\infty} \frac{(x-T)^2}{d^2}\, \mathrm{d}F(x) = \left(\frac{\sigma}{d}\right)^2 + \left(\frac{\mu - T}{d}\right)^2, \qquad (4)$$

where $F(x)$ is the cumulative distribution function of the measured characteristic. If we denote the first term $(\sigma/d)^2$ as L_{pe} and the second term $[(\mu - T)/d^2]$ as L_{ot}, then L_e can be rewritten as $L_e = L_{pe} + L_{ot}$. We note that the mathematical relationships $L_e = (3C_{pm})^{-2}$, $L_{ot} = (1 - C_a)^2$ and $L_{pe} = (3C_p)^{-2}$ can be established. The advantage of using L_e over C_{pm} is that the estimator of the former has better statistical properties than that of the latter, as the former does not involve a reciprocal transformation of the process mean and variance. Also it provides an uncontaminated separation between information concerning the process precision and process accuracy. The process accuracy reflects the departure of the process mean from the target value, and the process precision reflects the overall process variability. The separation suggests which parameters practitioners may consider to improve process quality. Some commonly used values of L_e, namely 1.00 (process is incapable), 0.44 (process is incapable), 0.11 (process is normally called capable), 0.06 (process is called satisfactory), 0.05 (process is normally called good), 0.04 (process is normally called excellent) and 0.03 (process is normally called super), and the corresponding C_{pm} values are listed in table 1.

The subindex L_{ot} measures the relative process departure, which has been referred to as the relative off-target loss index. On the other hand, the subindex L_{pe} measures process variation relative to the specification tolerance, which has been referred to as the relative inconsistency loss index. Some commonly used values of L_{pe}, namely 0.11, 0.06, 0.05, 0.04 and 0.03, and the corresponding quality conditions are listed in table 2. Note that those values of L_{pe} are equivalent to $C_p = 1.00$, 1.33, 1.50, 1.67 and 2.00 respectively, covering a wide range of the precision requirements used for most real-world applications.

Ever since Shewhart introduced control charts, it has become a common practice for practitioners to use various control charts to monitor different processes on a routine basis. For example, when dealing with variable data, one usually employs a chart (such as the $\bar{X}$ chart) to monitor the process mean, and a chart (such as the R chart or S chart) to monitor

Table 1. Some commonly used L_e and equivalent C_{pm} values.

Condition	L_e	C_{pm}
Incapable	1.00	0.33
Incapable	0.44	0.50
Capable	0.11	1.00
Satisfactory	0.06	1.33
Good	0.05	1.50
Excellent	0.04	1.67
Super	0.03	2.00

Table 2. Some commonly used precision requirements.

Quality condition	Precision requirement
Incapable	$0.11 < L_{pe}$
Capable	$0.06 < L_{pe} \le 0.11$
Satisfactory	$0.05 < L_{pe} \le 0.06$
Good	$0.04 < L_{pe} \le 0.05$
Excellent	$0.03 < L_{pe} \le 0.04$
Super	$L_{pe} \le 0.03$

process spread. Those charts are essential tools for quality control. In the multiple-manufacturing-lines environment where a group of processes need to be controlled, it could be difficult and time consuming for factory engineers or supervisors to analyse each individual chart in order to evaluate overall factory performance. To evaluate the performance of a group of multiple processes with symmetric specifications, Singhal (1990, 1991) introduced a multiple-process performance analysis chart (MPPAC) using process capability index C_{pm}.

Pearn and Chen (1997) proposed a modification to the C_{pk} MPPAC combining the more-advanced process capability indices C_{pm} or C_{pmk}, to identify problems causing the processes that fail to centre around the target. Chen *et al.* (2001) considered an extension of the MPPAC to processes with multiple characteristics. In current practice of implementing those charts, practitioners simply plot the estimated index values on the chart and then draw conclusions on whether processes meet the capability requirement and modifications need to be made for capability improvement. Unfortunately, those approaches (Singhal 1990, 1991, Pearn and Chen 1997, Chen *et al.* 2001) are highly unreliable since the estimated index values are random variables and sampling errors are ignored. Therefore, process information conveyed from those charts is often misleading.

Traditional $(\bar{X}, R)$ or $(\bar{X}, S)$ control charts are online statistical process control techniques for monitoring and surveillance of the process. However, process capability analysis is a vital part of an overall quality-improvement program. In this paper, we introduce the L_{e} MPPAC based on the subindices L_{pe} and L_{ot}. The L_{e} MPPAC chart is an offline technique for evaluating the performance of multiple processes, which sets priority activity to be taken for process improvement (reducing process variability or process departure). The L_{e} MPPAC displays process variability relative to their specification tolerances (in relative inconsistency L_{pe}) and process departure (in relative off-target loss L_{ot}) for multiple processes on one single chart. We propose a reliable approach by first converting the estimated index values to the upper confidence bounds and then plotting the corresponding upper confidence bounds on the L_{e} MPPAC. The upper confidence bounds not only provide us with a clue on the minimal actual process performance but are also useful in making decisions for capability testing. We demonstrate the applicability of the L_{e} MPPAC by presenting a case study of a group of liquid-crystal display module manufacturing processes, to evaluate the factory performance.

2. Estimations and upper confidence bounds of L_{pe}, L_{ot} and L_{e}

2.1 *Estimations of L_{pe}, L_{ot} and L_{e}*

To estimate the process relative inconsistency loss, we consider the estimator $\hat{L}_{\mathrm{pe}}$ defined as follows, where $S_n^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/n$ is the maximum-likelihood estimator (MLE) of the process variance σ^2:

$$\hat{L}_{\mathrm{pe}} = \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i - \bar{X})^2}{d^2} = \frac{S_n^2}{d^2}. \tag{5}$$

The estimator $\hat{L}_{\mathrm{pe}}$ can be rewritten as

$$\hat{L}_{\mathrm{pe}} = \frac{L_{\mathrm{pe}}}{n}\frac{n\hat{L}_{\mathrm{pe}}}{L_{\mathrm{pe}}} = \frac{L_{\mathrm{pe}}}{n}\sum_{i=1}^{n}\frac{(X_i - \bar{X})^2}{\sigma^2}. \tag{6}$$

If the process follows the normal distribution, the estimator $\hat{L}_{\mathrm{pe}}$ is distributed as $(L_{\mathrm{pe}}/n)\chi^2_{n-1}$, where χ^2_{n-1} is a chi-squared distribution with $n-1$ degrees of freedom. Pearn *et al.* (2004) showed that the estimator $\hat{L}_{\mathrm{pe}}$ is the MLE of L_{pe}, which is consistent, asymptotically unbiased and efficient. To estimate the relative off-target loss, we consider the natural estimator $\hat{L}_{\mathrm{ot}}$ defined as follows, where $\bar{X} = \sum_{i=1}^{n} X_i/n$ is the conventional estimator of the process mean μ:

$$\hat{L}_{\mathrm{ot}} = \frac{(\bar{X} - T)^2}{d^2}. \tag{7}$$

We note that the estimator $\hat{L}_{\mathrm{ot}}$ can also be written as a function of L_{pe}:

$$L_{\mathrm{ot}} = \frac{L_{\mathrm{pe}}}{n}\frac{n\hat{L}_{\mathrm{ot}}}{L_{\mathrm{pe}}} = \frac{L_{\mathrm{pe}}}{n}\frac{n(\bar{X} - T)^2}{\sigma^2}. \tag{8}$$

If the process characteristic is normally distributed, Pearn *et al.* (2004) showed that the estimator $\hat{L}_{\mathrm{ot}}$ is distributed as $(L_{\mathrm{pe}}/n)\chi_1^2(\delta)$, where $\chi_1^2(\delta)$ is a non-central chi-squared distribution with one degree of freedom and non-centrality parameter $\delta = n(\mu - T)^2/\sigma^2 = nL_{\mathrm{ot}}/L_{\mathrm{pe}}$. Since $\bar{X}$ is the MLE of μ, then, by the invariance property of MLE, the natural estimator $\hat{L}_{\mathrm{ot}}$ is the MLE of L_{ot}. To estimate the expected relative loss of the process (a combined measure of the relative inconsistency loss of the process and the relative off-target loss of the process), we consider the natural estimator $\hat{L}_{\mathrm{e}}$ defined as follows:

$$\hat{L}_{\mathrm{e}} = \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i - T)^2}{d^2} = \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i - \bar{X})^2}{d^2} + \frac{(\bar{X} - T)^2}{d^2}. \tag{9}$$

We note that the estimator $\hat{L}_e$ can also be written as a function of L_{pe}:

$$\hat{L}_e = \frac{L_{pe}}{n}\frac{n\hat{L}_e}{L_{pe}} = \frac{L_{pe}}{n}\sum_{i=1}^{n}\frac{(X_i - T)^2}{\sigma^2}. \quad (10)$$

If the process characteristic is normally distributed, then the estimator $\hat{L}_e$ is distributed as $(L_{pe}/n)\chi_n^2(\delta)$, where $\chi_n^2(\delta)$ is a non-central chi-squared distribution with n degrees of freedom and non-centrality parameter δ. Pearn *et al.* (2004) showed that $\hat{L}_e$ is the MLE, which is also the uniformly minimum variance unbiased estimator of L_e. The statistic $\hat{L}_e$ is consistent and asymptotically efficient. Since the estimator has all the desired statistical properties, in practice using $\hat{L}_e$ to estimate the expected relative loss of the process would be reasonable. Some distributional and inferential properties of the process loss indices have been provided by Pearn *et al.* (2004).

2.2 *Upper confidence bounds of L_{pe}, L_{ot} and L_e*

In the following, we derive the upper confidence bounds of the three process loss indices L_{pe}, L_{ot} and L_e respectively. We note the expression

$$\begin{aligned}\hat{L}_e &= \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i - T)^2}{d^2}\\ &= \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i - \bar{X})^2}{d^2} + \frac{(\bar{X} - T)^2}{d^2} = \hat{L}_{pe} + \hat{L}_{ot}, \quad (11)\end{aligned}$$

where $\hat{L}_{pe}$ and $\hat{L}_{ot}$ are the MLEs of L_{pe} and L_{ot} respectively. By estimating the mean μ of the unknown parameters by the sample mean $\bar{X}$, and the variance σ^2 by S_n^2, the relationship $\hat{L}_e = \hat{L}_{pe} + \hat{L}_{ot}$ may be established. This expression provides an uncontaminated separation between calculated information concerning the relative inconsistency loss $\hat{L}_{pe}$ of the process and the relative off-target loss $\hat{L}_{ot}$ of the process.

Under the normality assumption, $n\hat{L}_{pe}/L_{pe}$ is distributed as χ_{n-1}^2, a chi-squared distribution with $n-1$ degrees of freedom. A $100(1-\alpha)\%$ upper confidence bound for L_{pe} can be expressed, in terms of $\hat{L}_{pe}$, as

$$U_{pe} = \frac{n\hat{L}_{pe}}{\chi_{n-1}^2(\alpha)}, \quad (12)$$

where $\chi_{n-1}^2(\alpha)$ is the (lower) αth percentile of the χ_{n-1}^2 distribution. Under the normality assumption, $\delta\hat{L}_{ot}/L_{ot}$ is distributed as $\chi_1^2(\delta)$, a non-central chi-squared distribution with one degree of freedom and non-centrality parameter δ. We note that $P(\delta\hat{L}_{ot}/L_{ot} \geq \chi_1^2(\alpha,\delta)) = 1-\alpha$. A $100(1-\alpha)\%$ upper confidence bound on L_{ot} can be expressed, in terms of $\hat{L}_{ot}$, as

$$U_{ot} = \frac{\delta\hat{L}_{ot}}{\chi_1^2(\alpha,\delta)}, \quad (13)$$

where $\chi_1^2(\alpha,\delta)$ is the (lower) αth percentile of the $\chi_1^2(\delta)$ distribution. Under the normality assumption, $(n+\delta)\hat{L}_e/L_e$ is distributed as $\chi_n^2(\delta)$, a non-central chi-squared distribution with n degrees of freedom and non-centrality parameter δ. We note that $P((n+\delta)\hat{L}_e/L_e \geq \chi_n^2(\alpha,\delta)) = 1-\alpha$. A $100(1-\alpha)\%$ upper confidence bound for L_e can be expressed, in terms of $\hat{L}_e$, as

$$U_e = \frac{(n+\delta)\hat{L}_e}{\chi_n^2(\alpha,\delta)}, \quad (14)$$

where $\chi_n^2(\alpha,\delta)$ is the (lower) αth percentile of the $\chi_n^2(\delta)$ distribution.

3. The L_e MPPAC

Statistical process control charts, such as the $\bar{X}$, R, S^2, S and MR charts, have been widely used in the manufacturing industries for controlling and/or monitoring process performance and are essential tools for any quality improvement activities. These charts are easy to understand and effectively communicate critical process information without using words and formulae. However, they are applicable only for a single process (one process at a time). Thus, using these charts in a multiple-process environment can be a difficult and time-consuming task for the supervisor or shop engineer to analyse each individual chart to evaluate the overall status of shop process control activity.

The MPPAC can be used to evaluate the performance of a single process as well as multiple processes, to set the priorities among multiple processes for quality improvement, to indicate whether reducing the variability or the departure of the process mean should be the focus and to provide an easy way to quantify the process improvement by comparing the locations on the chart of the processes before and after the improvement effort. The MPPAC is an efficient tool for communicating between the product designer, the manufacturers and the quality engineers, and between the management departments.

Based on the definition $L_e = (\mu - T)^2/d^2 + \sigma^2/d^2$, we first set $L_e = k$, for various k values, and then a set of (μ, σ) values satisfying the equation $(\mu - T)^2 + \sigma^2 = kd^2$ can be plotted on the contour (a curve) of $L_e = k$. These contours are semicircles centred at $(\mu, \sigma) = (T, 0)$ with radius $k^{1/2}d$. The more capable the process, the smaller the semicircle is. We plot the seven contours on the L_e MPPAC for the seven L_e values listed in table 1, as shown in figure 1. On the L_e MPPAC, we note the following.

(i) As the point gets closer to $(\mu, \sigma) = (T, 0)$, the value of L_e becomes smaller, and the process performance is better.
(ii) For the points inside the semicircle of contour $L_e = k$, the corresponding L_e values are smaller than k. For the points outside the semicircle of contour $L_e = k$, the corresponding L_e values are greater than k.
(iii) When the processes have fixed values of L_e, for points within the envelope of the two 45° lines, the variability is contributed mainly by the variance of the process.
(iv) When processes have fixed values of L_e, for points outside the envelope of the two 45° lines, the variability is contributed mainly by the departure of the process mean from the target.
(v) The perpendicular line and parallel line through the plotted point intersecting the horizontal axis and vertical axis at points represent its L_{ot} and L_{pe} respectively. For example, the point (0.11, 0.44) represents $L_{ot} = 0.11$ and $L_{pe} = 0.44$.
(vi) The distance between T and the point at which the perpendicular line through the plotted point intersects the horizontal axis denotes the departure of process mean from target. For example, the point (0.11, 0.44) represents the departure of the process mean from target given by $\mu - T = \pm(0.11)^{1/2}d$.
(vii) The distance between T and the point at which the parallel line through the plotted point intersects the vertical axis denotes the departure of the standard deviation of the process. For example, the point $(0.11, 0.44)$ represents the departure of the standard deviation of the process given by $\sigma = (0.44)^{1/2}d$.

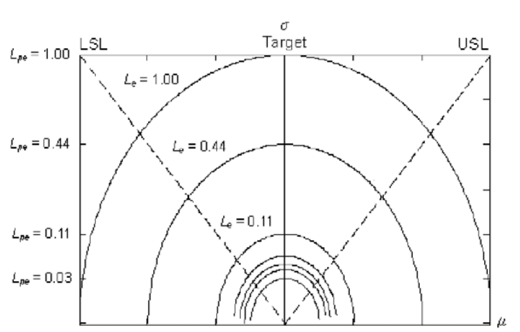

Figure 1. The L_e MPPAC.

4. An application example

In the following, we consider a liquid-crystal display module manufacturing process. Three key components make the liquid-crystal display module function properly. Those include the liquid-crystal display, the back lighting and the peripheral (interface) system. The liquid-crystal display module is one of the key components used in many high-technology electronic commercial devices, such as cellular phones, the personal digital assistants, pocket calculators, digital watches and automobile accessory visual displays. Currently, the mounting technology of the chip on glass, which makes the exposed particle overturned with the side of the circuits facing downwards, is the best manufacturing technology for the liquid-crystal display module in terms of the mounting density. Conduction of electricity occurs between the IC and the panel of the liquid-crystal display through the mounting material. We note that different mounting materials requires different mounting technologies of the chip on glass.

We consider the following case taken from a manufacturing factory making liquid-crystal display modules and located at the Science-Based Industrial Park in Taiwan. With a focus on the main bonding process (i.e. the stages of manufacturing the chip on the glass), the bonding precision is essentially a process key parameter. We investigated eight specific types of liquid-crystal display module requiring different bonding precision standards.

A random sample of size 100 is taken from each of the eight main bonding processes. With the target value T set to zero (i.e. $T = 0$), their required tolerances, bonding precision specifications are displayed in table 3. If the characteristic data do not fall within the tolerance (LSL, USL), the lifetime or reliability of the liquid-crystal display module product will be discounted. The calculated sample mean, standard deviation, the index values and 95% upper confidence bounds of L_e, L_{ot} and L_{pe} are shown in table 4. Figure 2 plots the L_e MPPAC for the eight processes listed in table 4. We analyse the process

Table 3. The bonding specifications of the liquid-crystal display module.

Code	Tolerance (μm)	LSL	T	USL
A	±25	−25	0	25
B	±25	−25	0	25
C	±15	−15	0	15
D	±15	−15	0	15
E	±20	−20	0	20
F	±5	−5	0	5
G	±10	−10	0	10
H	±30	−30	0	30

Table 4. The calculated statistics and upper confidence bounds.

Process	$\bar{x}$	s_n	$\hat{L}_{pe}$	$\hat{L}_{ot}$	$\hat{L}_e$	U_{pe}	U_{ot}	U_e
A	0.542	12.711	0.259	0.001	0.259	0.336	0.018	0.332
B	0.731	8.785	0.124	0.001	0.124	0.160	0.075	0.160
C	–0.627	6.824	0.207	0.002	0.209	0.269	0.161	0.268
D	4.502	3.554	0.056	0.090	0.146	0.073	0.119	0.178
E	–5.921	4.644	0.054	0.088	0.142	0.070	0.116	0.172
F	1.118	1.175	0.055	0.050	0.105	0.072	0.073	0.131
G	–1.057	2.561	0.066	0.011	0.077	0.085	0.031	0.098
H	1.271	3.947	0.017	0.002	0.019	0.023	0.008	0.025

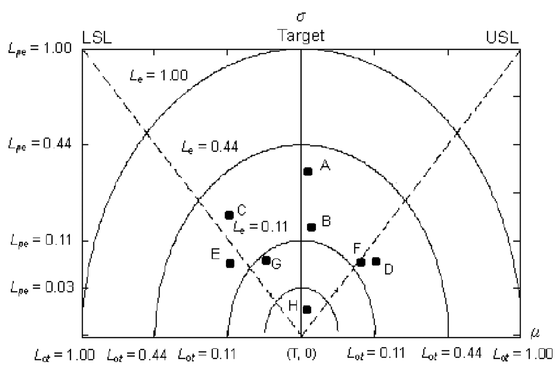

Figure 2. The L_e MPPAC for the example.

points in figure 2, and obtain the following summary of quality conditions.

(i) The plotted point A is very close to the contour $L_e = 0.44$; this indicates that the process has a low capability. Since the point A is close to the target line, it demonstrates that the poor capability is mainly contributed by the process variation. Thus, it calls for immediate quality improvement action to reduce the variance of the process.

(ii) The plotted points B and C lie outside the contour $L_e = 0.11$; this indicates that their L_e values are higher than 0.11. Since these points lie inside the envelope of the two 45° lines, it demonstrates that their L_{pe} values must be higher than their L_{ot} values. Thus, reducing their process variances has higher priority than reducing the departures of the process means.

(iii) The plotted points D and E lie outside the contour $L_e = 0.11$ and the envelope of the two 45° lines; therefore their L_{ot} values must be higher than their L_{pe} values. Quality improvement efforts for these processes should be first focused on reducing the departures of process means from the target value.

(iv) The plotted point F is close to the 45° line and is outside the contour of $L_e = 0.11$. This indicates that the variability of the process is contributed equally by the mean departure and the process variance.

(v) The plotted point G lies inside the contour $L_e = 0.11$; this means that its L_e value is lower than 0.11. The capability of this process is considered to be satisfactory, but it will be a candidate for lower-priority quality improvement efforts.

(vi) Process H is very close to $(\mu, \sigma) = (T, 0)$ and its L_e is small; so the process H is considered to perform well.

5. Conclusions

Existing research on manufacturing capability control ignores sampling errors. In this paper, we develop a MPPAC, using the process loss index L_e to control the product quality for multiple manufacturing processes. Taking into account the sampling errors, we obtained the upper confidence bounds. We applied the upper confidence bounds to the L_e MPPAC to ensure that the capability groupings are accurate, which is essential to product quality assurance. The L_e MPPAC is an effective tool for multiple-process control, which displays multiple processes with the relative inconsistency of the process and relative off-target degree of the process on one single chart. An application example is given to demonstrate the applicability of the proposed L_e MPPAC, which incorporates the upper confidence bounds, to evaluate the factory performance.

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