Unusual Superconducting transition in Topological Insulators

2014 J. Phys.: Conf. Ser. 568 022027

(http://iopscience.iop.org/1742-6596/568/2/022027)

Journal of Physics: Conference Series **568** (2014) 022027 doi:10.1088/1742-6596/568/2/022027

Unusual Superconducting transition in Topological Insulators

Dingping Li[1,2], B. Rosenstein[3,4], B. Ya. Shapiro[5] and I. Shapiro[5]

[1]School of Physics, Peking University, Beijing 100871, China
[2]Collaborative Innovation Center of Quantum Matter, Beijing, China
[3]Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, R.O.C.
[4]Applied Physics Department, Ariel University Center of Samaria, Ariel 40700, Israel
[5]Department of Physics, Institute of Superconductivity, Bar-Ilan University, Ramat-Gan 52900, Israel

Abstract. Superconducting transition generally belongs to the $U(1)$ class of phase transitions. However it was pointed out long time ago that if the normal state dispersion relation is "ultra-relativistic" the transition is unusual: even the mean field critical exponents are different from the standard ones leading to a number of observable effects. Attempts to experimentally discover such a system included chiral condensate in graphene. Recently it was found that some 3D topological insulators (that possess the ultrarelativistic metal on its surface) exhibit surface superconductivity.Starting from microscopic TI Hamiltonian with local four fermions interaction, we calculated the total set of the Gor'kov equations allowing to build the Ginzburg - Landau (GL) theory including the magnetic field effects. It was shown that the GL equations reflect the novel chiral universality class, very different from original GL equations. For example the temperature dependence of the coherence length diverges at the critical temperature with critical exponent $\nu = -1$ in rather than customary $\nu = -1/2$, magnetization near the upper critical magnetic field is quadratic as a function of deviation from the upper critical field while the Superfluid density is $\psi^2 = (T_c - T)^\beta$, $\beta = 2$, not $\beta = 1$.

1. Introduction

Since best studied Topological insulator (TI) possess a quite standard phonon spectrum [1], it was predicted recently [2] that they become superconducting TI (STI) (this should be distinguished from "topological superconductors", TSC, in which superconductivity appears in the bulk[3]). The predicted critical temperature of order of $1K$ is rather low (despite a fortunate suppression of the Coulomb repulsion due to a large dielectric constant $\varepsilon \sim 50$), the nature of the "normal" state, the 2D Weyl semi-metal, might make the superconducting properties of the system unusual. Especially interesting is the case (that actually was originally predicted for the [111] surface of Bi_2Te_3 and Bi_2Se_3[4]) when the chemical potential coincides with the Weyl point. Although subsequent ARPES experiments show the location of the cone of surface states order tenths of eV off the Fermi surface; there are experimental means to shift the chemical potential, for example by the bias voltage [5].

Unlike the more customary poor 2D metals with several small pockets of electrons/holes on the Fermi surface (in semiconductor systems or even some high T_c materials), STI has two peculiarities especially important when pairing is contemplated. The first is the bipolar nature of the Weyl spectrum: there is no energy gap between the upper and lower cones. The second is that the spin degree of freedom is a major player in the quasiparticle dynamics. This degree of freedom determines the pairing channel.

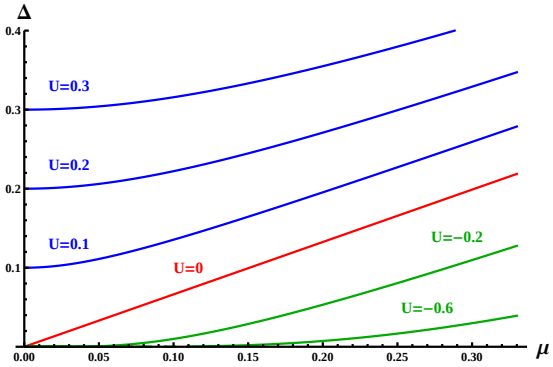

Figure 1. Order parameter at zero temperature as function of chemical potential of the TI surface Weyl semi-metal at various values of coupling parametrized by the renormalized energy U, Eq.(2). For positive U (blue lines) the superconductivity is strong and does not vanish even for zero chemical potential. There exists the critical coupling, $U = 0$ (red line), at which the second order transition occurs at quantum critical point $\mu = 0$. For negative U the superconductivity still exists at $\mu > 0$, but is exponentially weak.

In this paper we study the thermodynamic and magnetic properties of the surface superconductivity in TI with local attraction pairing Hamiltonian characterized by the coupling strength g and cutoff parameter T_D within the self-consistent approximation. The phase diagram is obtained for arbitrary temperature T and chemical potential $\mu < T_D$ We found a quantum critical point at $T = \mu = 0$ when the coupling strength g reaches a critical value g_c dependent on the cutoff parameter. We concentrate on properties of the superconducting state in a part of the phase diagram that is dominated by the QCP. Various critical exponents are obtained. In particular, the coupling strength dependence of the coherence length is $\xi \propto (g - g_c)^{-\nu}$ with $\nu = 1$, the order parameter scales as $\Delta \propto (g - g_c)^{\beta}$, $\beta = 1$. It is found that near the QCP the Ginzburg - Landau effective model is rather unconventional. The structure of the single vortex core is different from the usual Abrikosov vortex, while the magnetization curve near the upper critical magnetic field H_{c2} is quadratic: $M = (H - H_{c2})^2$, not linear.

2. TI with a local pairing interaction.

Electrons on the surface of a TI are described by a Pauli spinor $\psi_\alpha(\mathbf{r})$, where the upper plane, $\mathbf{r} = \{x, y\}$, is considered. The Hamiltonian for electrons in TI, $H = \int d^2 r \psi^+_\alpha \left(-i\hbar v_F \varepsilon_{ij} \nabla_i \sigma^j_{\alpha\beta} - \mu \delta_{\alpha\beta}\right) \psi_\beta$ where σ^j are the Pauli matrices. and interacting via four-Fermi local coupling of strength g. The effective local interaction might be generated by a phonon exchange or perhaps other mechanisms and will be assumed to be weak coupling. Therefore the BCS type approximation can be employed. In the homogeneous case, the matrix gap function can be chosen as (Δ real) $\widehat{\Delta}_{\beta\gamma} = g \langle \psi_\gamma(0) \psi_\beta(0) \rangle = i\sigma_y \Delta$. The matrix gap equation

$$\widehat{\Delta}^{*t} = -g \sum_{\omega q} D^t \widehat{\Delta}^{*t} \left(D^{-1} - \widehat{\Delta} D^t \widehat{\Delta}^{*t}\right)^{-1}. \quad (1)$$

where $D^{-1}_{\gamma\beta} = (i\omega - \mu)\delta_{\gamma\beta} - v_F \varepsilon_{ij} p_i \sigma^j_{\alpha\beta}$. The spectrum of elementary excitations, $E_p = \pm\sqrt{\Delta^2 + (v_F p - \mu)^2}$, coincides with that found within the Bogoliubov - de Gennes approach [6].

3. Zero temperature phase diagram and QCP.

At zero temperature the integrations over frequency and momentum limited by the UV cutoff Λ result in

$$U = \sqrt{\Delta^2 + \mu^2} - \frac{\mu}{2} \log \frac{\sqrt{\Delta^2 + \mu^2} + \mu}{\sqrt{\Delta^2 + \mu^2} - \mu}, \tag{2}$$

where the dependence on the cutoff is incorporated in the renormalized coupling with dimension of energy defined as $U = v_F \Lambda \left(1 - g_c/g\right)$ with $g_c = 4\pi\hbar^2 v_F/\Lambda$. Of course the superconducting solution exists only for $g > 0$. In Fig. 1 the dependence of the gap Δ as function of the chemical potential μ is presented for different values of U.

For an attractive coupling g stronger than the critical one, (when $U > 0$), blue lines in Fig. 1, there are two qualitatively different cases.

(i). When $\mu << U$ the dependence of Δ on the chemical potential is parabolic $\Delta/U \approx 1 + (\mu/U)^2$.

In particular, when $\mu = 0$, the gap equals U. As can be seen from Fig. 1, the chemical potential makes a very limited impact in the large portion of the phase diagram.

(ii) For the attraction just stronger than critical, $g > g_c$, namely for small positive U, the dependence becomes linear, see red line in Fig. 1, $\Delta = 0.663\,\mu$. So that the already weak condensate becomes sensitive to μ.

The case (i) is more interesting than (ii) since it exhibits stronger superconductivity (larger T_c, see below). Finally for $g < g_c$, namely negative U (green lines in Fig. 1), the superconductivity is very weak with exponential dependence similar to the BCS one, $\Delta \approx \mu \exp[-\left(|U|/\mu - 1\right)]$. Therefore it is possible to neglect the effect of weak doping and consider directly the $\mu = 0$ particle-hole symmetric case.

At zero temperature $\Delta = U$, while $\Delta \to 0$ as a power of the parameter $U \propto g - g_c$ describing the deviation from quantum criticality $T_c \propto U^{z\nu}$; $z\nu = 1$. Here z is the dynamical critical exponent[8]. Therefore, as expected, the renormalized coupling describing the deviation from the QCP is proportional to the temperature at which the created condensate disappears. The temperature dependence of the gap reads, $\Delta(T) = 2T \cosh^{-1}\left(\frac{1}{2}\exp\frac{U}{2T}\right)$, typical for chiral universality classes [8, 9].

4. Ginzburg - Landau effective theory and magnetic properties of the superconductor near QCP

The quadratic term of the Ginzburg-Landau energy $F_2 = \sum_{\mathbf{p}} \Delta_p^* \Gamma(p) \Delta_p$ is obtained exactly from expanding the gap equation to linear terms in Δ for arbitrary external momentum: $\Gamma(p) = -U/4\pi\hbar^2 v_F^2 + |p|/16 v_F \hbar^2$. The dependence on $\mathbf{p}$ is non-analytic and within our approximation higher powers of p do not appear. The second term is very different from the quadratic term in the GL functional for conventional phase transitions at finite temperature or even quantum phase transitions in models without Weyl fermions [8] and has a number of qualitative consequences. The coherence length as a power of parameter $U \propto g - g_c$ describing the deviation from criticality: $\xi(U) = \frac{\pi}{4} v_F \hbar U^{-\nu}$; $\nu = 1$. This is different from the dependence in non-chiral universality classes that is $\xi(T) \propto (T_c - T)^{-\nu}$, $\nu = 1/2$ in mean field. Of course in the regime of critical fluctuations this exponent is corrected in both non-chiral and chiral[9] universality classes.

Local terms in the GL energy density are calculable exactly: $f_{cond} =\propto -U|\Delta|^2 + \frac{2}{3}|\Delta|^3$. It is quite nonstandard compared to customary quartic term in conventional universality classes. The GL equations in the homogeneous case for the condensate gives $\Delta_0 = U^\beta$ with critical exponent $\beta = 1$, different from the mean field value $\beta = 1/2$ for the $U(1)$ universality class. The condensation energy density is $f_0 \propto U^{2-\alpha}$ with $\alpha = -1$. The free energy critical exponent at QCP therefore is also different from the classical $\alpha = 0$. In the present case the equation for the order parameter is nonlocal and

nonanalytic. Near the upper critical field H_{c2} the Abrikosov hexagonal lattice is formed. The optimal Δ at external field H close to H_{c2} is $\Delta \propto (1 - H/H_{c2})^{\sigma}$, $\sigma = 1$, different from the ordinary Abrikosov lattice value of $\sigma = 1/2$. The magnetization is $M \propto -(1 - H/H_{c2})^{\tau}$, $\tau = 2$ in contrast to linear dependence, $\tau = 1$.

5. Discussion and conclusions

To estimate the pairing efficiency due to phonons, one should rely on recent studies of surface phonons in TI [2]. The coupling constant is obtained from the exchange of acoustic (Rayleigh) surface phonons $g = \lambda v_F^2 \hbar^2 / 2\pi\mu$, where λ is the dimensionless effective electron - electron interaction constant of order 0.1. It was shown in ref. [2] that at zero temperature the ratio of λ and μ is constant with well defined $\mu \to 0$ limit with value $g = 0.23\ eV\ nm^2$ for $v_F \approx 7\ \cdot 10^5 m/s$ (for Bi_2Se_3). The critical coupling constant g_c, can be estimated from the Debye cutoff $T_D = 200K$ determining the momentum cutoff $\Lambda = T_D/c_s$, where c_s is the sound velocity. Taking value to be $c_s = 2 \cdot 10^3 m/s$ (for Bi_2Se_3), one obtains $g_c = 4\pi v_F c_s \hbar^2 / T_D = 0.20\ eV\ nm^2$. Therefore the stronger superconductivity, $g > g_c$, is realized.

In this paper we focused on the qualitatively distinct case of Weyl fermions with small chemical potential. Note that a reasonable electron density of $n = 3 \cdot 10^{11} cm^{-2}$ in Bi_2Te_3 already conforms to the requirement that chemical potential $\mu = \sqrt{n}\hbar v_F / 2\pi = 100K$ is smaller that the Debye cutoff energy $T_D = 200K$. The concept of QCP at zero temperature and varying doping constitutes a very useful language for describing the microscopic origin of superconductivity in high T_c cuprates and other "unconventional" superconductors. Superconducting transitions generally belong to the $U(1)$ class of second order phase transitions, however it was pointed out a long time ago that, if the normal state dispersion relation is "ultra-relativistic", the transition at zero temperature as function of parameters like the pairing interaction strength is qualitatively distinct and belongs to chiral universality classes classified in ref. [9]. Attempts to experimentally identify second order transitions governed by QCP included quantum magnets [8], superconductor - insulator transitions and more recently chiral condensate in graphene.

Acknowledgements. We are indebted to C.W. Luo, J.J. Lin for valuable discussions. Work of D.L. and B.R. was supported by NSC of R.O.C. Grants No. 98-2112-M-009-014-MY3 and MOE ATU program. The work of D.L. also is supported by National Natural Science Foundation of China (No. 11274018).

[1] Luo C W et al 2013 *Nano Lett.* **13**, 5797.
[2] Das Sarma S and Li Qiuzi 2013 *Phys. Rev. B* **88**, 081404(R).
[3] X.-L. Qi and S.-C. Zhang 2011 *Rev. Mod. Phys.* **83**.
[4] Zhang H et al 2009 *Nat. Phys.* 5, 438.
[5] Kim D et al 2012 *Nat. Phys.* **8**, 459.
[6] Lu Chi-Ken and Herbut I F 2010 *Phys. Rev. B* **82**, 144505.
[7] Cheng M, Lutchyn R M, and Das Sarma S 2012 *Phys. Rev B* **85**, 165124.
[8] Sachdev S 2011 *Quantum Phase Transitions* (Cambridge: Cambridge University Press)
[9] Rosenstein B, Warr B J and Park S H 1989 *Phys. Rev. Lett.* **62**, 1433; Gat G, Kovner A and Rosenstein B 1992 *Nucl. Phys.[FS] B* **385**, 76.