

# Sliding mode grey speed control of synchronous reluctance motor current sensorless drive

Chien-An Chen<sup>1</sup>, Huann-Keng Chiang<sup>2</sup>, Bor-Ren Lin<sup>2</sup> and Chih-Huang Tseng<sup>3</sup>

<sup>1</sup>Graduate School of Engineering Science & Technology, National Yunlin University of Science & Technology, Douliu, Yunlin 640, Taiwan

<sup>2</sup>Department of Electrical Engineering, National Yunlin University of Science & Technology, Douliu, Yunlin 640, Taiwan      [chianghk@yuntech.edu.tw](mailto:chianghk@yuntech.edu.tw)

<sup>3</sup>Department of Electrical Engineering, Nan Jeon Institute of Technology, Yen-Shui, Tainan 178, Taiwan

## Abstract

*A synchronous reluctance motor driven by an integral variable structure grey controller for current sensorless is presented. We adopt a maximum torque control strategy. The integral variable structure controller is used to reject the uncertain bounded disturbances and parameter variations. The grey prediction can reduce the high frequency chattering phenomenon or eliminate steady state error due to use the boundary layer. The voltage reference equation generates the require voltage from the torque command and the motor speed. Simulation results show that the proposed controller has a fast response and a good disturbance rejection capability.*

## 1. Introduction

The Synchronous Reluctance Motor (SynRM) has a mechanically simple and robust structure. It can rotate at high speeds in high temperature environments. Since last two decades, many researchers have focus their attention on the speed control of SynRM using sliding mode control strategies [1,2]. There are also some researches focusing their attention on the current sensorless using different control strategies [3-5]. In [3], an ideal model of SynRM speed control with current sensorless and low resolution position sensor was discussed. In [4], an ideal model of SynRM current sensorless speed control using sliding mode controller was proposed for maximum torque control (MTC), maximum power factor control (MPFC), maximum rate of change of torque (MRCTC) and constant current in inductive control (CCIAC) strategies. In [5], SynRM includes core losses for integral variable structure controller in MTC without current sensor was discussed.

One of the popular methods about robust control is the so-called variable structure control [6,7]. It has been proven as an effective and robust control

technology in SynRM [1,2]. The integral variable structure control can offer fast dynamic response, insensitivity to parameter variations and external disturbances rejection when the boundaries of the system uncertainties are known. However, specific and reliable system uncertainty boundaries are difficult to obtain for practical applications. In real applications, uncertainty boundaries can easily exceed the assumed magnitude range, under which the existence-conditions of the sliding mode are broken and control performance deteriorates severely. Using high gain control to improve disturbance rejection has been proposed [8] which causes large amounts of chattering in the control system. Serious chattering can reduce by using the boundary layer which the signum function is replaced by the saturation function. However, it produces the steady state error.

The grey theory proposed by Deng [9] has been successfully employed in motor control and control systems [10,11]. Grey prediction methodology requires only several output data to develop a grey model and to forecast a future value, without complex calculation, and is suitable in on-line control systems. Hence, a current sensorless of the SynRM model using integral variable structure grey control is considered for MTC strategy in this paper.

## 2. Modeling of the SynRM

The d-q equivalent voltage equations of the ideal model SynRM with the synchronously rotating rotor reference frame are represented as

$$V_d = R_s i_d - \omega_r L_q i_q + L_d \frac{di_d}{dt} \quad (1a)$$

$$V_q = R_s i_q + \omega_r L_d i_d + L_q \frac{di_q}{dt} \quad (1b)$$

where the  $V_d$  and  $V_q$  are direct and quadrature axis terminal voltages, respectively. The  $i_d$  and  $i_q$  are, respectively, direct axis and quadrature axis terminal

currents or the torque producing currents. The  $L_d$  and  $L_q$  are the direct and quadrature axis magnetizing inductances, respectively. The  $R_s$  is the stator resistance and  $\omega_r$  is the speed of the rotor.

The corresponding electromagnetic torque  $T_e$  and motor dynamic equation are given as following

$$T_e = \frac{3}{2} \frac{P}{2} (L_d - L_q) i_d i_q \quad (2)$$

$$T_e - T_L = J_m \frac{d\omega_r}{dt} + B_m \omega_r \quad (3)$$

where  $P$ ,  $T_L$ ,  $J_m$  and  $B_m$  are the number of poles, the torque load, the inertia moment of rotor and the viscous friction coefficient, respectively.

### 3. The Integral Variable Structure Grey Controller (IVSGC)

#### 3.1. Integral Variable Structure Controller (IVSC)

Defining the velocity error  $e(t) = \omega_r - \omega_{ref}$ ,  $\omega_{ref}$  is the velocity command. Assume the time required to change the velocity command is much longer than the velocity response time (i.e.  $d\omega_{ref}/dt = 0$ ). The velocity error differential equation is expressed as

$$\begin{aligned} \frac{de(t)}{dt} &= \left(-\frac{B_m}{J_m}\right)\omega_r + \left(\frac{1}{J_m}\right)T_e - \left(\frac{1}{J_m}\right)T_L \\ &= \tilde{a}e(t) + \tilde{a}\omega_{ref} + \tilde{b}u(t) - \tilde{b}T_L(t) \end{aligned} \quad (4)$$

where  $\tilde{a} \equiv -\frac{B_m}{J_m} = a_0 + \Delta a$

$$\tilde{b} \equiv \frac{1}{J_m} = b_0 + \Delta b$$

$$u \equiv T_e$$

The subscript index ‘‘o’’ indicates nominal system value; ‘‘ $\Delta$ ’’ symbol indicates uncertainty. The sliding function  $S$  is combined with the integration of the state as

$$S = e(t) + c \int_{-\infty}^t e(\tau) d\tau, \quad c > 0 \quad (5)$$

To eliminate the sliding error and reduce the serious chattering, traditional input control  $u(t)$  (the electromagnetic torque  $T_e$ ) can be defined as

$$u(t) = u_{eq}(t) + u_n(t) \quad (6)$$

where  $u_{eq} = -\frac{1}{b_0}[(a_0 + c)e + a_0\omega_{ref}]$

$$u_n = \Phi_1 e(t) + \Phi_2$$

$$\Phi_1 = -\text{sat}(Se) \left[ \max\left(\frac{|k_1|}{b}\right) + \delta \right], \quad \delta > 0$$

$$\Phi_2 = -\text{sat}(S) \left[ \max\left(\frac{|k_2|}{b}\right) + \delta \right], \quad \delta > 0$$

$$k_1 = \Delta a - \frac{\Delta b}{b_0} (a_0 + c)$$

$$k_2 = \Delta a \omega_{ref} - \frac{\Delta b}{b_0} a_0 \omega_{ref} - \tilde{b} T_L$$

The saturation function  $\text{sat}()$  in  $\Phi_1, \Phi_2$  is used to replace the signum function of the conventional sliding mode control to reduce chattering. However, this situation causes the velocity error  $e$  toward the neighborhood of zero for  $t \rightarrow \infty$ .

#### 3.2. Grey Model [10]

Let  $x^{(0)}$  be the original data sequence

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} \quad (7)$$

where  $n$  is the sampling size of the recorded data. We map the original sequence  $x^{(0)}$  into the non-negative sequence  $x_m^{(0)}$ .

$$x_m^{(0)} = \{x_m^{(0)}(1), x_m^{(0)}(2), \dots, x_m^{(0)}(n)\} \quad (8)$$

The relationship of  $x^{(0)}$  and  $x_m^{(0)}$  are represented as

$$x_m^{(0)}(k) = \alpha + \beta x^{(0)}(k), \quad \alpha, \beta > 0 \quad (9)$$

The first order accumulated generating operation (AGO) sequence of  $x_m^{(0)}$  is

$$x_m^{(1)} = \{x_m^{(1)}(1), \dots, x_m^{(1)}(n)\} = \left\{ \sum_{j=1}^k x_m^{(0)}(j) \right\} \quad (10)$$

We define

$$z_m^{(1)}(k) = \frac{1}{2} (x_m^{(1)}(k-1) + x_m^{(1)}(k)), \quad k = 2, \dots, n \quad (11)$$

From  $z_m^{(1)}(k)$ , we formed a  $GM(1,1)$  first order whitening grey difference equation as

$$x_m^{(0)}(k) + a z_m^{(1)}(k) = b, \quad k = 2, 3, \dots, n \quad (12)$$

The parameters  $a$  and  $b$  can be solved using the least square method as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N \quad (13)$$

$$\text{where } B = \begin{bmatrix} -z_m^{(1)}(2) & 1 \\ -z_m^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z_m^{(1)}(n) & 1 \end{bmatrix}, \quad Y_N = \begin{bmatrix} x_m^{(0)}(2) \\ x_m^{(0)}(3) \\ \vdots \\ x_m^{(0)}(n) \end{bmatrix}$$

Based on the whitening equation, the prediction value of the  $GM(1,1)$  model is obtained by

$$\hat{x}_m(k+1) = (x_m^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \quad (14)$$

where  $\hat{\cdot}$  denotes the predicted values. The forecasted value of  $x_m(k+1)$  can be expressed as

$$\hat{x}_m(k+1) = (x_m^{(0)}(1) - \frac{b}{a})(e^{-ak} - e^{-a(k-1)}) \quad (15)$$

Similarly, the predicted value of the original data sequence  $\hat{x}^{(0)}$  is

$$\hat{x}^{(0)}(k+1) = \frac{1}{\beta}[(x_m^{(0)}(1) - \frac{b}{a})(e^{-ak} - e^{-a(k-1)}) - \alpha] \quad (16)$$

### 3.2. Integral Variable Structure Grey Controller (IVSGC)

To eliminate the steady state error of (6) due to the sat() in  $\Phi_1$  and  $\Phi_2$ , the control input  $u(t)$  of the proposal IVSGC is designed as

$$u(t) = u_{eq}(t) + u_n(t) + u_{gc}(t) \quad (17)$$

where  $u_{eq}$ ,  $u_n$  are unnecessary exact known the parameter variations and external disturbance in (6) and

$$u_{gc} = \begin{cases} -\rho \text{ sign}(S\hat{S}), & \rho > 0 \text{ for } |S| > \Phi \\ 0, & \text{otherwise} \end{cases}$$

The  $\Phi$  is grey prediction boundary layer which makes the velocity error to move toward the neighborhood of the origin.

### 4. Voltage reference calculation of current sensorless for the MTC strategy

The electromagnetic torque of (2) can be represented as

$$T_e = \frac{3}{2} \frac{P}{2} \frac{I_s^2}{2} (L_d - L_q) \sin 2\phi \quad (18)$$

where  $I_s = \sqrt{i_d^2 + i_q^2}$  and  $\phi = \tan^{-1} \frac{i_q}{i_d}$ . When the current angle is  $\phi = \pm\pi/4$ , the SynRM has the maximum torque[4]. Hence, we can get the torque current  $i_d$  and  $i_q$  as

$$i_d = \sqrt{\frac{|T_e|}{\frac{3P}{4}(L_d - L_q)}} \quad (19)$$

$$i_q = \pm \sqrt{\frac{|T_e|}{\frac{3P}{4}(L_d - L_q)}} \quad (20)$$

In (20),  $i_q$  is positive when  $T_e > 0$  and  $i_q$  is negative when  $T_e < 0$ .

A current sensorless control scheme includes the voltage reference calculator which generates the required voltage from the torque command and the motor speed. By inserting (19), (20) into (1), the required voltages  $V_d$  and  $V_q$  are

$$V_d = \sqrt{\frac{1}{\frac{3P}{4}(L_d - L_q)}} \left( R_s \sqrt{|T_e|} + L_d \frac{d\sqrt{|T_e|}}{dt} \mp L_q \omega_r \sqrt{|T_e|} \right) \quad (21)$$

$$V_q = \sqrt{\frac{1}{\frac{3P}{4}(L_d - L_q)}} \left( \pm R_s \sqrt{|T_e|} \pm L_q \frac{d\sqrt{|T_e|}}{dt} + L_d \omega_r \sqrt{|T_e|} \right) \quad (22)$$

## 5. Simulation results

The block diagram of the experimental SynRM system is shown in Fig. 2. The 0.25hp three-phase SynRM of nominal parameters and the proposal IVSGC parameters are shown in Table 1. The external disturbance is more important effect than the parameter variations. Hence, we only consider the disturbance effect in simulation.

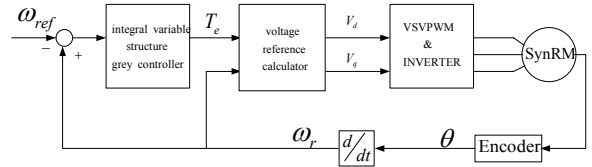


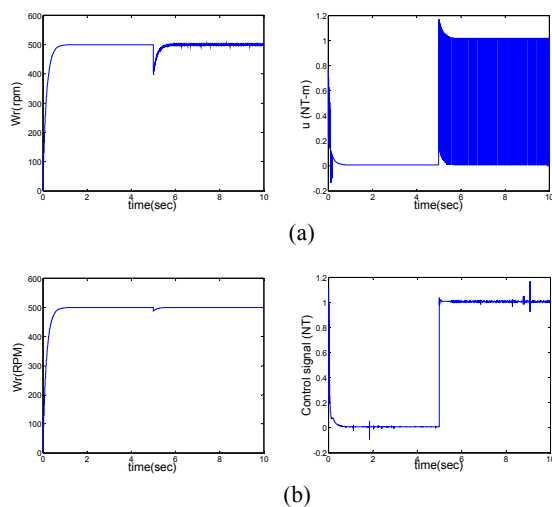
Fig. 2 Simulation structure of current sensorless SynRM

Table 1 The parameters of SynRM (0.37KW) IVSGC

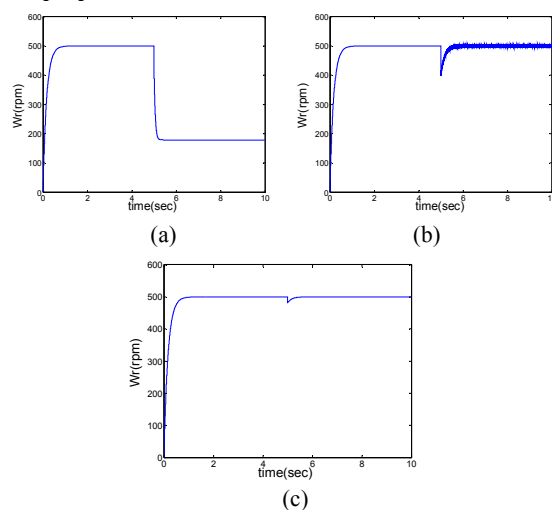
$R_s$	$R_c$	$L_d$	$L_q$
4.2 $\Omega$	50 $\Omega$ (f=60Hz)	0.328 H	0.181 H
$J_m$	$B_m$	$P$	$k_1$
0.00076 kg-m <sup>2</sup>	0.00012 Nt-m.s	2	0
$k_2$	$\delta$	$\rho$	$\Phi$
1 Nt-m	0.01	0.75	10rad/sec

Fig. 3 shows the speed responses  $\omega_r$  and control signal  $u$  for 500rev/min reference command with a 1 NT-m external disturbance at  $t=5$  sec using sliding mode controller and the proposed IVSGC, respectively. In Fig. 3(a), the speed response has a chattering phenomenon when the external disturbance is added. In Fig. 3(b), the speed response is better than the

sliding mode control when the external disturbance is added. The speed response for 500rev/min reference command with a 1.5 NT-m external disturbance at  $t = 5$  sec is shown in Fig. 4. In Fig. 4(a), the speed response has steady state error when the maximum practice uncertainty 1.5Nt-m is unexpectedly larger than the designed 1 Nt-m maximum bounded in sliding mode control. If we magnify the design maximum uncertainty 2Nt-m to overcome the unexpectedly larger disturbance, the system will have a serious chattering phenomenon when the disturbance is added shown as Fig. 4(b). Fig. 4(c) shows the good speed response without steady state error and less chattering using the proposed IVSGC.



**Fig. 3** The speed response  $\omega_r$  and control signal  $u$  for speed command 500rpm with a 1 NT-m external disturbance at  $t = 5$  sec using (a) sliding mode control, (b) the proposal IVSGC.



**Fig. 4** The speed responses  $\omega_r$  for speed command 500rpm with a 1.5 NT-m external disturbance at  $t = 5$  sec using (a) sliding mode control when external disturbance is larger than the designed maximum uncertainty, (b) sliding mode control when we magnify the design maximum uncertainty to overcome the unexpectedly larger disturbance, (c) the proposal IVSGC.

## 6. Conclusions

A complete model development and analysis for the current sensorless of synchronous reluctance motor speed control is presented in this paper. We adopt a MTC strategy of the current sensorless integral variable structure grey controller. This control scheme does not use current sensor which reduces the system cost. Finally, we employ the simulations to validate the proposed method.

## 7. References

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