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ORIGINAL ARTICLE

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Incorporating process capability index and quality loss function into analyzing the process capability for qualitative data

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Abstract Process capability analysis (*PCA*) is frequently employed to evaluate a product or a process if it can meet the customer's requirement. In general, process capability analysis can be represented by using the process capability index (*PCI*). Until now, the *PCI* was frequently used for processes with quantitative characteristics. However, for process quality with the qualitative characteristic, the data's type and single specification caused limitations of using the *PCI*. When the product can not meet the target, even if it lies in the specified range, it should lead to the corresponding quality loss. Taguchi developed a quadratic quality loss function (*QLF*) to address such issues. In this study, we intend to construct a measurable index which incorporates the *PCI* philosophy and *QLF* concept to analyze the process capability with the consideration of the qualitative response data. The manufacturers can not only employ the proposed index to self-assess the process capability, but they also can make comparisons with the other competitors.

Keywords Process capability analysis (*PCA*) · Process capability indexes (*PCI*s) · Qualitative data · Quality loss function (*QLF*).

1 Introduction

Process capability analysis (*PCA*) [1, 4, 5, 11] is frequently employed by the manufacturers to evaluate if the capability of process can meet the customer's requirement. Process capability indexes (*PCIs*) [1, 4] are a quantitative measurement of the process capability in most manufacturing industries. *PCIs,* such as C_a, C_p and C_{pk} are commonly used for most manufactures [3, 7–11], can frequently measure the process capability for the quantitative response. Herein, C_a evaluates the related scale of the process mean with the tolerance specification (i.e. the difference between the upper tolerance limit and the lower tolerance limit). C_p evaluates the related scale of the specification's tolerance with process's tolerance. While C_{pk} simultaneously evaluates the centering degree and the dispersion degree. These *PCIs* will make some adjustments if there are necessary particulars like the unilateral specification [10]. For the quantitative type, the theories on *PCA* and *PCIs* are well developed [1, 8, 11]. The qualitative data type may exist during the manufacturing environment, e.g. the integrated circuit (*IC*) manufacturing industry uses the defect count on a wafer to analyze their product's yield and control their process, the process capability analysis for qualitative data will be an important issue to study. However, most studies only focus on the *PCA* application for the quantitative response data, and the qualitative response data is seldom mentioned [6, 7]. Several difficulties can be mentioned as: (1) the target of the qualitative data may lead to unobvious centering evaluation, e.g. the target will be set as zero defect, (2) the limitation of the unilateral specification, especially only the upper specification exist, e.g. the defect rate may be less than 1% and (3) the quantitative data utilizes the process mean (μ) and process deviation (σ) to compute the *PCIs*, however, the qualitative data can not directly utilize them to compute the *PCIs*.

Under the global market environment, to realize the process capability comparison with other competitions can provide helpful information for enhancing organizational competence or making strategic decisions. Especially, the *PCA* for the different manufacturers will be a significant factor to seek for the collaborators during the consideration of supply chain management (*SCM*). In this study, we intend to construct a process capability index, the *PCI* on qualitative response data, to evaluate the process capability for the qualitative response data. The logical idea is to combine the *PCI* philosophy and *QLF* concept. The rest of this study is organized as follows. Sect. 2 clearly demonstrates the construction procedure of the quantitative measurement we proposed. Sect. 3 will employ the numerical examples to demon-

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strate the effectiveness of the proposed approach. Concluding remarks are finally made in Sect. 4.

2 Construction of *PCI* for the qualitative response data

2.1 Construction concept of *PCI*

The product can not meet the target, even if it lies in the specified range, and it should lead to quality loss. Taguchi developed a quadratic quality loss function to address such a case [2]. The quadratic quality loss function is defined as follows:

$$L(y) = k(y - T)^2 \tag{1}$$

and the expected quality loss can be described as:

$$\begin{aligned} QL &= E[L(y)] = E[k\,(y - T)^2] \\ &= kE[(y - \mu + \mu - T)^2] = k((\mu - T)^2 + \sigma^2) \end{aligned} \tag{2}$$

where y denotes the response data, T denotes the target value or the nominal value, k denotes the constant of the quality loss when the process is within the allowable tolerance, μ denotes the process mean and σ denotes the process deviation.

For the qualitative response data set, the target situation should be zero defect ($T = 0$) or not non-conforming. Hence, applying it into the quality loss function, Eq. 2 can be modified as:

$$QL = k(\mu^2 + \sigma^2) \tag{3}$$

Then, the quality loss function can be represented as $QL = QL(\theta)$ if the process parameter θ is involved. For the qualitative data, according to the concept of the quality loss function, the quantitative measurement of the process capability can be constructed. That is, we can take the ratio of the customer's allowable quality loss and the actual quality loss. Hence, the generalized *PCI* of the attribute data can be defined as:

$$PCI = \frac{QL(\theta_c)}{QL(\theta)} \tag{4}$$

where θ denotes the process parameter of the actual process and θ_c denotes the process parameter of the customer's expectation. In fact, the qualitative data can be described well by several distributions like the binomial and Poisson distributions. In this section, we will clearly describe the *PCI* value of both distributions, and the features hidden in the proposed *PCI* also are explained in the next section.

2.2 Binomial distribution

First, let $y \sim \mathrm{Ber}(1, p)$, $\mathrm{Ber}(1, p)$ denotes the Bernoulli's trial,

$$\theta_c = p_c,\ \theta = p,\ \mu = p,\ \sigma^2 = p(1-p),$$

hence,

$$PCI = \frac{QL(\theta_c)}{QL(\theta)} = \frac{K[p_c^2 + (p_c - p_c^2)]}{k[p^2 + (p - p^2)]} = \frac{p_c}{p} \tag{5}$$

where p denotes the non-conforming rate (the parameter of the binomial distribution), p_c denotes the acceptable quality level of the customer for the non-conforming rate and n denotes inspection count.

The features include:

1. When $p_c < p$, it means the capability of the process can not meet the customer's requirement; that is, it is a "bad" process, $PCI < 1$.
2. When $p_c = p$, it means that the capability of the process exactly meets the customer's requirement, $PCI = 1$.
3. When $p_c > p$, it means that the capability of the process absolutely satisfies the customer's requirement, $PCI > 1$.
4. Figure 1 graphically depicts the relationship between process parameter p and the *PCI* value with different p_c.

Then, let $y \sim \mathrm{Ber}(1,\ p)$, and if the lot count is n, the related parameters can be denoted as:

$$D = \sum_{i=1}^{n} y_i \underset{i.i.d.}{\sim} B(n,\ p),$$

$$\theta_c = p_c,\ \theta = p,\ \mu = np,\ \sigma^2 = np(1-p)$$

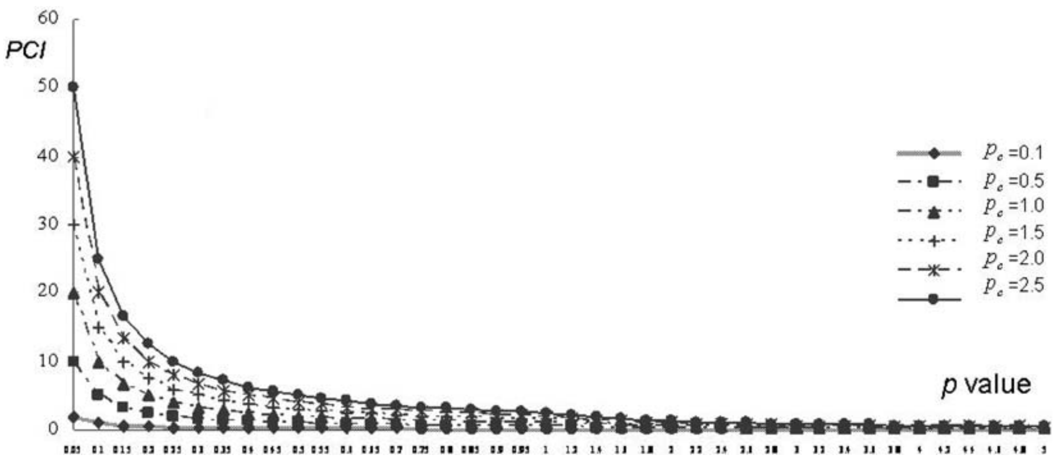

Fig. 1. The diagram for Bernoulli distribution with different p_c, p and PCI

where $B(n, p)$ will denote the binominal distribution. Hence, the *PCI* formula for binominal distribution can be represented as follows:

$$PCI = \frac{QL(\theta_c)}{QL(\theta)} = \frac{K[(np_c)^2 + n(p_c - p_c^2)]}{k[(np)^2 + n(p - p^2)]} = \frac{[(n-1)p_c^2 + p_c]}{[(n-1)p^2 + p]} \tag{6}$$

Features:

1. When $p_c < p$, it means the capability of the process can not meet the customer's requirement; the *PCI* will decrease with respect to the inspection count n.
2. When $p_c = p$, it means that the capability of the process exactly meets the customer's requirement.
3. When $p_c > p$, it means that the capability of the process absolutely satisfies the customer's requirement; the *PCI* will increase with respect to the lot count n.
4. From Fig. 2a, when the process capability can meet the customer's requirement, the *PCI* will increase with respect to the lot count n; while the process capability can not meet the customer's requirement, the *PCI* will decrease with respect to the lot count n; the $p_c = p$ will be the saddle point in such cases. Furthermore, from Fig. 2a and b, we can clearly realize the change of the *PCI* curve when the customer's allowable parameter changes.

2.2.1 Poisson distribution

Let $y \sim P(\lambda)$, herein, P denotes the Poisson distribution, and the related distribution's parameters can be represented as: $\theta_c = \lambda_c,\ \theta = \lambda,\ \mu = \lambda,\ \sigma^2 = \lambda$,

$$PCI = \frac{QL(\theta_c)}{QL(\theta)} = \frac{K[\lambda_c^2 + \lambda_c]}{k[\lambda^2 + \lambda]} = \frac{[\lambda_c^2 + \lambda_c]}{[\lambda^2 + \lambda]} \tag{7}$$

where λ denotes the defect rate of the Poisson distribution for the actual process and λ_c denotes the acceptable quality level of the customer for the defect rate of the Poisson distribution.

The features include:

1. When $\lambda_c < \lambda$, it means the process capability can not meet the customer's requirement; that is, it is a "bad" process, that is, $PCI < 1$.

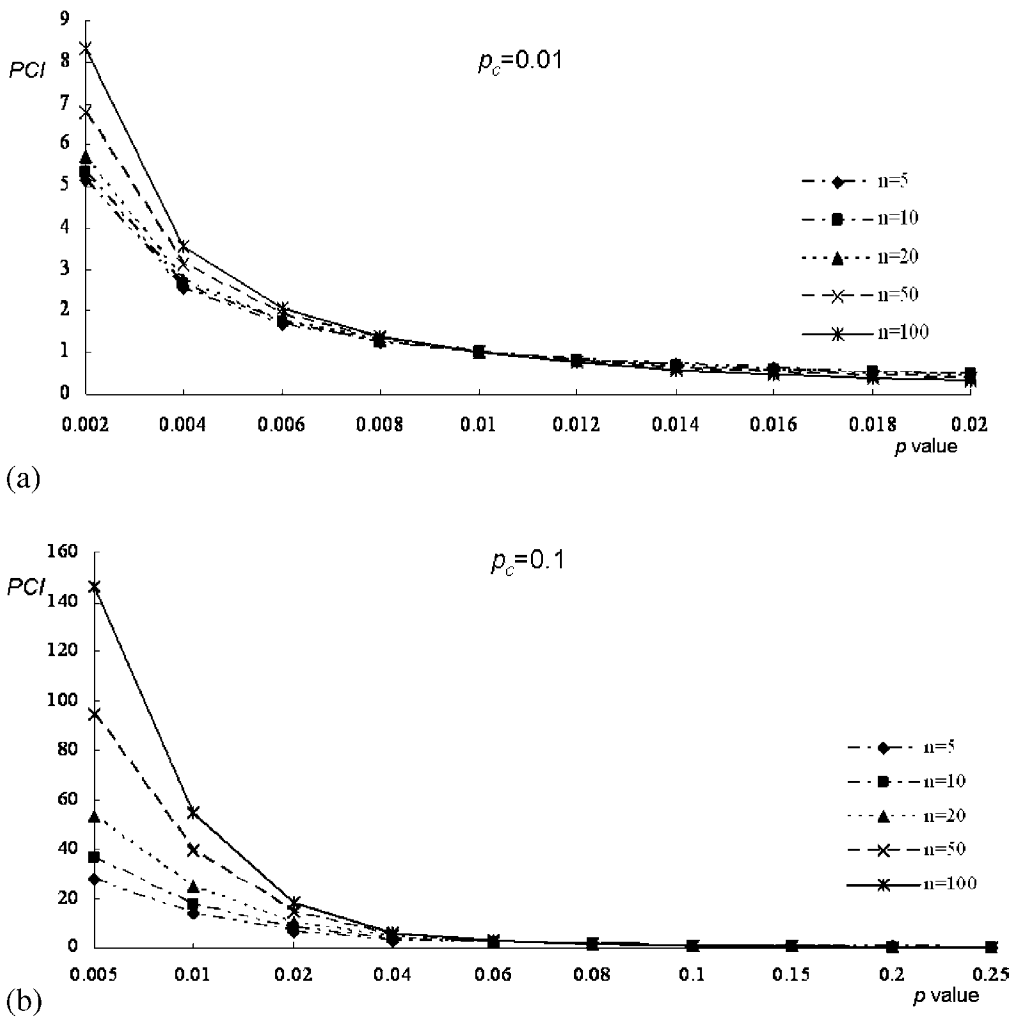

Fig. 2. a The *PCI* curve diagram for different inspection count when $p_c = 0.01$ **b** The *PCI* curve diagram for different inspection count when $p_c = 0.1$

2. When $\lambda_c = \lambda$, it means that the process capability exactly meets the customer's requirement; that is, $PCI = 1$.
3. When $\lambda_c > \lambda$, it means that the process capability absolutely satisfies the customer's requirement; that is, $PCI > 1$.
4. Figure 3 depicts graphically the relationship between the process parameter $\lambda$and PCI with different λ_c.

If the inspection count be m and it is denoted as the case of the average defect rate, $y \sim P(\lambda)$, P denotes the Poisson distribution, and the related parameter can be denoted as: $\theta_c = \lambda_c,\ \theta = \lambda,\ \mu = \frac{\lambda}{m},\ \sigma^2 = \frac{\lambda}{m^2}$, and the PCI formula is:

$$PCI = \frac{QL(\theta_c)}{QL(\theta)} = \frac{K[\frac{\lambda_c^2}{m} + \frac{\lambda_c}{m}]}{k[\frac{\lambda^2}{m} + \frac{\lambda}{m}]} = \frac{[\lambda_c^2 + \lambda_c]}{[\lambda^2 + \lambda]} \tag{8}$$

where μ denotes the average defect rate and μ_c denotes the acceptable quality level of the customer for the average defect rate. We can find out that it has the same structure as the PCI of the Poisson distribution. Hence, the features are also the same as Poisson's.

2.3 Construction of *PCI* for comparison with different competitors

We will make some integration according to the qualitative PCI we previously proposed for the case of comparison between different competitors. The concept is to substitute the manufacturer's quality loss by the competitors' quality loss. Hence, the constructed comparison PCI is given as follows:

$$PCI_{\text{competitor: manufacturer}} = \frac{QL(\theta_{\text{competitor}})}{QL(\theta_{\text{manufacturer}})} \tag{9}$$

Features:

1. $PCI_{\text{competitor: manufacturer}} < 1$, it means the competitor's process capability is better than the manufacturer's process capability.
2. $PCI_{\text{competitor: manufacturer}} = 1$, it means the competitor's process capability is equal to the manufacturer's process capability.
3. $PCI_{\text{competitor: manufacturer}} > 1$, it means the manufacturer's process capability is better than the competitor's process capability.

3 Numerical analysis and the conclusions

3.1 Illustrative example 1

A lead frame manufacturer in Taiwan expects to realize if their process capability can meet the customer's requirement (the packaging fabrication). Several hundreds of lead frame types are produced in lead frame manufacturing. The packaging fabrications expect the defect count of the lead frame in their in-line quality control (IQC) must be less than ten strips per 500 inspection strips, and then the yield of the packaging product can be enhanced. Hence, the lead frame manufacturer plans to study their process capability and make a suitable compromise with the customers. The following data listed in Table 1 are collected for

Table 1. The non-conforming count

Day	Non-conforming count per 500 strips	Day	Non-conforming count per 500 strips
1	7	16	10
2	5	17	7
3	13	18	9
4	11	19	14
5	12	20	12
6	9	21	11
7	10	22	8
8	14	23	9
9	10	24	12
10	6	25	8
11	13	26	10
12	9	27	9
13	12	28	7
14	8	29	8
15	12	30	10

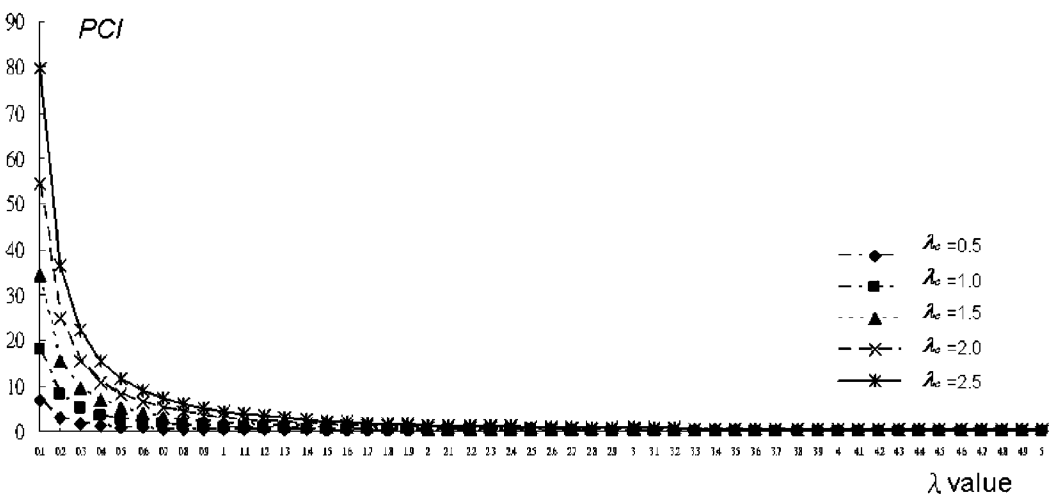

Fig. 3. The PCI curve diagram for the process parameter λ_c, λ and PCI

30 lots as the same *QFP* lead frame, and the constant k of the manufacturer is 180 (that is the necessary cost for rework). The quality (result) of the product can be divided into two categories: conforming and non-conforming. Each inspection is independent of the others. About 30 records were collected in Table 1. Obviously, the data obey the binomial distribution.

First, we must make sure that the process is in-control. The data type is owing to the qualitative type. Hence, the *np*-chart is applied to process control (see Fig. 4). The total non-conforming count can be computed as 295. Hence, we can use the following formula to estimate the non-conforming rate p.

$$\bar{p} = \frac{\text{total number of non-conforming units}}{\text{total number inspected}} = \frac{295}{30 \times 500} = 0.0197$$

Then, the *np*-chart is constructed as follows.

$$\begin{aligned} UCL &= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 19.17 \\ CL &= n\bar{p} = 9.85 \\ LCL &= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 0.53. \end{aligned}$$

It is in-control for screening out the *np*-chart. Then, we will employ the proposed *PCI* formula to study the process capability of the lead frame manufacturer. As the customer's requirement, the parameter $\theta_c(p_c)$ is 0.02, and the manufacture's estimated parameter θ (p) is 0.0197. When the inspection count is 500, the average count of the non-conforming unit is 9.85 (0.0197×500). Then, the proposed *PCI* value can be computed as follows:

$$\begin{aligned} PCI &= \frac{QL(\theta_c)}{QL(\theta)} = \frac{K[(np_c)^2 + n(p_c - p_c^2)]}{k[(np)^2 + n(p - p^2)]} \\ &= \frac{[(n-1)p_c^2 + p_c]}{[(n-1)p^2 + p]} = \frac{499 \times 0.02^2 + 0.02}{499 \times 0.0197^2 + 0.0197} = 1.0393. \end{aligned}$$

We can find out that the *PCI* value exceeds 1, and it means the current process capability can meet the customer's requirement. However, the ratio is not significantly larger than 1, so the lead frame manufacturer still need to pay more attention to their process.

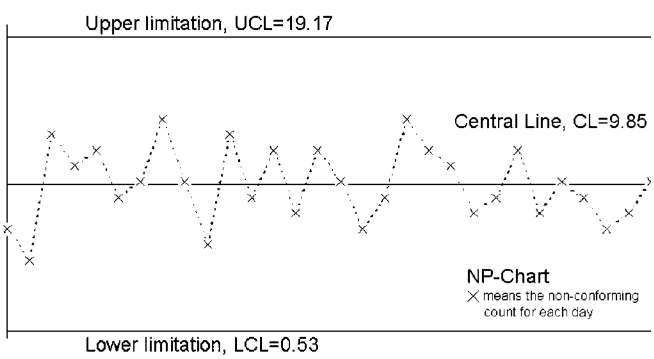

Fig. 4. The constructed NP-chart

3.2 Numerical example 2: Comparison with different competitors

For the same numerical example, besides, the manufacturer also collected the related competitor's information. They also expect to realize the difference between their process capability and competitors' process capability. The collected information is given as follows.

For competitor A: the parameter θ_a (the non-conforming rate p_a) is 0.015, the inspection count is 600 strips and the constant k is 200.

For competitor B: the parameter θ_b (the non-conforming rate p_b) is 0.025, the inspection count is 400 strips and the constant k is 250.

For manufacturer: the parameter θ_b (the non-conforming rate p_b) is 0.0197, the inspection count is 500 strips and the constant k is 180.

Compare with competitor A:

$$\begin{aligned} PCI_{A:M} &= \frac{QL(\theta_A)}{QL(\theta_M)} \\ &= \frac{k_A[(n_A p_A)^2 + n_A(p_A - p_A^2)]}{k_M[(n_M p_M)^2 + n_M(p_M - p_M^2)]} \\ &= \frac{200[(0.015 \times 600)^2 + 600 \times (0.015 - 0.015^2)]}{180[(0.0197 \times 500)^2 + 500 \times (0.0197 - 0.0197^2)]} \\ &= \frac{17973}{19202.22} = 0.936 < 1 \end{aligned}$$

Conclusion: $PCI_{A:M} < 1$, it means the competitor's process capability is better than the manufacturers' process capability. That is, competitor A's quality loss is less than the manufacturer's quality loss.

Compare with competitor B:

$$\begin{aligned} PCI_{B:M} &= \frac{QL(\theta_B)}{QL(\theta_M)} \\ &= \frac{k_B[(n_B p_B)^2 + n_B(p_B - p_B^2)]}{k_M[(n_M p_M)^2 + n_M(p_M - p_M^2)]} \\ &= \frac{250[(0.025 \times 400)^2 + 400 \times (0.025 - 0.025^2)]}{18[(0.0197 \times 500)^2 + 500 \times (0.0197 - 0.0197^2)]} \\ &= \frac{27437.5}{19202.22} = 1.429 > 1 \end{aligned}$$

Conclusion: $PCI_{B:M} > 1$, it means the competitor's process capability is worse than the manufacturer's process capability. That is, competitor B's quality loss is larger than the manufacturers' quality loss.

According to the comparison, we can make the conclusion: "Competitor B's process capability is worse than the manufacturer, while competitor A's process capability is significantly better than the manufacturer. Furthermore, the sequence of process capability from the best to the worst is Competitor A → Manufacturer → Competitor B". Restated, the manufacturer should work hard to enhance their process capability by performing the necessary quality improvement.

4 Concluding and remarks

In this study, we construct a quantitative measurement *PCI* for the qualitative response. The quantitative measurement is based on the Taguchi's quality loss function philosophy and *PCI* concept. It is a ratio deriving from the customer's quality loss with respect to the actual process's quality loss. By employing the proposed *PCI*, the manufacturers can employ it to assess if the process capability can meet the customer's requirement. Besides, the constructed *PCI* can also be employed to make the comparison between the manufacturer and the competitors. The *PCI* formulas for different quality data obeying the binomial distribution or Poisson distribution are proposed in this study. The other advantage is that the practitioners do not need complicated computation to obtain the attribute *PCIs* by using the proposed *PCIs*.

References

1. Montgomery DC (2001) Introduction to statistical quality control, 4th edn. Wiley, New York, NY
2. Phadke MS (1989) Quality engineering using robust design. Prentice-Hall International, New Jersey, NJ
3. Chen KS, Pearn WL (1997) An application of non-normal process capability indices. Qual Reliab Eng Int 13:355–360
4. Kane VE (1986) Process capability indices. J Qual Technol 18:41–52
5. Bissell AF (1990) How reliable is your capability index? Appl Stat 39:331–340
6. Kaminsky FC, Davis RD (1989) Statistical measures of process capability and their relationship to non-conforming product. In: Proc Third Biennial International Manufacturing Research Forum
7. Kaminsky FC, Dovich RA, Burke RJ (1998)Process capability indices: now and in the future. Qual Eng 10:445–453
8. Kureková K (2001) Measurement process capability – trends and approaches. Meas Sci Rev 1(1):43–46
9. Tong LI, Chenn KS, Chen HT (2001) Statistical testing for assessing the performance of lifetime index of elcetronic component with exponential distribution. Int J Qual Reliab Manage 19(6/7):812–824
10. Chen JP, Ding CG (2001) A new process capability index for non-normal distribution. Int J Qual Reliab Manage 18(7):762–770
11. Kotz S, Lovelace CL (1998) Process capability indices in theory and practice. Arnold, London