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Strategy of packet detection for burst-mode OFDM systems

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Abstract: An efficient method on selecting threshold values according to minimax test for packet detection in burst-mode OFDM systems is proposed. Packet detection decides whether a packet is coming or not by comparing a threshold value in the wireless receiver. Related with sliding window size and SNR, the threshold value affects receiving performance including probabilities of false alarm and miss. The minimax test for detection based on empirical CDF and survival functions is proposed. Also the performances of two general used detection methods are surveyed and compared.

Keywords: packet detection, frame detection, OFDM, WLAN
Classification: Science and engineering for electronics

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1 Introduction

Some burst-mode wireless communication systems such as WLAN and WPAN transmit information in each packet with three segments: preamble, header and data signal [1]. Ahead of any other operations in the receiver, packet detection shall decide whether a packet is coming or not. Some packet detection methods only use receiving signal power and easily suffer from a drawback: threshold values involving with received power and gain control in radio-frequency circuits [2]. For a specific communication system supporting repeated preambles, correlation properties can be used for packet detection and carrier synchronization in OFDM system, and furthermore these correlation values are divided by power value to eliminate problems of variant threshold [3]. The correlation and power functions used in the detection algorithm are defined as

$$C(n) = \sum_{i=0}^{L-1} r_{n+i} r^*_{n+i+D}, \tag{1}$$

and

$$P(n) = \sum_{i=0}^{L-1} |r_{n+i+D}|^2, \tag{2}$$

where r_k implies kth complex-valued received sample, L is the sliding window size and D is the interval of two repeated preambles. Due to that $C(n)$ is complex-valued and $P(n)$ is real-valued, packet detection has two normalized functions according to practical implementation methods:

$$M2(n) = \frac{|C(n)|^2}{P(n)^2}, \tag{3}$$

and

$$M1(n) = \frac{|C(n)|}{P(n)}. \tag{4}$$

Derivative of the function $M1(n)$ needs an extra square root operation, whereas derivative of $M2(n)$ needs an extra square operation and greater precision representation in practice. In consideration of circuit implementation, these extra requirements are small enough compared to a full receiver. Thus we leave aside implement complexity and only consider which one performs better in the detection. On the other hand, the selected threshold values affect probabilities of detection and false alarm. We adopt a Bayes test, or called minimax test, to select a proper threshold value by minimizing the maximum possible risk according to different assumptions of hypothesis probability and risk [4]. Since $M1(n)$ and $M2(n)$ are random variables (RV) combined with multiple complex-valued RVs in numerators and denominators, their probability density function (PDF) are very difficult to be calculated in simple deterministic form. Moreover, because only cumulative distribution functions (CDF) are required in the minimax test, an empirical CDF can be used to estimate ideal CDF values from a statistical viewpoint. We adopt the product-limit (PL) method (Kaplan and Meier method) to

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calculate empirical CDFs and use interpolation method to acquire any cumulative probability corresponding to different threshold values [5].

Most OFDM systems transmit through frequency-selective fading channels, which can be viewed as time-variant linear filters with random coefficients of amplitude, phase and delay. The received equivalent baseband signal can be viewed as

$$x_b(t) = \sum_{k=1}^{N_p} \alpha_k(t) \exp(-j\theta_k(t)) s_b(t - \tau_k(t)), \tag{5}$$

where $s_b(t)$ is the transmitted baseband signal, α_k, θ_k, τ_k are time-variant coefficients, and N_p is the number of resolvable paths. After passing through the fading channel, the signal is disturbed at the receiver by additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2. Then the instantaneous signal-to-noise ratio (SNR) is defined as

$$\gamma(t) = E[|x_b(t)|^2]/\sigma_n^2. \tag{6}$$

The received discrete samples after analog-to-digital converter are represented as the sum of $x_b(t)$ and AWGN both multiplied by a RF gain G_{RF} and sampled by T_s:

$$r_k = G_{RF}[x_b(kT_s) + w_k], \tag{7}$$

The AWGN is given as $w_k = w_{I,k} + jw_{Q,k}$ and $w_{I,k}$, $w_{Q,k} \in N(0, \sigma_n^2/2)$. The RF gain G_{RF} will be canceled in (3) and (4), which is the main purpose of the normalization. Thus the detection function is no longer relative to G_{RF}.

Define two hypotheses for two detection conditions:

$$\begin{cases} \mathrm{H}_1: \text{ a packet has been transmitted} \\ \mathrm{H}_0: \text{ no packet has been transmitted} \end{cases}. \tag{8}$$

Let $N_L(0, \sigma_x^2)$ be an independent Gaussian vector whose components are independent Gaussian RVs with zero mean and equal variance σ_x^2. Define two complex-valued independent Gaussian vectors as $W^{(i)} = W_I^{(i)} + jW_Q^{(i)}$, $i = 1, 2$ with $W_I^{(i)}$ and $W_Q^{(i)} \in N_L(0, \sigma_n^2/2)$. Therefore, for H_0 is true, the power function $P(n)$ can be represented as the squared norm of the Gaussian vector:

H_0 is true:

$$P(n) = \sum_{i=1}^{L} |w_{n+i+D}|^2 = \left\|W^{(2)}\right\|^2 = \left\|W_I^{(2)}\right\|^2 + \left\|W_Q^{(2)}\right\|^2 \quad , \tag{9}$$

where $\left\|W_I^{(2)}\right\|^2$ and $\left\|W_Q^{(2)}\right\|^2$ are central chi-square RVs with L degrees of freedom. Thus $P(n)$ in (9) is a central chi-square RV with $2L$ degrees of freedom:

$$p_{P(n)|H_0}(y|H_0) = \frac{1}{\sigma_n^2 \Gamma(L)} \left(\frac{y}{\sigma_n^2}\right)^{L-1} \exp\left(-\frac{y}{\sigma_n^2}\right), \quad y \geq 0, \tag{10}$$

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where $\Gamma(x)$ is a Gamma function [6]. For H_0 is true, the correlation function $C(n)$ can be viewed as an inner product of two complex-valued independent Gaussian vectors:

H_0 is true:

$$\begin{aligned} C(n) &= \sum_{i=0}^{L-1} w_{n+i} w_{n+i+D}^{*} = W^{(1)} \cdot W^{(2)*} \\ &= (W_I^{(1)} + jW_Q^{(1)}) \cdot (W_I^{(2)} - jW_Q^{(2)}) \\ &= [W_I^{(1)} \cdot W_I^{(2)} + W_Q^{(1)} \cdot W_Q^{(2)}] + j[W_Q^{(1)} \cdot W_I^{(2)} - W_I^{(1)} \cdot W_Q^{(2)}] \\ &\equiv [\mathrm{K}_L^{(1)} + \mathrm{K}_L^{(2)}] + j[\mathrm{K}_L^{(3)} - \mathrm{K}_L^{(4)}] \equiv \mathrm{K}_{2L}^{(1)} + jK_{2L}^{(2)} \end{aligned}, \quad (11)$$

where $K_n^{(i)}$ is a RV as inner product of two independent Gaussian vectors with identical variance $\sigma^2 = \sigma_n^2/2$, whose PDF is given below (for $n = 2m$) [6]:

$$p_K(x) = \frac{1}{\sigma^2 \Gamma(m)} \exp\left(-\frac{|x|}{\sigma^2}\right) \sum_{i=0}^{m-1} \frac{\Gamma(m+i)}{2^{m+i}\Gamma(i+1)\Gamma(m-i)} \left(\frac{|x|}{\sigma^2}\right)^{m-1-i}. \quad (12)$$

For H_1 is true, the indoor time-variant channel can be viewed static within D samples because of low Doppler frequency. Thus assume the two complex-valued signal vectors are almost the same:

$$X_b^{(1)} = [x_{b,n} \ldots x_{b,n+L-1}]^{\mathrm{T}} \cong X_{b,}^{(2)} = [x_{b,n+D} \ldots x_{b,n+D+L-1}]^{\mathrm{T}}, \quad (13)$$

Then $P(n)$ given H_1 is true can be represented as

H_1 is true:

$$\begin{aligned} P(n) &= \sum_{i=1}^{L} |x_{b,n+i+D} + w_{n+i+D}|^2 = \left\| X_b^{(2)} + W^{(2)} \right\|^2 \\ &= \left\| X_{b,I}^{(2)} + W_I^{(2)} \right\|^2 + \left\| X_{b,Q}^{(2)} + W_Q^{(2)} \right\|^2 \end{aligned}, \quad (14)$$

which can be viewed as a noncentral chi-square RV with $2L$ degrees of freedom:

$$p_{P(n)|H1}(y|H_1) = \frac{1}{2\sigma^2} \left(\frac{y}{a^2}\right)^{(L-1)/2} \exp\left(-\frac{y+a^2}{2\sigma^2}\right) I_{L-1}\left(\sqrt{\frac{a^2 y}{\sigma^4}}\right), \quad y \geq 0, \quad (15)$$

where

$$\begin{aligned} \sigma^2 &= \sigma_n^2/2 \\ a^2 &= \left\| X_{b,I}^{(2)} \right\|^2 + \left\| X_{b,Q}^{(2)} \right\|^2 = \left\| X_b^{(2)} \right\|^2 = \sum_{i=0}^{L-1} |x_{b,n+i+D}|^2 \end{aligned}. \quad (16)$$

From (6) the noncentral parameter a^2 can be represented by instantaneous SNR as $a^2 = \gamma L \sigma_n^2$. Therefore the noncentral chi-square RV in (15) is not related to the distribution of X_b, but only involved with SNR value and noise power instead. For H_1 is true, the correlation $C(n)$ is given as

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H_1 is true:

$$\begin{aligned} C(n) &= \sum_{i=0}^{L-1} (x_{b,n+i} + w_{n+i})(x_{b,n+i+D} + w_{n+i+D})^* \\ &= (X_b^{(1)} + W^{(1)}) \cdot (X_b^{(2)} + W^{(2)})^* \\ &= X_b^{(1)} \cdot X_b^{(2)*} + X_b^{(1)} \cdot W^{(2)*} + X_b^{(2)*} \cdot W^{(1)} + W^{(1)} \cdot W^{(2)*} \\ &= \gamma L \sigma_n^2 + (X_{b,I}^{(1)} + jX_{b,Q}^{(1)}) \cdot (W_I^{(2)} - jW_Q^{(2)}) \\ &+ (X_{b,I}^{(2)} - jX_{b,Q}^{(2)}) \cdot (W_I^{(1)} + jW_Q^{(1)}) + W^{(1)} \cdot W^{(2)*} \\ &= \left[\gamma L \sigma_n^2 + X_{b,I}^{(1)} \cdot W_I^{(2)} + X_{b,Q}^{(1)} \cdot W_Q^{(2)} + X_{b,I}^{(2)} \cdot W_I^{(1)} \right. \\ &\quad \left. + X_{b,Q}^{(2)} \cdot W_Q^{(2)} + K_{2L}^{(1)}\right] \\ &+ j[X_{b,Q}^{(1)} \cdot W_I^{(2)} - X_{b,I}^{(1)} \cdot W_Q^{(2)} + X_{b,I}^{(2)} \cdot W_Q^{(1)} - X_{b,Q}^{(2)} \cdot W_I^{(1)} \\ &\quad + K_{2L}^{(2)}] \end{aligned} \quad , \tag{17}$$

The inner product of a constant vector and a Gaussian vector is still a Gaussian RV with zero mean and variance equal to the original variance multiplied by the norm of the constant vector, e.g. the term $X_{b,I}^{(1)} \cdot W_I^{(2)}$ in (17) is a Gaussian RV with variance equal to $\left\|X_{b,I}^{(1)}\right\|^2 \cdot \sigma_n^2/2$. Therefore the middle four terms in the real part of (17) integrate into a Gaussian RV with variance equal to $\left\|X_b^{(1)}\right\|^2 \sigma_n^2 = \gamma L \sigma_n^4$. The first four terms in the imaginary part has the same variance. Thus (17) can be simplified as

$$C(n) = \left(Z^{(1)} + K_{2L}^{(1)}\right) + j\left(Z^{(2)} + K_{2L}^{(2)}\right), \tag{18}$$

where $Z^{(1)} \in N(\gamma L \sigma_n^2, \gamma L \sigma_n^4)$ and $Z^{(2)} \in N(0, \gamma L \sigma_n^4)$. When SNR is large enough, $C(n)$ is close to a complex-valued Gaussian RV. The PDF of image and imaginary parts in (18) can be acquired by joint PDF:

$$p_{C_I(n)|H1}(y|H_1) = \int_{-\infty}^{\infty} p_{Z^{(1)}}(x) p_{K_{2L}^{(1)}}(y-x) dx. \tag{19}$$

Although the PDFs of (1) and (2) for two hypotheses H_0 and H_1 are derived, the distribution of $M1(n)$ and $M2(n)$ still can not be derived because of the dependence between the numerator and denominator. But we can conclude from (10) (12) (15) (18) that the detection functions only involve with SNR and window size Linstead of received signals and channels. This helps to build a simulation of RVs and to clarify relative parameters.

2 Empirical CDF and minimax test for threshold values

According to different window sizes and SNR values, we build a RV simulation with N_s tests for the detection functions $M1(n)$ and $M2(n)$ and use nonparametric method to estimate empirical CDFs. The PL method is used here to acquire CDF and survival functions [5]. Assume there are N_s samples observed and we sort them in ascending order such that $s_{(1)} \le s_{(2)} \ldots \le s_{(N_s-1)} \le s_{(N_s)}$. The survival function is given as:

$$\hat{f}_s(s_{(i)}) = \hat{f}_s(s_{(i-1)}) \frac{N_s - i}{N_s - i + 1}, \; i = 1, 2, \ldots, N_s, \tag{20}$$

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where $\hat{f}_s(s_{(0)}) = 1$ is assumed. The precision of estimates in (20) are dependent on the number of samples Ns, i.e. the minimum precision is $1/Ns$. For a specific value λ, $\hat{f}_s(\lambda)$ can be acquired by linear interpolation:

$$\hat{f}_s(\lambda) = \begin{cases} 1, & \lambda < s_{(0)} \\ \text{interpolation}, & s_{(0)} \leq \lambda \leq s_{(N_s)} \\ 0, & \lambda > s_{(N_s)} \end{cases} . \tag{21}$$

The CDF can be acquired from the survival function:

$$\hat{f}_c(\lambda) = 1 - \hat{f}_s(\lambda). \tag{22}$$

Figure 1 shows the empirical CDF and survival curves of $M1(n)$ for two hypotheses with $N_s = 2 \times 10^6$, L=16, 32, 64, and SNR=0, 2, 4, 6 dB, and also reveals that the survival curves of $M1(n)$ given H_0 are only dependent on window size L. Once a threshold value λ is assigned, we denote some useful probabilities:

$$\begin{aligned} P_F &= \int_{\lambda}^{\infty} p_{M|H_0}(R|H_0)dR = \hat{f}_{s|H_0}(\lambda) \\ P_D &= \int_{\lambda}^{\infty} p_{M|H_1}(R|H_1)dR = \hat{f}_{c|H_1}(\lambda) \\ P_M &= 1 - P_D = 1 - \hat{f}_{c|H_1}(\lambda) \end{aligned} . \tag{23}$$

Also we denote C_F and C_M as the costs of false alarm and miss, respectively, and P_1 and P_0 for the a priori probabilities of H_1 and H_0.

Then the Bayes risk function is given as [4]:

$$R_B(\lambda) = P_0 C_F P_F(\lambda) + P_1 C_M P_M(\lambda). \tag{24}$$

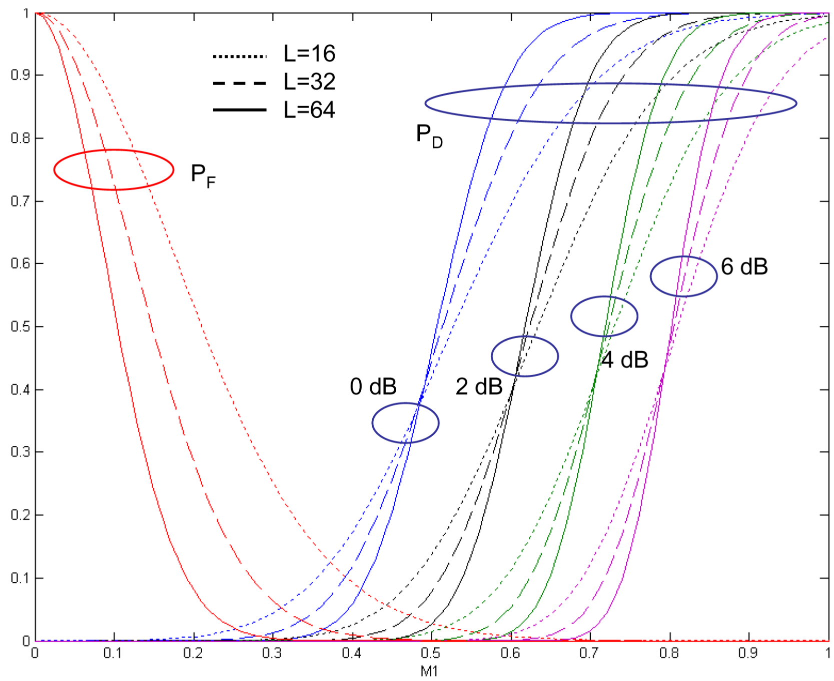

Fig. 1. Empirical CDF and survival curves of packet detection function $M1$

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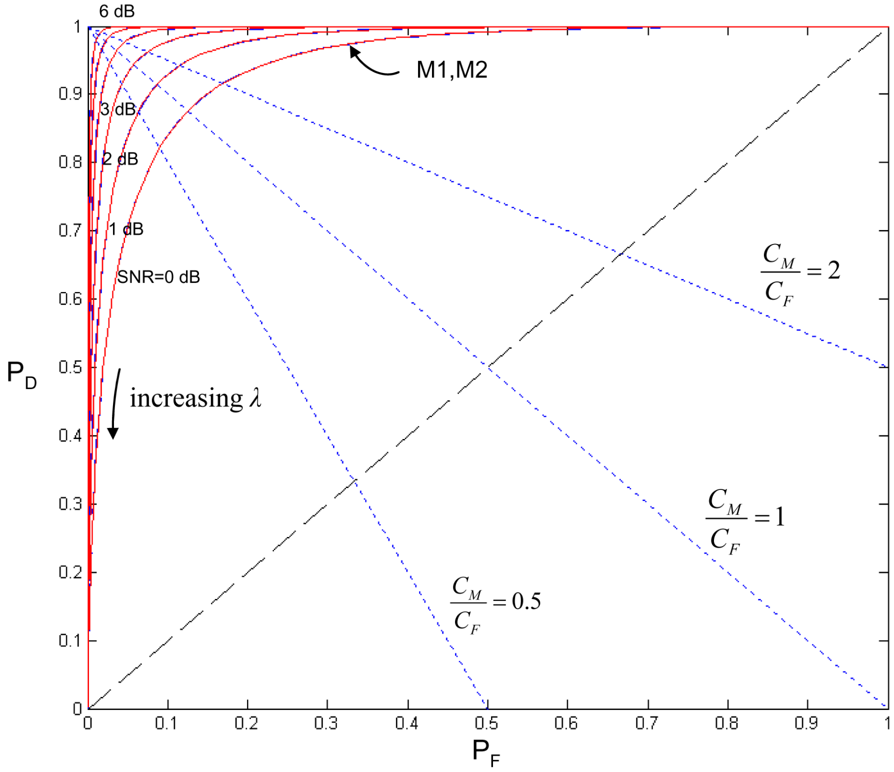

Fig. 2. Receiver operating characteristic and minimax operating points (L=16, SNR=0, 1, 2, . . . 6 dB)

Figure 2 illustrates the receiver operating characteristic (ROC) of both detection functions $M1$ and $M2$. ROC reveals the relationship between P_F and P_D as λ varies. As SNR value decreases or window size increases, the ROC curves moves toward left-top, which implies higher P_D and lower P_F can be achieved. That the ROC curves of $M1$ and $M2$ are overlapping implies the performances of $M1$ and $M2$ are equal. Assume $P_1 = P_0$, the minimax equation is

$$C_F P_F(\lambda) = C_M P_M(\lambda), \tag{25}$$

which has solutions of λ corresponding to the intersection points for different C_M and C_F ratios in Figure 2. Another approach to find the threshold value of minimum risk is directly drawing the risk functions as shown in Figure 3. Obviously the minimum points move as conditions change. Thus it is important to assign C_F, C_M, P_1 and P_0 for the selection of threshold values. Regarded as a watchdog in the OFDM receivers, packet detection shall operate at lower SNR required by lowest data transmission rate. In wireless LAN OFDM systems, the SNR required for 6 Mbps for PER=0.1 is about 5 dB [7]. Therefore SNR values lower than 5 dB shall be considered in the design of packet detection.

3 Conclusions

The paper verifies that the distributions of both detection functions $M1$ and $M2$ are not related to channels and preamble signals, but only involved with SNR and the sliding window size. The performances of $M1$ and $M2$ are the

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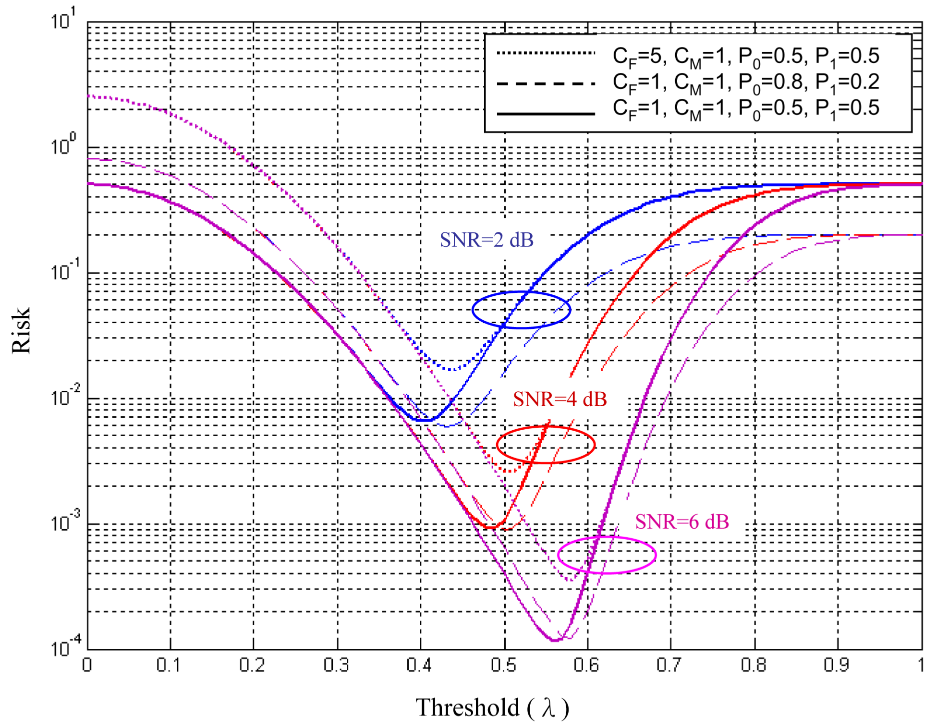

Fig. 3. Risk functions of $M1$ under three different conditions (L=32)

same according to ROC curves. A strategy to decide threshold values in the detection is proposed:

1) decide SNR and window size according to system requirement,

2) build RV simulations for two hypotheses H_0 and H_1,

3) generate empirical CDF and survival functions,

4) select a priori probabilities of hypotheses and costs of false alarm and miss,

5) draw risk functions and find threshold values corresponding to minimum points.

Sufficient performance can be obtained with the newly proposed packet detection strategy.

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