



Anti-control of chaos of single time scale brushless dc motors and chaos synchronization of different order systems

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Abstract

Anti-control of chaos of single time scale brushless dc motors (BLDCM) and chaos synchronization of different order systems are studied in this paper. By addition of an external nonlinear term, we can obtain anti-control of chaos. Then, by addition of the coupling terms, by the use of Lyapunov stability theorem and by the linearization of the error dynamics, chaos synchronization between a third-order BLDCM and a second-order Duffing system are presented. © 2005 Elsevier Ltd. All rights reserved.

1. Introduction

Chaos is undesirable in most engineering applications. Many researchers have devoted themselves to find new ways to suppress and control chaos more efficiently. However, chaos is desirable under certain circumstances. Chaotic phenomena are quite useful in many applications such as fluid mixing [1], human brain [2], and heart beat regulation [3], etc. Therefore, making a regular dynamical system chaotic, or preserving chaos of a chaotic dynamical system, is meaningful and worth to be investigated.

Chaos synchronization has been applied in many fields such as secure communication [4,5], chemical and biological systems [6,7] and others [8–16] etc. A lot of researchers have studied synchronization between two identical chaotic systems. But, seldom researchers study synchronization of different order chaotic systems. This motivates us to investigate this absorbing and challenging research topic.

The theme of this paper is brushless dc motor. The major advantage of BLDCM is the elimination of the physical contact between the brushes and the commutators. BLDCM has been widely applied in direct-drive applications such as robotics [17], aerospace [18], etc. In this paper, we investigate chaos anti-control of BLDCM and chaos synchronization of different order systems. In order to verify periodic and chaotic phenomena of investigated systems, several numerical techniques such as time history, phase portrait, bifurcation diagram and Lyapunov exponents are employed.

This paper is organized as follows. Section 2 contains the dynamic characteristics of BLDCM [19–22]. First, the system model is described. Second, the system equations are transformed to a compact form. Finally, the numerical results

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of periodic and chaotic phenomena are presented. In Section 3, one method is investigated to achieve anti-control of chaos: the addition of a nonlinear term [23]. Chaos synchronization of different order systems [24] is discussed in Section 4. Two different chaotic dynamical systems, Duffing system and BLDCM, are applied in this section. Three methods are investigated to achieve chaos synchronization: the addition of the coupling terms, the use of Lyapunov stability theorem and the linearization of the error dynamics [25]. Finally, the conclusions of the whole paper are briefly stated.

2. Regular and chaotic dynamics of brushless dc motor

BLDCM is an electromechanical system [19–21]. By using an affine transformation and a single time scale transformation [22], its governing equations can be transformed into a dimensionless form as following:

$$\begin{aligned} \frac{d}{dt} \hat{x}_1 &= \hat{v}_q - \hat{x}_1 - \hat{x}_2 \hat{x}_3 + \rho \hat{x}_3 \\ \frac{d}{dt} \hat{x}_2 &= \hat{v}_d - \delta \hat{x}_2 + \hat{x}_1 \hat{x}_3 \\ \frac{d}{dt} \hat{x}_3 &= \sigma(\hat{x}_1 - \hat{x}_3) + \eta \hat{x}_1 \hat{x}_2 - \hat{T}_L \end{aligned} \tag{2.1}$$

where $\rho = 60$, $\hat{v}_q = 0.168$, $\hat{v}_d = 20.66$, $\delta = 0.875$, $\eta = 0.26$, $\hat{T}_L = 0.53$ and the initial condition is $\hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0.01$.

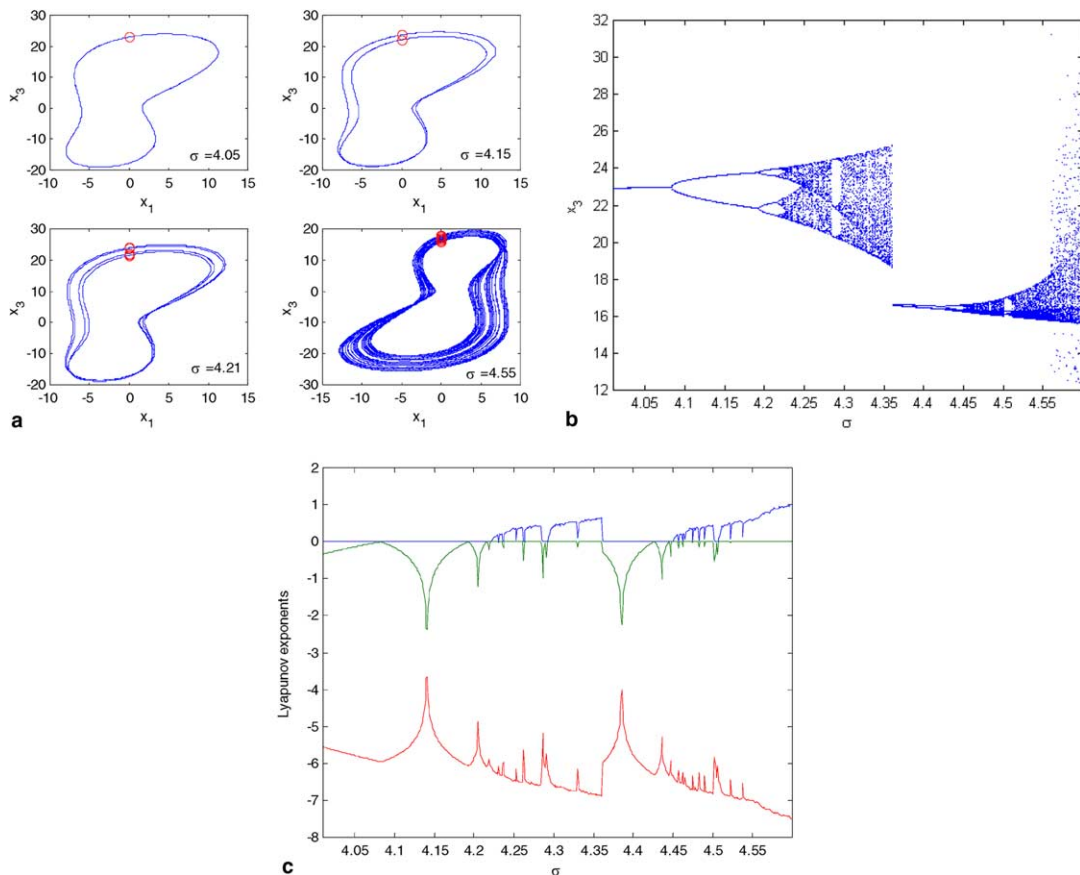


Fig. 1. (a) Phase portrait. (b) Bifurcation diagram. (c) Lyapunov exponents for BLDCM.

In addition, BLDCM is an autonomous system. It means that the period of the system is not explicitly known, so different choice of Poincaré section would lead to different bifurcation diagrams. In the sections below, adding control inputs changes the dynamics of the system, thus we have to modify the choice of Poincaré section. Modifying Poincaré section, we obtain almost the same bifurcation diagram. The only difference is the shift in \hat{x}_3 axis. Therefore, we just present the original bifurcation diagram.

The phase portrait, bifurcation diagram, and Lyapunov exponents are shown in Fig. 1. It can be observed that the motion is period 1 for $\sigma = 4.05$, period 2 for $\sigma = 4.15$, and period 4 for $\sigma = 4.21$. For $\sigma = 4.55$, the motion is chaotic.

3. Anti-control of chaos

In order to preserve chaotic phenomena of BLDCM, a nonlinear term $x|x|$ is added [23].

3.1. Adding one term of $x|x|$

First, we add an external nonlinear input $k_1\hat{x}_1|\hat{x}_1|$ to the first equation of (2.1). When $k_1 > 0$, the process of choice and the numerical results are shown in Fig. 2, it is quite clear that the chaotic phenomenon is not increased for $k_1 = 0.052$.

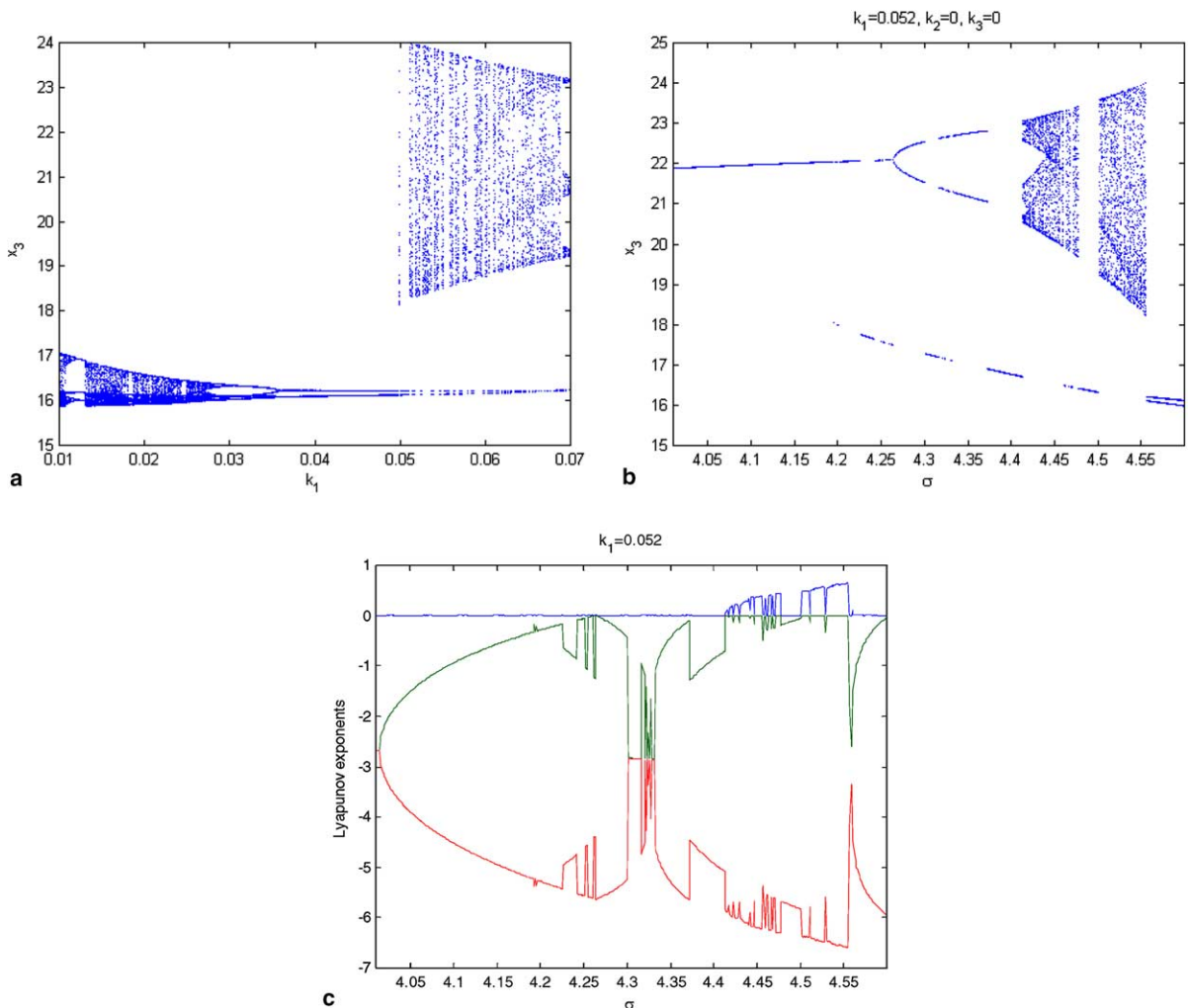


Fig. 2. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.01-0.07$. (b) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.052$. (c) Lyapunov exponents for $k_1 = 0.052$.

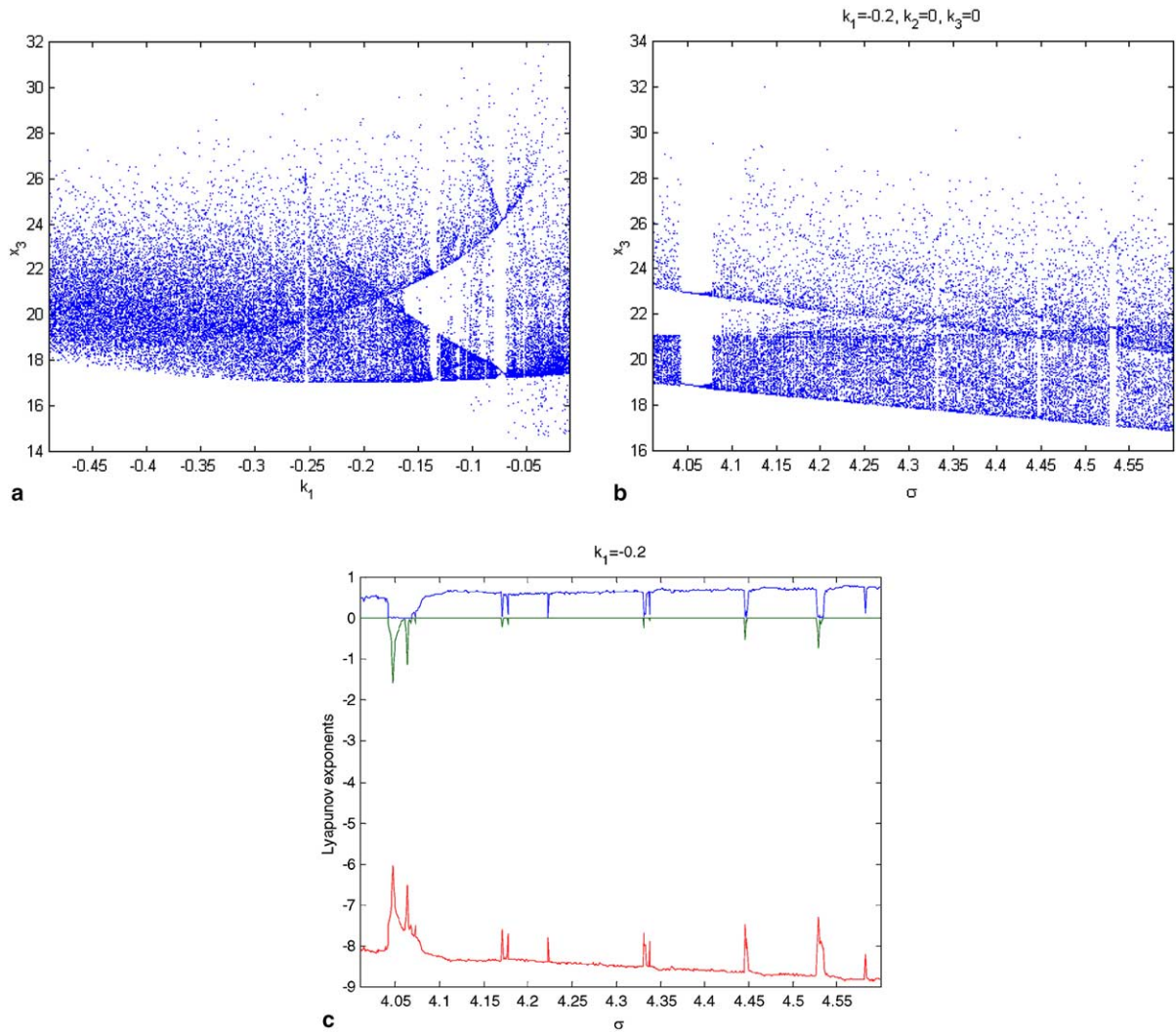


Fig. 3. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.49$ to -0.01 . (b) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.2$. (c) Lyapunov exponents for $k_1 = -0.2$.

When $k_1 < 0$, the process of choice and the numerical results are shown in Fig. 3, it is clear that the chaotic phenomenon is increased for $k_1 = -0.2$ by comparison of Figs. 1(c) and 3(c).

Second, we add an external nonlinear input $k_2\hat{x}_2|\hat{x}_2|$ to the second equation of (2.1). When $k_2 > 0$, the process of choice and the numerical results are shown in Fig. 4, it is clear that the chaotic phenomenon is increased for $k_2 = 0.0051$. When $k_2 < 0$, the process of choice and the numerical results are shown in Fig. 5, it is clear that the chaotic phenomenon is not increased for $k_2 = -0.0011$.

Third, we add an external nonlinear input $k_3\hat{x}_3|\hat{x}_3|$ to the third equation of (2.1). When $k_3 > 0$, the process of choice and the numerical results are shown in Fig. 6, it is clear that the chaotic phenomenon is not increased for $k_3 = 0.001$. When $k_3 < 0$, the process of choice and the numerical results are shown in Fig. 7, it is clear that the chaotic phenomenon is increased for $k_3 = -0.6$.

From above numerical results, we can get some comments. First, when we choose positive value of k_1, k_2, k_3 , only the choice of k_2 is successful. On the other hand, when we choose negative values of k_1, k_2, k_3 , only the choice of k_2 fails. The effect of negative k_3 is better than that of negative k_1 , and effect of negative k_1 is better than that of positive k_2 .

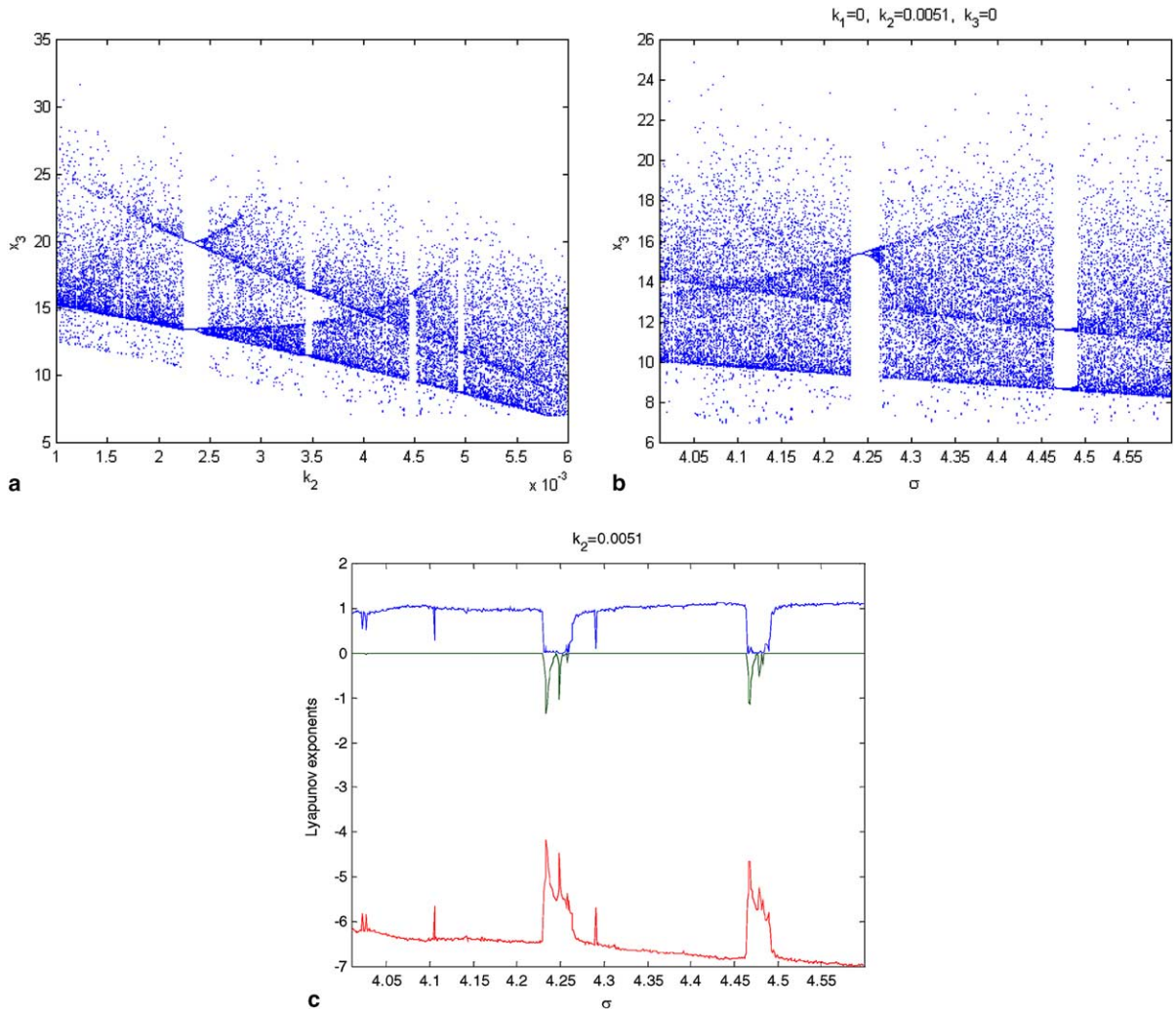


Fig. 4. (a) Bifurcation diagram of \hat{x}_3 for $k_2 = 0.001\text{--}0.006$. (b) Bifurcation diagram of \hat{x}_3 for $k_2 = 0.0051$. (c) Lyapunov exponents for $k_2 = 0.0051$.

3.2. Adding two terms of $x|x|$

First, we choose two positive values of k_1 and k_2 , the process of choice and the numerical results are shown in Fig. 8, it is clear that the chaotic phenomenon is increased. We also investigate two negative values of k_1 and k_2 , the process of choice and the numerical results are shown in Fig. 9, it is clear that the chaotic phenomenon is also increased.

Second, we choose two positive values of k_1 and k_3 , the process of choice and the numerical results are shown in Fig. 10, it is clear that the chaotic phenomenon is not increased. We also investigate two negative values of k_1 and k_3 , the process of choice and the numerical results are shown in Fig. 11, it is clear that the chaotic phenomenon is increased.

Third, we choose two positive values of k_2 and k_3 , the process of choice and the numerical results are shown in Fig. 12, it is clear that the chaotic phenomenon is increased. We also investigate two negative values of k_2 and k_3 , the process of choice and the numerical results are shown in Fig. 13, it is clear that the chaotic phenomenon is increased.

From above numerical results, we can get some comments. When we choose two positive values of k_1 and k_3 , the result fails. For the other choices, the results are all successful. The effects of two negative values of k_1, k_3 and two negative values of k_2, k_3 are the best.

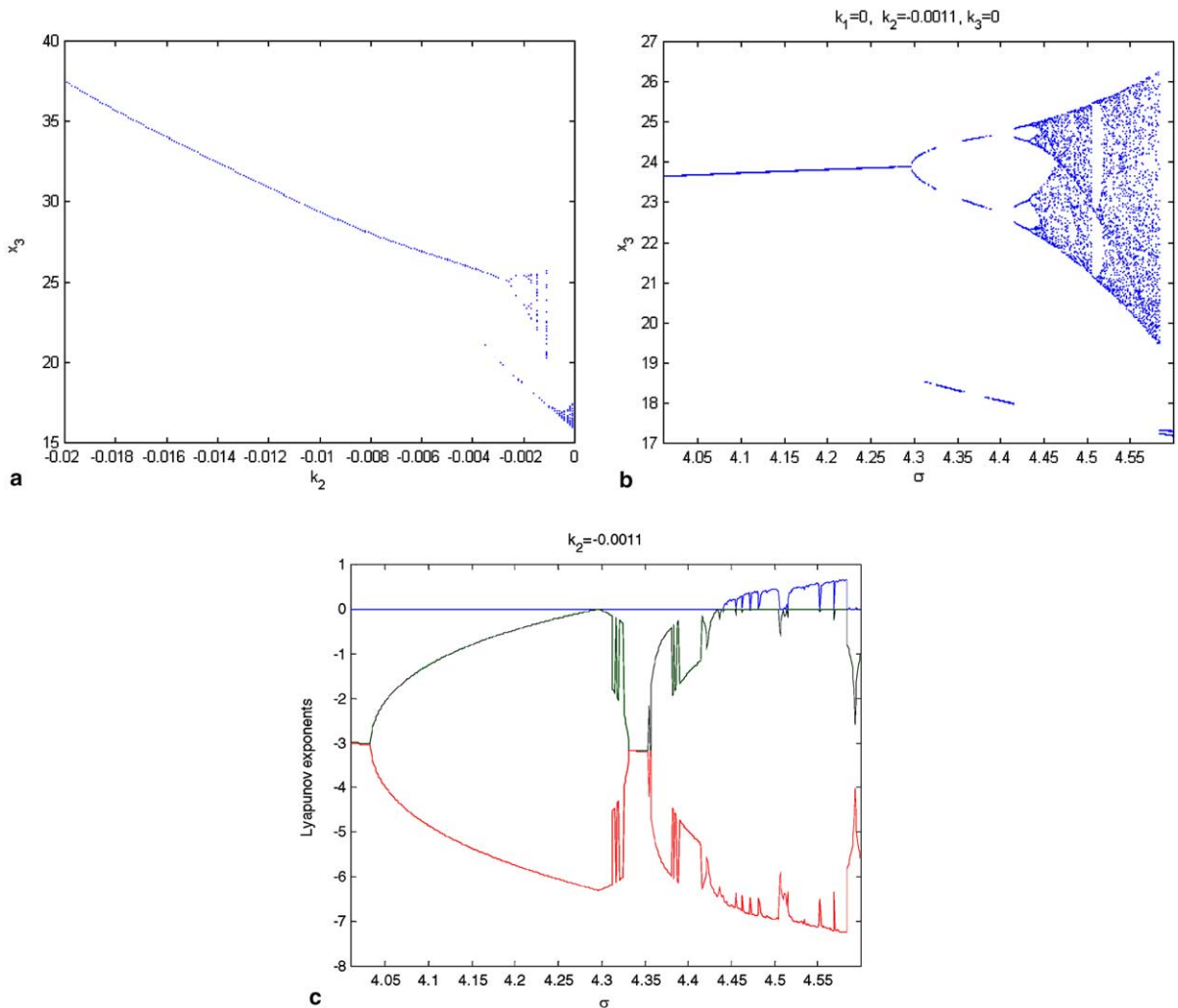


Fig. 5. (a) Bifurcation diagram of \hat{x}_3 for $k_2 = -0.02$ to 0.0. (b) Bifurcation diagram of \hat{x}_3 for $k_2 = -0.0011$. (c) Lyapunov exponents for $k_2 = -0.0011$.

From the results in this section, we find that the necessary condition for success of increase of chaos is that there exists at least one successful addition of nonlinear term.

4. Chaos synchronization of different order systems

We discuss chaos synchronization of two different order systems [24] in this section. These two systems are the autonomous third-order BLDCM system and the nonautonomous second-order Duffing system. Three methods are applied: the addition of the coupling terms, the Lyapunov stability theorem, and the linearization of the error dynamics [25].

BLDCM is described by

$$\begin{aligned}
 \dot{x}_1 &= V_q - x_1 - x_2x_3 + px_3 \\
 \dot{x}_2 &= V_d - Bx_2 + x_1x_3 \\
 \dot{x}_3 &= a(x_1 - x_3) + h, x_1x_2 - T_3
 \end{aligned}
 \tag{4.1}$$

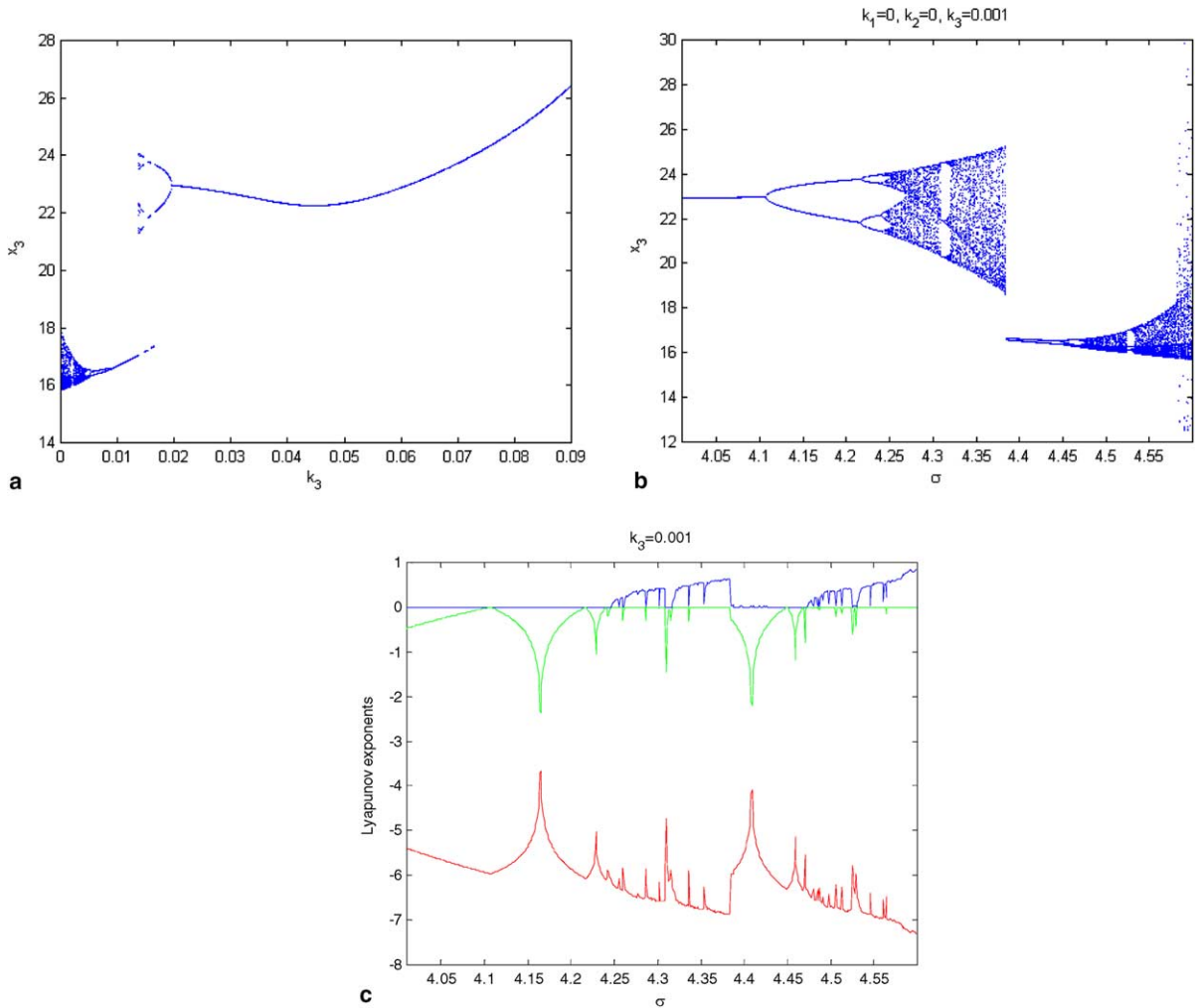


Fig. 6. (a) Bifurcation diagram of \hat{x}_3 for $k_3 = 0.0-0.09$. (b) Bifurcation diagram of \hat{x}_3 for $k_3 = 0.001$. (c) Lyapunov exponents for $k_3 = 0.001$.

where $V_q = 0.168$, $p = 60$, $V_d = 20.66$, $B = 0.875$, $a = 4.55$, $h = 0.26$, $T_3 = 0.53$, and the initial condition is $x_1(0) = x_2(0) = x_3(0) = 0.01$.

Duffing system is described by

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_1 - y_1^3 - \delta y_2 + \alpha \cos \omega t \end{aligned} \tag{4.2}$$

where $\delta = 0.15$, $\alpha = 0.3$, $\omega = 1.0$, and the initial condition is $y_1(0) = y_2(0) = 0$.

4.1. Chaos synchronization of coupled different order chaotic systems

First, we choose BLDCM as the master system and Duffing system as the slave system. For leading (y_1, y_2) to (x_1, x_2) , we add two coupling terms, $k_1(x_1 - y_1)$ and $k_2(x_2 - y_2)$, to the first and second equation of (4.2), respectively.

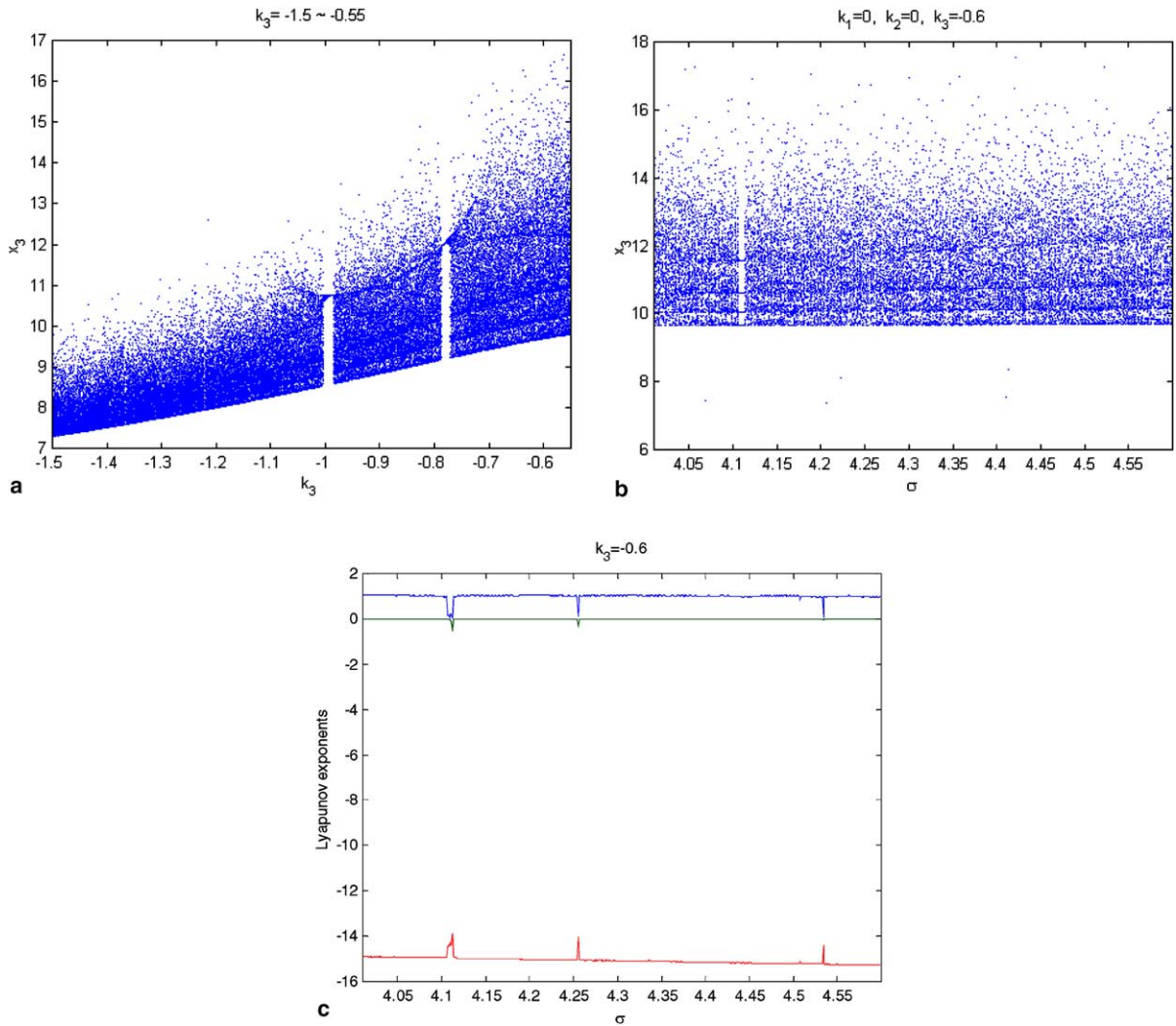


Fig. 7. (a) Bifurcation diagram of \hat{x}_3 for $k_3 = -1.5$ to -0.05 . (b) Bifurcation diagram of \hat{x}_3 for $k_3 = -0.6$. (c) Lyapunov exponents for $k_3 = -0.6$.

We use the random optimization method [26] to find the critical coupling strength. If the critical coupling strength does exist, the coupling strength should converge to some constant value, and the difference U should be zero.

Define U by

$$U = \int_{0.9T}^T |\mathbf{x} - \mathbf{y}|^2 dt \tag{4.3}$$

where $\mathbf{x} = [x_1 \ x_2]^T$, $\mathbf{y} = [y_1 \ y_2]^T$, and T is the simulation time.

In numerical simulation, the larger the coupling strength, the better the synchronization is. The difference U can be rather small but not zero, this means that chaos synchronization of different order systems can be practically achieved. The numerical results are shown in Figs. 14 and 15.

Second, we choose Duffing system as the master system and BLDCM as the slave system. For leading (x_1, x_2) to (y_1, y_2) , we add two coupling terms, $k_1(y_1 - x_1)$ and $k_2(y_2 - x_2)$, to the first and second equation of (4.1), respectively.

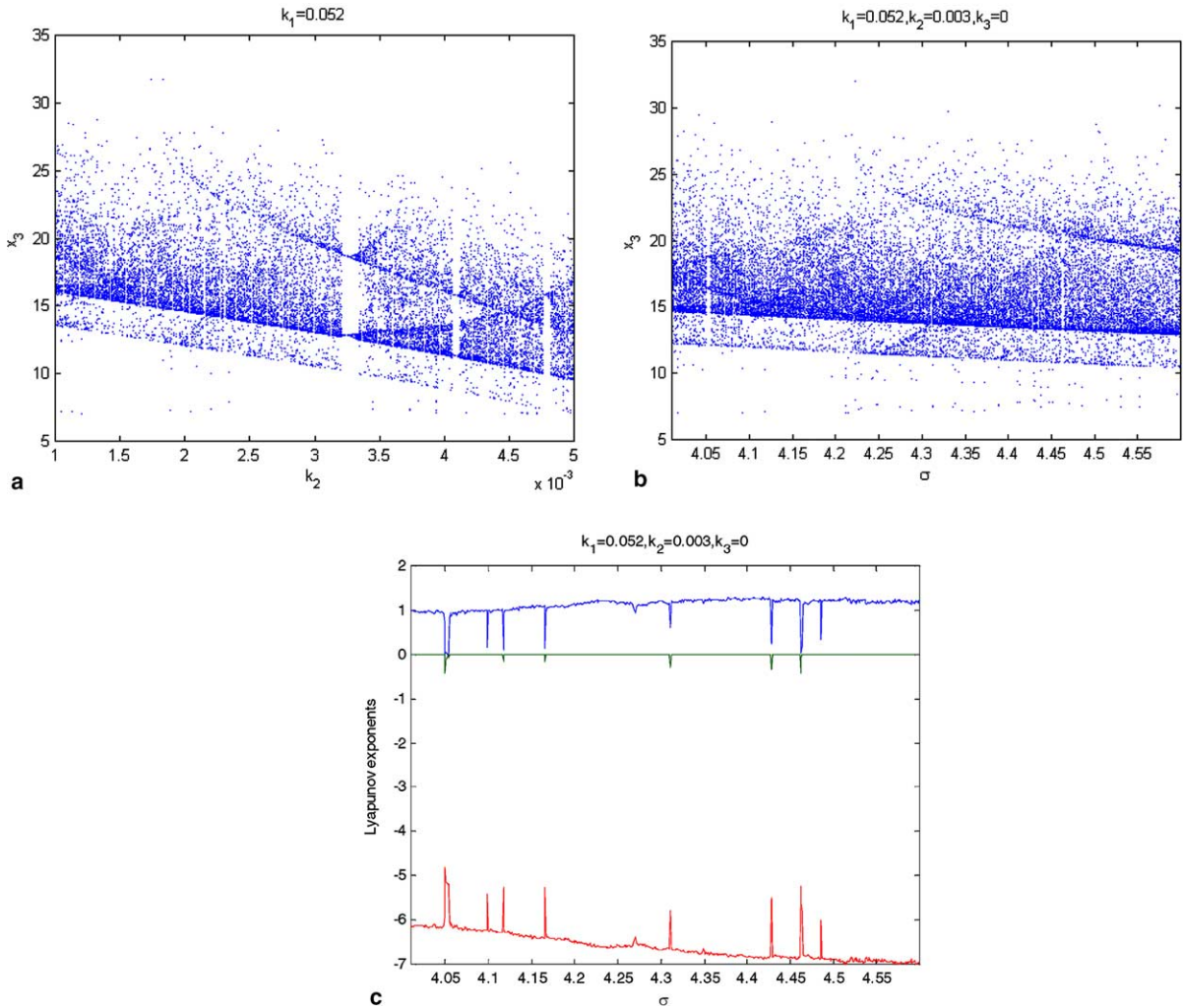


Fig. 8. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.052$, $k_2 = 0.001$ – 0.005 . (b) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.052$, $k_2 = 0.003$. (c) Lyapunov exponents for $k_1 = 0.052$, $k_2 = 0.003$.

The numerical results are shown in Figs. 16 and 17. Chaos synchronization of different order systems can be practically achieved.

Up to now, the coupling methods are both uni-directional. The bi-directional coupling method is studied now. We add two coupling terms, $k_1(y_1 - x_1)$ and $k_2(y_2 - x_2)$, to the first and second equation of (4.1). We also add two coupling terms, $k_1(x_1 - y_1)$ and $k_2(x_2 - y_2)$, to the first and second equation of (4.2). The numerical results are shown in Figs. 18 and 19. Chaos synchronization of different order systems can be practically achieved.

4.2. Chaos synchronization of different order systems by Lyapunov stability theorem

BLDCM is chosen as the master system and Duffing system is chosen as the slave system. For leading (y_1, y_2) to (x_1, x_2) , we add u_1 and u_2 on the first and second equation of (4.2), respectively.

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_1 - y_1^3 - \delta y_2 + \alpha \cos \omega t + u_2 \end{aligned} \tag{4.4}$$

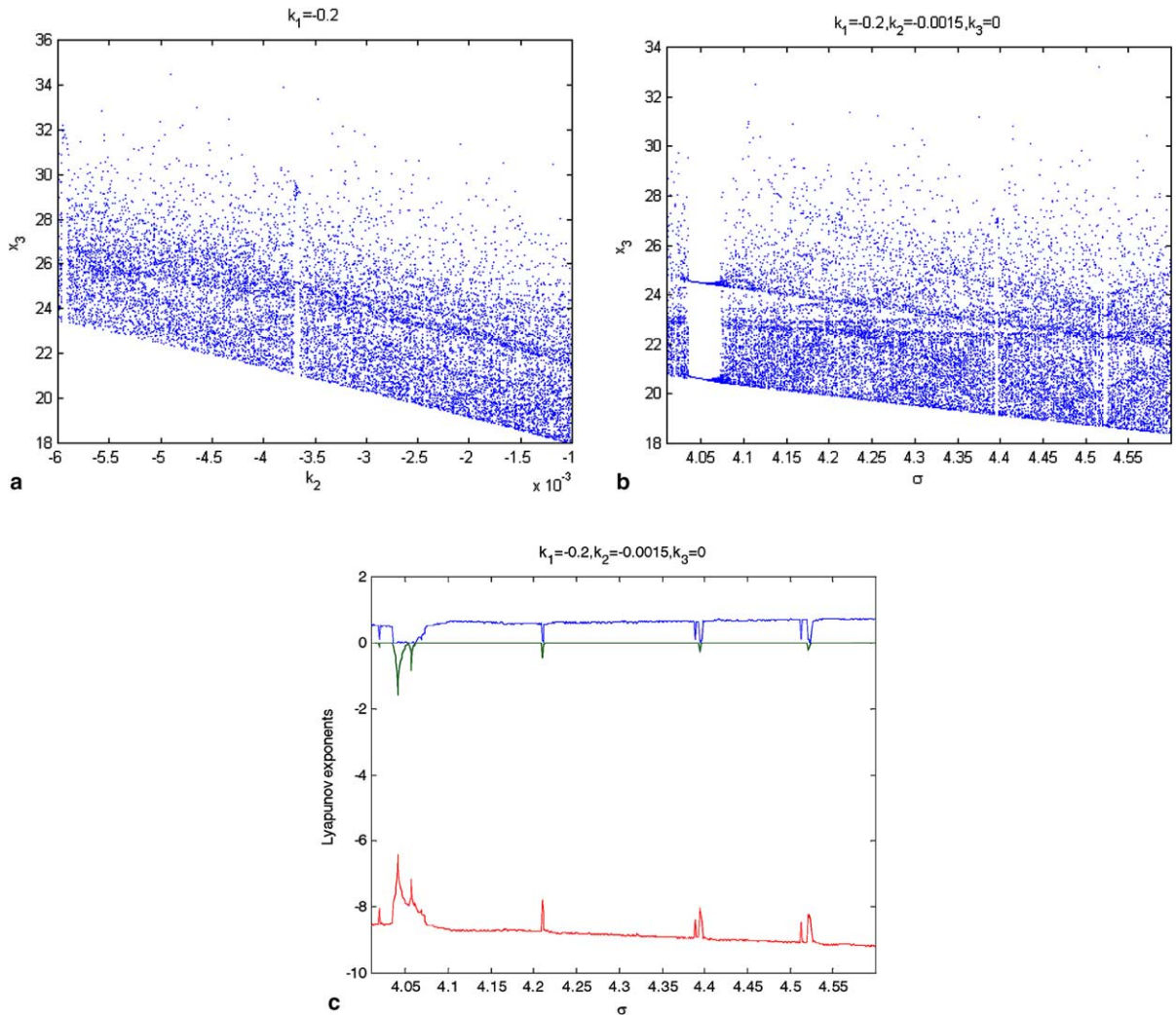


Fig. 9. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.2$, $k_2 = -0.006$ to -0.001 . (b) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.2$, $k_2 = -0.0015$. (c) Lyapunov exponents for $k_1 = -0.2$, $k_2 = -0.0015$.

Subtracting Eq. (4.4) from the first two equations of (4.2), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_1 &= V_q - x_1 - x_2x_3 + px_3 - y_2 - u_1 \\ \dot{e}_2 &= V_d - Bx_2 + x_1x_3 - y_1 + y_1^3 + \delta y_2 - \alpha \cos \omega t - u_2 \end{aligned} \tag{4.5}$$

where $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$.

Choose a Lyapunov function of the form

$$V(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2) \tag{4.6}$$

its derivative along the solution of Eq. (4.5) is

$$\dot{V} = e_1(V_q - e_1 - y_1 - x_2x_3 + px_3 - y_2 - u_1) + e_2(V_d - Be_2 - Bx_2 + x_1x_3 - y_1 + y_1^3 + \delta y_2 - \alpha \cos \omega t - u_2) \tag{4.7}$$

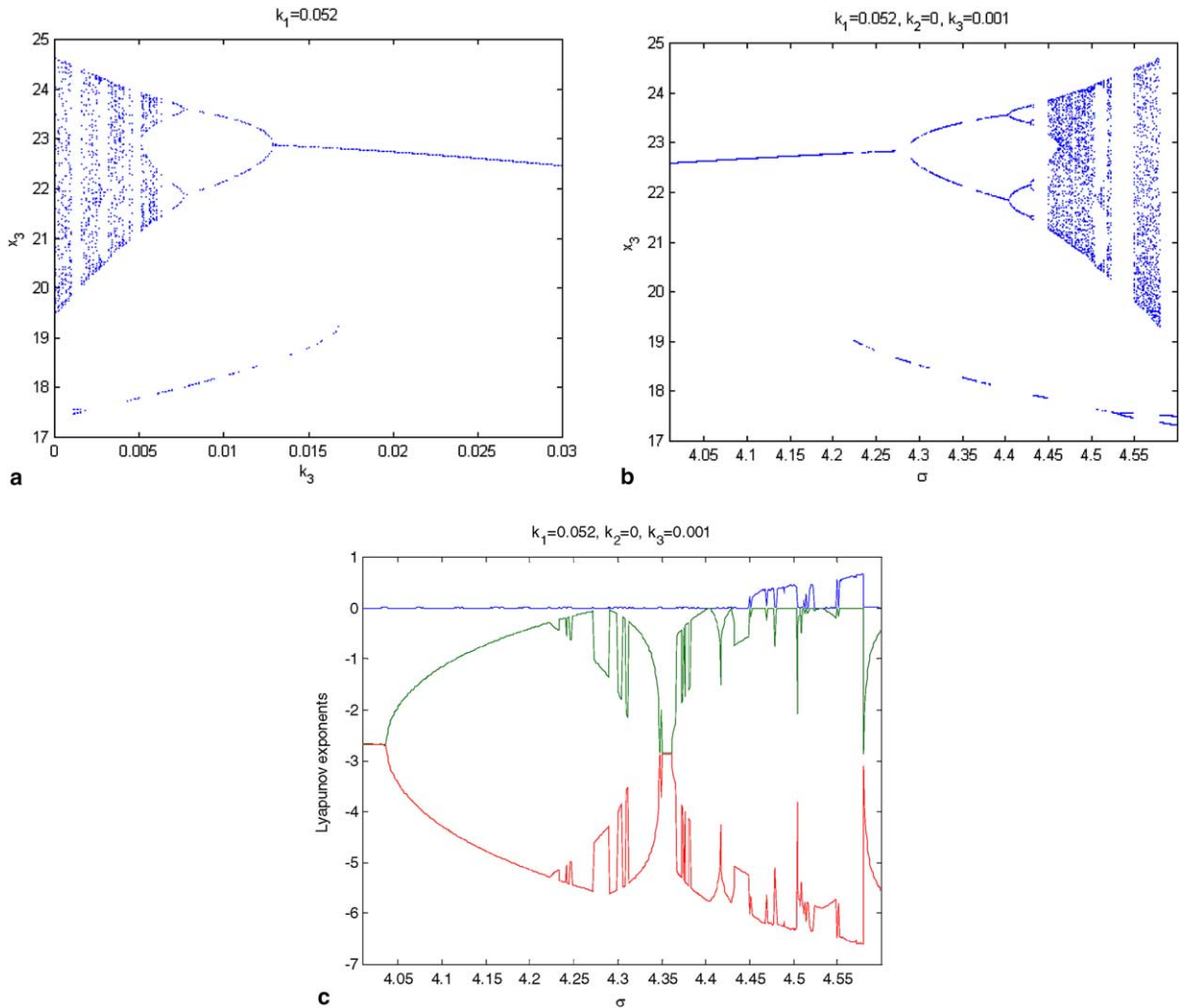


Fig. 10. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.052$, $k_3 = 0.0-0.03$. (b) Bifurcation diagram of \hat{x}_3 for $k_1 = 0.052$, $k_3 = 0.001$. (c) Lyapunov exponents for $k_1 = 0.052$, $k_3 = 0.001$.

Choose

$$\begin{aligned}
 u_1 &= V_q - y_1 - x_2x_3 + px_3 - y_2 \\
 u_2 &= V_d - Bx_2 + x_1x_3 - y_1 + x_1^3 + \delta y_2 - \alpha \cos \omega t
 \end{aligned}$$

Eq. (4.7) can be rewritten as

$$\dot{V} = -e_1^2 - Be_2^2 < 0$$

this means that chaos synchronization between different order systems, Duffing system and BLDCM, can be achieved. The numerical results are shown in Fig. 20.

4.3. Chaos synchronization of different order systems by linearization of error dynamics

BLDCM is chosen as the master system and Duffing system is chosen as the slave system. For leading (y_1, y_2) to (x_1, x_2) , we add u_1 and u_2 on the first and second equation of (4.2), respectively.

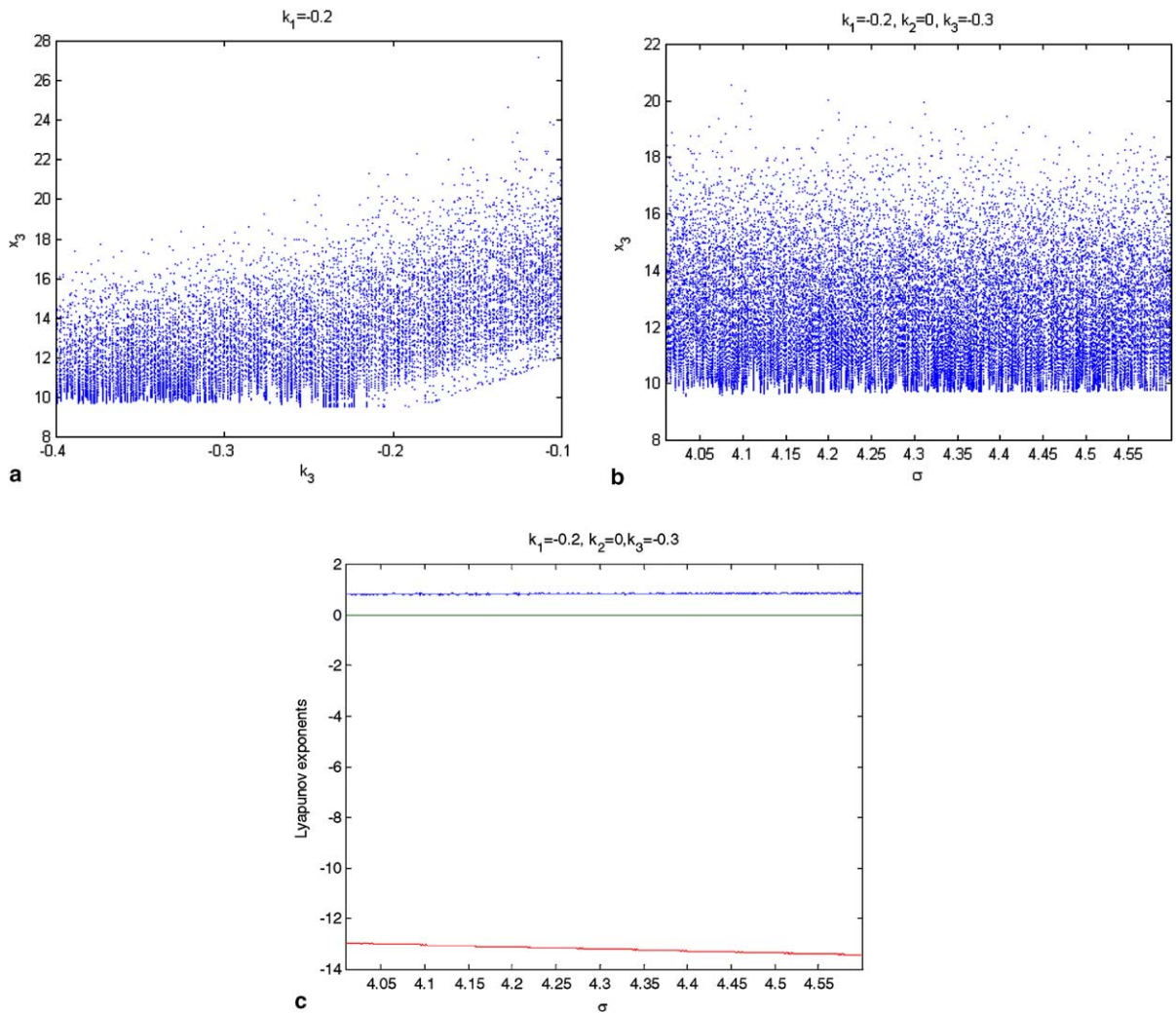


Fig. 11. (a) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.2, k_3 = -0.4$ to -0.1 . (b) Bifurcation diagram of \hat{x}_3 for $k_1 = -0.2, k_3 = -0.3$. (c) Lyapunov exponents for $k_1 = -0.2, k_3 = -0.3$.

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_1 - y_1^3 - \delta y_2 + \alpha \cos \omega t + u_2 \end{aligned} \tag{4.8}$$

Define

$$\begin{aligned} e_1 &= x_1 - y_1 \\ e_2 &= x_2 - y_2 \end{aligned}$$

Subtracting Eq. (4.8) from the first two equations of (4.1), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_1 &= -e_1 + e_2 + V_q - x_2 x_3 + p x_3 - y_1 - x_2 - u_1 \\ \dot{e}_2 &= e_1 - (B + \delta) e_2 + V_d + x_1 x_3 + y_1^3 - \alpha \cos \omega t - B y_2 - x_1 + \delta x_2 - u_2 \end{aligned} \tag{4.9}$$

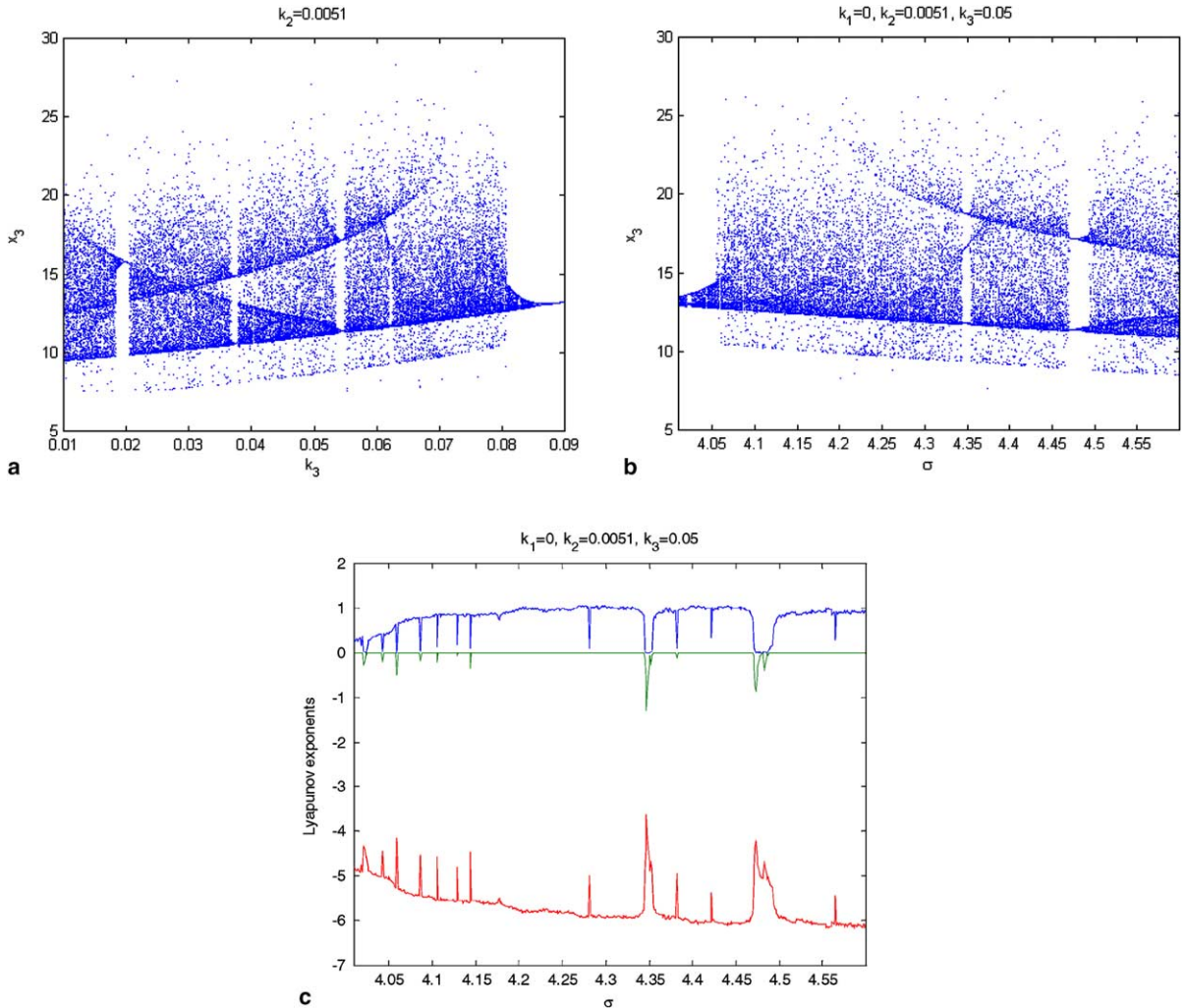


Fig. 12. (a) Bifurcation diagram of \hat{x}_3 for $k_2 = 0.0051$, $k_3 = 0.01-0.09$. (b) Bifurcation diagram of \hat{x}_3 for $k_2 = 0.0051$, $k_3 = 0.05$. (c) Lyapunov exponents for $k_2 = 0.0051$, $k_3 = 0.05$.

To delete the nonlinear terms in Eq. (4.9) [25], we design u_1 and u_2 as

$$u_1 = V_q - x_2x_3 + px_3 - y_1 - x_2 + k_{11}e_1 + k_{12}e_2$$

$$u_2 = V_d + x_1x_3 + y_1^3 - \alpha \cos \omega t - By_2 - x_1 + \delta x_2 + k_{21}e_1 + k_{22}e_2$$

Eq. (4.9) can be rewritten as

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 - k_{11} & 1 - k_{12} \\ 1 - k_{21} & -(B + \delta + k_{22}) \end{bmatrix}$$

If each eigenvalue of \mathbf{A} is negative, \mathbf{e} would converge to zero. By designing $k_{11} = 0$, $k_{12} = 0$, $k_{21} = 2$, and $k_{22} = -0.025$, we can get two negative eigenvalues of \mathbf{A} : -1 and -1 . Thus u_1 and u_2 can be obtained, and \mathbf{e} would converge to zero. This means that chaos synchronization between different order systems, Duffing system and BLDCM, can be achieved. The numerical results are shown in Fig. 21.

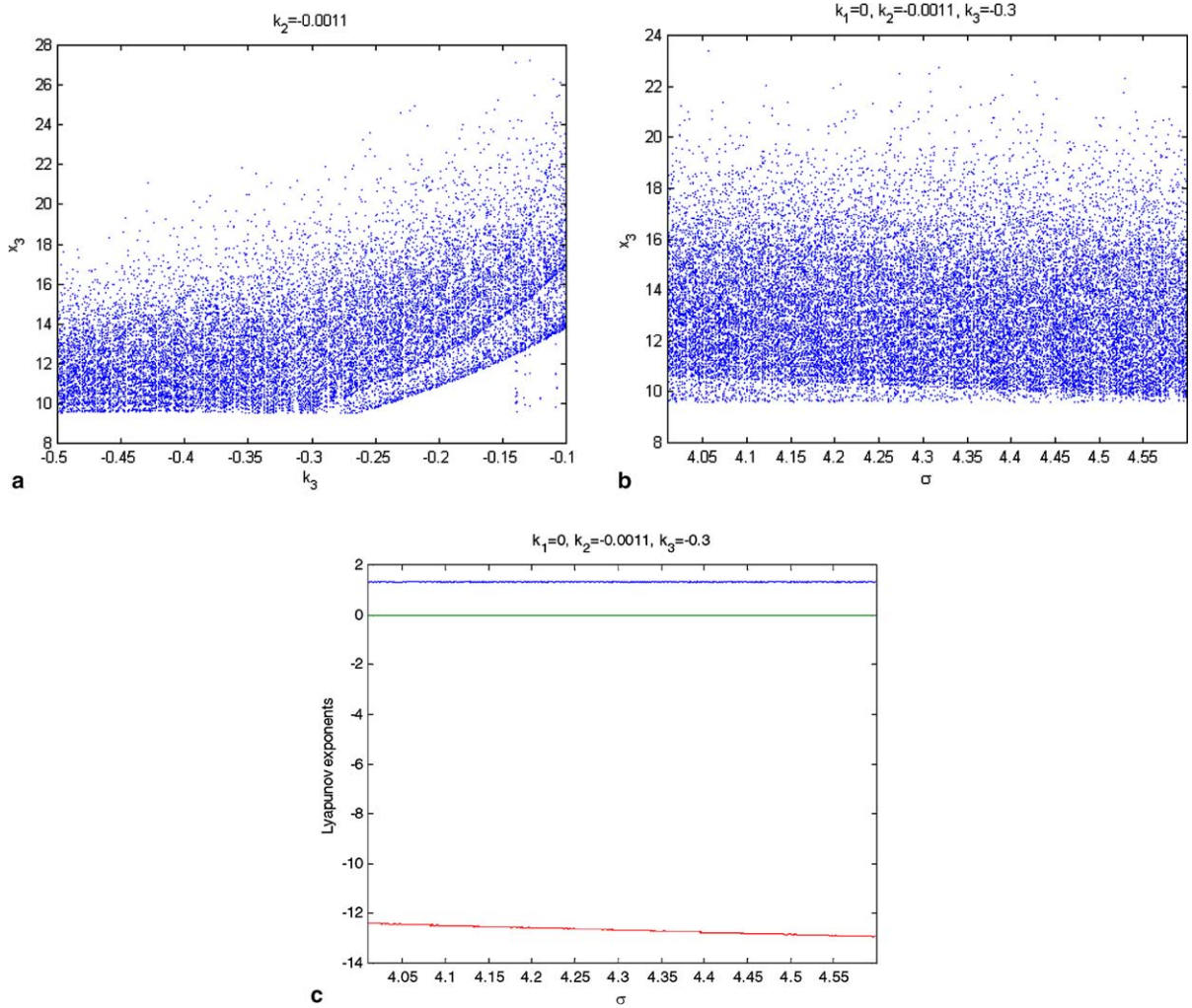


Fig. 13. (a) Bifurcation diagram of \hat{x}_3 for $k_2 = -0.0011, k_3 = -0.5$ to -0.1 . (b) Bifurcation diagram of \hat{x}_3 for $k_2 = -0.0011, k_3 = -0.3$. (c) Lyapunov exponents for $k_2 = -0.0011, k_3 = -0.3$.

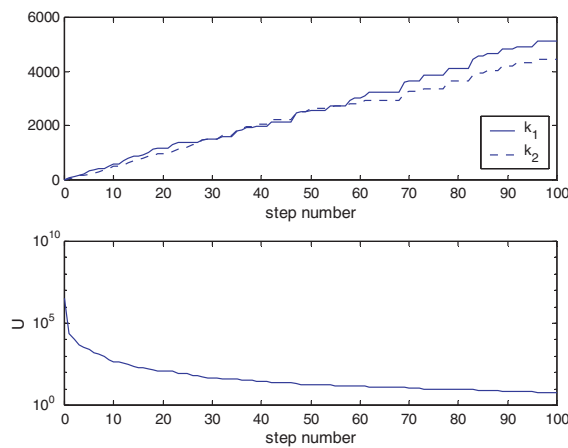


Fig. 14. Time evolution of k_1 and k_2 by random optimization process.

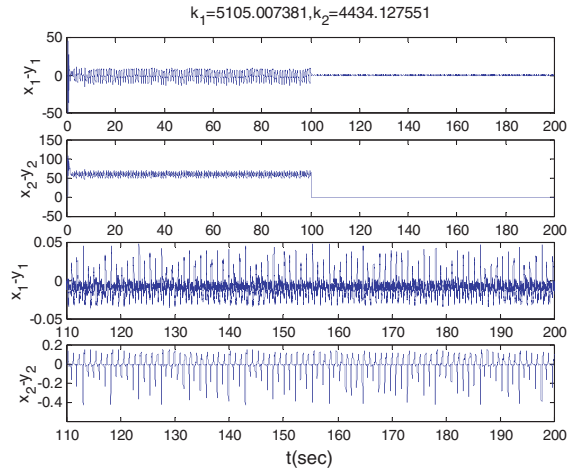


Fig. 15. Time history of errors for $k_1 = 5105.007381$, and $k_2 = 4434.127551$.

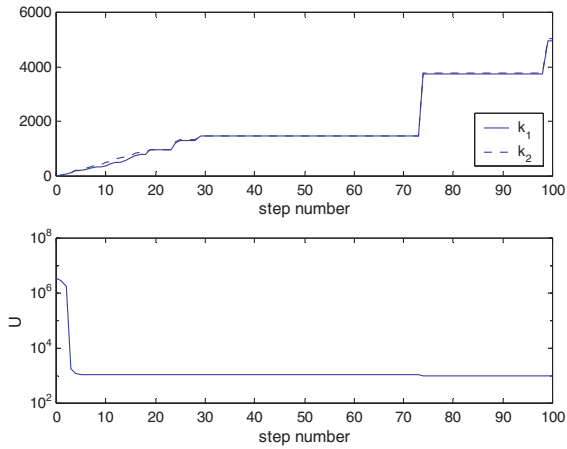


Fig. 16. Time evolution of k_1 and k_2 by random optimization process.

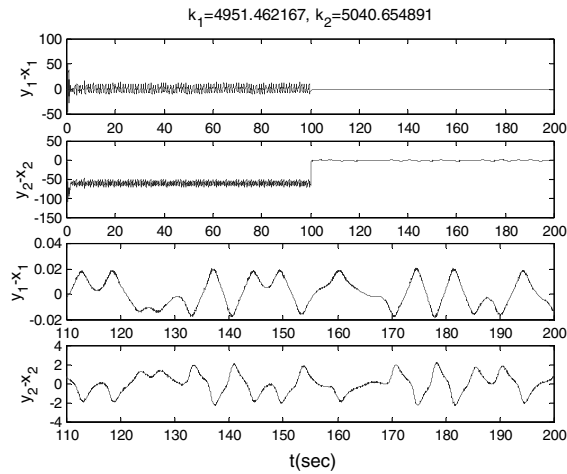


Fig. 17. Time history of errors for $k_1 = 4951.462167$, $k_2 = 5040.654891$.

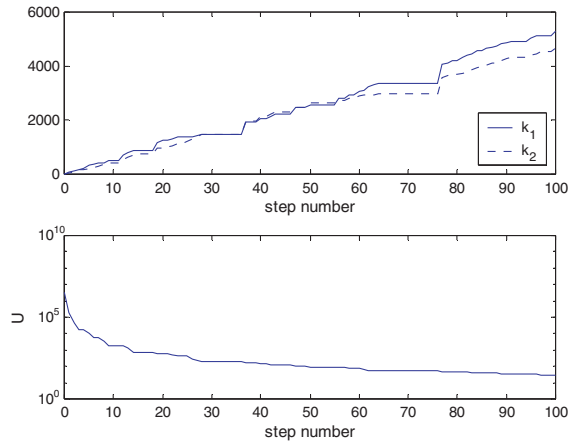


Fig. 18. Time evolution of k_1 and k_2 by random optimization process.

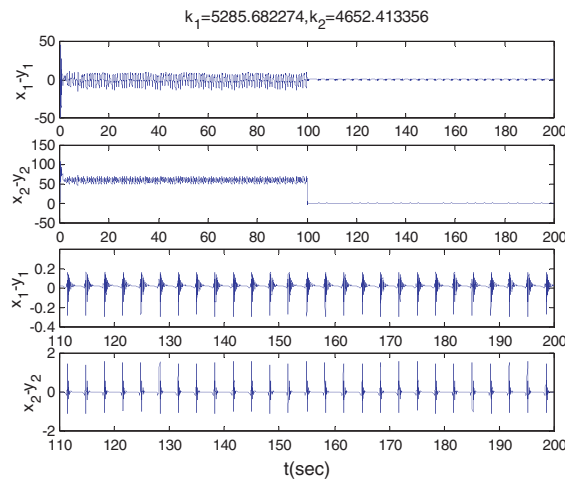


Fig. 19. Time history of errors for $k_1 = 5285.682274$, $k_2 = 4652.413356$.

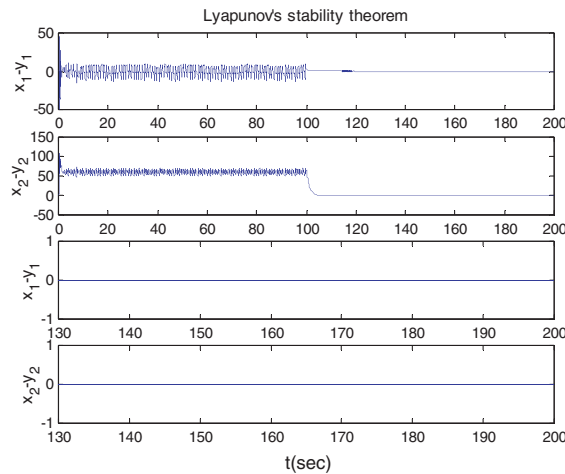


Fig. 20. Time history of errors.

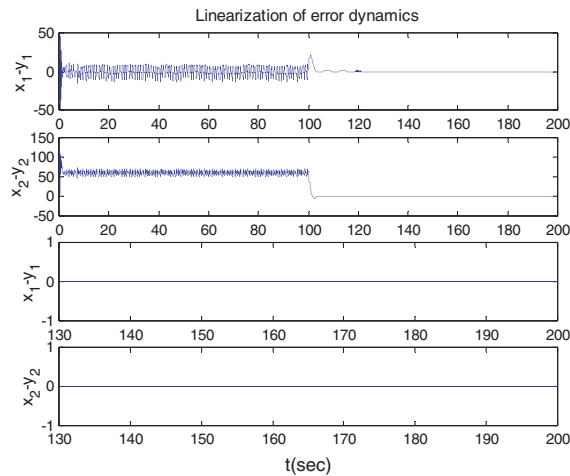


Fig. 21. Time history of errors for $k_{11} = 0$, $k_{12} = 0$, $k_{21} = 2$ and $k_{22} = -0.025$.

5. Conclusions

Brushless dc motor (BLDCM) is studied in this paper. It is an autonomous third-order electromechanical system. In order to verify periodic and chaotic phenomena of investigated systems, several numerical techniques such as time history, phase portrait, bifurcation diagram, and Lyapunov exponents are employed.

The dynamic characteristics of BLDCM are discussed in Section 2. The system model is described, and the numerical results of periodic and chaotic phenomenon are presented.

In Section 3, the nonlinear term $k\hat{x}|\hat{x}|$ added to achieve anti-control of chaotic BLDCM. When one nonlinear positive term is added, only the choice of k_2 succeeds. However, when one nonlinear negative term is added, only the choice of k_2 fails. The effect of negative k_3 is better than that of negative k_1 , and effect of negative k_1 is better than that of positive k_2 . When we add two nonlinear terms, the necessary condition for success of increase of chaos is that there exists at least one successful addition of nonlinear term.

Three methods to achieve chaos synchronization of different order systems are investigated in Section 4. Two different chaotic dynamical systems, Duffing system and BLDCM, are applied. First, the coupling terms are added. We study two kinds of uni-directional coupling methods and a bi-directional coupling method. The larger the coupling strength, the better the synchronization is. The difference can be rather small but not zero, this means that chaos synchronization of different order systems can be practically achieved. Second, Lyapunov stability theorem is used. Chaos synchronization between different order systems can be achieved. Third, linearization of the error dynamics is used. Chaos synchronization between different order systems can be achieved.

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