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Recurrent neural network for dynamic portfolio selection

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Abstract

In this paper, the dynamic portfolio selection problem is considered. The Elman network is first designed to simulate the dynamic security behavior. Then, the dynamic covariance matrix is estimated by the cross-covariance matrices. Finally, the dynamic portfolio selection model is formulated. In addition, a numerical example is used to demonstrate the proposed method and compare with the vector autoregression (VAR) model. On the basis of the numerical example, we can conclude that the proposed method outperform to the VAR model and provide the accurate dynamic portfolio selection.
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1. Introduction

The mean–variance approach was proposed by Markowitz to deal with the portfolio selection problem [1]. A decision-maker can determine the optimal investing ratio to each security based on the sequent return rate. The formulation of the mean–variance method can be described as follows [1–3]:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \\
\text{s.t.} \quad & \sum_{i=1}^{n}\mu_i x_i \geqslant E, \\
& \sum_{i=1}^{n} x_i = 1, \\
& x_i \geqslant 0 \quad \forall i = 1,\ldots,n,
\end{aligned} \tag{1}$$

where μ_i denotes the expected return rate of the ith security, σ_{ij} denotes the covariance coefficient between the ith security and the jth security, and E denotes the acceptable least rate of the expected return.

On the basis of Eq. (1), it can be seen that the conventional portfolio selection problem above is considered as a static situation. However, this assumption is truly against our intuition i.e. we always vary our optimal portfolio selection with time. Although many methods including vector autoregression (VAR) [3,4] and generalize autoregressive conditional heteroshedastic (GARCH) [5–7] has been proposed to deal with the dynamic portfolio selection problem, several restricted assumptions, such as stationary time series, independent variables, and the linear relationship among variables, make these models impractical. The purpose of this paper is to propose a non-parameter and non-linear method to deal with the dynamic portfolio selection problem.

In this paper, a dynamic portfolio selection model is proposed by incorporating the recurrent neural network (RNN) [8,9] and the cross-covariance matrices [4]. The dynamic expected return rate is first derived using the Elman network [8]. Then, the cross-covariance matrices are calculated to estimate the covariance matrix among securities.

The remainder of this paper is organized as follows. The dynamic portfolio selection model is proposed in Section 2. A numerical example, which is used to illustrate the proposed method and compare with the VAR method, is presented in Section 3. Discussions and conclusions are in the last section.

2. Dynamic portfolio selection model

Consider a market contains n securities. The dynamic behavior of the securities can be depicted as shown in Fig. 1. The return rates/risk of securities in the period $t-1$ will affect the return rates/risk of the securities in the period t, and so forth.

In order to simulate the dynamic behavior of the security market, the Elman network is employed here. The Elman network [8] is one kind of the recurrent neural networks and widely used to deal with time-varying problems [10,11]. Recently, the recurrent neural networks have been reported the better accuracy than the conventional timer series approaches such as autoregression integrated moving-average (ARIMA), VAR, and GARCH [12,13]. In this paper, the Elman network is constructed as shown in Fig. 2.

After obtaining the forecasting expected return rates, we should calculate the covariance matrix among securities. In this paper, the cross-covariance matrices are used to derive the covariance matrix with different periods. The contents of the cross-covariance matrices can be described as follows.

Consider the multivariate time series $\boldsymbol{Z}_t$, and the mean vector $\boldsymbol{\mu}$, then the cross-covariance matrices at the lth lag can be defined as

$$\begin{aligned}\boldsymbol{\Sigma}_l^{t+1} &= [\sigma_{ij}^{t+1}] = \mathrm{Cov}(\boldsymbol{Z}_t, \boldsymbol{Z}_{t-l}) = E[(\boldsymbol{Z}_t - \boldsymbol{\mu})(\boldsymbol{Z}_{t-l} - \boldsymbol{\mu})'] \\ &= E\begin{bmatrix} z_{1t} - \mu_1 \\ z_{2t} - \mu_2 \\ \vdots \\ z_{kt} - \mu_k \end{bmatrix} [z_{1(t-l)} - \mu_1, z_{2(t-l)} - \mu_2, \ldots, z_{k(t-l)} - \mu_k] \\ &= \begin{bmatrix} \sigma_{11}^{t+1}(l) & \sigma_{12}^{t+1}(l) & \cdots & \sigma_{1k}^{t+1}(l) \\ \sigma_{21}^{t+1}(l) & \sigma_{22}^{t+1}(l) & \cdots & \sigma_{2k}^{t+1}(l) \\ \vdots & \vdots & \ldots & \vdots \\ \sigma_{k1}^{t+1}(l) & \sigma_{k2}^{t+1}(l) & \cdots & \sigma_{kk}^{t+1}(l) \end{bmatrix}. \end{aligned} \tag{2}$$

Now, we can reformulate the conventional portfolio selection to consider the time-varying portfolio selection problem as follows:

$$\begin{aligned} \min\ & \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}^{t+1} x_i^{t+1} x_j^{t+1} \\ \text{s.t.}\ & \sum_{i=1}^{n} \mu_i^{t+1} x_i^{t+1} \geqslant R, \\ & \sum_{i=1}^{n} x_i^{t+1} = 1, \\ & x_i^{t+1} \geqslant 0 \quad \forall i = 1, \ldots, n, \end{aligned} \tag{3}$$

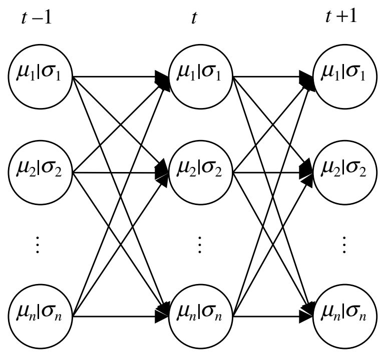

Fig. 1. The dynamic security market network.

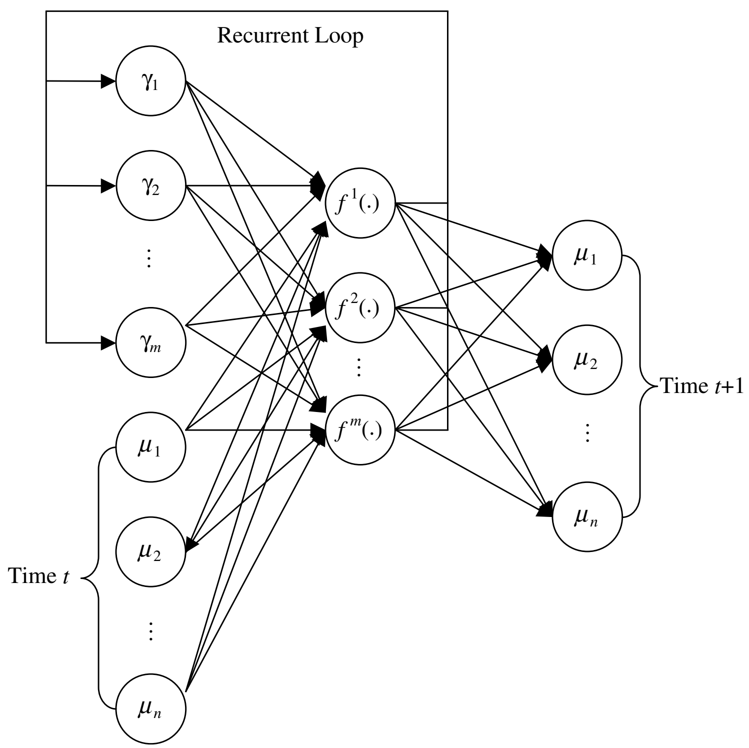

Fig. 2. The structure of the Elman network in this paper.

where μ_i^{t+1} denotes the expected return rate of the ith security in the period $t+1$, σ_{ij}^{t+1} denotes the covariance coefficient between the ith security and the

jth security in the period $t+1$, and R denotes the acceptable least rate of the expected return.

In the next section, a numerical example is used to demonstrate the proposed method and compare the accuracy with the VAR model.

3. Numerical example

In this numerical example, five series with 100 data, which are collected form Taiwan's stock market, are used to demonstrate the proposed method. The return rates of the five stocks can be depicted as shown in Fig. 3.

Next, we perform VAR(1) and the Elman network to forecast the expected return rates of the five stocks. By calculating the mean-square error (MSE) as shown in Table 1, it can be seen that the Elman network outperform to VAR(1). That is, the Elman network can predict the more accurate results than VAR(1).

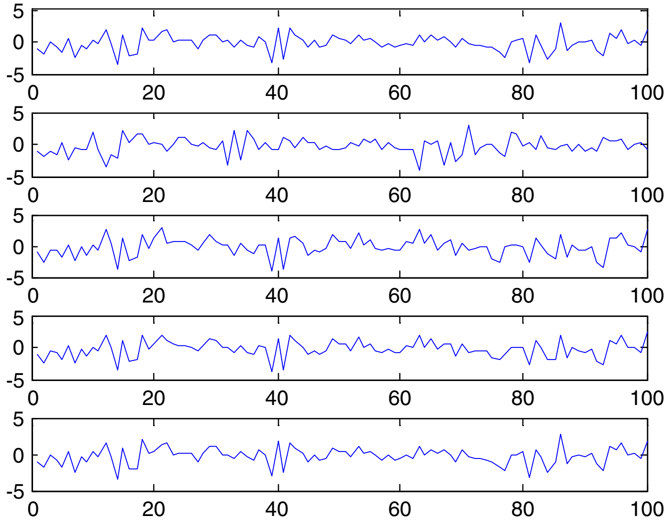

Fig. 3. Return rate chart (μ_1–μ_5).

Table 1
Mean-square error of the five stocks

MSE	μ_1	μ_2	μ_3	μ_4	μ_5
VAR(1)	11.9409	12.4489	14.2361	14.7871	11.5802
Elman's network	5.4637	9.2073	6.4914	5.1323	5.1610

Table 2
The expected return rates of the five stocks

Expected return rate (%)	μ_1	μ_2	μ_3	μ_4	μ_5
VAR(1)	−0.5327	0.0484	−0.0047	−0.2487	−0.4674
Elman's network	−0.3628	0.0258	−0.0283	−0.1395	−0.3098

We also use the Elman network and VAR(1) to predict the expected return rates of the five stocks as shown in Table 2 to be the inputs of the dynamic portfolio selection model.

From Table 2, we can conclude that although the expected return rates of the five stocks have the same direction, it can be seen that the values are much different. Since the tiny different results may change the optimal portfolio selection, we should predict the expected return rate as accuracy as possible.

In order to calculate the covariance matrix of the five stocks, the cross-covariance matrix at the 1st lag is used to estimate the covariance matrix. Note that other weighted methods like mean average or geometric average method can also be used to estimate the covariance matrix. The covariance matrix of the five stocks derived by VAR(1) and the Elman network can be shown as in Tables 3 and 4.

On the basis of the results above, we can obtain the optimal dynamic portfolio selection by solving Eq. (3). The comparison of the optimal portfolio

Table 3
The covariance matrix derived by VAR(1)

Covariance matrix (VAR(1))	μ_1	μ_2	μ_3	μ_4	μ_5
μ_1	1.4915	0.0288	1.5586	1.5067	1.4402
μ_2	0.0288	1.5800	−0.0744	−0.0243	0.0343
μ_3	1.5586	−0.0744	2.0940	1.8476	1.5259
μ_4	1.5067	−0.02427	1.8476	1.6945	1.4689
μ_5	1.4402	0.03434	1.5259	1.4689	1.3953

Table 4
The covariance matrix derived by the Elman network

Covariance matrix (Elman's network)	μ_1	μ_2	μ_3	μ_4	μ_5
μ_1	1.0402	−0.0179	1.1726	1.1004	1.0104
μ_2	−0.0179	0.4253	−0.0484	−0.0280	−0.0123
μ_3	1.1726	−0.0484	1.3473	1.2557	1.1408
μ_4	1.1004	−0.0280	1.2557	1.1736	1.0702
μ_5	1.0104	−0.0123	1.1408	1.0702	0.9819

Table 5
The optimal dynamic portfolio selection

Optimal portfolio selection	μ_1	μ_2	μ_3	μ_4	μ_5	Risk	Acceptable least return rate
VAR(1)	0	0.5695	0.3668	0.0637	0	0.8278	0.01
Elman's network	0	0.7719	0.1970	0.0311	0	0.3065	0.01
VAR(1)	0	0.5847	0.3893	0.0260	0	0.8600	0.02
Elman's network	0	0.8928	0.1072	0	0	0.3498	0.02

selection between VAR(1) and the proposed method can be presented as shown in Table 5.

From Table 5, it can be seen that the results of the optimal dynamic portfolio selection between VAR(1) and the Elman network is quite different. Since the accuracy of the Elman network outperform to VAR(1), we can conclude that the optimal dynamic portfolio selection of the Elman network should be the alternative.

4. Discussions and conclusions

Mean–variance is widely used in the finance area to deal with the portfolio selection problem. However, the conventional method only considers the static situation. The purpose of the mean–variance approach is to determine the $t+1$ period optimal investing rate to each security based on the sequent return rate. The key is to forecast the $t+1$ period return rate as accuracy as possible.

In this paper, the dynamic portfolio selection problem is considered. The Elman network is first designed to simulate the dynamic behavior of the security market to predict the expected return rate. Next, the cross-covariance matrices are used to estimate the covariance matrix among securities. Finally, the optimal dynamic portfolio selection is determined by the revised model. On the basis of the numerical example, we can conclude that the accuracy of the Elman network outperform to VAR(1) and provide the better solution for the dynamic portfolio selection problems.

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