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Extrusion analysis and workability prediction of three-layer composite hexagonal clad rods

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Abstract

This study uses upper bound method to predict the workability for the extrusion of three-layer composite hexagonal clad rods. A velocity field is generated with the assistance of a product's cross-section profile functions. The velocity component in the extrusion axis is expressed as a convex distribution. The simultaneous deformation of each constituent material with quite different mechanical properties may lead to fracture. A criterion for fracture based on the velocity deviation existing on the layer interface was proposed. Material fractures under extrusion are affected by a set of independent process parameters, including the initial area percentage of each layer, friction condition of die, mechanical characteristics of three constituent materials and reduction of area of billet. Finally, figures of extrusion limits containing safety and fracture zones for each process parameter are obtained for extrusion of three-layer composite hexagonal clad rods.

Keywords: Extrusion; Workability; Hexagon clad rod

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1. Introduction

Composite clad rods consisted of three different materials are extensively applied as conductors, electrodes and chemical devices. For instance, three-layer composite clad rods are commercially used as superconductor cables, with pure niobium as core, copper–tin alloy as sleeve and a Nb_3Sn diffusion layer in between. During the extrusion process, owing to the differences among mechanical properties with respect to the constituent materials and complexity of construction, three-layer composite clad rods frequently exhibit a non-homogeneous deformation in the extrusion process, subsequently leading to fracturing or defects within the core, mid-layer and sleeve, even each material is highly ductile. In general, if one constituent material is harder than the others, it will resist deformation, undergo a smaller reduction in area and lead to the other materials exerts a tensile stress on the harder material, sooner or later, the harder material fractures.

Extrusion of conventional composite clad rods consisting of two constituents has been thoroughly examined [1–5]. A criterion for the prevention of core and sleeve fracture during extrusion of bimetal rods was presented by Avitzur et al. [6,7]. Alcaraz and Sevillano [8] derived a criterion based on the geometric compatibility of the deformation of each material. These studies focused primarily on the bimetal rods and the axisymmetric extrusion, in which analytical models were subsequently developed. These studies are applicable only to two-layer composite clad rods. Studies involving the rolling of sandwich plate (which also has three layers) closely examined the problem's complexity [9]. Although our earlier work proposed an analytical model to extrude a three layer composite rod [10], the extrusion was confined to the axisymmetric round rods.

In this study, we refine the above model to extrude three-layer composite clad rods with hexagonal cross-section profiles. Also proposed herein is a three-dimensional velocity field which has a non-uniform velocity distribution in extrusion axis. In addition, the working limit with respect to independent process parameters like the initial area percentage of mid-layer, core and friction condition of die is examined as well. Those results are presented in the following section.

2. Discretisation of the plastic deformation zone

Fig. 1 illustrates a three-layer composite clad rod that is extruded through a linear converging die. Each constituent layer

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Nomenclature

A_s	area of sleeve at die exit
D_s, D_m, D_c	non-uniform velocity component in the extrusion axis of sleeve, mid-layer and core, respectively
J	total power consumption in extrusion
L	die length (dimensionless)
m, m_1, m_2	friction factor at the die surface, mid-layer/sleeve and core/mid-layer inter-surface, respectively
$P_{avg.}$	average extrusion pressure
r, ϕ, y	cylindrical coordinates
R.A.	reduction of area of billet
$R_f(\phi)$	product radius function
R_f, R_{mf}, R_{cf}	the radius of sleeve, mid-layer and core after extrusion, respectively
R_o, R_{mo}, R_{co}	the radius of sleeve, mid-layer and core before extrusion, respectively
R_{ss}, R_{sm}, R_{sc}	the function represents die surface, mid-layer/sleeve and core/mid-layer inter-surface profile, respectively
U_s, U_m, U_c	uniform velocity component in the extrusion axis of sleeve, mid-layer and core, respectively
$V_o, V_{fs}, V_{fm}, V_{fc}$	entrance velocity of the billet, exit velocity of sleeve, mid-layer and core of extruded product, respectively
V_p	volume of the plastic region
$V_{rm}, V_{\phi m}, V_{ym}$	velocity components of mid-layer in the cylindrical coordinates (r, ϕ, y), respectively
$V_{rs}, V_{\phi s}, V_{ys}$	velocity components of sleeve in the cylindrical coordinates (r, ϕ, y), respectively
$\Delta V_{\Gamma s}, \Delta V_{\Gamma f}$	the relative slip velocity on Γs and Γf surfaces, respectively
$\dot{W}_{im}, \dot{W}_{sm}, \dot{W}_{f1}$	power dissipation due to internal deformation, internal shear of the mid-layer and friction at mid-layer/sleeve surface, respectively
$\dot{W}_{is}, \dot{W}_{ss}, \dot{W}_{fd}$	power dissipation due to internal deformation, internal shear of the sleeve and friction at die surface, respectively
$\dot{W}_{ic}, \dot{W}_{sc}, \dot{W}_{f2}$	power dissipation due to internal deformation, internal shear of the core and friction at core/mid-layer inter-surface, respectively
$\dot{W}_i, \dot{W}_s, \dot{W}_f$	power dissipation due to internal deformation, internal shear and friction, respectively
Y_{ss}, Y_{sm}, Y_{sc}	yield stress of the sleeve, mid-layer and core, respectively

Greek letters

Γ_s, Γ_f	surface of shear velocity discontinuities and friction, respectively
α	semi-die angle
$\delta, \lambda, \beta_s, \beta_m, \beta_c$	optimization parameters introduced in the velocity fields
$\phi_f(y)$	angle of geometric symmetry surface
$\sigma_0, \dot{\bar{\varepsilon}}$	the yield stress and effective strain rate of materials, respectively
$\omega_s, \omega_m, \omega_c$	angular velocity of sleeve, mid-layer and core, respectively

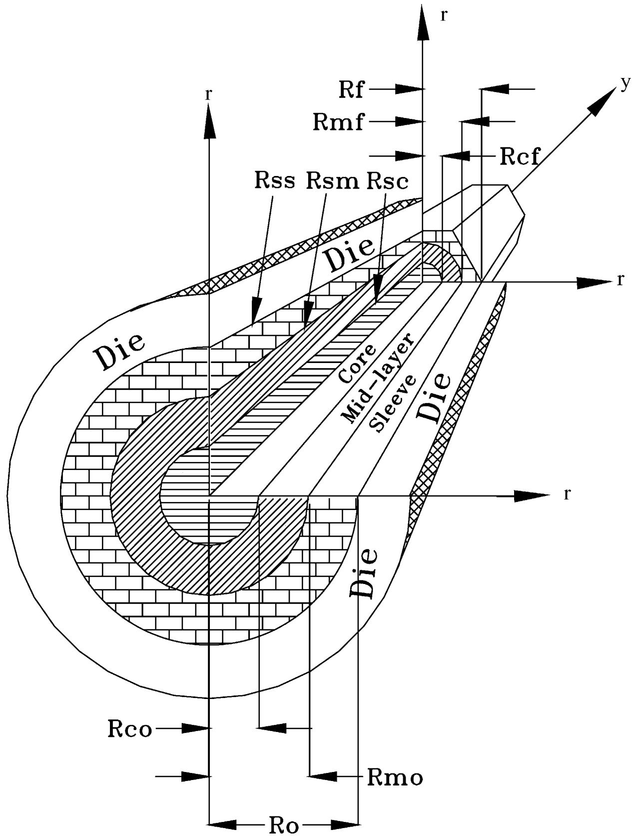

Fig. 1. Schematic representation of extrusion of a three-layer composite clad rod with a hexagonal cross-section profile.

has its own material characteristics, accounting for why a particular velocity field must be developed for each layer. Notably, $R_{ss}(\phi,y)$, $R_{sm}(\phi,y)$ and $R_{sc}(\phi,y)$ are functions which describe the die surface, mid-layer/sleeve and core/mid-layer inter-surfaces, respectively. In the case of linear converging die, the following

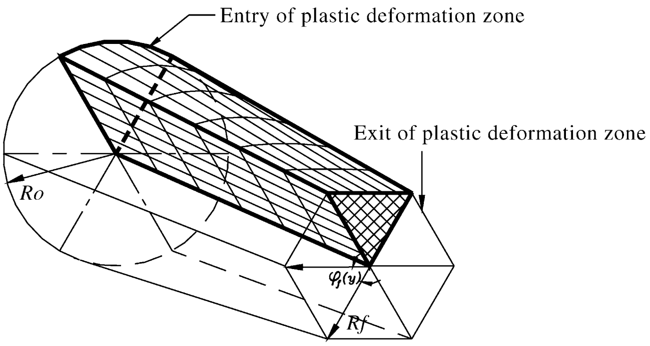

Fig. 2. Extrusion die for a hexagonal cross-section.

equations represent these functions:

$$R_{\mathrm{ss}}(\phi, y) = \left\{R_{\mathrm{o}} - \left[(R_{\mathrm{o}} - R_{\mathrm{f}}(\phi)) \cdot \frac{y}{L}\right]\right\} \tag{1}$$

$$R_{\mathrm{sm}}(\phi, y) = \left\{R_{\mathrm{mo}} - \left[(R_{\mathrm{mo}} - R_{\mathrm{mf}}) \cdot \frac{y}{L}\right]\right\} \tag{2}$$

$$R_{\mathrm{sc}}(\phi, y) = \left\{R_{\mathrm{co}} - \left[(R_{\mathrm{co}} - R_{\mathrm{cf}}) \cdot \frac{y}{L}\right]\right\} \tag{3}$$

In the above equations, R_{o}, R_{mo} and R_{co} denote the radius of sleeve, mid-layer and core before extrusion, respectively. During the extrusion process, since core and mid-layer are assumed to retain a circular cross-section, R_{mf} and R_{cf} is the radius of the mid-layer and core after extrusion, respectively. In addition, R_{f} is the dimension of the hexagonal length after extrusion as shown in Fig. 2. Where L denotes the die length.

3. Kinematically admissible velocity field

Fig. 3 schematically depicts the extrusion of a hexagonal three-layer composite clad rod. The round billet deforms to the final hexagonal cross-section through the die, which is defined by an envelope of a number of straight lines. The composite clad rod consists of a core, mid-layer and outer sleeve. Each layer of the rod has a circular cross-section before extrusion. Following extrusion, the die used dictates the cross-sectional shape of the composite clad rod. In this study, we thoroughly examine a non-axisymmetric product with a hexagonal cross-section. For simplicity, material behavior is assumed to be rigid-plastic, isotropic, homogeneous, isothermal and the die is assumed to be rigid throughout the extrusion process.

In order to postulate a kinematically admissible velocity field, the plastic deformation occurs only within zones bounded by the die entrance plane (A–A′), die exit plane (B–B′) and the die surface as indicated in Fig. 3. Before entering the die, each constituent material of the composite clad rod moves as a rigid body with the same velocity V_{o}, in the extrusion direction; after extrusion, each layer of the composite clad rod maybe moves with the same velocity V_{f} if the sound extrusion occurs or moves with the different velocity V_{fs}, V_{fm} and V_{fc} if non-homogeneous deformation occurs. Initially, polar coordinate system (r, ϕ, y) is imposed with its origin selected at the center of the billet at die entrance plane. Y-axis is aligned with the extrusion axis. Herein, for each constituent layer, we introduce the convex distributed non-uniform V_{y} with specific parameter β_{s}, β_{m}, β_{c} to control its convexity. On the other hand, a rotational velocity component V_{ϕ} exists since the outer sleeve experiences the non-axisymmetric deformation. Furthermore, assume that V_{ϕ} is linearly distributed along radius, that is $V_{\phi} = r\omega(\phi,y)$. Meanwhile, angular velocity $\omega_{\mathrm{c}}(\phi,y)$, $\omega_{\mathrm{m}}(\phi,y)$ of core and mid-layer are zero owing to that the core and mid-layer remain circular after extrusion, and thus both are axisymmetrical. In addition, the three-dimensional admissible velocity fields for core, mid-layer and sleeve can be derived with the assistance of incompressibility of materials or constant volume flow.

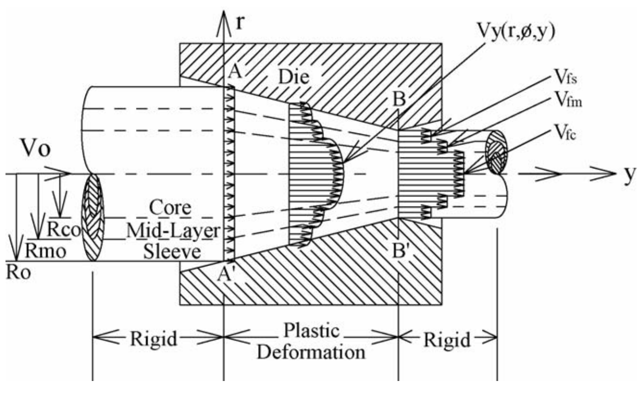

Fig. 3. Velocity components in extrusion direction (y-axis).

Herein, velocity component for core, mid-layer and sleeve in the extrusion axis, V_{yc}, V_{ym} and V_{ys} are assumed to consist of a uniform component U_{s}, U_{m}, U_{c}, and a non-uniform component D_{s}, D_{m}, D_{c}, respectively. Obviously, $V_{\mathrm{ys}}(r,\phi,y) = U_{\mathrm{s}}(y)$, $V_{\mathrm{ym}}(r,\phi,y) = U_{\mathrm{m}}(y)$, $V_{\mathrm{yc}}(r,\phi,y) = U_{\mathrm{c}}(y)$ at $y = 0, L$. Restated, the velocity components in y-axis for all layers in both die entrance and die exit are uniformly distributed. Only in the plastic deformation zone, that $D_{\mathrm{s}}(r,\phi,y)$, $D_{\mathrm{m}}(r,\phi,y)$ and $D_{\mathrm{c}}(r,\phi,y)$ can be nonzero functions. One characteristic of this velocity field is each velocity component can be directly calculated once the functions $R_{\mathrm{ss}}(\phi,y)$, $R_{\mathrm{sm}}(\phi,y)$, $R_{\mathrm{sc}}(\phi,y)$ are defined. During the analysis, constant friction factors m_1, m_2 are adopted to calculate the friction energy loss on the core/mid-layer and mid-layer/sleeve inter-surfaces, respectively.

4. Power consumption

While extruding a hexagonal three-layer composite clad rod, power consumption includes the power deemed necessary to overcome the resistance to deformation of sleeve, mid-layer and core, $\dot{W}_{\mathrm{is}}$, $\dot{W}_{\mathrm{im}}$, $\dot{W}_{\mathrm{ic}}$; shear power losses, $\dot{W}_{\mathrm{ss}}$, $\dot{W}_{\mathrm{sm}}$, $\dot{W}_{\mathrm{sc}}$ over boundaries of velocity discontinuities (slip), A–A′, B–B′; frictional power consumed along the die surface, mid-layer/sleeve and core/mid-layer inter-surfaces, $\dot{W}_{\mathrm{fd}}$, $\dot{W}_{\mathrm{f1}}$, $\dot{W}_{\mathrm{f2}}$. This power consumption can be estimated by integrating the strain rate and the yield stress over the entire deformation volume. Formulations for the above mentioned power items are expressed as follows:

$$\dot{W}_{\mathrm{i}} = \int_{V_{\mathrm{p}}} \sigma_0 \cdot \dot{\bar{\varepsilon}}\, \mathrm{d}v \tag{4}$$

$$\dot{W}_{\mathrm{s}} = \int_{\Gamma \mathrm{s}} \frac{1}{\sqrt{3}} \cdot \sigma_0 \cdot \Delta V_{\Gamma \mathrm{s}}\, \mathrm{d}s \tag{5}$$

$$\dot{W}_{\mathrm{f}} = \int_{\Gamma \mathrm{f}} \frac{m}{\sqrt{3}} \cdot \sigma_0 \cdot \Delta V_{\Gamma \mathrm{f}}\, \mathrm{d}A \tag{6}$$

where σ_0 denotes the yield stress of the constituent materials; $\dot{\bar{\varepsilon}}$ represents the effective strain rate of materials; V_{p} is the volume of plastic deformation region; $\Delta V_{\Gamma \mathrm{s}}$ denotes the relative slip velocity at velocity discontinue surfaces; m represents the friction factor and $\Delta V_{\Gamma \mathrm{f}}$ is the relative slip velocity at frictional surfaces.

As mentioned earlier, each constituent layer may not deform homogeneously owing to dissimilar mechanical properties.

Under this circumstance, the reduction of each constituent layer may be different from each other. Thus, the exit radius ratios of the core/sleeve and mid-layer/sleeve differ from those of the assembled billet. Herein, parameters δ, λ are introduced to account for this non-homogeneous deformation. That is:

$$R_{\mathrm{mf}} = \left[\frac{(1+\delta)A_{\mathrm{s}}}{\pi}\right]^{1/2}\frac{R_{\mathrm{mo}}}{R_{\mathrm{o}}}, \quad R_{\mathrm{cf}} = \left[\frac{(1+\lambda)A_{\mathrm{s}}}{\pi}\right]^{1/2}\frac{R_{\mathrm{co}}}{R_{\mathrm{o}}} \tag{7}$$

A_{s} is the area of sleeve at die exit.

Minimizing the total extrusion power determines the value of parameters δ, λ. On the other hand, non-uniform velocity components in the y-axis, $D_{\mathrm{s}}(r, \phi, y)$, $D_{\mathrm{m}}(r, \phi, y)$, $D_{\mathrm{c}}(r, \phi, y)$ are specified by the parameters β_{s}, β_{m}, β_{c}, respectively. Thus, δ, λ, β_{s}, β_{m}, β_{c} are considered as optimal parameters and their values are mathematically optimized by minimizing the total power J consumed in the extrusion process.

5. Results and discussion

This study closely examines extrusions of hexagonal three-layer composite clad rods from round billets. All the dies used in the analysis are linearly connected and equal angle divided, as indicated in Fig. 2. Herein, semi die angles are defined as the angle of die surface inclination. Owing to the differences among mechanical properties with respect to the constituent materials and complexity of construction, three-layer composite clad rods frequently exhibit a non-homogeneous deformation in the extrusion process. When one material is softer than the others, it tends to undergo larger reduction and the velocity is faster. Then, the material which flow fast will exert a tensile load on the material which flow slowly and, sooner or later, the harder material fractures. The larger the deviation of velocity between two different materials, the possibility of fracture is larger. Considering the computation error, we assume the harder material will fracture if the deviation velocity between two materials is larger than 5%.

Fig. 4 depicts the working limit (semi die angle) against the initial area percentage of mid-layer, $A_{\mathrm{mo}}/A_{\mathrm{bo}} \times 100$. Herein, A_{mo}, A_{bo} is the initial area of mid-layer and billet, respectively. In this case, the core is the hardest and the sleeve is the softest. As mention before, the sleeve will exert a front tensile load on mid-layer, and the core will exert a back tensile load on mid-layer. The lower the area percentage of mid-layer is, the fracture possibility is larger, the workability is small.

Fig. 5 depicts how the initial area percentage of core ($A_{\mathrm{co}}/A_{\mathrm{bo}} \times 100$) influences the working limit, semi die angle. The larger the area percentage of core is, the mid-layer is thinner, the mid-layer fractures more possibly. If the radius of core is small, extreme the radius is zero, the three-layer composite material is like bimetal composite material, the possibility of sound extrusion is larger.

Fig. 6 plots that the die surface friction condition affect the working limit (semi die angle). This figure indicates that the working limit generally increases with an increasing friction factor. In this case, the sleeve which is softest flows fastest. If

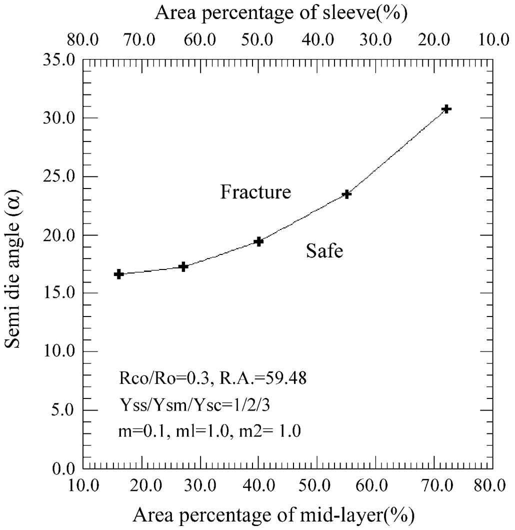

Fig. 4. Effects of area percentage of mid-layer ($A_{\mathrm{mo}}/A_{\mathrm{bo}}$) on the working limit (semi die angle).

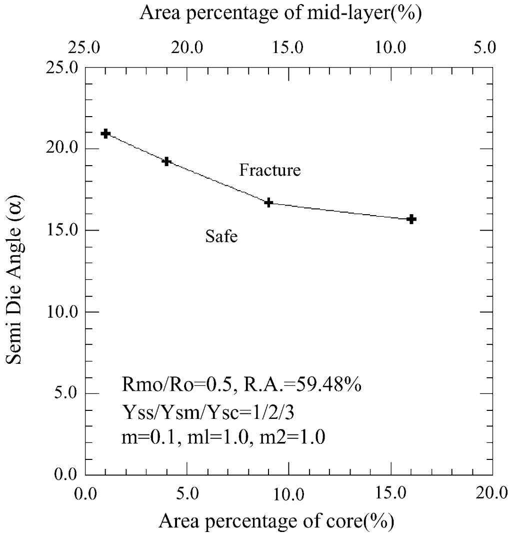

Fig. 5. Effects of area percentage of core ($A_{\mathrm{co}}/A_{\mathrm{bo}}$) on the working limit (semi die angle).

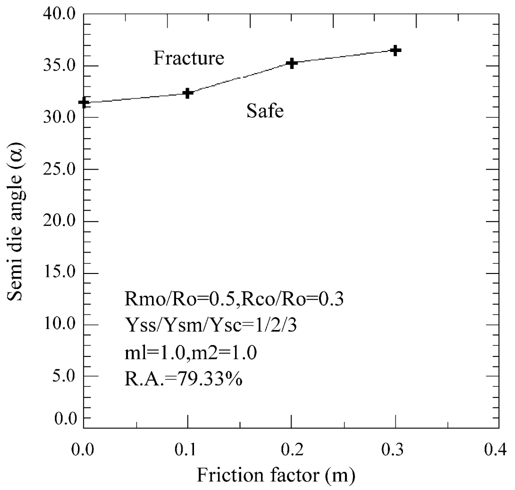

Fig. 6. Effects of friction factor of die on the working limit (semi die angle).

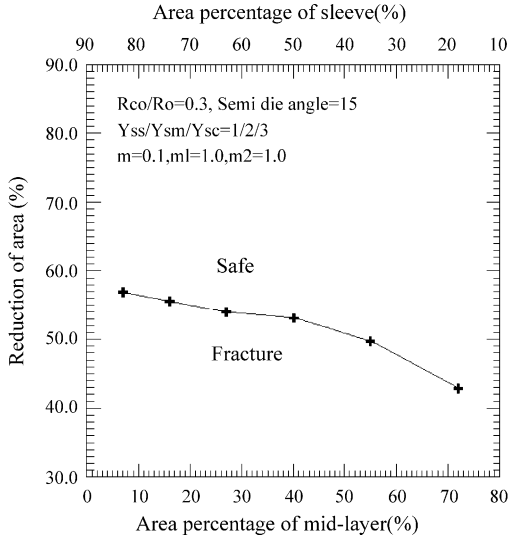

Fig. 7. Effects of area percentage of mid-layer ($A_{\mathrm{mo}}/A_{\mathrm{bo}}$) on the working limit (reduction of area).

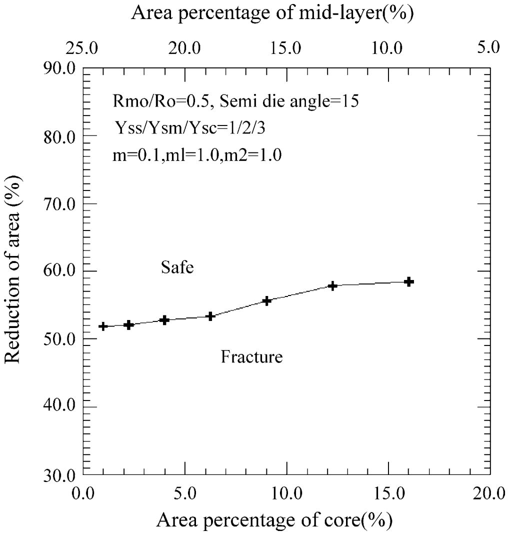

Fig. 8. Effects of area percentage of core ($A_{\mathrm{co}}/A_{\mathrm{bo}}$) on the working limit (reduction of area).

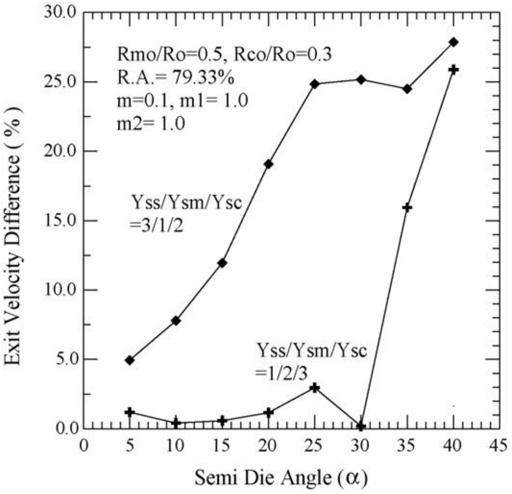

Fig. 9. Effects of the combination of the three-layer constituent materials on the exit velocity difference.

the friction factor is larger, the sleeve flows along die surface is more difficult, the deviation of velocity between sleeve and mid-layer will be small. But high friction factor causes product appearance will be defaced.

Fig. 7 displays the working limit (reduction of area) against the initial area percentage of mid-layer. If the reduction of area is small, and the radius of mid-layer is too small than the radius of exit, the mid-layer and core would not deform because the mid-layer and core are harder than sleeve. Meanwhile, the sleeve undergoes total deformation. The deviation of velocity between sleeve and mid-layer is large, the billet tend to fracture. On the other hand, if the reduction of area is large, the sleeve will force mid-layer to flow inside to reduce the area of mid-layer. Then, the deviation of velocity between sleeve and mid-layer is small, the processing of extrusion will be safe.

Fig. 8 reveals that the initial area of core affect the working limit (reduction of area). Increasing the initial area of core, the initial area of mid-layer decreases. As mention before, the possibility of fracture increases with a decreasing initial area of mid-layer. But the influence of the initial area of core is less dominant than the initial area of mid-layer.

As shown in Fig. 9, the combination of a hardest core $Y_{\mathrm{ss}}/Y_{\mathrm{sm}}/Y_{\mathrm{sc}} = 1/2/3$ demonstrates less exit velocity differences among the constituent layers for a semi die angle smaller than 30. On the other hand, for $Y_{\mathrm{ss}}/Y_{\mathrm{sm}}/Y_{\mathrm{sc}} = 3/1/2$, the exit velocity difference is obvious for a semi die angle larger than 5 and increases rapidly. That is to say the hardest core combines with the most soft sleeve layer promotes more homogeneous deformation and is highly desired for extrusion.

Table 1 demonstrates that the strength combination conditions affect the working limit, reduction of area and semi die angle. In general, the material in the center is flow faster than in the periphery. If the strength combination condition is

Table 1
Effects of the strength combination conditions on the working limit (reduction of area and semi die angle)

Relative strength (sleeve/mid-layer/core)	Working limit (reduction of area)	Working limit (semi die angle)
1/2/3	55.58% ↑ save	32.33° ↓ save
1/3/2	58.92% ↑ save	27.55° ↓ save
2/3/1	62.13% ↑ save	25.68° ↓ save
2/1/3	74.70% ↑ save	18.01° ↓ save
3/2/1	78.82% ↑ save	15.06° ↓ save
3/1/2	86.77% ↑ save	9.06° ↓ save

$Y_{ss}/Y_{sm}/Y_{sc} = 3/1/2$ or 3/2/1, mid-layer and core are more easily deformed, particularly at larger die angles where plastic flows are more severe, the deviation of velocity is relatively large. Therefore, this strength combination condition is most prejudicial as shown in Table 1. On the other hand, for yield stress combination of $Y_{ss}/Y_{sm}/Y_{sc} = 1/2/3$, the possibility of sound extrusion is largest.

6. Conclusions

This work presents a three-dimensional kinematically admissible velocity field to extrude a hexagonal three-layer composite clad rod. Also examined herein are factors that dominate the working limit of the rod include the initial area percentage of mid-layer and core, semi die angle, friction on die surface, strength combinations of constituent materials and reduction of area of the billet. Based on the results presented herein, we can conclude the following:

1. The larger the initial area percentage of mid-layer is, the working range (semi die angle and reduction of area of billet) is more spacious. One the other hand, the larger the initial area percentage of core is, it is harder to obtain a sound extrusion.
2. A higher frictional factor promotes the working limit range. But high friction factor causes product appearance will be defaced.
3. Combining the three-layer composite clad rod with hardest core/softest sleeve is highly desired for extrusion.
4. The proposed model can be applied to extrude a composite clad rod with an irregular cross-section if a function can mathematically express its cross-section profile.

Finally, from the results of this paper, it can be concluded that the fracture condition is promoted by an increase in the yield stress ratio and the initial area percentage of core. On the other hand, an increase in the initial area percentage of mid-layer, in the reduction of area of the billet and in the friction factor on die surface.

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