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Innovative Applications of O.R.

House selection via the internet by considering homebuyers' risk attitudes with S-shaped utility functions

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ABSTRACT

The widespread use of the Internet has significantly changed the behavior of homebuyers. Using online real estate agents, homebuyers can rapidly find some modern houses that meet their needs; however, most current online housing systems provide limit features. In particular, existing systems fail to consider homebuyers' housing goals and risk attitudes. To increase effectiveness, online real estate agents should provide an efficient matching mechanism, personalized service and house ranking with the aim of increasing both buyers' satisfaction and deal rate. An efficient online real estate agent should provide an easy way for homebuyers to find (rank) a suitable house (alternatives) with consideration of their different housing philosophies and risk attitudes. In order to comprehend these ambiguous housing goals and risk attitudes, it is also indispensable to determine a satisfaction level for each fuzzy goal and constraint.

In this study, we propose fuzzy goal programming with an S-shaped utility function as a decision aid to help homebuyers in choosing their preferred house via the Internet in an easy way. With the use of a decision aid, homebuyers can specify their housing goals and constraints with different priority levels and thresholds as a matching mechanism for a fuzzy search, while the matching mechanism can be translated into a standard query language for a regular relational database. Moreover, a laboratory experiment is conducted on a real case to demonstrate the effectiveness of the proposed approach. The results indicate that the proposed method provides better customer satisfaction than manual systems in housing selection service.

1. Introduction

Many buyers' experiences of using search tools through the Internet to find an appropriate house may not reduce their search time (D'Urso, 2002; Leonard, Ken, & Randy, 2003). This is due to the difficulty of evaluating the multitude of factors, such as emotional priorities, financial situations and arbitrary preferences at the same time. For the sake of easy illustration, we consider the following example throughout this paper. A young couple, Alice and John, decides to buy a house with emphasized consideration of children's education. Alice would like to buy a house near the best high school in the city. In order to gather as much housing information as possible within the shortest time, Alice turns to the Internet. By using "real estate" as a keyword, she receives more than one million related links from Google. However, these real estate websites can only screen out houses that exactly match specific constraints (e.g., city/state, price range and number of bedrooms) given by Alice. Alice is disappointed with the result because she cannot input appropriate criteria into the system to meet her needs, such as a house of "about" 250 square meters or "not too far" from her workplace. She scrutinizes the housing information listed on the Internet and eliminates the unqualified houses by herself. Since there are a huge number of alternatives, it is difficult for Alice to evaluate and rank them, and none of the online agents can provide a good ranking service.

How can someone become a successful real estate agent? They should provide an efficient and flexible search tool for homebuyers with different ages, housing considerations and risk attitudes. Usually, risk tolerance increases with age when other variables are controlled (Wang & Hanna, 1997). Young buyers, who have less money, may engage in less risk by selecting an apartment. Middle-aged buyers are risk lovers, with more money and more experience. Thus, they may choose bigger houses. With decreasing income, elders who are usually risk averters will choose houses with less risk such as countryside houses.

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Table 1
Housing attributes.

Housing attributes	Sub items of housing attributes	Sources
Housing value	Price	Lindberg, Garling, & Montgomery (1989) and Michaelides (2011)
	Owner's estimate of annual housing value	Arimah (1992)
Structure attributes	Lot size	Stull (1970), King (1976), and Lindberg et al. (1989)
	Number of floors	Arimah (1992)
	Number of rooms	Stull (1970), Arimah (1992)
Neighborhood attributes	Pollution level	Lindberg et al. (1989), Arimah (1992), Kim, Yang, Yeo, & Kim, 2005, Natividade-Jesus et al. (2007)
	Safety	Kim et al. (2005) and Natividade-Jesus et al. (2007)
	Landscaping	Kim et al. (2005) and Waltert & Schlapfer (2010)
	Recreational facilities in the neighborhood	Lindberg et al. (1989) and Arimah (1992)
Location attributes	Distance to Central Business District (CBD)	Stull (1970), King (1976), Lindberg et al. (1989), Arimah (1992), and Natividade-Jesus et al. (2007)
	Distance to workplace of head of household	Lindberg et al. (1989) and Arimah (1992)
	Average distance to children's school	Lindberg et al. (1989) and Arimah (1992)
	Public transportation	Kim et al. (2005)

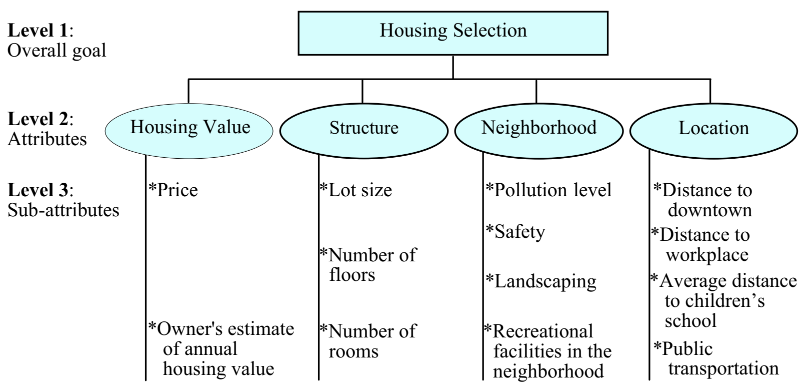

Fig. 1. AHP hierarchy for housing selection.

Most online real estate agents, such as Yahoo Real Estate (http://realestate.yahoo.com/) and Realtor.com (http://www.realtor.com/), provide a common search tool with basic constraints for homebuyers to list all houses which exactly match their requirements from the database. In this case, some potential houses with slight deviations from the constraints will be excluded by using the search tool. That is, a good match between buyer and potential house is difficult to reach because everyone has his/her own preferences. With multiple housing goals and different priorities for each individual, comparing similar houses is very complicated work for agents. For example, a homebuyer would like to buy a suburban house with consideration of convenient transportation, beautiful environment and at least two bedrooms. However, so far, there is no online real estate agent providing appropriate tools to apply the functions of multi-goal and multi-criteria searches with fuzzy preferences.

2. Important concerns for online agents and homebuyers

We list some important concerns for online agents and homebuyers as follows:

(1) There is a lot of fuzzy information on the Internet such as "great quiet neighborhood with excellent schools" or "close to world-class shopping, dining and entertainment at nearby Santana Row and Valley Fair Plaza". Liu and Zhang (2009) present a fuzzy evaluation method for residential real estate electronic marketing based on network DEA which uses linguistic variables to evaluate the factors. However, thus far, online agents do not provide any appropriate search tool to aid buyers in describing their ambiguous criteria, such as "comfortable" environment and "nice" neighborhood for housing. Moreover, most of this information is considered as extra descriptions of houses and cannot be processed as a standard query search in a database system.

(2) Online real estate agents should provide a tool for homebuyers to prioritize housing constraints. Customers also need a flexible method to determine the relative weights between constraints and rank housing alternatives. To increase the probability of finding the most suitable house, real estate agents should provide a better matching mechanism by taking buyers' preemptive priority preferences into account (Yuan, Lee, Kim, & Kim, 2013).

(3) Ford, Rutherford, and Yavas (2005) pointed out that online homebuyers have to evaluate more houses in order to ultimately find a better match. This leads to higher transactional costs. Personalized service is an essential factor in increasing the competitiveness of online real estate agents (Hamilton & Selen, 2004). However, tools of current agents do not provide the necessary personalization for housing evaluation and ranking. In reality, homebuyers need this service very much. Moreover, it is necessary to offer a user-friendly information search system so as to save busy customers' time.

(4) Online real estate agents should provide a tool to match housing alternatives for buyers according to their housing philosophies and risk attitudes. With different risk attitudes,

buyers choose different houses according to the future value of a house (Zhang and Yang, 2012). Based on historical prices and utility functions, homebuyers can predict the future value of the house. This is another important function of the tool.

To the best of our knowledge, there is no single tool provided by current real estate agents to handle all the above-mentioned problems. Therefore, in this paper, we try to develop a decision support aid to quantify ambiguous search criteria and rank houses for buyers by considering the above factors. Such a system also allows customers to specify housing constraints with thresholds for standard fuzzy queries. All the constraints and fuzzy queries can then be translated into a series of precise queries for a regular relational database. Finally, in order to demonstrate the effectiveness of the proposed decision aid system, a laboratory experiment is conducted on a real case and detailed corresponding analysis is also provided.

3. Method and materials

3.1. Housing attributes classification

Bond, Seiler, Seiler, and Blake (2000) stated that the types of online property information provided by most online real estate agents include geographic region, asked price, neighborhood, structural features and a picture of the house. Real-time listings and virtual home tours make real estate websites rich in content and help homebuyers to be better informed throughout the search and purchase process (Kummerow & Lun, 2005). Internet real estate agents, e.g., Yahoo Real Estate, Realtor.com, and Century 21 Real Estate (http://www.century21.com/home.aspx), usually allow homebuyers to specify characteristics of their target house such as the city, location, price range, number of bedrooms, and number of bathrooms. Then, homebuyers can receive a list of suggested houses based on their given constraints.

Obviously, these characteristics are important considerations for homebuyers. Because the Internet can increase search intensity, its prescreening capability allows homebuyers to discover and visit more appropriate properties in a short period (Zumpano, Johnson, & Anderson, 2003). However, the current searching functions provided by online real estate agents seem too simple to meet buyers' goals and preferences. In order to provide sufficient considerations for customers, this study collects important housing attributes from previous studies and interviews 10 house buyers and 10 senior real estate agents in Taiwan. Some duplicate or irrelevant attributes are eliminated and the selected list is depicted in Table 1.

In order to elicit important housing attributes for buyers, this study also constructs an Analytical Hierarchy Process (**AHP**) (Saaty, 1980). AHP provides solutions for decision problems in multi-criteria environments (Forman & Gass, 2001). This study constructs an AHP hierarchy of housing selection as shown in Fig. 1. We invite twenty homebuyers to evaluate these housing attributes using an AHP questionnaire which is partially listed in Appendix A. Finally, the overall relative weights of attributes and sub-attributes are obtained, as shown in Table 2. As seen, price is the most important factor. The second important consideration is the lot size. In addition, distance to children's schools and safety are also important sub-attributes for housing choices.

In this paper, the Fuzzy Goal Programming (**FGP**) method with an S-shaped utility function is adopted to develop a decision support system to help homebuyers search for appropriate houses on the Internet in consideration of their housing risk attitudes and satisfaction levels.

3.2. Data representation

The parameters which define the size of the problem are listed as follows:

$\mathbf{x}$	an n-vector with components $x_1, x_2, \ldots, x_n$
B	the number of achieved fuzzy constraints
x_i	house alternatives, $i = 1, \ldots, n$
Index sets:	
k	the kth goals
i	the ith alternative
j	the jth attribute
r	the rth priority level, $r = 1, 2, \ldots, i-1$
Problem data:	
$f_k(\mathbf{x})$	the linear function of the kth goal
g_k	the aspiration level of the kth goal
l_k	lower limits for the kth goal
u_k	upper limits for the kth goal
A_{ij}	the jth attribute of the ith alternative
μ_{Aij}	utility function of the jth attribute and the ith alternative
$\mu_{Aij}(\mathbf{x})$	the utility function of the decision maker's satisfaction level
$\mu_{attribute}(AV_j)$	the average satisfaction level for attribute j
$\mu_{at\ least}$	the utility functions of meeting the buyer's "at least" level constraints
$\mu_{at\ most}$	the utility functions of meeting the buyer's "at most" level constraints
$\mu_{about\ Y}$	the utility functions of meeting the buyer's "about" level constraints
C_r	the binary variable for determining the preemptive priority of the r-th fuzzy constraint
w_{ks}	the weights attached to the bounded positive deviations p_{ks} for the sth break point in the kth goal
p_{ks}	the bounded positive deviations from the target value b_{ks} for the sth break point in the kth goal
b_{ks}	the utility value of the break points in the kth goal's utility function
S_{ks}	the slope of the deviation between b_{ks}
λ_k	the additional continuous variable that represents the utility value in the kth goal
e_k^+	positive deviations from the highest possible value of the utility function for the kth goal
e_k^-	negative deviations from the highest possible value of the utility function for the kth goal
α_k	the positive weights attached to the sum of the deviations of $\lvert\lambda_k - 1\rvert$
z_k	the linear function of the kth goal
$\mu_{ks}(z_k(\mathbf{x}))$	a membership function of the kth goal
β_k	the positive weights obtained from AHP attached to each goal

3.3. Goal programming and fuzzy goal programming

The housing choice, which involves homebuyers' heterogeneous preferences, is a typical multi-criteria and multi-objective decision-making problem. Buyers usually have different satisfaction levels for various housing criteria, such as the number of bedrooms, quality of environment and convenience of transportation. Furthermore, they often expect some conflicting housing goals, such as minimizing house price while maximizing lot size and

Table 2
Composite priority weights for attributes and sub-attributes.

Attributes	Local weights	Sub-attributes	Local weights	Global weights	Priority order
Housing value	0.23	Price	0.70	0.161	1
		Owner's estimate of annual housing value	0.30	0.069	8
Structure attributes	0.22	Lot size	0.50	0.110	2
		Number of floors	0.10	0.022	13
		Number of rooms	0.40	0.088	5
Neighborhood attributes	0.28	Pollution level	0.30	0.084	7
		Safety	0.32	0.090	4
		Landscaping	0.23	0.064	9
		Recreational facilities in the neighborhood	0.15	0.042	11
Location attributes	0.27	Distance to downtown	0.16	0.043	10
		Distance to workplace	0.32	0.086	6
		Average distance to children's school	0.40	0.108	3
		Public transportation	0.12	0.032	12

location utility. In order to pursue aspirations-maximization, Charnes and Cooper (1961) proposed Goal Programming (**GP**) to model real world problems. GP is especially useful for multi-criteria and multi-objective decision problems. The mathematical formulation of GP is introduced as follows:

(GP)

$$\text{Minimize} \quad \sum_{k=1}^{m} |f_k(\mathbf{x}) - g_k| \tag{1}$$

$$\text{Subject to} \quad \mathbf{x} \in F, (F \text{ is a feasible set}).$$

where $f_k(\mathbf{x})$ is the function of the kth goal and g_k is the aspiration level of the kth goal.

In order to resolve the imprecise aspiration level of the Decision maker's (**DM**'s) goals, Narasinhan (1980) utilized the fuzzy weights approach to describe linguistic priorities in the utility functions. The conventional form of FGP can be expressed as follows:

(FGP)

$$f_k(\mathbf{x}) \gtrsim g_k \quad (\text{or} f_k(\mathbf{x}) \lesssim g_k) \quad k = 1, 2, \ldots n$$
$$\text{Subject to} \quad \mathbf{x} \in F, \quad (F \text{ is a feasible set}) \tag{2}$$

where $f_k(\mathbf{x}) \gtrsim (\lesssim g_k)$ indicates the kth fuzzy goal approximately greater or equal to (approximately less or equal to) the aspiration level g_k; other variables are defined as in GP.

Fuzzy goals and fuzzy constraints can be defined as fuzzy sets in the space of alternatives (Bellman & Zadeh, 1970). This study adopts the fuzzy logic to deal with the linguistic words in fuzzy constraints, such as the safety of the house should be good. For the sake of simplicity, the preference-based utility functions are expressed as follows:

$$\mu_k(f_k(\mathbf{x})) = \begin{cases} 1, & \text{if } f_k(\mathbf{x}) \geq g_k, \\ \frac{(f_k(\mathbf{x}) - l_k)}{g_k - l_k}, & \text{if } l_k < f_k(\mathbf{x}) < g_k \quad \text{for } f_k(\mathbf{x}) \gtrsim g_k, \\ 0, & \text{if } f_k(\mathbf{x}) \leq l_k \end{cases} \tag{3}$$

$$\mu_k(f_k(\mathbf{x})) = \begin{cases} 1, & \text{if } f_k(\mathbf{x}) \leq g_k, \\ \frac{(u_k - f_k(\mathbf{x}))}{u_k - g_k}, & \text{if } g_k < f_k(\mathbf{x}) < u_k \quad \text{for } f_k(\mathbf{x}) \lesssim g_k, \\ 0, & \text{if } f_k(\mathbf{x}) \geq u_k \end{cases} \tag{4}$$

where l_k and u_k are, respectively, lower and upper limits for the kth goal; $f_k(\mathbf{x})$ and g_k are defined as in GP.

Online housing decision aids should not only consider the priority weight of each goal but also the homebuyers' fuzzy preferences. However, it is usually not easy to describe housing goals and criteria precisely. Housing decisions are laced with subjective human values that are usually neither crisp nor deterministic. In 2005, Mohanty and Bhasker (2005) proposed a fuzzy approach for solving production classification problems on the Internet. A DM usually searches for the best satisfactory product that fulfills "most" of the attributes rather than all attributes. Therefore, they defined the linguistic quantifier "most" as a key element for vague aspiration as follows:

$$\mu_{most}(x) = \begin{bmatrix} 1 & x \geq 0.8 \\ (x - 0.3)/(0.5) & 0.3 \leq x \leq 0.8 \\ 0 & x \leq 0. \end{bmatrix} \tag{5}$$

Other solutions include the weighted additive model, provided by Tiwari, Dharmar, and Rao (1987), and the weighted max–min model, provided by Lin (2004). However, with a preemptive priority setting, unless a particular goal is achieved, other goals should not be considered. The inexperienced setting of weights in the formulation of GP can lead to incorrect results (Tamiz, Jones, & Romero, 1998).

Buckles and Petry (1983) developed a fuzzy relational model to incorporate fuzzy information in a relational database. To extend database management systems functions for the expression of flexible queries, Bosc and Pivert (1995) introduced a SQLf language which is a fuzzy extension of standard query language (SQL). Shenoi and Melton (1999) extended Buckles and Petry's model to incorporate with proximity relations for scalar domains. Yazici and Cibiceli (1999) utilized a multi-dimensional data structure, Multi Level Grid File, to access both crisp and fuzzy data from a fuzzy database. Ma and Yan (2007) presented generic fuzzy queries for a regular relationship database.

In order to handle DM's fuzzy preferences, Fan, Ma, and Zhang (2002) proposed a method to solve multiple attribute decision making problems by considering the fuzzy relations of alternatives. Rasmy, Lee, Abd El-Wahed, Ragab, and El-Sherbiny (2002) established a fuzzy expert system based on the DM's linguistic preferences for multiple objective decision making problems. Cheng, Chan, and Lin (2006) derived a fuzzy inference system as a negotiation agent to search for a mutually acceptable contract in an e-market.

Chang (2010) presented an approach to formulate an S-shaped utility function without adding extra binary variables. The utility function describes the risk attitudes of DMs, including risk aversion and risk seeking. With different risk attitudes in gain or loss situations, homebuyers can find ideal houses with consideration of their housing preferences. In order to comprehend ambiguous housing goals and risk attitudes from with conflicting preferences, it is indispensable to determine the satisfaction level for each fuzzy goal and constraint.

There are several studies that integrated the AHP and GP (Badri, 2001; Ho, Chang, & Ku, 2013; Ramanathan & Ganesh, 1995; Schniederjans & Garvin, 1997). Ramanathan and Ganesh (1995) derived AHP weights for the qualitative criteria and employing them as coefficients of the decision variables in the objective functions of the GP model in solving energy resource allocation problem. Badri (2001) implemented the AHP weights for the quality control instruments on each alternative as constraints in GP to reflect the preferences for the different instruments. Ho et al. (2013) obtained weights from AHP and implement it upon each corresponding goal using multi-choice goal programming for the location selection problem.

3.4. The proposed method

With prospect theory (Kahneman & Tversky, 1979), we can find the varying risk attitudes of DMs in different situations. DMs intend to avoid risk in choices involving sure gains and to seek risk in choices involving sure losses. Similarly, homebuyers exhibit more risk aversion in gain situations as a concave function. On the other hand, homebuyers prefer to be risk lovers in loss situations as a convex function. Therefore, in uncertain situations, each homebuyer should have his/her own S-shape utility function to represent at their risk attitudes.

The combination of the above mentioned function may lead to a more effective approach with many advantages. Moreover, it can solve some or all of the shortcomings of each individual approach. Therefore, we integrate FGP with an S-shaped utility function as a decision aid to help with Internet housing choices as follows.

This study formulates the buyer's housing preference among alternatives with Eq. (6). There are K goals and each goal has with attributes A_{ij}, (A_{ij}, μ_{Aij}). The average satisfaction level for attribute j is given as

$$\mu_{attribute}(AV_j) = \frac{1}{K}\sum_{i=1}^{K}\mu_{Aij}(\mathbf{x}) \tag{6}$$

and the utility function of DM $\mu_{Aij}(\mathbf{x})$ is defined as in FGP.

This study constructs the aspiration-maximization of the buyer's housing goals in consideration of their risk attitudes (Eqs. (7)–(11)) which are represented by the S-shaped utility function (Chang, 2010), while the homebuyer's preferences, such as price, expected lot size and so on, are represented by Eq. (12). There are two housing goals about future value of the house, the maximization of the expected gain and the minimization of the expected loss. With the slope increase/decrease, Eqs. (7)–(11) can formulate these two goals as a concave/convex function with homebuyers' risk attitudes (risk averter/lover) in different situations. The approach described above leads to the following formulation:

$$\text{Minimize} \quad \beta_k * (w_{k1}p_{k1} + w_{k2}p_{k2} + w_{k3}p_{k3} + \alpha_k(e_k^+ + e_k^-))$$

$$\text{Subject to} \quad \lambda_k = [\mu_{ks}(b_{k2}) - \mu_{ks}(b_{k1})]\frac{p_{k1}}{b_{k2}-b_{k1}} + [\mu_{ks}(b_{k3}) - \mu_{ks}(b_{k2})]\frac{p_{k2}}{b_{k3}-b_{k2}} + [\mu_{ks}(b_{k4}) - \mu_{ks}(b_{k3})]\frac{p_{k3}}{b_{k4}-b_{k3}}, \tag{7}$$

$$\lambda_k - e_k^+ + e_k^- = 1, \tag{8}$$

$$z_k(\mathbf{x}) - p_{k1} - p_{k2} - p_{k3} \le b_{k1}, \tag{9}$$

$$w_{k1} < w_{k2} < w_{k3}, \tag{10}$$

$$0 \le p_{k1} \le b_{k2} - b_{k1}, \quad 0 \le p_{k2} \le b_{k3} - b_{k2}, \quad 0 \le p_{k3} \le b_{k4} - b_{k3}, \tag{11}$$

$$\mu_{Aij}(\mathbf{x}) \ge \mu_{attribute}(AV_j)C_r, \quad r = 1, 2, \ldots m \tag{12}$$

$$\sum_{r=1}^{m} C_r \ge B, \tag{13}$$

$$\mathbf{x} \in F \quad (F \text{ is a feasible set})$$

where β_k are positive weights obtained from AHP attached to each goal. With AHP method, the relative importance (the relative weights) between attributes will be translated as weights β_k on each corresponding goal in the FGP. w_{ks} are the weights attached to positive deviations, p_{ks} (s = 1, 2, 3). p_{ks} are the positive deviations from the target value b_{ks} for the s th break point in the kth goal. λ_k is the additional continuous variable that represents the utility value of the S-shaped utility function in Eq. (7). $z_k(\mathbf{x})$ is the linear function of the kth goal. x is an n-vector with components x_1, x_2, ..., x_n. $\mu_{ks}(z_k(\mathbf{x}))$ is a membership function of the kth goal. C_r(r = 1, 2, ..., m) are binary variables for determining the preemptive priority of the rth fuzzy constraint. In the proposed model, a DM can choose different weights w_{ks} on each deviation to determine the priority of deviations p_{ks}. The risk attitudes of DMs can be described as risk averse (a concave utility function) and risk seeking (a convex utility function). In this study, we formulate these two housing risk attitudes in gain and loss situations as shown in Figs. 2–7.

Figs. 2 and 4 present the concave utility function of a risk averter in gain and loss situations, respectively. As shown in Figs. 2 and 4, the slope decreases from $|S_{k1}|$, $|S_{k2}|$ to $|S_{k3}|$. This means that with the increased risk of expected gain/loss $z_k(\mathbf{x})$, the average accumulated satisfaction level $\mu_{ks}(z_k(\mathbf{x}))$ of the DM decreases. The slope $|S_{i1}| > |S_{i2}| > |S_{i3}|$ indicates that the DM is a risk averter. Figs. 3 and 5 show a convex utility function of a risk lover in gain and loss situations, respectively. As seen in Figs. 3 and 5, the slope increases from $|S_{k1}|$, $|S_{k2}|$ to $|S_{k3}|$. This means that with the increased risk of expected gain/loss $z_k(\mathbf{x})$, the average accumulated satisfaction level $\mu_{ks}(z_k(\mathbf{x}))$ of the DM increases. The slope $|S_{k1}| < |S_{k2}| < |S_{k3}|$ shows that the DM is a risk lover.

This study formulates homebuyers' risk attitudes in gain situations with an S-shaped utility function as shown in Fig. 6. Where the average accumulated satisfaction level $\mu_{ks}(z_k(\mathbf{x}))$ is a convex function (risk lover) for $0 \leqslant z_k(\mathbf{x}) \leqslant \varepsilon_k$ and is a concave function (risk averter) for $z_k(\mathbf{x}) \geqslant \varepsilon_k$. Similarly, the homebuyer's two risk attitudes in loss situations are formulated with an S-shaped utility function as shown in Fig. 7, where the average accumulated satisfaction level $\mu_{ks}(z_k(\mathbf{x}))$ is a concave function (risk averter) for $0 \leqslant z_k(\mathbf{x}) \leqslant \varepsilon_k$ and is a convex function (risk lover) for $z_k(\mathbf{x}) \geqslant \varepsilon_k$.

Sometimes, a homebuyer cannot find a suitable house when too many constraints are requested. For instance, he/she may set many constraints such as distance to a market and distance to the nearest major hospital at the same time. He/she may find no house meeting these criteria due to excessive specificity. In contrast, if a preemptive priority is set for each constraint or the relationship between constraints is determined, the suitable house could be found more easily from their criteria, and the probability of finding a satisfactory house would increase. The preemptive priority structure can be stated as $C_r \ggg C_{r+}$ meaning that the constraint in the rth evaluation criteria has higher priority than the (r + 1)-th evaluation criteria. With Eqs. (12) and (13), a homebuyer can set a preemptive priority for each constraint to obtain the best

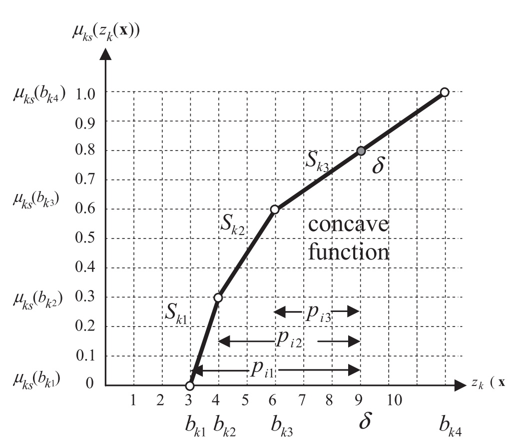

Fig. 2. A concave utility function as a risk averter in gain situation.

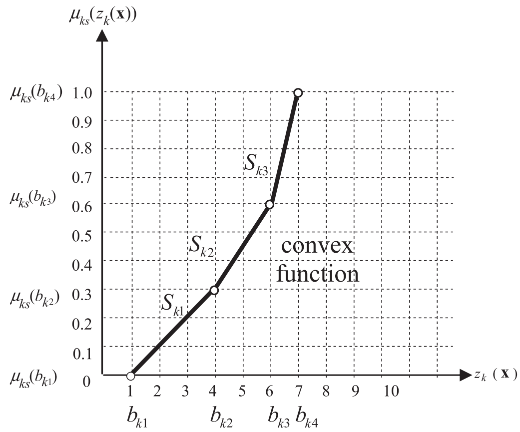

Fig. 3. A convex utility function as a risk lover in gain situation.

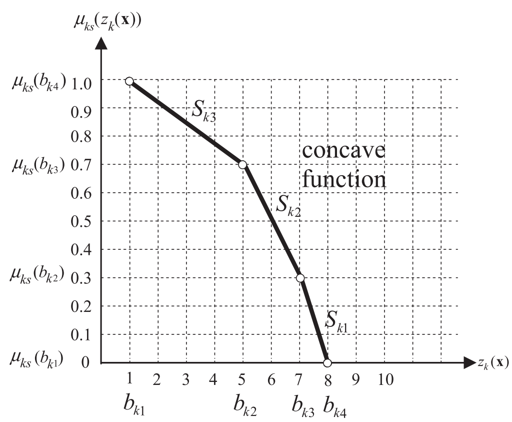

Fig. 4. A concave utility function as a risk averter in loss situation.

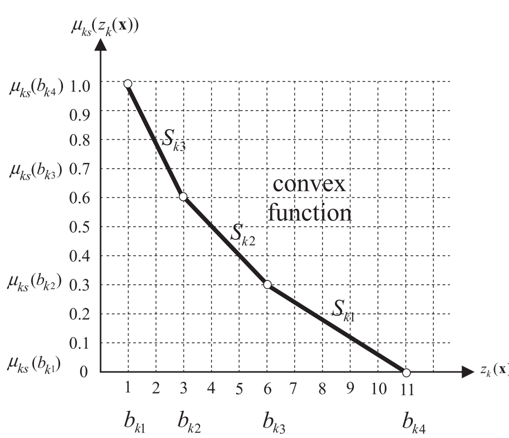

Fig. 5. A convex utility function as a risk lover in loss situation.

available housing options. This modified FGP can determine the most appropriate constraints and recommend a suitable ranking list. In contract, for classic FGP methods, setting relationships among each constraint would be almost impossible.

Let us consider a simple modified FGP example with preemptive priority to demonstrate the above-mentioned idea. A homebuyer, Alice, sets three constraints in Eqs. (14)–(16) as: (i) the safety should be good at least so and so, (ii) the pollution level should be low at least so and so, and (iii) the view should be good at least so and so, and specifies that only one of these needs should be achieved. The problem can be formulated as the following achievement function.

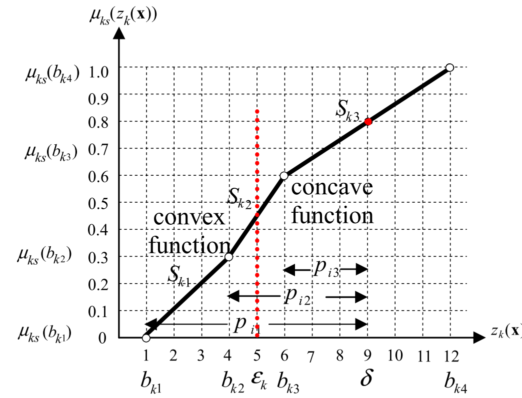

Fig. 6. A right S-shaped utility function represents risk attitudes in gain situation.

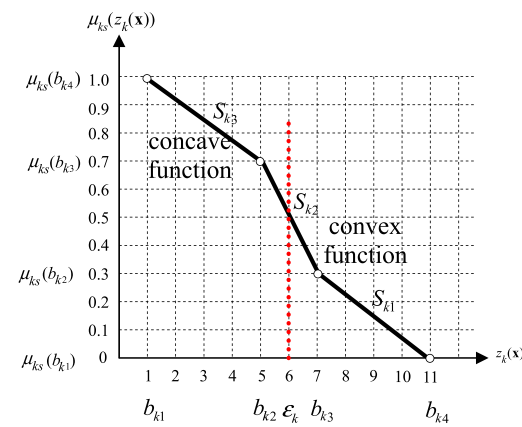

Fig. 7. A left S-shaped utility function represents risk attitudes in loss situation.

$$\begin{aligned} &\text{Minimize} && w_{k1}p_{k1} + w_{k2}p_{k2} + w_{k3}p_{k3} + \alpha_k(e_k^+ + e_k^-) \\ &\text{Subject to} && \mu_{Aij}(\mathbf{x}) \geq \mu_{\text{safety}}(AV_j)C_1 && (14) \\ &&& \mu_{Aij}(\mathbf{x}) \geq \mu_{\text{pollution}}(AV_j)C_2 && (15) \\ &&& \mu_{Aij}(\mathbf{x}) \geq \mu_{\text{view}}(AV_j)C_3 && (16) \\ &&& C_1 + C_2 + C_3 = 1 && (17) \end{aligned}$$

where x_i ($i = 1, \ldots, 9$) and C_r($r = 1, 2, 3$) are binary variables

Because $C_r(r = 1, 2, 3)$ are binary variables, thus, Eq. (17) dictates that only one constraint is fulfilled in Eqs. (14)–(16). Accordingly, Alice can set different preemptive weights for her constraints according to her preferences.

In order to implement the fuzzy concept, this study combines FGP and homebuyer's fuzzy constraints with linguistic quantifiers, such as "at least", "at most" or "about". For example, we replace $\mu_{attribute}(AV_j)$ with $\mu_{atleast}(q)$ in Eq. (18) to meet the homebuyer's constraints with "at least" Other utility functions of the fuzzy constraints such as "at least Y", "at most Y" and "about Y" are defined as in the model proposed by Ma and Yan (2007).

$$\mu_{atleast}(q) = \begin{cases} 0, & \text{if } q \leq a, \\ \frac{(q-a)}{Y-a}, & \text{if } a < q < Y, \\ 1, & \text{if } q \geq Y \end{cases} \tag{18}$$

$$\mu_{at\ most}(q) = \begin{cases} 1, & \\ \frac{(b-q)}{b-Y}, & \text{if } Y < q < b \\ 0, & \end{cases} \tag{19}$$

$$\mu_{about\ Y}(q) = \frac{1}{1 + \left(\frac{q-Y}{\beta}\right)^2}, \tag{20}$$

where larger values of β correspond to a wide curve and a and b are, respectively, lower and upper limits for each fuzzy constraint.

DMs can determine the housing constraints with different thresholds for fuzzy queries, and then these fuzzy queries are translated into precise SQL for a regular relational database as follows.

SELECT housing alternative
FROM housing table
WHERE Attribute at least/
most WITH matching rate(fuzzy query) (21)

Fuzzy query Eq. (21) can be substituted by precise query Eq. (22) when implemented in the relational database.

$$\text{WHERE } A \geq a \quad \text{AND} \quad A \leq b(\text{precise query}) \tag{22}$$

In short, the main contributions of the proposed method are as follows.

1. Homebuyers can easily describe and quantify ambiguous housing preferences with fuzzy satisfaction levels. Moreover, online real estate agents can even convert this approach into a utility function.
2. DMs can decide suitable weights for their risk attitudes in different situations. To express the risk attitudes in different situations, they assign different expected gains or losses on individual house alternatives. The proposed approach can transform these risk attitudes into weights for each target and present different housing ranks.
3. DMs can set preemptive priorities for each constraint according to different situations and obtain different housing ranks which are closer to their preferences.
4. The proposed approach can deal with fuzzy searches in related databases on the Internet for buyers. In order to meet their constraints with linguistic quantifiers, this model evaluates the houses by giving preferential weights according to these fuzzy satisfaction levels.

The approach involves inputting the homebuyer's preferences, goals and criteria, and developing a modified FGP model to obtain individual solutions for each objective function as in the following six steps: *Step* 1: Identify the homebuyer's housing goals with suitable risk attitude and roughly determine his/her housing criteria. *Step* 2: Define the homebuyer's satisfaction level for each housing goal and criterion. This process allows a homebuyer to develop their own utility function for the fuzzy goals and ambiguous criteria. *Step* 3: Search for possible alternatives in the database on the Internet using linguistic quantifiers such as "at least", "at most" or "about." *Step* 4: Establish the FGP model with an S-shaped utility function, and aggregate all the homebuyer's fuzzy goals and criteria. *Step* 5: Solve the FGP model with an S-shaped utility function, which evaluates each alternative according to the homebuyers' risk attitude and the scoring attribute set by the fuzzy preferences. *Step* 6: Rank the house alternatives based on obtained scores, with which the customer can finally choose the utility-maximizing house.

4. Results and discussion

4.1. An illustrative real case

A real case is presented to illustrate how a personalized ranking method can create more accurate list of ideal houses for homebuyers. Alice, who works for a computer company in San Jose, would like to buy a house for her family. Considering her children's education, she would prefer a house located in a neighborhood with good high schools – Monta Vista High School, Gunn High School or San Jose High Academy. In addition to location, price is her second most important concern. Based on these considerations, Alice offers her housing goals and criteria to Google to search for suitable houses. However, the searching results are quite frustrating because she obtains too many alternatives. She has to expend a lot of time to screen the alternatives. The current tools of online agents only provide explicit inputs that cannot deal with buyers' fuzzy priorities. Moreover, most of online real estate agents do not provide a landmark searching choice. This makes it even more difficult for Alice to find an appropriate house in a desired location.

The proposed method can solve the above-mentioned problems and exclude most unacceptable alternatives. Furthermore, it also creates a personalized ranking list according to the scoring attributes of her fuzzy preferences. The interface of this housing decision aid is presented in Fig. 8. The proposed system can consider multiple constraints in regard to the "distance of the house to some places". In Fig. 8, the real-time fuzzy utility functions are provided to help homebuyers estimate their preferences more accurately.

First, Alice gets the relative weights with the AHP questionnaire. The overall relative weights of attributes and sub-attributes are obtained, as shown in Table 3.

From the result of AHP in Table 3, we can find Alice's top two important attributes are Owner's estimate of annual housing value and Lot size. Therefore, Alice selects three housing goals (G1, G2, G3, $K = 3$) about the potential gain, the potential loss and the lot size of a house. The objective is to find houses closest to her preferences. The satisfaction levels for each goal are expressed by an S-shaped utility function as shown in Figs. 9–11. The expected gains and losses of twenty house alternatives ($n = 20$) are listed in Table 4.

(G1) The potential gain should be over 20 thousand dollars and the more the better.

According to the prospect theory (Kahneman & Tversky, 1979), a DM will be more risk averse in a gain situation. Vise versa, in a loss situation, a DM will be more risk seeking. We interview Alice and formulate the satisfaction level of her expected gain for houses as shown in Fig. 9. Obviously, she is a risk lover when the expected gain is lower than 45 thousand dollars (as a convex function) and a risk averter when the expected gain is more than 45 thousand dollars (as a concave function) in a gain situation. The spot line indicates that the turning point of 45 thousand dollars separates the convex and concave function in Fig. 9. The bold line indicates the right S-shaped utility function which is established by both convex and concave function.

Based on Alice's requirements, the problem can be formulated as follows. In this illustrative case, we set $\alpha_k = 7000$, a relative large number, in order to increase the influence of $(e_k^+ + e_k^-)$.

$$\begin{aligned}
\text{Minimize} \quad & p_{11} + 2p_{12} + 3p_{13} + 4p_{14} + 5p_{15} + 7000(e_1^+ + e_1^-) \\
\text{Subject to} \quad & \lambda_1 = [0.15 - 0]\frac{p_{11}}{30-5} + [0.3 - 0.15]\frac{p_{12}}{40-30} \\
& \quad + [0.6 - 0.3]\frac{p_{13}}{50-40} \\
& \quad + [0.8 - 0.6]\frac{p_{14}}{65-50} \\
& \quad + [1 - 0.8]\frac{p_{15}}{100-65}, \quad \lambda_1 - e_1^+ + e_1^- = 1, \\
& z_1(\mathbf{x}) - p_{11} - p_{12} - p_{13} - p_{14} - p_{15} = 5, \\
& \sum_{i=1}^{20} x_i = 1, \quad 0 \leq p_{11} \leq 30 - 5, \quad 0 \leq p_{12} \leq 40 - 30, \\
& 0 \leq p_{13} \leq 50 - 40, \quad 0 \leq p_{14} \leq 65 - 50, \\
& 0 \leq p_{15} \leq 100 - 65, \\
& z_1(\mathbf{x}) = 100x_1 + 80x_2 + 70x_3 + 60x_4 + 55x_5 + 40x_6 \\
& \quad + 50x_7 + 45x_8 + 50x_9 + 90x_{10} \\
& \quad + 85x_{11} + 70x_{12} + 20x_{13} + 35x_{14} + 30x_{15} \\
& \quad + 40x_{16} + 25x_{17} + 20x_{18} + 10x_{19} + 5x_{20},
\end{aligned}$$

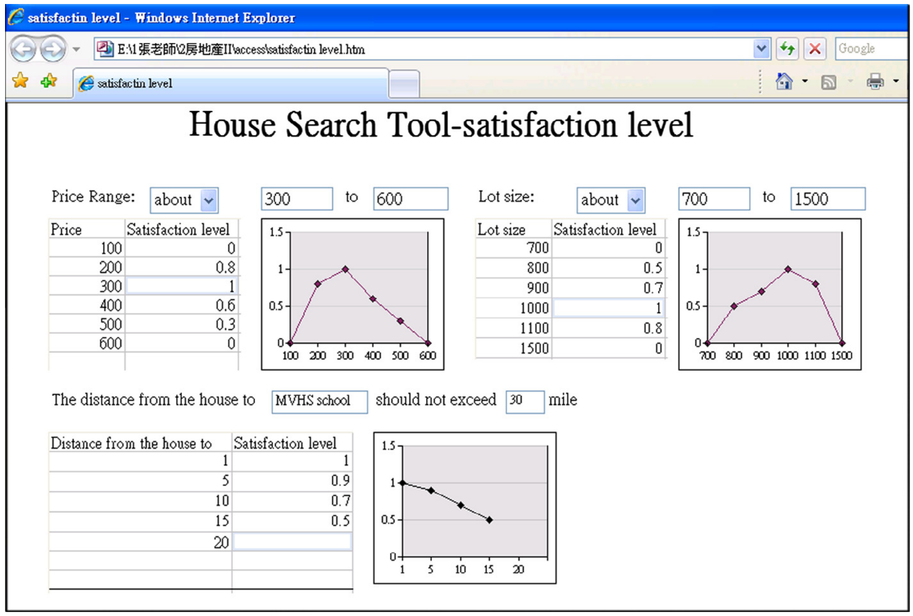

Fig. 8. The interface of the housing decision aid - satisfaction level setting.

Table 3
Composite priority weights for attributes and sub-attribute from Alice.

Attributes	Local weights	Sub-attributes	Local weights	Global weights	Priority order
Housing value	0.32	Price	0.4	0.128	3
		Owner's estimate of annual housing value	0.6	**0.192**	**1**
Structure attributes	0.28	Lot size	0.55	**0.154**	**2**
		Number of floors	0.15	0.042	10
		Number of rooms	0.3	0.084	5
Neighborhood attributes	0.26	Pollution level	0.21	0.0546	8
		Safety	0.33	0.0858	4
		Landscaping	0.32	0.0832	6
		Recreational facilities in the neighborhood	0.14	0.0364	11
Location attributes	0.14	Distance to downtown	0.14	0.0196	12
		Distance to workplace	0.42	0.0588	7
		Average distance to children's school	0.33	0.0462	9
		Public transportation	0.11	0.0154	13

Note: The bold values in Table 3 are the top two highest values among all global weights.

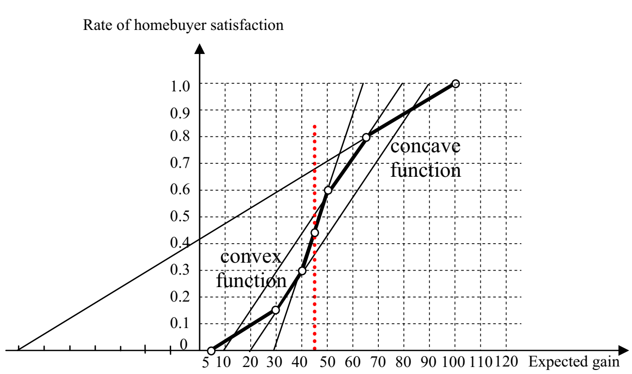

Fig. 9. Alice's right S-shaped utility function in gain situation.

This problem is solved by using LINGO (Schrage, 2002) to obtain the solution as (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}) = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), (p_{11}, p_{12}, p_{13}, p_{14}, p_{15}) = (25, 10, 10, 15, 35) and the utility value $\lambda = 1$ (i.e., the rate of homebuyer satisfaction is 100%). The recommended alternative is house x_1 and the expected gain of this house is 100 thousand dollars.

Table 4
The expected gain and loss of the twenty house alternatives.

House alternatives	Expected gain (thousand dollars)	Expected loss (thousand dollars)
x_1	100	80
x_2	80	50
x_3	70	50
x_4	60	40
x_5	55	30
x_6	40	45
x_7	50	55
x_8	45	90
x_9	50	45
x_{10}	90	20
x_{11}	85	20
x_{12}	70	30
x_{13}	20	40
x_{14}	35	30
x_{15}	30	45
x_{16}	40	40
x_{17}	25	10
x_{18}	20	20
x_{19}	10	10
x_{20}	5	5

(G2) The potential loss should not be over 100 thousand dollars and the less the better.

In a loss situation, Alice becomes more risk seeking. Assume that the satisfaction level of her expected loss is shown in Fig. 10. As seen, she is a risk averter when the expected loss of the house is lower than 50 thousand dollars (a concave function) and a risk lover when the expected loss of the house is more than 50 thousand dollars (a convex function). In Fig. 10, the spot line indicates that the turning point of 50 thousand dollars separates the convex and concave functions. The bold line indicates the left S-shaped utility function is established by both convex and concave functions.

This case can be expressed as follows:

$$\text{Minimize}\quad 4p_{21}+3p_{22}+2p_{23}+p_{24}+7000(e_2^+ + e_2^-)$$

$$\text{Subject to}\quad \lambda_2 = 1-\Big([1-0.8]\frac{p_{21}}{30-0}+[0.8-0.4]\frac{p_{22}}{50-30} + [0.4-0.13]\frac{p_{23}}{90-50}+[0.13-0]\frac{p_{24}}{128-90}\Big),$$

$$\lambda_2 - e_2^+ + e_2^- = 1,\quad z_2(\mathbf{x}) - p_{21} - p_{22} - p_{23} - p_{24} \le 0,$$

$$\sum_{i=1}^{20} x_i = 1,$$

$$0 \le p_{21} \le 30-0,\quad 0 \le p_{22} \le 50-30,$$

$$0 \le p_{23} \le 90-50,\quad 0 \le p_{24} \le 128-90,$$

$$z_2(\mathbf{x}) = 80x_1+50x_2+50x_3+40x_4+30x_5 +45x_6+55x_7+90x_8+45x_9+20x_{10} +20x_{11}+30x_{12}+40x_{13}+30x_{14}+45x_{15} +40x_{16}+10x_{17}+20x_{18}+10x_{19}+5x_{20},$$

This problem is solved by using LINGO (Schrage, 2002) to obtain the solution as (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1), (p_{21}, p_{22}, p_{23}, p_{24}) = (0, 0, 0, 5) and the utility value $\lambda_2 = 0.9829$ (i.e., the rate of homebuyer satisfaction is 98.29%). The recommended alternative is house x_{20}. The expected gain of this house is 5 thousand dollars and the expected loss is also 5 thousand dollars.

(**G3**) The lot size should be around 1000 square meters and must be over 700 but not over 1500 with the more the better. Alice does not want to buy too big of a house because of the cost and time needed for maintenance. Hence, the satisfaction level reaches 0 if the lot size is over 1500. In this case, the house size utility function can be expressed as a concave function in Fig. 11.

This problem is formulated in Appendix B (the part of G3) and is solved by using LINGO (Schrage, 2002) to obtain the solution as (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}) = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), (p_{31}, p_{32}, p_{33}) = (50, 100, 300) and the utility value $\lambda_3 = 1$ (i.e., the rate of homebuyer satisfaction is 100%). The recommended alternative is house x_2. The expected gain of this house is 80 thousand dollars, the expected loss is 50 thousand dollars and the lot size is 1249 square meters.

Considering three goals of expected gain and loss simultaneously and also the lot size, we formulate this problem in the appendix B again and it is solved by using LINGO (Schrage, 2002) to obtain the solution as (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}) = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), (p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{21}, p_{22}, p_{23}, p_{24}, p_{31}, p_{32}, p_{33}) = (25, 10, 10, 15, 15, 0, 0, 12, 38, 50, 100, 300) with the utility values $\lambda_1 = 0.8857$, $\lambda_2 = 0.789$ and $\lambda_3 = 1$. The recommended alternative is house x_2. The expected gain of x_2 is 80 thousand dollars, the expected loss is 50 thousand dollars and the lot size of the house is 1249 square meters.

Comparison of the results in the above four situations is shown in Table 5. If we consider the potential gain (G1) alone, house x_1, which has the highest expected gain is the best choice. If we consider the potential loss (G2) alone, house x_{20}, which has the lowest expected loss, is selected. If we consider the lot size (G3) alone, house x_2, which has the largest lot size, is the best choice. However, if we consider the three goals, potential gain, loss and lot size, simultaneously, house x_2, which has relatively high expected gain, low loss and largest lot size is the best choice. In this case, the rate of homebuyer satisfaction for lot size (G3) is the highest utility value among the three goals. The rates of homebuyer satisfaction for the potential gain (G1) and for the potential loss (G2) decrease. This may be because the houses with big lot size also have relatively high potential loss.

(Constraints).

In order to better suit the real world, seven constraints are specified as follows. For general constraints, the price, number of bedrooms, distance from house to work, and the reliability of house information must be achieved. As for environmental constraints, safety, pollution level and view are considered. At least two of these constraints should be satisfied. The preferences for each constraint are expressed in Table 6. (1) The house price should be around 300 thousand dollars but should not exceed 600 thousand dollars. If the house price is lower than 100 thousand dollars which is far under the market price, Alice thinks it may have a quality issue. (2) The safety of the house should at least be good. (3) The pollution level should be low at least. (4) The view from the house should be good at least. (5) There must be at least 2 bedrooms, and 4 bedrooms are desired. (6) The distance from house to work is not too far and at most 13 miles. (7) The reliability of house information should at least be average.

Alice would like to find the qualified houses that at least reach her housing constraints at different levels with thresholds for the fuzzy queries. Hence, we use the utility function approach to translate the fuzzy range with linguistic quantifiers into a crisp range as shown in Table 7. With different matching rates for each

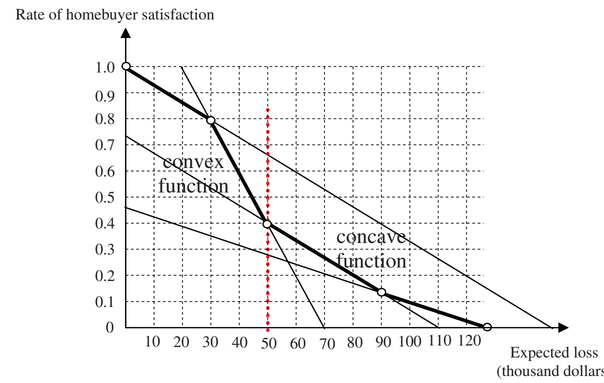

Fig. 10. Alice's left S-shaped utility function in loss situation.

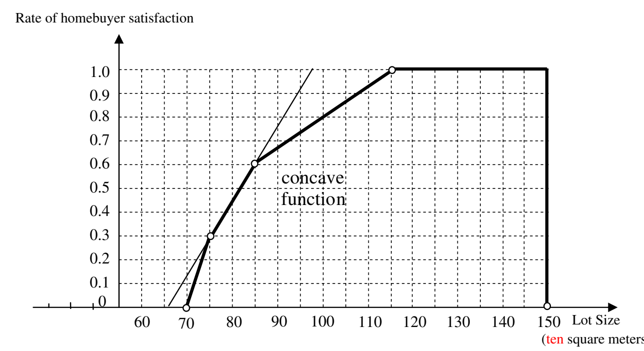

Fig. 11. Alice's S-shaped utility function for house size.

Table 5
The comparison of the results in four situations.

	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{21}	P_{22}	P_{23}	P_{24}	P_{31}	P_{32}	P_{33}	The selected house	λ_1	λ_2	λ_3	Expected gain (thousand dollars)	Expected loss (thousand dollars)	Lot size (square meters)
Goal 1	25	10	10	15	35								x_1	1			100	80	798
Goal 2						0	0	0	5				x_{20}		0.9829		5	5	850
Goal 3										50	100	300	x_2			1	80	50	1249
Goals 1–3	25	10	10	15	15	0	0	12	38	50	100	300	x_2	0.8857	0.7890	1	80	50	1249

Table 6
Alice's housing constraints and the satisfaction level for each constraint.

Price (thousand dollars)	Satisfaction level	Safety	Satisfaction level l	Pollution level	Satisfaction level l	View	Satisfaction level l	Number of bedrooms	Satisfaction level	The distance from house to work (mile)	Satisfaction level	The reliability of house information	Satisfaction level
100	0	Very good	1	Very high	0	Very good	1	1	0	4	1	Very good	1
200	0.8	Good	0.8	High	0.4	Good	0.9	2	0.8	5	0.9	Good	0.9
300	1	Average	0.6	Average	0.7	Average	0.7	3	0.9	13	0.8	Average	0.7
400	0.6	Bad	0.4	Low	0.9	Bad	0.4	4	1	15	0.5	Bad	0.4
500	0.3	Very bad	0	Very low	1	Very bad	0	5	1	20	0.3	Very bad	0

constraint, the search results provide twenty available houses from the Yahoo Real Estate database as shown in Fig. 12. The housing parameters and satisfaction levels of these twenty houses are listed in Table 8. It is of note that the distance from house to work is calculated by using Google Maps (http://maps.google.com/). Also, Alice can get the distances from available houses to a specific point from Google Maps and then input the data into the proposed decision support system to find appropriate houses.

To calculate the average satisfaction level for each attribute of the houses with Eq. (6), we have AV_{room} = 0.835, AV_{work} = 0.815, AV_{view} = 0.83. According to Alice's preferences, the real estate agent evaluates the available houses by assigning weights to maximize her expected satisfaction with three goals subject to all constraints. This problem is formulated in the Appendix B. The relative weights obtained from AHP in Table 3 are attached on each corresponding goal in the FGP as follows.

$$\begin{aligned}\text{Minimize}\quad & 0.192 * (p_{11} + 2p_{12} + 3p_{13} + 4p_{14} + 5p_{15} + 7000(e_1^+ \\ & + e_1^-)) + 0.192 * (4p_{21} + 3p_{22} + 2p_{23} + 1p_{24} + 7000(e_2^+ + e_2^-)) \\ & + 0.154^*(p_{31} + 2p_{32} + 3p_{33} + 7000(e_3^+ + e_3^-))\end{aligned}$$

From Table 3, the weight value of owner's estimate of annual housing is 0.192 which is attached on G1 and G2. Also, the weight value of lot size is 0.154 which is attached on G3.

The problem is solved by using LINGO (Schrage, 2002) to obtain the solution as (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}) = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), (p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{21}, p_{22}, p_{23}, p_{24}, p_{31}, p_{32}, p_{33}) = (20, 10, 10, 15, 0, 0, 0, 2, 38, 50, 100, 145). As seen, G1 = 60 (5 + 20 + 10 + 10 + 15 = 60) can be observed from Fig. 9, i.e., Alice's expected gain is 60 thousand dollars for the new house, x_4, with the utility value $\lambda_1 = 0.77$. G2 = 40 (0 + 0 + 2 + 38 = 40) can be observed from Fig. 10, i.e., Alice's expected loss is 40 thousand dollars for the new house, x_4, with the utility value $\lambda_2 = 0.8565$. G3 = 1094 (700 + 50 + 100 + 145 + 99 = 1094) can be observed from Fig. 11, i.e., the lot size is 1094 square meters with the utility value $\lambda_3 = 0.7933$. House x_4 with the relative high expected gain and low loss is the best choice for Alice. The rates of homebuyer satisfaction for all three goals are above 77%.

In order to discover more suitable houses, Alice adjusts different preemptive priorities on constraints 2–6 with Eq. (12) and then the best alternative is derived accordingly in Table 9. From Table 9, houses x_2, x_4 and x_{10} are the three best choices for Alice. If she determines that some of constraints 1–3 (safety, pollution level and view) should be achieved, house x_{10}, which has very good safety, an average pollution level and a good view, would be the best choice. However, when constraints 1–3 are all need to be achieved, the best choice becomes house x_2, which has good safety, a low pollution level and a very good view. When all five

Table 7
Translation of fuzzy range into crisp range of housing constraints.

Constraints	Fuzzy range of housing constraints	Crisp range of housing constraints
Constraint 2: Safety	The safety should at least be good	The description of the house should include the word "safety", and the satisfaction level of "the safety of the house" should at least be 80%
Constraint 3: Pollution level	The pollution level should at least be low	The description of the house should not include the word "pollution", and the satisfaction level of "the pollution of the house" should at be least 90%
Constraint 4: View	The view should at least be good	The description of the house should include the word "view", and the satisfaction level of "the view of the house" should at least be 90%
Constraint 5: Number of bedrooms	There must be at least 2 bedrooms, and 4 bedrooms will be good	There must be at least 2 bedrooms, and the satisfaction level of "the number of bedrooms" should at least be 80%
Constraint 6: The distance from the house to the workplace	The distance from the house to the workplace is "not too far" and at most 13 mile	The distance from the house to the workplace do not exceed 13 mile and the satisfaction level of "The distance from the house to the workplace" should at least be 84%
Constraint 7: The reliability of the house information	The reliability of the house information should at least be average	The description of the house should make the homebuyer feel reasonable, and the satisfaction level of "the reliability of the house information" should at least be 70%

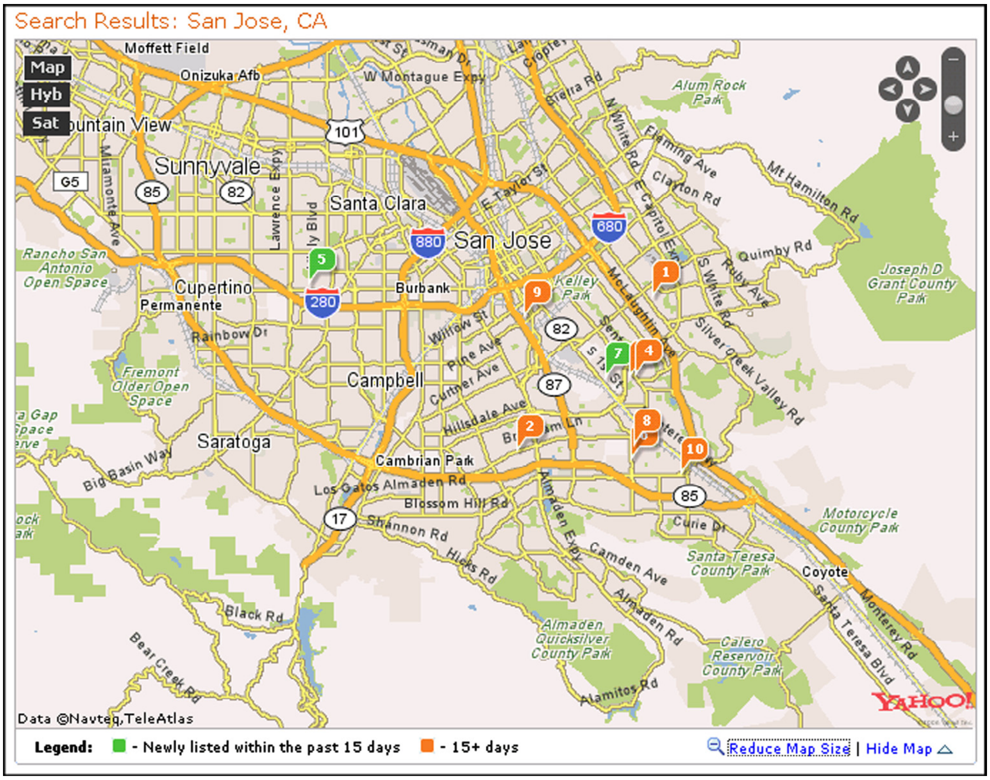

Fig. 12. Twenty house alternatives (http://realestate.yahoo.com/California/San_Jose/Homes_for_sale/result.html).

Table 8
House parameters and the satisfaction levels of 20 alternatives.

House alternatives	Price	μ_{price}	Lot Size (square meters)	Safety	μ_{safety}	Pollution level	$\mu_{pollution}$	View	μ_{view}	Number of bedrooms	μ_{room}	Distance to the workplace (mile)	μ_{work}	Reliability of the house information	$\mu_{reliability}$
x_1	$330,000	0.88	798	Very Good	1	Average	0.7	Good	0.9	2	0.8	10.4	0.833	Good	0.9
x_2	$320,000	0.92	1249	Good	0.8	Low	0.9	Very Good	1	3	0.9	10.1	0.836	Very good	1
x_3	$316,000	0.94	1148	Very Good	1	Low	0.9	Average	0.7	2	0.8	9.7	0.841	Average	0.7
x_4	$307,000	0.972	1094	Good	0.8	Low	0.9	Good	0.9	3	0.9	6.2	0.885	Average	0.7
x_5	$295,000	0.99	871	Good	0.8	Low	0.9	Good	0.9	2	0.8	11.4	0.82	Good	0.9
x_6	$285,000	0.97	924	Average	0.6	Low	0.9	Average	0.7	2	0.8	12.9	0.813	Average	0.7
x_7	$275,000	0.95	997	Very Good	1	Low	0.9	Good	0.9	2	0.8	12.1	0.811	Good	0.9
x_8	$262,888	0.926	770	Bad	0.4	Average	0.7	Very Good	1	2	0.8	5.4	0.895	Very good	1
x_9	$249,950	0.9	903	Good	0.8	High	0.4	Bad	0.4	2	0.8	10.8	0.828	Bad	0.4
x_{10}	$275,000	0.95	990	Very Good	1	Average	0.7	Good	0.9	2	0.8	6.2	0.885	Good	0.9
x_{11}	$300,000	1	950	Good	0.8	Low	0.9	Average	0.7	3	0.9	9.7	0.841	Very good	1
x_{12}	$250,000	0.9	850	Good	0.8	Low	0.9	Good	0.9	2	0.8	5.4	0.895	Average	0.7
x_{13}	$263,000	0.926	800	Good	0.8	Average	0.7	Average	0.7	3	0.9	10.1	0.836	Average	0.7
x_{14}	$316,000	0.94	1100	Good	0.8	Low	0.9	Average	0.7	3	0.9	11.4	0.82	Good	0.9
x_{15}	$300,000	1	900	Good	0.8	Low	0.9	Average	0.7	2	0.8	12.9	0.813	Very good	1
x_{16}	$307,000	0.972	1000	Good	0.8	Average	0.7	Good	0.9	3	0.9	5.4	0.895	Average	0.7
x_{17}	$250,000	0.9	840	Good	0.8	Low	0.9	Average	0.7	2	0.8	10.8	0.828	Average	0.7
x_{18}	$250,000	0.9	850	Very Good	1	Low	0.9	Very Good	1	3	0.9	13	0.8	Very good	1
x_{19}	$200,000	0.8	800	Good	0.8	Low	0.9	Average	0.7	2	0.8	20	0.3	Good	0.9
x_{20}	$200,000	0.8	850	Good	0.8	Average	0.7	Good	0.9	2	0.8	11.4	0.82	Good	0.9

constraints need to be satisfied, house x_4 is chosen because it has low-price, a low-pollution level and is near her workplace.

In this way, Alice can easily find a better house ranking list chosen according to her personal preferences and constraints. The proposed method can also provide a better suggestion for homebuyers and increase the probability of making a good decision when searching on the Internet.

4.2. A laboratory experiment

In order to investigate the customer satisfaction of the proposed decision aid system, a laboratory quasi-experiment has been implemented using Active Server Pages and an Access database. The interface of the decision aid system is presented in Fig. 8. We adopt (Pereira's, 1999) questionnaire and use the modified research model as shown in Fig. 13. The experimental subjects are 250 middle-aged workers with house-buying experience in central Taiwan. They have used the Internet to search for housing information or buy houses. 125 subjects are instructed not to use the housing decision aid and the other 125 subjects use this system. Subjects are approximately distributed equally by gender and age. All subjects input their preferred price range and zip code. Then our decision aid system presents housing suggestions. Subjects without access to the housing decision aid have to decide

Table 9
Different preemptive priorities on each constraint and the derived best house.

Constraints	Constraint 2: Safety C_1	Constraint 3: Pollution level C_2	Constraint 4: View C_3	Constraint 5: Number of bedrooms C_4	Constraint 6: Distance to the workplace C_5	The best house
$C_1 + C_2 + C_3 \geqslant 1$, $C_4 + C_5 = 0$	1	0	0	0	0	x_{10}
$C_1 + C_2 + C_3 \geqslant 1$, $C_4 + C_5 = 1$	1	0	0	1	0	x_{10}
$C_1 + C_2 + C_3 \geqslant 1$, $C_4 + C_5 = 2$	1	0	0	1	1	x_{10}
$C_1 + C_2 + C_3 \geqslant 2$, $C_4 + C_5 = 0$	1	0	1	0	0	x_{10}
$C_1 + C_2 + C_3 \geqslant 2$, $C_4 + C_5 = 1$	1	0	1	1	0	x_{10}
$C_1 + C_2 + C_3 \geqslant 2$, $C_4 + C_5 = 2$	1	0	1	1	1	x_{10}
$C_1 + C_2 + C_3 \geqslant 3$, $C_4 + C_5 = 0$	1	1	1	0	0	x_2
$C_1 + C_2 + C_3 \geqslant 3$, $C_4 + C_5 = 1$	1	1	1	1	0	x_2
$C_1 + C_2 + C_3 \geqslant 3$, $C_4 + C_5 = 2$	1	1	1	1	1	x_4

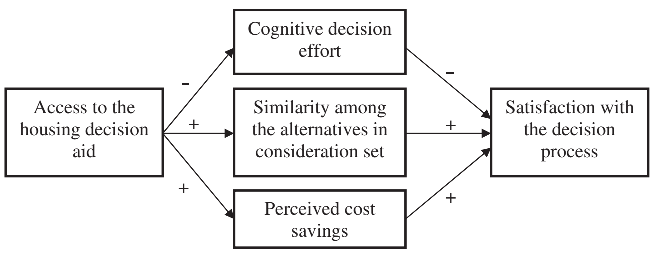

Fig. 13. Influence of housing decision aid on satisfaction with the decision process.

Table 10
Reliability of the used measures.

Construct	Measure	Cronbach's α
Effort	Cognitive decision effort	0.87
Similarity	Similarity among the alternatives in consideration set	0.83
Savings	Perceived cost savings	0.88
Satisfaction	Satisfaction with the decision process	0.84
Housing decision aid	Access to the housing decision aid	

Table 11
Result of single factor ANOVA tests.

Dependent variable	Mean of samples with access to decision aid	Mean of samples without access to decision aid	F	Significance level
Effort	3.12	3.38	9.324	0.012**
Similarity	82.24	70.28	12.018	0.001**
Savings	4.46	3.42	3.125	0.026**
Satisfaction	4.638	3.712	3.746	0.024**

** Significance at the 0.05 level of significance ($p < 0.05$).

which house is the best choice for them. Subjects with access to the decision aid system have to identify the housing goals with risk attitude and define their satisfaction level for each goal and criterion with an S-shaped utility function. Then our system calculates and aggregates all of the subject's fuzzy housing goals and criteria using the FGP model. Finally, the rank of house alternatives is derived. Table 10 illustrates the values of Cronbach's α for the used measures indicating that the measures have high reliability.

A single factor Analysis of variance (ANOVA) test is conducted to examine the influence of the variable "Housing decision aid" on mediating and dependent variables. The factor "Housing decision aid" is coded as a dummy variable, i.e., present or absent. The constructs Effort, Savings and Satisfaction are represented as the mean-centered scores on a seven-point Likert scale. The system calculates a similarity score, with a range from 0 (completely different) to 100 (completely similar) for each alternative based on the fuzzy queries and preferences of the DM. The results of the experiment are listed in Tables 11 and 12. The "Housing decision aid" variable has a significant influence on the satisfaction variable. The mean value of Satisfaction for users with access to the decision aid system (4.638) is higher than that for those with no access to the decision aid system (3.712). This indicates that use of the housing decision aid significantly increases the satisfaction levels of customers. The single factor ANOVA test of the influence of the housing decision aid on Satisfaction shows a significant relationship ($F = 3.746$; $p < 0.05$).

Furthermore, a single factor ANOVA test is performed with regression analysis of Satisfaction related to Effort, Savings and Similarity. We find a significant explanation of variation for Satisfaction in Table 11. Savings ($\beta = 0.386$; $t = 4.424$), Similarity ($\beta = 0.328$; $t = 3.315$) and Effort ($\beta = -0.204$; $t = -3.142$) have significant influence on Satisfaction.

After conducting the laboratory experiment, we have found some challenges for our housing decision aid. First, subjects sometimes obtain too many or too few alternatives from the decision aid system because of restricted criteria. Fortunately, this aid can rank the house alternatives according to the aggregation of the buyer's fuzzy goals and criteria using the FGP model. The ranking list helps subjects avoid confusion about similar houses. Second, it is not easy for customers to identify their housing goals with suitable risk attitude and determine the expected gain and loss of alternatives. The proposed decision aid can collect recent prices of houses which

Table 12
Result of regression analysis.

Dependent variable	R Square	Adjusted R square	F-Statistic significance level	β Coefficient for effort t-statistic significance level	β Coefficient for similarity t-statistic significance level	β Coefficient for savings t-statistic significance level
Satisfaction	0.412	0.322**	$F_{3,50} = 9.455$ 0.001**	$\beta = -0.204$ t = -3.142 0.0012**	$\beta = 0.328$ $t = 3.315$ 0.026**	$\beta = 0.386$ $t = 4.424$ 0.004**

** Significance at the 0.05 level of significance ($p < 0.05$).

are similar to the alternatives in order to determine the expected gain and loss. During the searching procedure, homebuyers usually spend 20–30 min on the traditional real estate site, Yahoo Real Estate, to find desired houses. However, it only takes 8–10 min for customers using our housing decision aid to obtain target houses. It is clear that the proposed decision aid system is more efficient than traditional search tools.

5. Conclusions

Creating an online search tool with a user-friendly interface for house searches is the key success factor for winning consumers' trust and preference. Nevertheless, current online agents cannot provide powerful search tools to meet homebuyers' possible conflicting goals and heterogeneous preferences. This study presents an integrated approach to support homebuyers in their online evaluation process. The proposed approach screens available houses according to homebuyers' risk attitudes in loss or gain situations. In this way, the proposed approach maximizes the sum of satisfaction levels with given weighted goals. Available houses with some important advantages but slight deviations from the search specifications are not retrieved by current systems. This issue can be solved by the proposed decision aid system. Also, DMs can determine the appropriate constraints with different thresholds for fuzzy queries. In order to meet the buyer's constraints with linguistic quantifiers, this method evaluates available houses by translating the DM's queries into precise SQL queries.

The proposed approach transforms homebuyers' fuzzy satisfaction levels into a fixed form. Then the ranking results of houses can be created for homebuyers. Personalized ranking is provided by the proposed system. Homebuyers can adjust their fuzzy goals or set different preemptive priorities on each constraint with ease to derive the different ranking lists. This can help buyers clarify their thoughts about the ideal house. A good ranking list can dramatically reduce search time and increase the matching rate.

In the competitive market of real estate, it is important to provide a user-friendly interface for customers to input fuzzy criteria and then derive a ranking list based on buyers' preferences and risk attitudes. This study finds that customer satisfaction is significantly increased by the use of the proposed housing decision aid.

Appendix A. AHP questionnaire

Level 1	Absolute importance			Strong importance				Equal importance			Strong importance			Absolute importance				Level 1
Housing value	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Structure attributes
	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Neighborhood Attributes
	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Location attributes
Structure attributes	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Neighborhood attributes
	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Location attributes
Neighborhood attributes	9:1	8:1	7:1	6:1	5:1	4:1	3:1	2:1	1:1	1:2	1:3	1:4	1:5	1:6	1:7	1:8	1:9	Location attributes

Appendix B. Model formulation

$$
\begin{aligned}
\text{Minimize}\quad & 0.192*(p_{11}+2p_{12}+3p_{13}+4p_{14}+5p_{15}+7000(e_1^+ + e_1^-))\\
& +0.192*(4p_{21}+3p_{22}+2p_{23}+1p_{24}+7000(e_2^+ + e_2^-))\\
& +0.154*(p_{31}+2p_{32}+3p_{33}+7000(e_3^+ + e_3^-))\\
\text{Subject to}\quad & \lambda_1=[0.15-0]\frac{p_{11}}{30-5}+[0.3-0.15]\frac{p_{12}}{40-30}\\
& \quad +[0.6-0.3]\frac{p_{13}}{50-40}\\
& \quad +[0.8-0.6]\frac{p_{14}}{65-50}+[1-0.8]\frac{p_{15}}{100-65}, \qquad \text{(for G1)}\\
& \lambda_1-e_1^+ + e_1^- = 1,\\
& z_1(\mathbf{x})-p_{11}-p_{12}-p_{13}-p_{14}-p_{15}=5,\\
& 0\le p_{11}\le 30-5,\ \ 0\le p_{12}\le 40-30,\\
& 0\le p_{13}\le 50-40,\ \ 0\le p_{14}\le 65-50,\\
& 0\le p_{15}\le 100-65,\\
& z_1(\mathbf{x})=100x_1+80x_2+70x_3+60x_4+55x_5+40x_6\\
& \qquad +50x_7+45x_8+50x_9+90x_{10}\\
& \qquad +85x_{11}+70x_{12}+20x_{13}+35x_{14}+30x_{15}+40x_{16}\\
& \qquad +25x_{17}+20x_{18}+10x_{19}+5x_{20},\\
& \lambda_2=1-\Big([1-0.8]\frac{p_{21}}{30-0}+[0.8-0.4]\frac{p_{22}}{50-30}\\
& \qquad +[0.4-0.13]\frac{p_{23}}{90-50}+[0.13-0]\frac{p_{24}}{128-90}\Big), \qquad \text{(for G2)}\\
& \lambda_2-e_2^+ + e_2^- = 1,\\
& z_2(\mathbf{x})-p_{21}-p_{22}-p_{23}-p_{24}\le 0,\\
& 0\le p_{21}\le 30-0,\ \ 0\le p_{22}\le 50-30,\\
& 0\le p_{23}\le 90-50,\ \ 0\le p_{24}\le 128-90,\\
& z_2(\mathbf{x})=80x_1+50x_2+50x_3+40x_4+30x_5+45x_6\\
& \quad +55x_7+90x_8+45x_9+20x_{10}\\
& \quad +20x_{11}+30x_{12}+40x_{13}+30x_{14}+45x_{15}+40x_{16}\\
& \quad +10x_{17}+20x_{18}+10x_{19}+5x_{20},\\
& \lambda_3=1-([0.3-0]\frac{p_{31}}{75-70}+[0.6-0.3]\frac{p_{32}}{85-75}\\
& \quad +[1-0.6]\frac{p_{33}}{115-85}),\ \lambda_2-e_2^+ + e_2^- = 1, \qquad \text{(for G3)}
\end{aligned}
$$

$$
\begin{aligned}
& z_3(\mathbf{x}) - p_{31} - p_{32} - p_{33} \le 799,\\
& 0 \le p_{31} \le 75-70,\quad 0 \le p_{32} \le 85-75,\\
& 0 \le p_{33} \le 115-85,\\
& z_3(\mathbf{x}) = 798x_1 + 1249x_2 + 1148x_3 + 1094x_4 + 871x_5\\
& \qquad + 924x_6 + 997x_7 + 770x_8 + 903x_9\\
& \qquad + 990x_{10} + 950x_{11} + 850x_{12} + 800x_{13} + 1100x_{14}\\
& \qquad + 900x_{15} + 1000x_{16} + 840x_{17} + 850x_{18} + 800x_{19} + 850x_{20},\\
& x_1 + x_2 + \ldots + x_{20} = 1,\quad \text{(buy only one house)}\\
& 0.88x_1 + 0.92x_2 + 0.94x_3 + 0.972x_4 + 0.99x_5 + 0.97x_6\\
& \qquad + 0.95x_7 + 926x_8 + 0.9x_9 + 0.95x_{10}\\
& \qquad + x_{11} + 0.9x_{12} + 0.926x_{13} + 0.94x_{14} + x_{15} + 0.972x_{16}\\
& \qquad + 0.9x_{17} + 0.9x_{18} + 0.8x_{19} + 0.8x_{20} \ge 0.8,\quad \text{(house price)}\\
& x_1 + 0.8x_2 + x_3 + 0.8x_4 + 0.8x_5 + 0.6x_6 + x_7 + 0.4x_8\\
& \qquad + 0.8x_9 + x_{10} + 0.8x_{11} + 0.8x_{12}\\
& \qquad + 0.8x_{13} + 0.8x_{14} + 0.8x_{15} + 0.8x_{16} + 0.8x_{17} + x_{18} + 0.8x_{19}\\
& \qquad + 0.8x_{20} \ge 0.8^{*}C_1,\quad \text{(the safety should at least be good)}
\end{aligned}
$$

where $x_i (i = 1, 2, \ldots, 20)$ are binary variables

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