The Visual Computer

An intuitive and device-independent method of generating color atlases for electronic displays

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We propose a HCL mix color system that can describe accurate colors on different electronic displays. Algorithms for computing the threshold values of chroma and lightness are devised so that the HCL mix color system can create the 3D color gamut of a display. We propose an intuitive and device-independent method of generating a color atlas in the HCL mix color system. Users can find the desired colors in the 3D color gamut by using perceptually descriptive items for hue names, color tones, and lightness levels. The system is helpful for color schemes of computer-designed products. Simulation results of generated color atlases are given to show the feasibility and effectiveness of the method.

Key words: CIE UCS – CIE L*u*v* – 3D Color gamut – Color atlas – Color scheme – Device-independent color description – CRT – Displays, device colorimetric characterization

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1 Introduction

The trend toward greater use of cathode-ray tube (CRT) color displays has made colorimetric characterization of CRT performance an absolute necessity [1, 2]. In recent years, computer-aided design (CAD) software has been widely used to design and demonstrate products on color displays. It is important to derive accurate colors for color atlas generation and for color description of electronic displays. Color description is needed to determine the display's colorimetric characterization, and this includes: (a) the chromaticity coordinates of the three phosphors and the display's white point, (b) the maximum luminance of the three phosphors, and (c) the gamma correction expressions of the three phosphors. The gamma correction expressions are used to correct the proportion of the CRT's lightness L and input digital count values D become directly proportion [1–8]. The precise colorimetric characterization of a display can be obtained by the telespectroradiometer (TSR) or colorimeters with sufficient precision and accuracy [7,8]. Note that color atlas generation is intended to generate accurate colors according to the colorimetric characterization of electronic displays. Theoretically, there is an unlimited number of computer-generated color atlases that users can search for desired colors. There are two types of color systems that can be used to generate the color atlas of an electronic display: (a) device-dependent color systems, such as rgb, hue, saturation, value (HSV), and hue, saturation, lightness (HSL), and (b) device-independent color systems, such as Commission Internationale de l'Eclairage (CIE) XYZ, CIE xyY, CIE uniform chromaticity scale (UCS), CIE L*u*v*, and CIE L*a*b*) [9–11]. A relational diagram of color systems is shown in Fig. 1.

For electronic displays, the most commonly used color system is the rgb system in which the rgb coordinates can come directly from the red, green, and blue electron gun voltages. Although the rgb system is the industry standard for computer graphics, it has two disadvantages. First, it is a perceptually nonuniform color system. Second, users cannot intuitively imagine a color in terms of the rgb coordinates. Third, the rgb coordinates of colors depend on individual color displays. For example, for the same rgb coordinate, say, (1, 0, 0), different color displays may not produce the identical shade of red.

Though some color systems, such as CIE 1931 XYZ (XYZ) and CIE 1931 xyY (xyY), have pro-

The Visual Computer (1998) 14:328–342

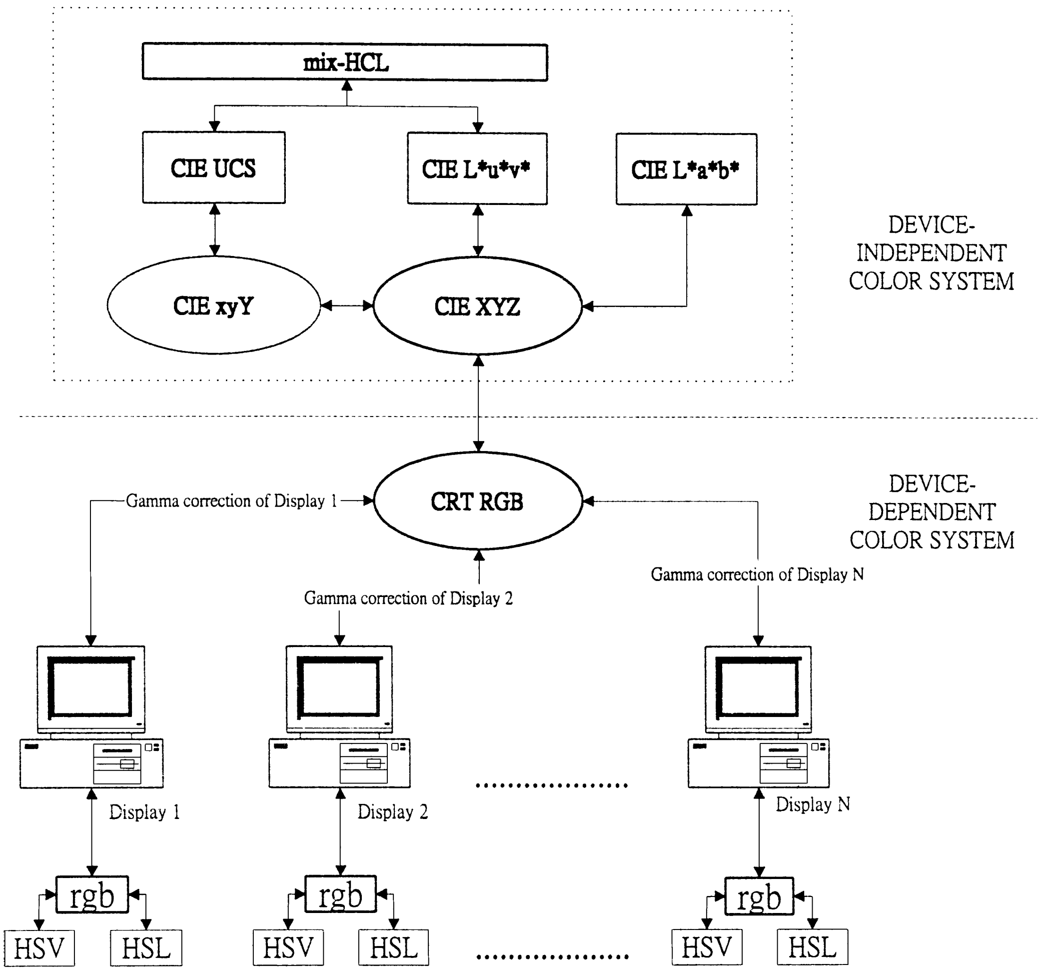


Fig. 1. Relational diagram of device-dependent color systems (rgb/HSV/HSL) and device-independent color systems (CIE XYZ/CIExyY/CIE UCS/L*u*v*/L*a*b*) for color displays

vided device-independent color generating characteristics [10], the (X, Y, Z) and (x, y, Y) coordinates are not perceptually uniform color dimensions. Users of such color systems still cannot intuitively predict the actual color appearance. Some efforts have been made to extend the CIE 1931 color systems to the CIE 1960 UCS (UCS), CIE 1976 L*u*v* (L*u*v*), and CIE 1976 L*a*b* (L*a*b*) systems. These systems all have the approximately perceptually uniform property [9, 10]. Usually, the L*a*b* system is recommended for modeling reflected light conditions (textiles, paints, plastics, etc.) [12, 13]. The L*u*v* system is recommended for modeling additive light source stimuli, photography, television (display), and graphic arts [12, 13]. The CIE tristimulus X, Y, Z values can be linearly translated into the CRT tristimulus R, G, B values of a display. The CRT R, G, B values can be converted into r,g,b values by the gamma correction expression of individual displays [9, 10]. Thus, each of the UCS, L*u*v*, or L*a*b* coordinates can be translated into an rgb coordinate (r, g, b) (see Fig. 1). When a color coordinate in any one of the XYZ, xyY, UCS, L*u*v*, or L*a*b* color systems is put into an electronic display, the way to reproduce accurate colors is to make sure the color coordinates fall within the 3D color gamut of a display. A 3D dis-

Table 1. A comparison of the implementary features of color systems

Implementary feature description	System					
	XYZ	xyY	UCS	L*u*v*	L*a*b*	HCL mix
Perceptually uniform dimensions	No	No	No	Yes	Yes	Yes
Easy to navigate	Poor	Poor	Fair	Fair	Fair	Good
Device-independent color coordinates	Yes	Yes	Yes	Yes	Yes	Yes
Checks input color coordinates	No	No	No	No	No	Yes
Generates 3D-display color gamut	No	No	No	No	No	Yes

play color gamut shows the limits or the range of colors that can accurately be reproduced by a display [14, 15].

Travis [14] points out that an ideal color system for electronic displays used on CAD applications should provide the following implementary features:

1. The color system should be built on *perceptually uniform dimensions*, such as hue, chroma, and lightness, so that the desired colors can be obtained by intuitive adjustment.
2. The color system should be *easy to navigate*, so users can intuitively move around.
3. *Accurate color specification* should be possible, so that all displays running the same application produce accurate colors (as long as the color selected is within the gamuts of both displays).

For the viewpoint of CAD applications, we suggest three further implementary features to enhance accurate color specification:

3.1 The color system should provide *device-independent color coordinates* for reproducing identical colors on different displays.
3.2 The color system should provide methods to *generate the 3D-display color gamut*.
3.3 The color system should provide *chroma and lightness threshold values* to check whether the input color coordinates are within the generated 3D-display color gamut.

When the UCS and L*u*v* color systems are used for electronic displays, we find that: (a) both the UCS and L*u*v* color systems have device-independent color coordinates, (b) the color coordinates of UCS are not perceptually uniform dimensions; however, the L*u*v* color system has defined approximately perceptually uniform dimensions [9, 10], (c) neither the UCS nor the L*u*v* color system individually provides methods to generate the 3D-display color gamut for checking the input color coordinates, and (d) neither the UCS nor the L*u*v* color system provide an intuitive method of generating color atlases for users to navigate among the desired colors.

By mixing the UCS and the L*u*v* color systems, we propose and develop color system with a mix of hue, chroma, and lightness (HCL) that has the implementary features just mentioned. We develop an intuitive and device-independent method for color atlas generation for electronic displays. Meanwhile, algorithms for computing the threshold values of chroma and lightness are also devised, so that, users can easyily describe the 3D-display color gamut. Hence, the computer can accurately reproduce the color atlas for each electronic display. A comparison of the implementary features of the color systems of XYZ, xyY, UCS, L*u*v*, L*a*b*, and the HCL mix is given in Table 1.

Though any color management system (CMS) makes it possible to create color gamuts for a display, the property of the closed system of most CMSs makes it hard for users to define and create color gamuts of an input or output device in a CMS [16]. In the proposed method, the users know clearly how the color gamut of a display is defined, and they also know the relationship between the CRT's colorimetric characterization and the proposed algorithms. That is, the method in this paper directly gives users open algorithms they can use for their applications. The color gamut of a display can also be generated by using Robertson's models [17, 18]. However, the algorithms for directly calculating the UCS/L*u*v* chroma and lightness threshold values are not found in Robertson's models. This paper presents a new approach that can be used to calculate the

UCS/L*u*v* chroma and lightness threshold values directly. Some systems have provided a user interface of device-independent color atlases [19–21]. However, the detailed methods and related algorithms for generating device-independent color atlases on CRTs are not found in [19–21]. This makes it hard for readers to improve or modify the generated color atlases for critical applications according to [19–21]. The hue names, color tones, and gray levels defined in this paper allow users to navigate the colors more intuitively.

This paper is organized as follows. First, the definitions of hue, chroma, and lightness in the UCS, the L*u*v*, and the HCL mix color systems are reviewed in Sects. 2 and 3. Section 4 presents the method for generating the 3D color gamut of the HCL mix system. Section 5 describes an intuitive method for accurate color atlas generation, which uses the HCL mix color system and shows a trial result. Finally, conclusions and suggestions are given.

2 Definition of hue, chroma, and lightness in the CIE UCS and L*u*v*

2.1 Definition of hue in the CIE UCS and L*u*v*

The uniform hue (wavelength) line of a given point $S(u, v)$ in the UCS chromaticity diagram [10] can be determined by drawing a straight line L_h from the white point $W(u_w, v_w)$, through the point $S(u, v)$ to the UCS spectrum locus (Fig. 2). The hue value of the hue line L_h can be determined by the wavelength of the intersecting point $\lambda(u_\lambda, v_\lambda)$,

$$L_h = \left\{ (u, v) \mid (u, v) \in \overline{W(u_w, v_w), \lambda(u_\lambda, v_\lambda)} \right\}, \qquad (1)$$

where h is the hue value and λ is the wavelength. The primary symbols and corresponding assumptions used in this paper are listed in Table 2.

Table 2. Symbols and assumptions used in developing the HCL mix color system

Symbols	Description and assumptions
xyY	xy Serve as the CIE 1931 (x, y) chromaticity coordinates; Y serves as the luminance of the CIE 1931 XYZ standard colorimetric system
XYZ	CIE 1931 XYZ chromaticity system
X,Y,Z	CIE 1931 tristimulus values
x,y,z	CIE 1931 chromaticity coordinates
Y	Tristimulus value of a given object-color stimulus defining the luminance of the object-color stimulus (the luminance of the CIE 1931 XYZ standard colorimetric system)
Y_w	Luminance of a display's standard illuminant (display's white point)
(x_r, y_r), (x_g, y_g), (x_b, y_b)	CIE 1931 chromaticity coordinates of a display's three phosphors
(x_w, y_w)	CIE 1931 chromaticity coordinates of a display's white point
UCS	CIE 1960 UCS chromaticity system
(u, v)	CIE 1960 UCS chromaticity coordinates
(u_r, v_r), (u_g, v_g), (u_b, v_b)	CIE 1960 UCS chromaticity coordinates of a display's three phosphors
(u_w, v_w)	CIE 1960 UCS chromaticity coordinates of a display's white point
$S_h(u_h, v_h)$	Chromaticity coordinates of the hue(h) reference point on the boundary of the CIE UCS chromaticity diagram
$C_{\text{ucs-trade-off}}(h)$	Threshold value of chroma in the UCS color system for a given UCS hue angle h
$L_{\text{ucs-trade-off}}(h, c)$	Threshold value of lightness in the UCS color system for a given UCS hue angle h and a given UCS chroma c
L*u*v*	CIE 1976 L*u*v* color system
$C^*_{\text{uv-trade-off}}(h)$	Threshold value of chroma in the L*u*v* color system for a given L*u*v* hue-angle h
$L^*_{\text{uv-trade-off}}(h, c^*)$	Threshold value of lightness in the L*u*v* color system for a given L*u*v* hue-angle h and a given L*u*v* chroma c^*
$L^*_{\text{max-trade-off}}(h, c^*)$	Maximum threshold value of lightness in the L*u*v* color system for a given L*u*v* hue-angle h and a given L*u*v* chroma c^*
$L^*_{\text{min-trade-off}}(h, c^*)$	Minimum threshold value of lightness in the L*u*v* color system for a given L*u*v* hue-angle h and a L*u*v* chroma c^*
R,G,B	Monitor tristimulus values
r,g,b	rgb Color coordinates (DAC input digital count values)

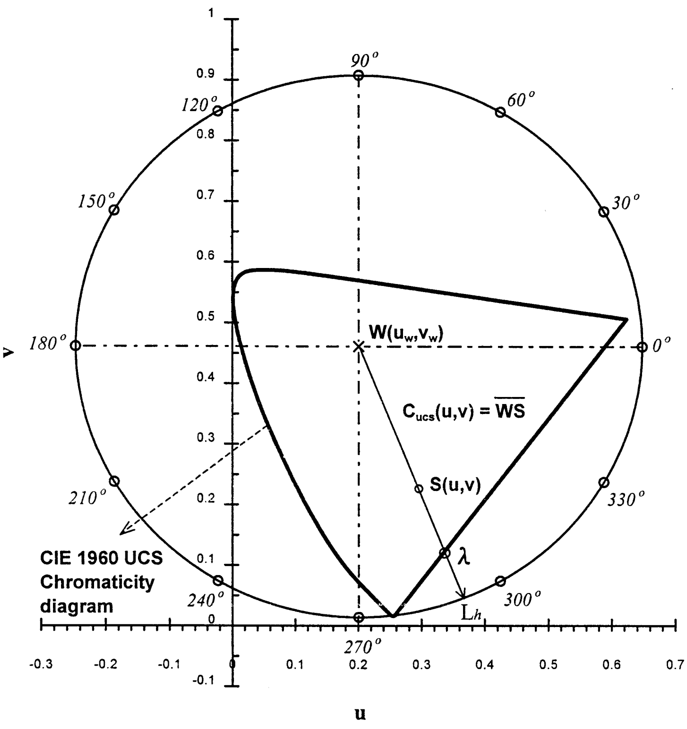


Fig. 2. The CIE 1960 UCS (u,v) chromaticity diagram. The hue and chroma dimensions of the UCS color system are set up for hues from 0º to 360º

All the points of a hue line have the same hue value. We used the display's white point $W(u_w, v_w)$ as the center of the UCS hue circle on the UCS chromaticity diagram. In Fig. 2, the radius of the UCS hue circle is set to the distance from white point $W(u_w, v_w)$ to point 360 (u_{360}, v_{360}) =(0.2589, 0.0176). Thus, the UCS hue angle, $\mathrm{H}^{\circ}_{\mathrm{ucs}}(u, v)$, for each point (u, v) can be defined around the circle every 1° counterclockwise (Fig. 2).

$$\mathrm{H}^{\circ}_{\mathrm{ucs}}(u, v) = \left(\frac{180^{\circ}}{\pi}\right) \arctan\left(\frac{v - v_w}{u - u_w}\right). \tag{2}$$

where u=4X/(X+15Y+3Z), v=9Y/(X+15Y+3Z), u_w=4X$_w$/(X$_w$+15Y$_w$+3Z$_w$), v_w=9Y$_w$/(X$_w$+15Y$_w$+3Z$_w$). Usually, the X$_w$, Y$_w$, Z$_w$ are the tristimulus values of the standard illuminant with Y$_w$ equal to 100 [9, 10].

In the L*u*v* color system, a metric hue angle value $\left(\mathrm{H}^{\circ}_{uv}\right)$ is calculated in accordance with Eq. 3 [9, 10].

$$\begin{aligned} \mathrm{H}^{\circ}_{uv}(u, v) &= \left(\frac{180^{\circ}}{\pi}\right) \arctan\left(\frac{v^*}{u^*}\right) \\ &= \left(\frac{180^{\circ}}{\pi}\right) \arctan\left(\frac{v - v_w}{u - u_w}\right), \end{aligned} \tag{3}$$

where u^*=13L*$(u-u_w)$, v^*=13L*$(v-v_w)$.

2.2 Definition of chroma in the CIE UCS and L*u*v*

The chroma value $\mathrm{C}_{\mathrm{ucs}}(u, v)$ of a given point $S(u, v)$ in the UCS color system is defined as the distance from point $S(u, v)$ to the white point $W(u_w, v_w)$. (Fig. 2.)

$$\mathrm{C}_{\mathrm{ucs}}(u, v) = \sqrt{(u - u_w)^2 + (v - v_w)^2}. \tag{4}$$

In the L*u*v* color system, a metric chroma value $\left(\mathrm{C}^*_{uv}\right)$ is calculated by Eq. 5 [9, 10].

$$\begin{aligned} C^*_{uv}(u,v) &= \sqrt{(u^*)^2+(v^*)^2} \\ &= 13\,L^*\sqrt{(u-u_w)^2+(v-v_w)^2}, \end{aligned} \tag{5}$$

where $u^*=13L^*(u-u_w)$, $v^*=13L^*(v-v_w)$, L^* is the metric lightness value of the L*u*v*.

2.3 Definition of luminance in the CIE UCS and lightness in the L*u*v*

In the CIE XYZ system, the luminance factor is the quotient of the luminance of the object divided by that of the reference white illuminant. Because the luminance factor in the UCS is the same as that in the XYZ [10, 12], we define the UCS luminance factor L_{ucs} to be the $(100\,Y/Y_0)$ value, that is $L_{ucs}=(100\,Y/Y_0)$, where Y_0 is the CIE tristimulus value Y of the reference white illuminant.

In the L*u*v* color system, a metric lightness value (L^*) is calculated by Eq. 6 [9, 10].

$$L^* = \begin{cases} 116(Y/Y_0)^{1/3}-16, & (Y/Y_0) > 0.01 \\ 903.3(Y/Y_0), & \text{elsewhere}, \end{cases} \tag{6}$$

where Y_0 is the CIE tristimulus value Y of the reference white illuminant.

In this paper, the CIE standard illuminant C is selected to be the reference white illuminant, and the Y_0 is set at 100. Meanwhile, the CIE XYZ chromaticity coordinate of the CIE standard illuminant D_{65} is used as the display's white point.

3 Definition of hue, chroma, and lightness in the HCL mix

Using Eqs. 2–6, one can derive Eq. 7 to transform the HCL coordinates between the UCS and the L*u*v* color systems.

$$\begin{aligned} &H^\circ_{uv}(u,v) = H^\circ_{ucs}(u,v), \\ &C^*_{uv}(u,v) = 13\,L^* C_{ucs}(u,v), \quad \text{and} \\ &L^* = \begin{cases} 116\left(L_{ucs}/100\right)^{1/3}-16, & L_{ucs} > 0.01 \\ 903.3(L_{ucs}/100), & \text{elsewhere}. \end{cases} \end{aligned} \tag{7}$$

Next, the HCL mix color system can be constructed by using the extended coordinates $\left(H^\circ_{ucs}, C_{ucs}, L_{ucs}\right)$ of the UCS and the extended coordinates $\left(H^\circ_{uv}, C^*_{uv}, L^*\right)$ of the L*u*v*. Since $\left(H^\circ_{ucs}, C_{ucs}, L_{ucs}\right)$ and $\left(H^\circ_{uv}, C^*_{uv}, L^*\right)$ are device-independent and approximately perceptually uniform color coordinates, they can be used to specify accurate colors on different color displays and to let the users find colors by intuitive adjustment. The following sections present methods for calculating the HCL mix threshold values of chroma and lightness and for generating the 3D-display color gamut in the HCL mix system.

4 Three-dimensional color gamut generation of the HCL mix

To generate a 3D-display color gamut of the HCL mix, we first derive the threshold values of chroma and lightness in the UCS and the L*u*v*. Next, we propose a method for generating a 3D color gamut based on the derived threshold values.

4.1 Three-dimensional color gamut generation of the UCS

4.1.1 Generating the threshold value of chroma in the UCS

A display can only accurately reproduce colors that lie within the display's color gamut. For each hue line in the UCS diagram, the maximum chroma will be on the boundary of the 2D-display color gamut as shown in Fig. 3. That is to say, for each hue line (a given h), the UCS chroma value of point S_h on the boundary of the 2D-display color gamut can be used as a threshold value of the UCS chroma, $C_{\text{ucs-trade-off}}(h)$ (Fig. 3), where parameter h is the UCS hue angle defined by Eq. 2. That is, the $C_{\text{ucs-trade-off}}(h)$ is the distance from the white point $W(u_w, v_w)$ to the point S_h. This threshold value can be used to determine the allowable reproduced chroma range for a display. An algorithm to compute the threshold value $C_{\text{ucs-trade-off}}(h)$ is given here.

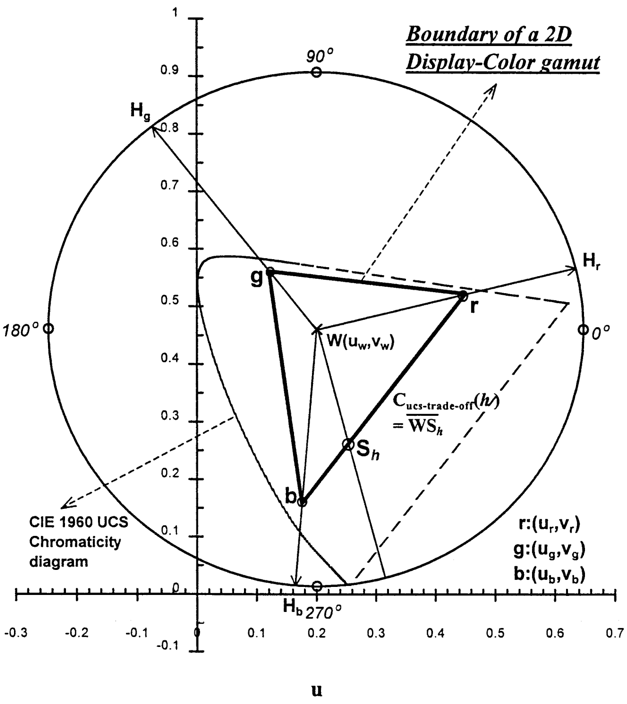


Fig. 3. The threshold value of chroma, C$_{\text{ucs-trade-off}}$(h), on the CIE 1960 UCS chromaticity diagram

Algorithm for computing ***Get_C$_{ucs\text{-}trade\text{-}off}$(h)***

```
(Input $h$; output $C_{\text{ucs-trade-off}}(h)$)
Method:
1. Setup: $(x_r, y_r)$, $(x_g, y_g)$, $(x_b, y_b)$ and $(x_w, y_w, Y_w)$.
2. Input: $h$.
3. Use: $\begin{cases} u = 4x/(-2x+12y+3) \\ v = 9y/(-2x+12y+3) \end{cases}$,   (8)
   translate $(x_r, y_r)$, $(x_g, y_g)$, $(x_b, y_b)$, $(x_w, y_w)$ to $(u_r, v_r)$, $(u_g, v_g)$, $(u_b, v_b)$, $(u_w, v_w)$, respectively.
   /* Steps 1 to 3: compute the UCS $(u, v)$ coordinates of the display's red, green, and blue points, respectively (Fig. 3). */
4. Find the UCS hue angles $H_r$, $H_g$ and $H_b$ of the points $(u_r, v_r)$, $(u_g, v_g)$ and $(u_b, v_b)$ by Eq. 2.
5. Calculate: $(u_h, v_h)=(C_{\max}\cos(h), C_{\max}\sin(h))$, and the following temporary variables of $m$ and $b$.
   $m_h=[(v_w-v_h)/(u_w-u_h)]$, $b_h=v_w-m_h u_h$,
   $m_{rg}=[(v_r-v_g)/(u_r-u_g)]$, $b_{rg}=v_r-m_{rg} u_r$,
   $m_{gb}=[(v_g-v_b)/(u_g-u_b)]$, $b_{gb}=v_g-m_{gb} u_g$,
   $m_{br}=[(v_b-v_r)/(u_b-u_r)]$, $b_{br}=v_b-m_{br} u_b$.
   Here, $C_{\max}$ is the distance from $W(u_w, v_w)$ to $360(u_{360}, v_{360})=(0.2589, 0.0176)$.
   /* Steps 4 to 5: compute the slopes of the lines $\overline{WS_h}(m_h)$, $\overline{rg}(m_{rg})$, $\overline{gb}(m_{gb})$, and $\overline{br}(m_{br})$ (Fig. 3). */
6. If $(H_r<=h<H_g)$ then $m'=m_{rg}$, $b'=b_{rg}$.
   If $(H_g<=h<H_b)$ then $m'=m_{gb}$, $b'=b_{gb}$.
   If $(H_b<=h<360°)$ or $(0°<=h<H_r)$ then $m'=m_{br}$, $b'=b_{br}$.
7. Calculate:
   $u'_h=(b'-b_h)/(m_h-m')$,
   $v'_h=(m_h b'-m_h)/(m_h-m')$.
   Get:
   $S_h=(u'_h, v'_h)$.
8. Calculate:
   $C'_h=\left[\left(u'_h-u_w\right)^2+\left(v'_h-v_w\right)^2\right]^{1/2}$.
9. Let: $C_{\text{ucs-trade-off}}(h)=C'_h$.
   /* Steps 6 to 9: compute the distance $(C'_h)$ from the white point $W(u_w, v_w)$ to the point $S_h$. (Fig. 3) */
10. Stop.
```

4.1.2 Generating the threshold value of lightness in the UCS

Generally, for an electronic display, the maximum value of the CRT (R, G, B) is set at (1, 1, 1), which will occur at the display's white point while the minimum value of the CRT (R,G,B) is set at (0,0,0). According to the relationship of

$$[C]\begin{bmatrix}R\\G\\B\end{bmatrix}=\begin{bmatrix}X\\Y\\Z\end{bmatrix}\quad\text{where }[C]\begin{bmatrix}X_r&X_g&X_b\\Y_r&Y_g&Y_b\\Z_r&Z_g&Z_b\end{bmatrix},$$

when an (X, Y, Z) coordinate is transformed to a CRT (R, G, B) coordinate [9, 10], if any one value of R, G, or B is outside the range of [0, 1], then the (X, Y, Z) point cannot be correctly reproduced by the display. The methods described in [6, 8] are used to calculate the tristimulus matrix C according to the display's colorimetric characterization and while the maximum value of the CRT (R, G, B) is set at (1, 1, 1). Such limitations in converting (X, Y, Z) to CRT (R, G, B) can be used to derive the threshold value of the UCS lightness, $L_{\text{ucs-trade-off}}(h, c)$, for each display, where parameter h is the UCS hue angle defined by Eq. 2 and the parameter c is the UCS chroma defined by Eq. 4. The algorithm for computing the $L_{\text{ucs-trade-off}}(h, c)$ value is given here.

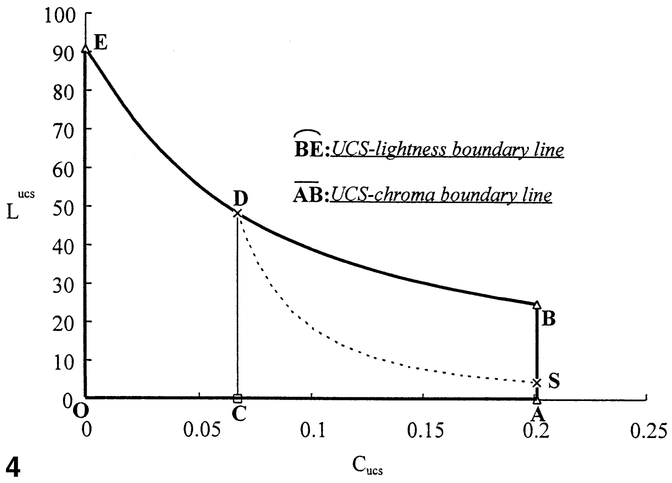
```
Algorithm for computing Get_Lucs-trade-off(h, c)
  (Input $h$, $c$; output $L_{\text{ucs-trade-off}}(h, c)$)
  Method:
  1. Setup:
     $(x_r, y_r)$, $(x_g, y_g)$, $(x_b, y_b)$ and $(x_w, y_w, Y_w)$.
  2. Input: $h$, $c$.
  3. Decide the $S_h(u, v)$ point that corresponds to the $h$ and $c$ values in Fig. 3.
  4. Convert $S_h(u, v)$ to $S_h(x, y)$ using:
     $x=4.5u/(3u-8v+6)$, and $y=2.0v/(3u-8v+6)$.    (9)
     Calculate $z=1-x-y$.
  5. Transform (X, Y, Z) to CRT (R, G, B) by
     $\begin{bmatrix}R\\G\\B\end{bmatrix}=[C^{-1}]\begin{bmatrix}X\\Y\\Z\end{bmatrix}.$    (10)
  6. Let: R=1, G=1, B=1 and substitute X=x(Y/y), Y=Y, Z=z(Y/y) in Eq. 10.
     Then:
     $\begin{bmatrix}1\\1\\1\end{bmatrix}=[C^{-1}]\begin{bmatrix}x(Y/y)\\Y\\z(Y/y)\end{bmatrix}=\begin{bmatrix}a11'&a12'&a13'\\a21'&a22'&a23'\\a31'&a32'&a33'\end{bmatrix}\begin{bmatrix}x(Y/y)\\Y\\z(Y/y)\end{bmatrix}.$
     Get:
     $L_{\text{r-trade-off}}(h, c)$
       $=1/[a11'(x/y)+a12'+a13'(z/y)]$,
     $L_{\text{g-trade-off}}(h, c)$
       $=1/[a21'(x/y)+a22'+a23'(z/y)]$, and
     $L_{\text{b-trade-off}}(h, c)$
       $=1/[a31'(x/y)+a32'+a33'(z/y)]$.
  7. If ($L_{\text{r-trade-off}}(h, c)<0$) or
     ($L_{\text{g-trade-off}}(h, c)<0$) or ($L_{\text{b-trade-off}}(h, c)<0$),
     then $(h, c)$ is out of the display color gamut; stop.
  8. Let: $L_{\text{ucs-trade-off}}(h, c)=\min[L_{\text{r-trade-off}}(h, c), L_{\text{g-trade-off}}(h, c), L_{\text{b-trade-off}}(h, c)]$.
  9. Stop.
```

According to points $\mathbf{A}(C_{\text{ucs-trade-off}}(h), 0)$ and $\mathbf{B}(C_{\text{ucs-trade-off}}(h), L_{\text{ucs-trade-off}}(h, C_{\text{ucs-trade-off}}(h)))$, one has the UCS chroma boundary line $\overline{AB}$ as shown in Fig. 4. Similarly, according to these points $(c, L_{\text{ucs-trade-off}}(h, c))$ with c from 0 to $C_{\text{ucs-trade-off}}(h)$, one has the UCS lightness boundary line $\widehat{BE}$. Thus, the 2D profile of a 3D color gamut for every UCS hue angle can be constructed by the boundary lines of the UCS chroma, UCS lightness, the C_{ucs} axis, and the L_{ucs} axis. Furthermore, the 3D UCS color gamut can be generated by composing the 2D color gamut profiles for the UCS hue angle from 0° to 360°.

4.2 Three-dimensional color gamut generation of the L*u*v*

4.2.1 Generating the threshold value of chroma in the L*u*v*

From Eq. 7, the chroma threshold value of the L*u*v*, $C^{*}_{\text{uv-trade-off}}(H^{\circ}_{uv})$, can be derived from the chroma threshold value of the UCS, $C_{\text{ucs-trade-off}}(h)$. The algorithm for computing the chroma threshold value $C^{*}_{\text{uv-trade-off}}(H^{\circ}_{uv})$ is given here.



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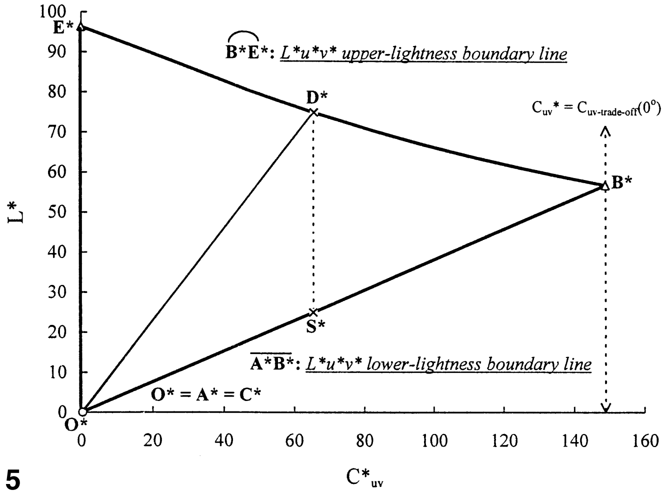

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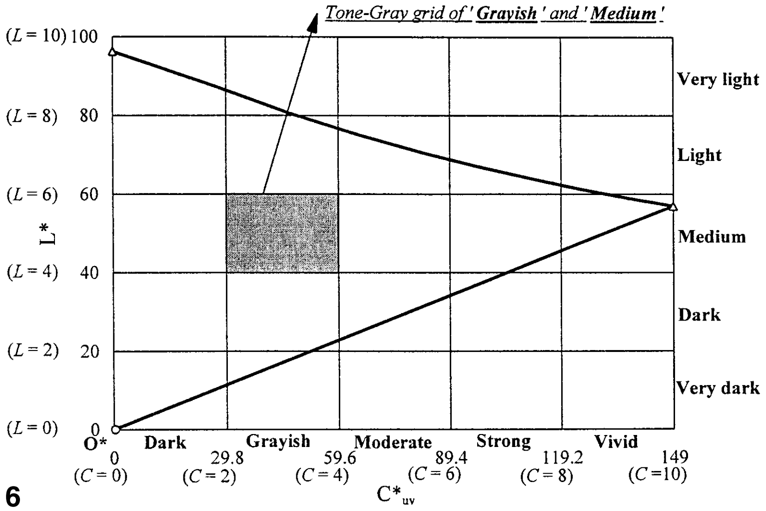

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Fig. 4. Illustration of the 2D UCS color gamut profile of a display constructed by the boundary lines of UCS-chroma, UCS-lightness, the C_{ucs} axis, and the L_{ucs}-axis when the UCS hue angle is 0°

Fig. 5. Illustration of the 2D L*u*v* color gamut profile constructed by the boundary lines of L*u*v* lower lightness, L*u*v* upper lightness, and the L*-axis when the L*u*v* metric hue angle is 0°

Fig. 6. Illustration of a generated color atlas page subdivided into 25 gray tone grids

Algorithm for computing $\boldsymbol{Get\_C^{*}_{uv-trade-off}}\left(\mathbf{H}^{\circ}_{uv}\right)$

```
(Input $H^{\circ}_{uv}$; output $C^{*}_{uv-trade-off}\left(H^{\circ}_{uv}\right)$)
Method:
1. Find the $C_t = C_{ucs-trade-off}\left(H^{\circ}_{uv}\right)$ by using algorithm $\boldsymbol{Get\_C_{ucs-trade-off}}\left(\mathbf{H}^{\circ}_{uv}\right)$.
2. Find the $L_t = C_{ucs-trade-off}\left(H^{\circ}_{uv}, C_t\right)$ by using algorithm $\boldsymbol{Get\_C_{ucs-trade-off}}\left(\mathbf{H}^{\circ}_{uv} C_t\right)$.
3. Find $C^{*}_{uv-trade-off}\left(H^{\circ}_{uv}\right) = 13(116(L_t/100)^{1/3} - 16)\,(C_t)$ by using Eq. 7.
4. Stop.
```

4.2.2 Generating the threshold value of lightness in the L*u*v*

From Eq. 7, the lightness threshold value of the L*u*v*, $L^{*}_{\text{uv-trade-off}}(h, c^{*})$, can be derived from the lightness threshold value of the UCS, $L_{\text{ucs-trade-off}}(h, c)$, where c^{*} is the L*u*v* chroma value. That is,

$$L^{*}_{\text{uv-trade-off}}(h, c^{*}) = \begin{cases} 116\left(L_{\text{ucs-trade-off}}(h, c)/100\right)^{1/3} - 16, \\ \qquad L_{\text{ucs-trade-off}}(h, c) > 0.01 \\ 903.3\left(L_{\text{ucs-trade-off}}(h, c)/100\right), \text{ elsewhere,} \end{cases}$$

where $c^{*} = 13L^{*}c$.
According to the two points $\mathbf{A}^{*}(0, 0)$ and $\mathbf{B}^{*}\left(C^{*}_{\text{uv-trade-off}}(h), L_{\text{ucs-trade-off}}\left(h, C^{*}_{\text{uv-trade-off}}(h)\right)\right)$, one has the L*u*v* lower-lightness boundary line $\overline{A^{*}B^{*}}$ as shown in Fig. 5. Similarly, corresponding to the points $\left(c^{*}, L^{*}_{\text{uv-trade-off}}(h, c^{*})\right)$ with c^{*} in $[0, C^{*}_{\text{uv-trade-off}}(h)]$, one has the L*u*v* upper-lightness boundary line $\widehat{B^{*}E^{*}}$, also shown in Fig. 5. Thus, the 2D profile of a 3D color gamut for a given L*u*v* hue angle can be constructed by the boundary lines of L*u*v* lower lightness, L*u*v* upper lightness, and the L* axis. Furthermore, the 3D L*u*v* color gamut can be generated by composing the 2D color gamut profiles for the L*u*v* hue angle from 0º to 360º.

5 Intuitive method for color atlas generation of the HCL mix

Usually, an application user can select desired colors from the color palette provided by CAD packages. Such a color palette may have two disadvantages: (a) it is generated by a device-dependent color system, and (b) it is not generated by an intuitive method; the desired colors cannot be obtained easily. Thus, we propose an intuitive method for color atlas generation of the HCL mix in which color appearance can be predicted intuitively and the desired colors can be obtained easily.

5.1 Color atlas generation of the HCL mix

In the HCL mix color system, each computer-generated color atlas page for a given hue angle can be a 2D profile of the 3D HCL mix color gamut as shown in Fig. 6. The method for generating the HCL mix color atlas is now described.

5.1.1 Generating the color atlas of the UCS

To generate a UCS color atlas page for every given UCS hue angle in [0º, 360º], first use the algorithm $\boldsymbol{Get\_C_{ucs\text{-}trade\text{-}off}(h)}$ to compute the accurately reproduced C_{ucs} values in the range $[0, C_{\text{ucs-trade-off}}(h)]$ for the display, and then use the algorithm $\boldsymbol{Get\_L_{ucs\text{-}trade\text{-}off}(h, c)}$ to compute the accurately reproduced L_{ucs} values in the range $[0, L_{\text{ucs-trade-off}}(h, c)]$ where c is in the range $[0, C_{\text{ucs-trade-off}}(h)]$. All the $\left(H^{\circ}_{\text{ucs}}, C_{\text{ucs}}, L_{\text{ucs}}\right)$ coordinates of the computer-generated UCS color atlas pages will lie within the color gamut of the 3D UCS display and can be reproduced precisely.

5.1.2 Generating the color atlas of the L*u*v*

When a color atlas page is generated by the L*u*v* color system, it is limited by the boundary lines of L*u*v* lower lightness, L*u*v* upper lightness, and the L_{ucs} axis as shown in Fig. 5. To generate a L*u*v* color atlas page for a given L*u*v* hue angle, we devise two algorithms for computing the correctly reproduced L*u*v* lightness value range, which corresponds to a given L*u*v* chroma value.
In Fig. 5, when a vertical line $C^{*}_{uv} = c$ with c in $[0, C^{*}_{\text{uv-trade-off}}(h)]$ on the $C^{*}_{uv}L^{*}$ plane is drawn, one can define (a) the minimum trade-off L*u*v* lightness, $L^{*}_{\text{min-trade-off}}$, to be the intersection point S^{*} of the line $C^{*}_{uv} = c$ and the L*u*v* lower-lightness boundary line, and (b) the maximum trade-off L*u*v* lightness, $L^{*}_{\text{max-trade-off}}$, to

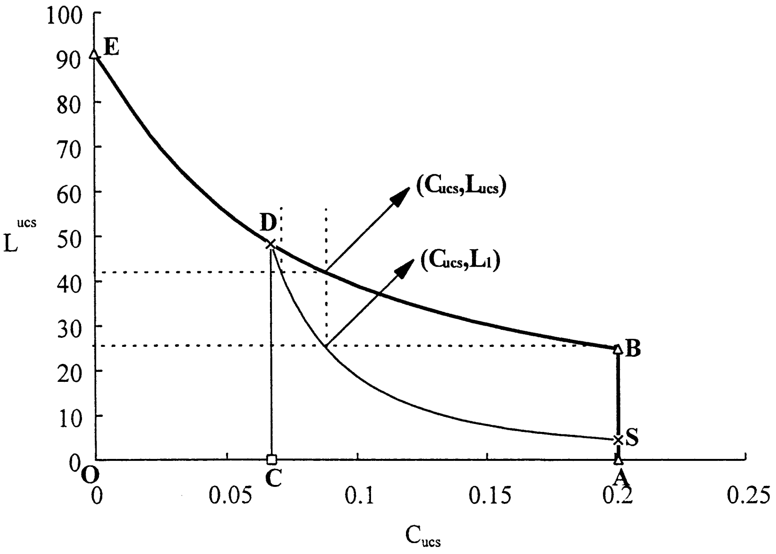

Fig. 7. A schematic diagram for computing the maximum L*u*v* lightness threshold value

be the intersection point D* of the line $C^*_{uv} = c$ and the L*u*v* upper-lightness boundary line. Hence, for a given coordinate $\left(H^\circ_{uv}, C^*_{uv}\right)$ with C^*_{uv} in $[0, C^*_{\text{uv-trade-off}}(h)]$, the correctly reproduced L*u*v* lightness value can be set in the range $\left[L^*_{\text{min-trade-off}}, L^*_{\text{max-trade-off}}\right]$.
By Eq. 7, one finds two characteristics between the UCS $\left(H^\circ_{\text{ucs}}, C_{\text{ucs}}, L_{\text{ucs}}\right)$ extension coordinates and the L*u*v* $\left(H^\circ_{uv}, C^*_{uv}, L^*\right)$ extension coordinates. The two characteristics are:

1. A vertical line on the $C_{\text{ucs}}L_{\text{ucs}}$ plane such as $\overline{AB}$ or $\overline{CD}$ in Fig. 4 will become a line through the point (0, 0) on the $C^*_{uv}L^*$ plane such as $\overline{A^*B^*}$ or $\overline{C^*D^*}$ in Fig. 5.
2. A vertical line on the $C^*_{uv}L^*$ plane such as $\overline{S^*D^*}$ in Fig. 5 will become a curve on the $C_{\text{ucs}}L_{\text{ucs}}$ plane such as the curve $\widehat{SD}$ in Fig. 4.

Thus, according to characteristic 1 and Eq. 7, the algorithm for computing the $L^*_{\text{min-trade-off}}\left(H^\circ_{uv}, C^*_{uv}\right)$ is given here.

```
Algorithm for computing
$\mathbf{Get\_L^*_{min-trade-off}}(\mathbf{H^\circ_{uv}}, \mathbf{C^*_{uv}})$
  (Input: $H^\circ_{uv}$, $C^*_{uv}$; output:
  $L^*_{min-trade-off}(H^\circ_{uv}, C^*_{uv})$)
  Method:
  1. Find $C_1 = C_{ucs-trade-off}(H^\circ_{uv})$ by using algorithm $\mathbf{Get\_C_{ucs-trade-off}}(\mathbf{H^\circ_{uv}})$.
  2. Find $L_1 = L_{ucs-trade-off}(H^\circ_{uv}, C_1)$ by using algorithm $\mathbf{Get\_L_{ucs-trade-off}}(\mathbf{H^\circ_{uv}}, \mathbf{C_1})$.
  3. Calculate $L^*_1 = (116(L_1/100)^{1/3} - 16)$ and $C^*_1 = 13(L^*_1)(C_1)$ by using Eq. 7.
  4. Find the L*u*v* lower-lightness boundary line $L^* = M_{H_{uv}} C^*_{uv}$ for $H^\circ_{uv}$, where $M_{H_{uv}} = (L^*_1/C^*_1)$ by using the points $B^*(C^*_1, L^*_1)$ and $O^*(0, 0)$.
  5. Set: $L^*_{min-trade-off}(H^\circ_{uv}, C^*_{uv}) = M_{H_{uv}} C^*_{uv}$.
  6. Stop.
```

Similarly, according to characteristic 2 and Eq. 7, the algorithm for computing the $L^*_{\text{max-trade-off}}\left(H^\circ_{uv}, C^*_{uv}\right)$ is given here. A graph for illustrating the iterative process of computing the maximum L*u*v* lightness-threshold value is given in Fig. 7.

```
Algorithm for computing
$\mathbf{Get\_L^*_{max-trade-off}}(\mathbf{H^\circ_{uv}}, \mathbf{C^*_{uv}})$
  (Input $H^\circ_{uv}$, $C^*_{uv}$; output $L^*_{max-trade-off}(H^\circ_{uv}, C^*_{uv})$)
  1. Find the $C_1 = C_{ucs-trade-off}(H^\circ_{uv})$ by using algorithm $\mathbf{Get\_C_{ucs-trade-off}}(\mathbf{H^\circ_{uv}})$.
  2. Find the $L_1 = L_{ucs-trade-off}(H^\circ_{uv}, C_1)$ by using algorithm $\mathbf{Get\_L_{ucs-trade-off}}(\mathbf{H^\circ_{uv}}, \mathbf{C_1})$.
  3. Find $C_{ucs} = C^*_{uv}/(1508(L_1/100)^{1/3} - 208)$ by using Eq. 7.
  4. Find $L_{ucs} = L_{ucs-trade-off}(H^\circ_{uv}, C_{ucs})$ by using algorithm $\mathbf{Get\_L_{ucs-trade-off}}(\mathbf{H^\circ_{uv}}, \mathbf{C_{ucs}})$.
```

5. If $\mathrm{L}_{\mathrm{ucs}}>L_1$, then set $L_1=\mathrm{L}_{\mathrm{ucs}}$ and go step 3;
6. Set: $\mathrm{L}^*_{max-trade-off}(\mathrm{H}^\circ_{\mathrm{uv}}, \mathrm{C}^*_{\mathrm{uv}}) = 116(\mathrm{L}_{\mathrm{ucs}}/100)^{1/3} - 16.$
7. Stop.

According to the two algorithms $\boldsymbol{Get\_L^*_{min-trade-off}}(\mathbf{H}^\circ_{\boldsymbol{uv}}, \mathbf{C}^*_{\boldsymbol{uv}})$ and $\boldsymbol{Get\_L^*_{max-trade-off}}(\mathbf{H}^\circ_{\boldsymbol{uv}}, \mathbf{C}^*_{\boldsymbol{uv}})$, one can be sure all the $(\mathrm{H}^\circ_{uv}, \mathrm{C}^*_{uv}, \mathrm{L}^*)$ coordinates of the computer-generated L*u*v* color atlas pages will lie within the color gamut of the 3D L*u*v* display and can be reproduced precisely.

5.2 Intuitive method for color atlas generation

Usually, when people want to describe a color, they will first name the hue, and then the chroma level (color tone) or lightness level (gray level). To let users search for the desired colors by using the intuitive descriptions of hue names, color tones, and gray levels in HCL mix color systems, we selected 19 commonly used hue names, 5 color tones, 5 gray levels, and their corresponding quantitative HCL mix color coordinates. The selected color descriptors are listed in Tables 3 to 5. These hue names, color tones, and gray levels were selected based on the Inter-Society Color Council-National Bureau of Standards (ISCC-NBS) system [22, 23]. The corresponding dominant wavelengths for the 19 hue names are given in [24].

Table 3. The selected descriptor of nineteen hue names and corresponding H°_{uv} values in the HCL mix

ISCC-NBS hue name	Notation	H°_{uv}	Dominant wavelength (nm)
Red	R	6º	493c
Reddish orange	rO	15º	606
Orange	O	28º	592
Yellowish orange	yO	47º	583
Yellow	Y	61º	578
Greenish yellow	gY	78º	573
Yellow green	YG	101º	565
Yellowish green	yG	132º	545
Green	G	156º	508
Bluish green	bG	180º	495
Blue green	BG	198º	490
Greenish blue	gB	221°	485
Blue	B	250º	476
Purplish blue	pB	271°	454
Bluish purple	bP	279º	566c
Purple	P	292º	560c
Reddish purple	rP	312º	545c
Red purple	RP	338°	506c
Purplish red	pR	357º	496c
Red	R	6º	493c

Table 4. The selected descriptor of five color tones and corresponding chroma values in the HCL mix[a]

Color tone	Chroma value range
Vivid	$C\in(8, 10]$
Strong	$C\in(6, 8]$
Moderate	$C\in(4, 6]$
Grayish	$C\in(2, 4]$
Dark	$C\in(0, 2]$

[a] According to the approximately perceptually uniform C^*_{uv}, the color tone is classified into five levels by the normalized 11-chroma scale C, $C = [\mathrm{C}^*_{uv}/\mathrm{C}_{\mathrm{uv-trade-off}}(h)] \times 10$

Table 5. The selected descriptor of five gray levels and corresponding lightness values in the HCL mix[a]

Gray level	Lightness value range
Very light gray	$L\in(8, 10]$
Light gray	$L\in(6, 8]$
Medium gray	$L\in(4, 6]$
Dark gray	$L\in(2, 4]$
Very dark gray	$L\in(0, 2]$

[a] According to the approximately perceptually uniform L*, the gray level is classified into five levels by the normalized 11-lightness scale L, L=(L*/10)

According to the five color tones and five gray levels, one can divide each page of the color atlas into 25 *tone-gray grids* for a selected hue name as shown in Fig. 6. Theoretically, each gray tonegrid will have $N\times N$ color chips. To ensure the HCL mix color atlas can be accurately reproduced for each electronic display, each $(\mathrm{H}^\circ_{uv}, \mathrm{C}^*_{uv}, \mathrm{L}^*)$ coordinate of such $N\times N$ color chips must be checked by using the derived threshold values of $\mathrm{C}_{\mathrm{ucs-trade-off}}(\mathrm{H}^\circ_{\mathrm{ucs}})$, $\mathrm{L}_{\mathrm{ucs-trade-off}}(\mathrm{H}^\circ_{\mathrm{ucs}}, \mathrm{C}_{\mathrm{ucs}})$ for the UCS, $\mathrm{C}^*_{\mathrm{uv-trade-off}}(\mathrm{H}^\circ_{\mathrm{uv}})$, $\mathrm{L}^*_{\mathrm{max-trade-off}}(\mathrm{H}^\circ_{\mathrm{uv}}, \mathrm{C}^*_{\mathrm{uv}})$, and $\mathrm{L}^*_{\mathrm{min-trade-off}}(\mathrm{H}^\circ_{\mathrm{uv}}, \mathrm{C}^*_{\mathrm{uv}})$ for the L*u*v*. Meanwhile, users can obtain desired colors intuitively based on the three perceptually descriptive items of hue names, color tones, and gray levels. Figure 8 shows the procedure for the intuitive color atlas generation by applying the HCL mix color system. Figure 9 shows an example of a practical, computer-generated, color atlas interface for which the HCL mix color system was used. 'Blue green'

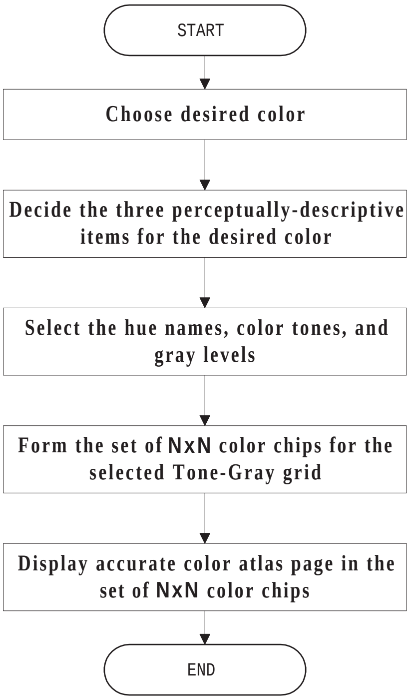

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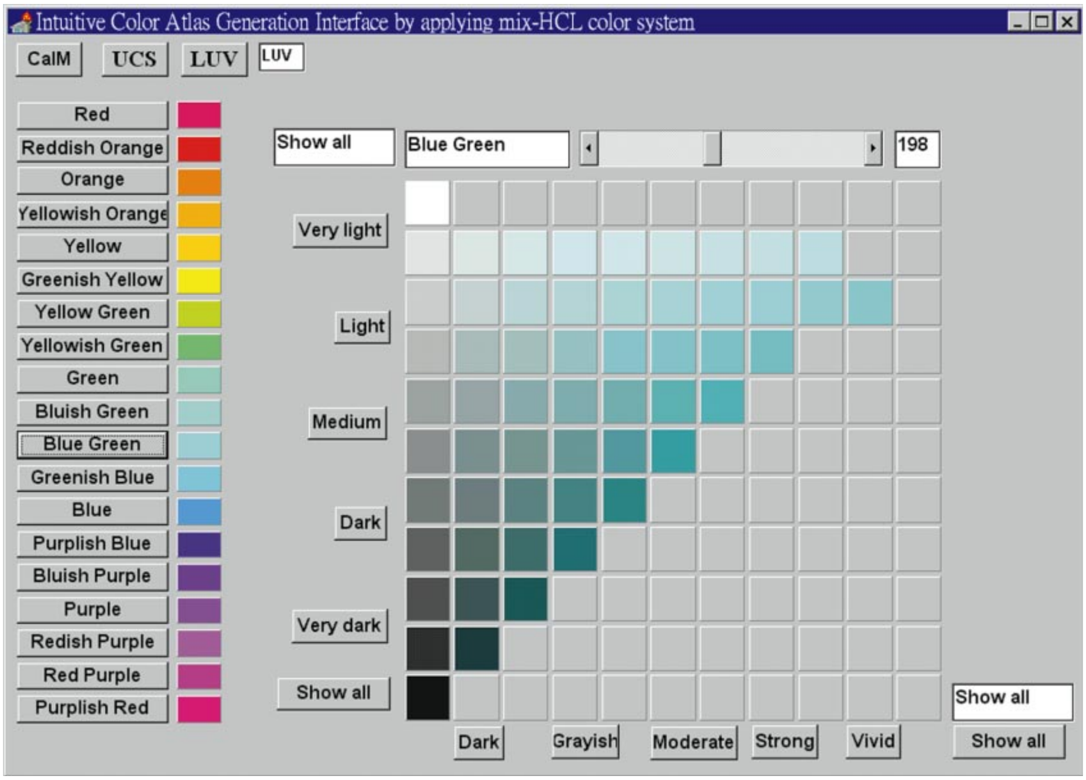
Fig. 8. Flowchart for intuitively generating a color atlas with the HCL mix color system

Fig. 9. Illustration of an interface for intuitively generating color atlases. The interface uses a HCL mix color system based on the three perceptually descriptive items



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and the "show all" modes of the 25 tone-gray grids are selected in Fig. 9.

6 Conclusions

We have proposed a HCL mix color system that can be used to describe accurate colors on different electronic displays. Algorithms have been devised in the HCL mix color system that compute the threshold values of chroma and lightness in creating the 3D color gamut of a display. The proposed HCL mix color system has the following implementary features:

1. Device-independent color coordinates are provided to specify colors on electronic displays.
2. A 3D display-color gamut can be quickly created.
3. Two threshold values of the chroma and lightness are derived to check the validity of input color coordinates.
4. Intuitive parameter interface of color specification is developed through the approximately perceptually uniform dimensions of HCL.

An intuitive and device-independent method for color atlas generation was proposed in the HCL mix color system. This method can help users find the desired or imaginary colors quickly and intuitively through the perceptually descriptive items of hue name, color tone, and gray level. Simulation results show that the proposed method is effective. For further research, an advanced linguistic or image-word color adjustment method (like "young" blue, "fashion-conscious" yellow, "passionate" red, etc.) based on the HCL mix may be useful for product design and be worthy of study. A method for reproducing a good approximate color between electronic devices (such as monitors, scanners) and hard-copy devices (such as printers) based on the HCL mix system is another future research topic.

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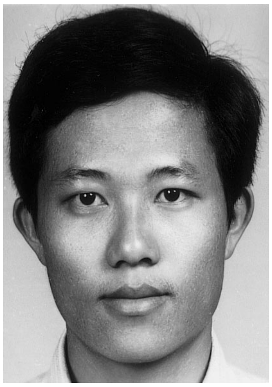

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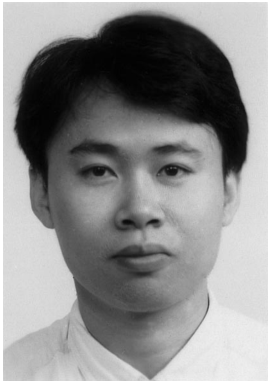

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