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Optimal fuzzy multi-criteria expansion of competence sets using multi-objectives evolutionary algorithms

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Abstract

Competence set is widely used to plan the optimal expansion process of skills, abilities or strategies. However, the conventional method is concerned only with one criterion rather than multi-criteria problems. In addition, the crisp value cannot reflect the ambiguity and the uncertainty in practice. In this paper, we propose the fuzzy criteria competence set analysis. In order to obtain Pareto solutions, multi-objective evolutionary algorithm (MOEA) is employed here. A numerical example with two fuzzy criteria is also used to illustrate the proposed method.

Keywords: Competence set; Multi-criteria problem; Pareto solutions; Multi-objective evolutionary algorithm (MOEA)

1. Introduction

Making good decisions involve the successive accumulation of the particular skills, ideas, information and knowledge. In order to efficiently and effectively acquire these abilities, competence set analysis was proposed (Yu, 1990; Yu & Zhang, 1990). Using the methods of competence set analysis such as the minimum spanning tree (Yu & Zhang, 1992) or the mathematical programming (Shi & Yu, 1996), we can obtain the optimal path (e.g. the minimum cost or time) to acquire the required competence.

In conventional competence set analysis, one criterion such as cost or benefit function is used to select the optimal expansion process. However, in practice we usually determine the optimal expansion process according to multi-criteria (e.g. cost, time, efficient, benefit, and so on) simultaneously. Additionally, in order to reflect the ambiguity and uncertainty in practice, we should incorporate the concept of fuzzy sets into competence set analysis.

In order to deal with multi-criteria problems, many methods such as goal programming (Charnes & Cooper, 1957; Ijiri, 1965), min–max optimization (Osyczka, 1978; Rao, 1986) and the ε-constraint method (Osyczka, 1984; Hwang et al., 1980) have been proposed. Recently, MOEA has been widely used in various multi-objective problems such as scheduling (Murata, Ishibuchi, & Tanaka, 1996), engineering (Fonseca & Fleming, 1998) and finance (Mardle, Pascoe, & Tamiz, 2000). Compared to these conventional methods, multi-objective evolutionary algorithm (MOEA) seems more suitable to solve multi-objective problems because it searches a set of possible solutions simultaneously. Therefore, we can obtain a set of Pareto solutions rather than a special solution as in conventional methods in a run. These solutions are very important for the decision-maker to chose the optimal expansion process because some objectives are intangible and cannot be form using the conventional mathematical model. In addition, another reason which we use MOEA in this paper is that it can deal well with concave and discountinuous objective functions and Pareto frontiers.

On the other hand, the concept of fuzzy sets was proposed by Zadeh (Kim, Modkowitz, & Koksalan, 1965) to represent the uncertain situations or the subjective judgments. Using the membership function, we can measure the degrees of the uncertainty and deal with the fuzzy problems. Recently, the concept of fuzzy sets has been incorporate into

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the conventional statistical or mathematical programming methods to reflect the ambiguity and uncertainty in practice (Zimmerman, 1978; Tanaka & Lee, 1998; Kim, Modkowitz, & Koksalan, 1996).

In this paper, we propose the fuzzy multi-criteria competence set analysis. A numerical example is demonstrated to select the optimal fuzzy multi-criteria expansion process. Two criteria (cost and benefit function) with fuzzy numbers are used to reflect the ambiguity and uncertainty in practice. By employing MOEA, we can obtain Pareto solutions. On the basis of Pareto solutions, decision-makers can determine the finial optimal expansion process based on his preferences or subjective judgments.

The remainder of this paper is organized as follows. The expansion process of competence set analysis and the proposed method are discussed in Section 2. Multi-objective evolutionary algorithm is proposed in Section 3 to describe its ideas and procedures. A numerical example is used to demonstrate the proposed method in Section 4. Discussions are presented in the Section 5 and conclusions are in the last section.

2. Expansion process of competence set

The concept of competence set was proposed by Yu (1990) to resolve a particular decision problem by acquiring the necessity of ideas, information, skills, and knowledge. The contents of competence set analysis are to identify the true competence set, the decision-maker's competence set, and the efficient expansion path to make good decisions.

Among these issues, the method to optimally expand the existing competence set is especially highlighted. Several methods, such as the minimum spanning tree (Yu & Zhang, 1992), the mathematical programming method (Shi & Yu, 1996), and the deduction graphs (Li & Yu, 1994), have been proposed to obtain the optimal path. The optimal expansion process from the existing competence set to the true competence set can be described as follows.

Let HD = SK∪T where HD (habitual domains) is all the related skills needed to solve a particular problem, *SK* denotes the already acquired competence set and *T* denotes the true required competence set. Therefore, the optimal expansion process can be obtained by minimizing the following equation

$$\min\{c(x_i, x_j),\quad x_i \in \mathrm{SK} \text{ and } x_j \in T\}, \tag{1}$$

where $c(x_i,x_j)$ denotes the cost of acquiring x_j from x_i. The corresponding graph can be represented as Fig. 1.

We can use an example to illustrate the optimal expansion process using the minimum spanning tree as follows. Let the SK = $\{a\}$, $T=\{a,b,c,d\}$, and the cost function can be given as shown in the following matrix

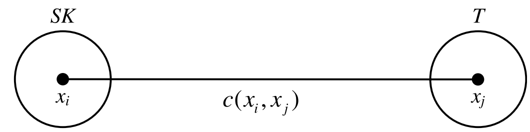

Fig. 1. The cost function of competence set.

Cost	*a*	*b*	*c*	*d*
a	0	2	6	8
b	8	0	1	4
c	8	2	0	1
d	1	2	3	0

In order to determine the first step of the expansion process, we must consider the cost information as follows:

Process	*b*	*c*	*d*
$c(a$, process)	2	6	8

Then, the first step is $a \rightarrow b$. Next, consider the following cost to determine the second step:

Process	*c*	*d*
$c(\{a \rightarrow b\}$, process)	1	4

Therefore, the optimal second expansion process is $b \rightarrow c$, and the optimal expansion process is $(a \rightarrow b \rightarrow c \rightarrow d)$.

The optimal expansion process of competence sets can be viewed as a special case of the network problems. We can employ the routing method (Shi & Yu, 1996) to select the optimal expansion process based on the following mathematical programming model

$$\begin{aligned} \min \quad & z = \sum c_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i=0}^{n} x_{ij} = 1,\ j = 1, 2, \ldots, n, \\ & u_i - u_j + (n+1)x_{ij} \le n,\ 1 \le i,j \le n,\ i \ne j, \\ & \forall x_{ij}, u_i \in \{0, 1, \ldots n\}. \end{aligned} \tag{2}$$

where $c_{ij}=c(x_i,x_j)$ denotes the cost of acquiring x_j from x_i and u_i denotes the subsidiary variable. Although many scholars extend competence set to consider further situations such as asymmetric acquiring cost (Shi & Yu, 1996), and group decisions (Li, 1997), these papers address only the single criterion and the crisp objective function. However, the decision-maker usually needs to consider the multi-criteria and the fuzzy situations in practice to select the optimal expansion process. Therefore, this paper extends competence set analysis to consider the fuzzy multi-criteria situation.

In order to formulate the fuzzy multi-criteria competence set, the fuzzy mathematical programming model is

```
procedure
GA
begin
    t = 0
    initialize P(t)
    evaluate P(t)
    transform fitness vectors into a scalar
    while not satisfy stopping rule do
    bigin
        t = t + 1
        select P(t) from P(t 1)
        alter P(t)
        evaluate P(t)
        transform fitness vectors into a scalar
    end
end
```

Fig. 2. The pseudo code of MOEA.

employed here. The fuzzy programming problem (Carlsson & Korhonen, 1986) can be represented as follows

$$\begin{aligned} \max \quad & \tilde{z} = \textstyle\sum_i \tilde{c}_i \tilde{x}_i \\ \text{s.t.} \quad & \tilde{X} = \{(x, \mu(x)) | (\tilde{A}x)_i \le \tilde{b}_i, \ \forall i, x \ge 0, \ \mu(x) \in [0,1]\} \end{aligned} \tag{3}$$

where $\mu(x)$ denotes the membership function of x. By setting the adequate membership function and α-cut, we can transform Eq. (3) into the following equation to derive the optimal solution of the fuzzy programming problem.

$$\begin{aligned} \max \quad & \tilde{z} = \sum_{j=1}^{n} \mu_{\tilde{c}_j}^{-1}(\alpha) x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} \mu_{\tilde{a}_{ij}}^{-1}(\alpha) x_j \le \sum_{j=1}^{n} \mu_{\tilde{b}_j}^{-1}(\alpha), \forall i = 1, \ldots, m, \\ & x_j \ge 0, \ \forall j = 1, \ldots, n. \end{aligned} \tag{4}$$

Now, based on the concepts above, we can formulate the optimal fuzzy multi-criteria expansion process as the following mathematical programming model

$$\begin{aligned} \text{min/max} \quad & \tilde{z}_1 = \textstyle\sum \mu_{\tilde{c}_{1ij}}^{-1}(\alpha) x_{1ij} \\ \text{min/max} \quad & \tilde{z}_2 = \textstyle\sum \mu_{\tilde{c}_{2ij}}^{-1}(\alpha) x_{2ij} \\ & \vdots \\ \text{min/max} \quad & \tilde{z}_m = \textstyle\sum \mu_{\tilde{c}_{mij}}^{-1}(\alpha) x_{mij} \\ \text{s.t.} \quad & \sum_{i=0}^{n} x_{ij} = 1, \ j = 1, 2, \ldots, n, \\ & u_i - u_j + (n+1) x_{ij} \le n, \ 1 \le i, j \le n, \ i \ne j, \\ & \forall x_{ij}, u_i \in \{0, 1, \ldots n\}. \end{aligned} \tag{5}$$

where $c_{ij} = c(x_i, x_j)$ denotes the cost of acquiring x_j from x_i and u_i denotes the subsidiary variable. As mentioned previously, because some criteria are in conflicting with each other and intangible, Pareto solutions can be derived by using MOEA. Then, decision-makers can select the finial optimal expansion process based on his preferences.

3. Multi-objective evolutionary algorithm

Multi-objective evolutionary algorithm (MOEA) has been widely used since the 1990's to resolve the combinational problem in various areas such as scheduling (Murata, 1996), engineering (Fonseca & Fleming, 1998) and finance (Mardle et al., 2000). The concept of MOEA is based on the method of genetic algorithm (GA). GA was pioneered in 1975 by Holland, and its concept is to mimic the natural evolution of a population by allowing solutions

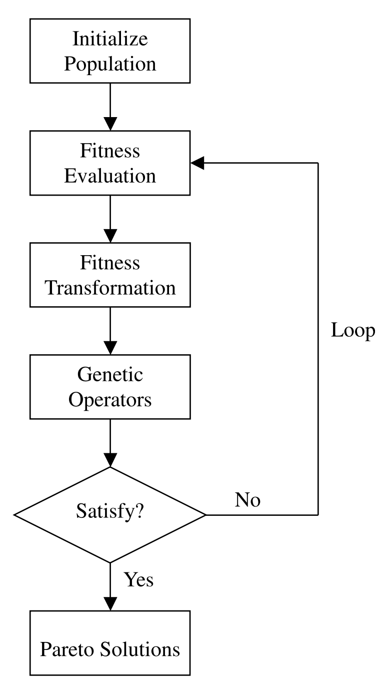

Fig. 3. The procedure graph of MOEA.

Table 1
Cost function of the fuzzy competence set

Cost	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_0	M	(4.9,5.9)	(6.1,7.1)	(2.7,3.7)	(2.6,3.6)	(3.2,4.2)	(5.5,6.5)	(3.4,4.4)
x_1	M	M	(3.9, 4.9)	(4.7,5.7)	(3.9,4.9)	(4.0,5.0)	(4.3,5.3)	(3.5,4.5)
x_2	M	(4.4,5.4)	M	(3.7,4.7)	(6.4,7.4)	(5.7,6.7)	(5.9,6.9)	(4.5,5.5)
x_3	M	(6.3,7.3)	(2.7,3.7)	M	(5.8, 6.8)	(6.4,7.4)	(6.4,7.4)	(3.9,4.9)
x_4	M	(5.5,7.5)	(3.9,4.9)	(5.6,6.6)	M	(3.3,4.3)	(2.8,3.8)	(2.8,3.8)
x_5	M	(4.4,5.4)	(4.5,5.5)	(4.0,5.0)	(6.4,7.4)	M	(6.3,7.3)	(2.8,3.8)
x_6	M	(4.0,5.0)	(2.9,3.9)	(4.9,5.9)	(6.1,7.1)	(6.0,7.0)	M	(4.7, 5.7)
x_7	M	(6.3,7.3)	(2.6,3.6)	(4.1,5.1)	(6.0,7.0)	(3.1,4.1)	(2.8,3.8)	M

Table 2
Benefit function of the fuzzy competence set

Benefit	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_0	M	(4.5,5.5)	(4.6,5.6)	(5.1,6.1)	(2.8,3.8)	(2.9,3.9)	(4.9,5.9)	(3.6,4.6)
x_1	M	M	(5.3,6.3)	(2.9,3.9)	(3.6,4.6)	(2.8,3.8)	(3.5,4.5)	(4.9,5.9)
x_2	M	(5.8,6.8)	M	(3.7,4.7)	(4.9,5.9)	(5.6,6.6)	(5.9,6.9)	(3.5,4.5)
x_3	M	(5.2,6.2)	(5.8,6.8)	M	(6.5,7.5)	(3.8,4.8)	(5.2,6.2)	(5.0,6.0)
x_4	M	(3.2,4.2)	(5.9,6.9)	(4.0,5.0)	M	(3.4,4.4)	(3.5,4.5)	(5.7,6.7)
x_5	M	(4.7,5.7)	(6.0,7.0)	(6.4,7.4)	(4.6,5.6)	M	(6.4,7.4)	(4.8,5.8)
x_6	M	(5.4,6.4)	(2.8,3.8)	(5.6,6.6)	(5.7,6.7)	(6.0,7.0)	M	(2.6,3.6)
x_7	M	(6.4,7.4)	(3.0,4.0)	(4.9,5.9)	(5.5,6.5)	(4.3,5.3)	(2.7,3.7)	M

to reproduce, create new solutions, and compete for surviving in the next iteration (Holland, 1975; Goldberg, 1989; Davis, 1991; Koza, 1992; Michalewicz, 1992). Then, the fitness is improved over generations and the best solution is finally achieved.

The procedures of MOEA are similar to GA. The initial population, $P(0)$, is encoded randomly by strings. In each generation, t, the more fit elements are selected for the mating pool. Then, three basic genetic operators, reproduction, crossover, and mutation, are processed to generate new offspring. On the basis of the principle of survival of the fittest, the best chromosome of a candidate solution is obtained. The pseudo codes and the corresponding procedure graph of MOEA can be represented as shown in Figs. 2 and 3.

The power of the evolution algorithm lies in its abilities to simultaneously search a population of points in parallel, not a single point. Therefore, the evolution algorithm can find the approximate optimum quickly without falling into a local optimum. In the conventional mathematical programming techniques, these methods generally assume small and enumerable search spaces (Coello Coello, David, & Gary, 2002). However, MOEA can handle various function problems such as discontinuous or concave form and scaling problems (Coello Coello et al., 2002; Deb, 2001; Gen & Cheng, 2000). In addition, we can obtain the Pareto optimal set rather than a special solution using the method of MOEA.

Next, we describe the three basic genetic operators used in MOEA as follows:

Crossover. The goal of crossover is to exchange information between two parent chromosomes in order to produce two new offspring for the next population. In this study, we use uniform crossover to generate the new offspring. The procedures of uniform crossover can be described as follows. Let two parents and a random template be represented by

$$\begin{aligned} \text{Template} &= 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1 \\ \text{Parent}_1 &= 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1 \\ \text{Parent}_2 &= 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0 \end{aligned}$$

Then, the two offspring that will be generated are represented as

$$\begin{aligned} \text{Offspring}_1 &= 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1 \\ \text{Offspring}_2 &= 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0 \end{aligned}$$

Mutation. Mutation is a random process where one genotype is replaced by another to generate a new chromosome. Each genotype has the probability of mutation, P_m, to change from 0 to 1, and vice versa.

Selection. The selection operator selects chromosomes from the mating pool using the ‘survival of the fittest’ concept, as in natural genetic systems. Thus, the best

Table 3
The parameter settings of MOEA

Parameter	Values
Population size	100
Selection strategy	Tournament
Maximum number of generation	1000
Crossover rate	0.9
Mutation rate	0.01

Table 4
Pareto solutions of a competence set

$\alpha=0.8$	Optimal expansion process							Cost	Benefit
Model 1	$0\to1$	$0\to3$	$1\to2$	$1\to4$	$1\to6$	$3\to7$	$6\to5$	31.0	38.6
Model 2	$0\to1$	$0\to4$	$0\to5$	$0\to7$	$4\to2$	$5\to3$	$5\to6$	30.0	38.1
Model 3	$0\to1$	$0\to3$	$0\to4$	$1\to6$	$4\to2$	$4\to7$	$7\to5$	25.7	37.4
Model 4	$0\to1$	$0\to3$	$0\to4$	$1\to6$	$3\to2$	$4\to7$	$7\to4$	24.5	37.3
Model 5	$0\to1$	$0\to3$	$0\to4$	$0\to5$	$0\to7$	$3\to4$	$7\to6$	24.0	33.0
Model 6	$0\to3$	$0\to4$	$0\to5$	$0\to7$	$5\to1$	$7\to2$	$7\to6$	23.4	30.4

chromosomes receive more copies, while the worst die off. The probability of variable selection is proportional to its fitness value in the population, according to the formula given by

$$P(x_i) = \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)} \quad (6)$$

where $f(x_i)$ represents the fitness value of the ith chromosome, and N is the population size.

In addition, one of the crucial procedures of MOEA is to determine the fitness function. In this paper, the crowding distance (Deb, Pratap, Agarwal, & Meyarivan, 2002; Jensen, 2003) is used to sort the chromosomes and determine Pareto solutions. In the next section, we use a numerical example to illustrate the proposed method.

4. Numerical example

In this numerical example, we will demonstrate a fuzzy two-criterion (i.e. cost and benefit) expansion of competence sets. Let $\mathrm{SK}=\{x_0\}$, $T\backslash\mathrm{SK}=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}$ and the fuzzy cost and the fuzzy benefit functions, which represent with interval values, are shown in Tables 1 and 2. Note that the symbol, M, denotes the infeasible route and will be treated as a minimum number in our fuzzy mathematical programming model. In addition, the membership of the cost and benefit functions are assumed to the triangular form.

Using Eq. (5) and let α-cut equal to 0.8 (other results which set $\alpha=0.2$ and 0.5 can also be obtained in Appendix A), we can formulate the optimal fuzzy multi-criteria expansion model based on the information from Tables 1 and 2. In order to obtain Pareto solutions, next we must first set the adequate parameters of MOEA as shown in Table 3.

After generating and calculating the optimal generations, we will then obtain six optimal expansion processes i.e. Pareto solutions as shown in Table 4.

For example, Model 1 depicts the optimal expansion process as shown in Fig. 4 to obtain the optimal costs equal to 31.0 and the optimal benefits equal to 38.6.

On the basis of the results, decision-makers can select one of the six paths based on his preferences or subjective judgments to determine the final optimal expansion process. Next, we provide the discussions about our numerical example in Section 5.

5. Discussions

Competence set analysis has been used for many applications, such as learning sequences for decision-makers (Hu, Chen, & Tzeng, 2002) and for consumer decision problems (Chen, 2001; Chen, 2002). However, these papers only consider the situation of using one criterion and the crisp function. In practice, decision-makers usually determine the optimal expansion process based on multi-criteria which may be conflicting with each other. Therefore, Pareto solutions should be derived for decision-makers to determine the final expansion process based on his preferences. In addition, due to the reason of uncertainty and subjective judgment, the concept of fuzzy sets should be incorporated into competence set analysis.

In this paper, the fuzzy multi-criteria expansion model is proposed to deal with the above problems. In order to obtain Pareto solutions efficiently and correctly, MOEA is employed here. A numerical example is also used to demonstrate the proposed method. On the basis of the simulated results, we can obtain six nondominated solutions. For the risk averse, Model 6 may be the optimal expansion process. However, Model 1 may be the optimal expansion process for a risk lover.

Compared to the conventional methods, the proposed method extends competence set analysis to consider the viewpoints of multi-criteria and fuzzy sets. On the basis of the first viewpoint, we can obtain Pareto solutions using MOEA and determine the expansion process based on our preferences. From the second viewpoint, we can reflect the degrees of uncertainty by adjusting α-cut. In summary, the proposed method can provide a more flexible and diverse model.

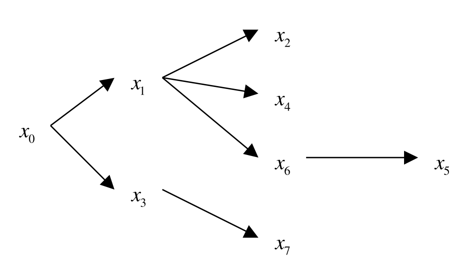

Fig. 4. The optimal fuzzy multi-criteria expansion process of model 1.

Table A1
Pareto solutions of a competence set ($\alpha=0.2$)

$\alpha=0.2$	Optimal expansion process							Cost	Benefit
Model 1	0→3	0→5	0→7	4→2	5→6	7→1	7→4	37.7	37.2
Model 2	0→3	0→7	1→4	5→2	5→6	6→1	7→5	33.5	35.8
Model 3	0→3	0→6	0→7	6→1	7→2	7→4	7→5	32.9	33.2
Model 4	0→3	0→7	4→6	7→1	7→2	7→4	7→5	32.5	32.8
Model 5	0→3	0→7	1→4	5→6	6→1	6→2	7→5	31.9	32.6
Model 6	0→3	0→7	2→1	4→5	7→2	7→4	7→5	30.6	32.2
Model 7	0→3	0→7	6→1	7→2	7→4	7→5	7→6	30.2	31.0

Table A2
Pareto solutions of a competence set ($\alpha=0.5$)

$\alpha=0.5$	Optimal expansion process							Cost	Benefit
Model 1	0→3	0→6	1→7	2→1	3→2	3→4	3→5	34.5	40.3
Model 2	0→3	0→6	0→7	2→1	3→2	3→4	3→5	34.4	39.0
Model 3	0→1	0→3	0→5	1→2	4→7	5→4	5→6	34.0	38.0
Model 4	0→3	0→7	2→1	3→2	3→4	3→5	4→6	31.7	37.6
Model 5	0→1	0→3	0→5	1→2	1→4	4→7	5→6	31.5	37.0
Model 6	0→1	0→3	0→5	1→2	1→4	2→6	4→7	31.1	36.5
Model 7	0→3	0→6	2→1	3→2	4→5	4→6	7→4	28.8	36.2
Model 8	0→1	0→3	0→4	4→2	4→5	4→7	7→6	26.5	33.6
Model 9	0→1	0→3	0→4	4→5	4→7	7→2	7→6	25.2	30.7

6. Conclusions

In this paper, we extend the conventional competence set analysis to consider the situation of multi-criteria and fuzzy number. In order to obtain Pareto solutions efficiently, MOEA is employed here. A numerical example is used to demonstrate the procedures of the proposed method. On the basis of the results, we can conclude that the proposed method can provide a more flexible and diverse model.

Appendix

By setting $\alpha=0.2$ and 0.5, we can obtain other two Pareto solutions as shown in Tables A1 and A2.

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