



A simulation-based evolutionary approach to LNA circuit design optimization

Yiming Li

Department of Communication Engineering, National Chaio Tung University, Hsinchu 300, Taiwan

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ABSTRACT

In this paper, we propose a simulation-based evolutionary approach for designing low noise amplifier (LNA) integrated circuits (ICs). Based on a genetic algorithm (GA), the Levenberg–Marquardt (LM) method, and a circuit simulator, the simulation-based evolutionary approach is developed for design optimization of LNA circuits. For a given LNA circuit, the simulation-based evolutionary approach simultaneously optimizes the electrical specifications, such as S_{11} , S_{12} , S_{21} , S_{22} , K factor, the noise figure, and the input third-order intercept point in the process between simulation and optimization. First of all, the necessary parameters of the LNA circuit for circuit simulation are loaded. By solving a set of nonlinear ordinary differential equations, the circuit simulator will then be performed for the circuit simulation and specification evaluation. Once the specification meets the aforementioned seven constraints, we output the optimized parameters. Otherwise, we activate the GA for the global optimization; in the meanwhile, the LM method searches the local optima according to the results of the GA. We then call circuit simulator to compute and evaluate newer results until the specification is matched. In numerical experiment, 10 parameters of the LNA circuit including device configuration and biasing condition are optimized with respect to the constraints. The design of LNA circuit is with $0.18 \mu\text{m}$ metal-oxide-silicon filed effect transistors. Benchmark results also computationally confirm the robustness and efficiency of the proposed method. This simulation-based evolutionary approach, in general can be applied to optimal design of other analog and radio frequency circuits. We believe that this systematical approach will help IC design optimization, and benefit computer-aided design of wireless communication system-on-a-chip.

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1. Introduction

Low noise amplifier (LNA) circuit plays an important role in radio frequency (RF) circuit design [1–6]. In modern integrated circuit (IC) design flow and chip implementation, designers perform a series of functional examination and analysis of electrical characteristics of a designed circuit by circuit simulation and electronic computer-aided design (ECAD) software to match specifications [7,8]. In order to achieve the specification, designers must continuously and repeatedly tune the design coefficients and perform the circuit simulation to get a set of optimized active device model parameters, passive device parameters, device size, circuit layout, width of wires, and biasing condition. It is in general requires experienced designers to accomplish such complicated works. Circuit simulation tool including ECAD software has manually been used in performing IC designs in the last decades, yet proper usage of optimization techniques will have a positive contribution to the communities of fabrication and design [6,9,10].

E-mail address: yml@faculty.nctu.edu.tw

In this paper, based on a genetic algorithm [6,9–14] (GA), the Levenberg–Marquardt [6,10,15–17] (LM) method, and a circuit simulator [18,19], we propose a simulation-based evolutionary approach for optimal design of LNA circuits. The basic concept of the simulation-based evolutionary approach has been proposed for model parameter extraction of sub-100 nm metal-oxide-silicon field effect transistors (MOSFETs) and optimal characterization of heterojunction bipolar transistor in our recent works [9,10]. We in this work successfully generalize this approach to IC design optimization; in particular, for analog and RF circuits. For a given LNA circuit, the simulation-based evolutionary approach simultaneously optimizes the electrical specifications [1,6] such as S_{11} , S_{12} , S_{21} , S_{22} , K factor, the noise figure, and the input third-order intercept point in the optimization process. First of all, the necessary parameters of the LNA circuit for circuit simulation, such as the macromodel of RF MOSFETs [18] and the netlist of explored LNA circuit are loaded [6,18]. A circuit simulator will then be performed for the circuit simulation and specification evaluation. In the circuit simulation, a set of nonlinear ordinary differential equations (ODEs) corresponding to the LNA circuit will be solved. Once the specification meets the aforementioned seven constraints, optimized parameters associated with the specified LNA circuit are then outputted. Otherwise, we activate the GA for the global optimization; in the meanwhile, the LM method searches the local optima according to the evolutionary results of the GA. This numerical optimization method does significantly accelerate the evolution process. We then call circuit simulator to compute and evaluate newer results until the specification is matched. We note that the well-known circuit simulation tool, HSPICE [18], is successfully integrated in our numerical implementation. To verify the validity of the proposed methodology for LNA circuits design optimization, more than 10 parameters including capacitance, inductance, resistance, and biasing conditions are optimized with respect to the aforementioned seven constraints. The design of LNA circuit is focus on the usage of the 0.18 μm MOSFETs, but it can be applied to other technology nodes. Benchmark results including the convergence property and the sensitivity of optimized parameters also computationally confirm the robustness and efficiency of the proposed simulation-based evolutionary approach. This approach not only works at the levels of device and a single circuit module but also can be applied to other analog and RF circuits; even an electronic system which is with a small number of transistors. It is because the proposed approach is mainly based upon a simulation-based technique. Therefore, once the netlist of circuit simulation for any specified circuits is generated the optimization technique works with the similar operation mechanism.

This paper is organized as follows: in Section 2, we introduce the proposed simulation-based optimization technique. In Section 3, the numerical experiment for this work is introduced. The achieved simulation results are discussed in Section 4. We also verify the validity, robustness, and efficiency of the method. Finally, we draw conclusions and suggest some future works.

2. The simulation-based evolutionary approach

It is difficult to find the solution of multidimensional global optimization problems in modern IC design by using either conventional numerical method or soft computing techniques. Typical genetic searching methods are plagued by problems such as rapid decrease in the population diversity and disproportionate exploitation and exploration of the solution space with multiple dimensions. The results are frequent premature convergence and inefficient search. Newton-based numerical methods find a solution rapidly compared with approaches of GA, but they are still a local method and are often trapped into local optimum. A basic idea of simulation-based evolutionary approach proposed in this paper mainly takes a GA to perform global search, and while the evolution seems to be saturated, the LM method is then enhancing the searching behavior to perform the local search.

For a circuit to be optimized, such as a LNA circuit, we automatically parse and generate the corresponding netlist of the circuit [19]. The generated script file will be inputted into an adopted circuit simulator for simulation and evaluation of results, where a set of circuit ODEs are solved. If the results meet the target, we then output the final optimized data. If the difference of errors between the target and result does not meet the convergence criterion, the established optimization kernel will enable the circuit parameter extraction in a global sense. The number of circuit parameters to be extracted depends upon the specification that we want to achieve. For examples, they could include active, passive, design window, and biasing conditions. The optimized results are used for automatically modifying the netlist and the next newer optimization process is performed. We note that the optimization technique implemented in this work is so-called the simulation-based evolutionary approach because an adopted circuit simulator is preformed repeatedly to calculate the cost function in the evolution loop.

An architecture of the optimization kernel for the proposed simulation-based evolutionary approach is shown below:

```

Initialize parameters extraction environment
Initialize GA
while fitness score of the best chromosome > tolerated score
  GA searches for better solution
  if the evolution seems to be saturated
    LM searches for currently local solution
End while

```

According to the procedure shown above, the GA firstly searches the entire problem space. During this period, the candidates that GA searched are passed to certain adopted circuit simulator [18] to retrieve the results of circuit simulation. For

the specified desired targets, the simulated results are then passed to the evaluation to measure the fitness score. The evaluated score is provided for the global optimization of GA. After a rough solution is obtained, the LM method simultaneously performs local searches and sets the local optimum as the initial values for the GA performing further optimizations. In the following sub-sections, the computational details of the GA and LM methods are described.

2.1. The proposed genetic algorithm

GA is a global search optimization method based on the mechanics of natural selection and natural genetics. It has been applied to different domain [6,9–14]. It works with a coded of parameters string called chromosome instead of the solutions themselves. Each chromosome represents a solution set, and the fitness functions used to measure the survival scores of all chromosomes in the population. Then the GA will accord its selection scheme to select several chromosomes for copulation, discard unwanted chromosomes, and adopt the crossover scheme to produce the new generation. Because the new chromosomes are made by better chromosomes, they may have higher probability to achieve better result. After crossover, one may apply mutation to change some genes in the new chromosomes to achieve higher diversity. Then the GA will apply fitness function for the new population again and loop this cycle until certain stop criteria is achieved.

In this work, the problem is defined as follows:

$$f(S_p, V_{in}, \vec{p}) = O_{result}, \quad (1)$$

where the function f can be regarded as a circuit simulator which solves a system of circuit nonlinear ODEs, the S_p is a netlist required by the circuit simulator, the V_{in} is input bias, and the \vec{p} is the parameters needs to be extracted. By feeding the three components into f , a series of result under different frequencies is generated in O_{result} , such as $S_{11}, S_{12}, S_{21}, S_{22}, \dots$. The GA has to optimize the \vec{p} under several S_p and V_{in} .

The typical GA has basic five genetic operators to perform the solution evolvement. It includes the gene encoding, the fitness evaluation, the chromosome selection, the sexual crossover, and the gene mutation. In the following, we describe each operator step by step.

2.1.1. Gene encoding

The gene encoding method strongly depends on the problem to be optimized. Encoding operator is the procedure that encodes the solution of the problem into the coded string format, and others genetic operators operate on the coded string directly instead of the solution itself. The design of gene encoding strategy strongly depends on the properties of the problem. There are 10 parameters in the LNA circuit. In the proposed GA for the simulation-based evolutionary algorithm, we transform these continuous floating-point numbers into discrete steps through step function as shown in the Eq. (2) instead of real numbers, and we encode the discrete steps as genes on chromosomes. The discrete steps show the strongly combinatorial properties, and we have found this representation has better results in crossover:

$$P_{value} = P_{min} + \frac{P_{max} - P_{min}}{\text{Resolution}}. \quad (2)$$

2.1.2. Fitness evaluation

The fitness evaluation evaluates the fitness score for each chromosome in the population. The fitness score can be seen as the accommodation status of each chromosome in current environment, and it is also an important reference for the selection procedure to judge the suitability of each chromosome. We consider the fitness function:

$$F = \sum_{\text{all target}} (W(\text{sim} - \text{spe})), \quad (3)$$

where the sim and the spe indicate the simulated result and target specification, respectively. The W is a weight function, when the sim is not matched with the spe , the weight increase to emphasize this problem.

2.1.3. Selection

After obtaining the fitness score of each chromosome, the selection method selects the better chromosomes by the specific schemes, such as the roulette wheel selection, the tournament selection, and the ranking selection, and different selection schemes may lead to the different convergence behavior. In this work, the tournament selection method is applied. The pseudocode of a tournament selection is shown below:

```

Set the number of contest up according to the selection rate
for each contest
    Randomly pick 2 competitors
    Select the competitor with better fitness score
End for

```

2.1.4. Crossover

Crossover combines the features of two parent chromosomes to form two similar off-springs by swapping corresponding segment of the parents. It is intuitive that the crossover operator is exchanging information between different potential solutions. We take a uniform crossover scheme in our developed GA; and based on our simulation experience, it is more effective than single and two-point cuts crossover schemes [9,10]. A pseudocode is shown below:

```

Randomly selects 2 parent
for each category of parameters of the new chromosome
    Randomly pick a parent and named it as  $p$ 
    Parameters in this category of the chromosome
    =parameters in this category of  $p$ 
End for

```

2.1.5. Mutation

Mutation arbitrarily changes one or more genes of a selected chromosome by a random variation with a probability factor so called the mutation rate. The mutation procedure may cause the entity unable to accommodate with the environment; however, a successful mutation may lead the evolutionary trend to achieve the better situation. A pseudoprocedure for each step in GA is described as:

```

GeneEncoding through Eq. (2)
do
    Chromosomes selection
    Offsprings reproduce
    Genes mutation
    Fitness evaluation
    if BestChromosome.error < Threshold
    then IfAchieveGoal = true
    else IfAchieveGoal = false
while IfAchieveGoal is false

```

2.2. The adopted Levenberg–Marquardt method

The LM method is a quasi-Newton method to accelerate the Gauss–Newton method [6,10,15–17]. The Gauss–Newton method is the basic algorithm for solving the nonlinear optimization problem. Due to the nonlinear property of the problem, a gradient for each variable can be obtained. It starts from an initial guess, and follows the direction of the normal of the gradient to find the optimal solution. Therefore, the initial guess must be chosen carefully, or the solution may fall into a local optima. Unlike the Gauss–Newton method has the fixed steps toward the solution, LM optimization method detects that some regions with monotonic variation property can be speed up by increasing the step size. On the other hand, when the optimization process encounters a sensitive region, the step should be shorten to avoid skipping the optimum. The procedure of the LM method is shown below:

```

Given:  $a$ ,  $I_D^M$ ,  $I_D^E$ , where “ $a$ ” is parameters set, “ $I_D^M$ ” is the optimization target, and “ $I_D^E$ ” is the initial guess of the solution.
 $\chi^2(a) = \sum \left( \frac{I_D^M - I_D^E(a)}{\sigma} \right)^2$ , where  $\sigma$  is the mean of the measured data.
 $\lambda = 0.0001$ .
Compute  $\alpha'_{jj} \equiv \alpha_{jj}(1 + \lambda)$ ,  $\alpha'_{jk} \equiv \alpha_{jk}$  ( $j \neq k$ )
while not converge
    Solve linear matrix problem:
     $\sum_{l=1}^M \alpha'_{kl} \delta a_l = \beta_k$ , where  $\alpha'_{kl} \equiv \frac{\partial^2 \chi^2}{2 \partial a_k \partial a_l}$ ,  $\beta_k \equiv -\frac{\partial \chi^2}{\partial a_k}$ 
    if  $\chi^2(a + \delta a) > \chi^2(a)$ ,
         $\lambda^{n+1} = \lambda^n + \frac{\chi^2(a + \delta a) - \chi^2(a)}{\alpha}$ ,
    else if  $\chi^2(a + \delta a) < \chi^2(a)$ ,
         $\lambda^{n+1} = \frac{\lambda^n}{\alpha + 1}$ ,
         $a^{n+1} = a^n + \delta a$ ,
End while

```

3. Application to LNA circuit design

The explored LNA circuit, shown in Fig. 1, focuses on the working frequencies ranging from 2.11 to 2.17 GHz. The LNA circuit is with two cascaded MOSFETs. We note that the L_{load} and R_{load} are the nonlinear functions in our LNA circuit [6].

$$\Delta = S_{11} \times S_{22} - S_{21} \times S_{12}, \quad (8)$$

then

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{12}|^2}{2 \times |S_{11}| * |S_{12}|}. \quad (9)$$

To evaluate the noise performance of a low noise amplifier, by the definition of noise factor F , the equation for noise figure (NF) is given by

$$NF = 10 \log(F) = 10 \log \left(\frac{SNR_{in}}{SNR_{out}} \right) = 10 \log \left(\frac{\frac{Signal}{Noise_{in}}}{\frac{Signal}{Noise_{out}}} \right) = 10 \log \left(\frac{Noise_{in} + Noise_{amp}}{Noise_{in}} \right) = 10 \log \left(1 + \frac{Noise_{amp}}{Noise_{in}} \right), \quad (10)$$

where SNR_{in} is the signal-to-noise ratio at the input and SNR_{out} is the signal-to-noise ratio at the output. $Noise_{in}$ is the noise from the previous stage, $Noise_{out}$ is the noise at the output which consists of the noise from amplifier ($Noise_{amp}$) plus the noise from $Noise_{in}$.

The definition of the third-order input intermodulation distortion (IIP3) is the input power in dBm where the fundamental output power and the third-order intermodulation output power are the same.

4. Results and discussion

By calculating several interested specifications, we firstly verify the feasibility of the proposed method. Fig. 2 shows the initial state (dash-dot) and an optimized result (line) of S_{11} parameter. The acceptable result is when $S_{11} < -10$ dB within the working frequency range. It is clearly that Fig. 2 states that the result has achieved to this goal. We note that the amplitude of the input sinusoidal signal within the working frequency range is with 1.0 V, $V_{B1} = 0.75$ V, and $V_{B2} = 2.7$ V, shown in Fig. 1. Fig. 3 shows the initial state and an optimized result of S_{12} parameter. The acceptable result is when $S_{12} < -25$ dB within the working frequency range, where Fig. 3 clearly confirms the achieved results with much more improvements than the original one. Fig. 4 shows a comparison between the initial state and an optimized result of S_{21} parameter, where a larger S_{21} is expected in the optimization process. Typically, we do not define an engineering specification for S_{21} in this testing case. However, a large value of S_{21} is good for optimal design of LNA circuits. Compared with the initial data, the obtained optimized result is improved. This phenomenon is due to a compromise among all physical constraints so that all characteristics can meet their targets at the same time. However, it can be further improved by performing more evolution generations. Fig. 5 shows the initial state and an optimized result of S_{22} parameter. The goal is the same with the parameter S_{11} , i.e., the result is acceptable if $S_{22} < -10$ dB within the working frequency range. Very good result, -20 dB is achieved when we refer to the setting of standard goal. Once the S parameters are optimized, with Eq. (9), we also calculate the K factor accordingly [1].

Fig. 6 shows a comparison between the initial state and an optimized result of the noise figure. The desired specification of the noise figure is that $NF < 2$ within the working frequency range. Both the initial state and an optimized result meet the target, but the obtained optimized result is a little bit shifted away due to the same reason of a global compromise among all electrical characteristics. Fig. 7 indicates the initial state and an optimized result of the input third-order intercept point. For the optimization criterion of IIP3, we do hope that the amplitude of output > -20 dB and is as large as possible. As shown in Fig. 7, the optimized IIP3 = -26 . Table 1 shows the optimized parameters of the investigated experiment and the Table 2 shows the corresponding optimized characteristics for the experiment. Results shown in both tables confirm the validity of the method.

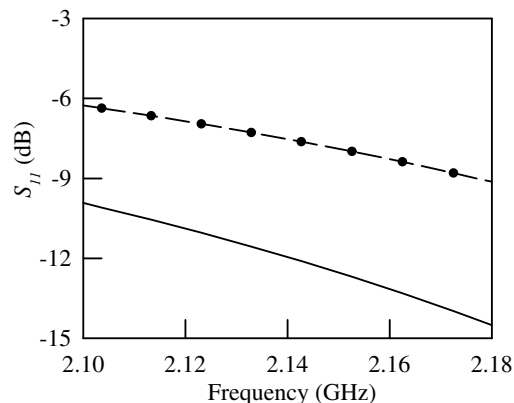


Fig. 2. The initial state (dash-dot) and an optimized result (line) for the parameter of S_{11} .

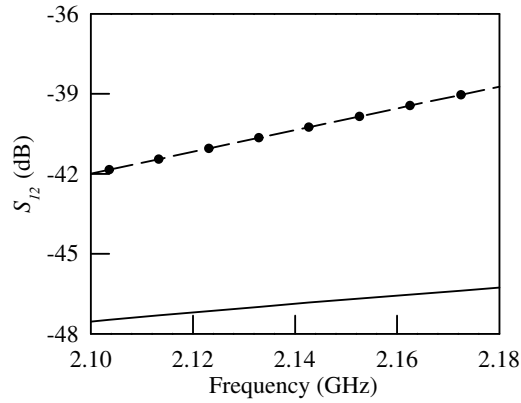


Fig. 3. The initial state (dash-dot) and an optimized result (line) for the parameter of S_{12} .

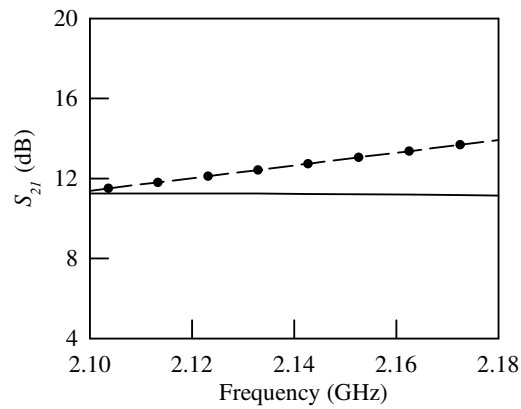


Fig. 4. The initial state (dash-dot) and an optimized result (line) for the parameter of S_{21} .

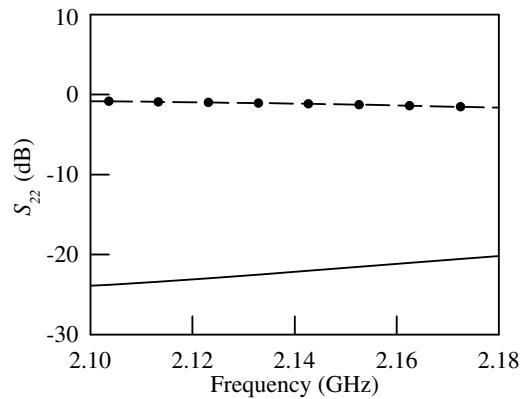


Fig. 5. The initial state (dash-dot) and an optimized result (line) for the parameter of S_{22} .

Inspecting the sensitivities of extracted parameters is an important work in IC design and performance analysis. The sensitivity examination of circuit parameters can point out which parameters affect behavior of the performance the most and which ones barely make effect. This experiment is designed as follow. The proposed simulation-based evolutionary approach optimizes single parameters category meanwhile locks other parameters. The LNA circuit parameters to be optimized are classified into three categories illustrated in Table 3. As shown in Fig. 8, it reveals that the geometry parameters would make the most improvement, while the input and output categories makes little improvement after 120 generations. This phenomenon indicates that the geometry of the active devices is more difficult to be optimized than passive device parameters.

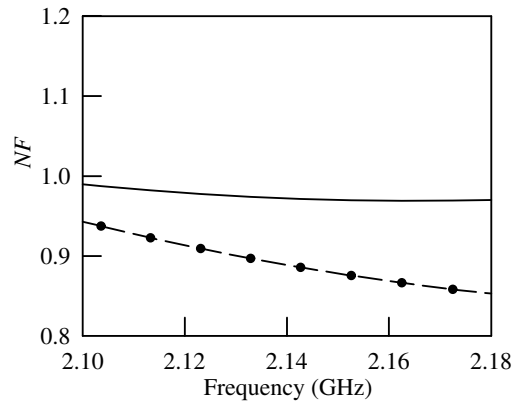


Fig. 6. The initial state (dash-dot) and an optimized result (line) of the parameter NF.

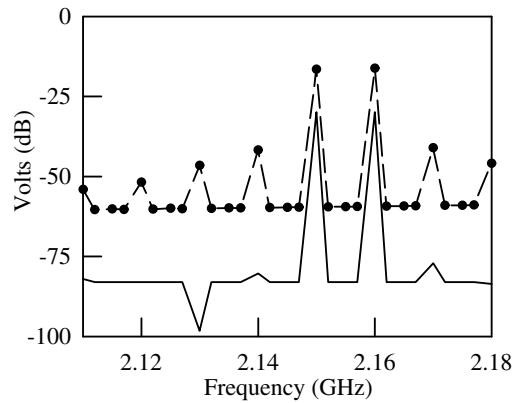


Fig. 7. The initial state (dash-dot) and an optimized result (line) of the parameter IIP3.

Table 1

A list of the optimized parameters for the testing experiment

Element	Unit	Range	Result
C_{match1}	fF	300–800	512.132
C_{match2}	pF	1–10	4.6104
C_{match3}	pF	1–10	4.5511
L_{bond}	nH	1–10	1.0782
L_{deg}	nH	0.1–5	1.145
L_{match1}	nH	1–10	6.202
R_{load}	Ω	1.5–5.5	3.5
L_{load}	H	1.5–5.5	3.5
V_{B1}	V	0.5–1.5	0.75
V_{B2}	V	0.5–5	2.7

Table 2

The final achieved result

Specification	Target	Result
S_{11}	< -10 dB	-14.1 dB
S_{22}	< -10 dB	-22.6 dB
S_{12}	< -25 dB	-39.3 dB
S_{21}	As large as possible	12.7 dB
K	< 1	10.7
NF	< 2	0.979
IIP3	< -10	-1.3

Table 3
Three categories of the circuit parameter of the LNA circuit

Category	Parameters
Geometry	$L_1, W_1, L_2,$ and W_2
Input	$C_{match1}, L_{match1}, L_{bond}, L_{choke}, C_{in}, L_{deg}, V_{B1},$ and V_{B2}
Output	$L_{load}, R_{load}, C_{match2},$ and C_{match3}

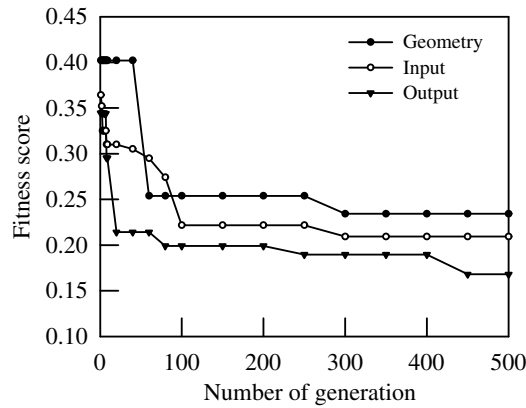


Fig. 8. A verification of the sensitivity analysis for the three cataloged parameters.

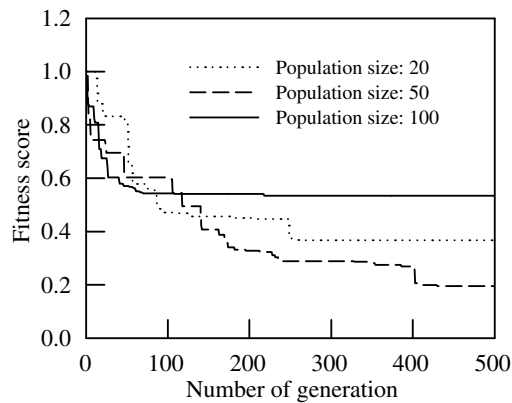


Fig. 9. The fitness score versus the number of generations with respect to different population sizes.

By considering the experiment, Fig. 9 shows a comparison of the score convergence behavior among population sizes, where the mutation rate is fixed at 0.5. The fitness score versus the number of generation suggests that the score convergence behavior does not have a satisfied result if the population size is too small. According to our experience, the population size = 50 is good for the optimal design of LNA circuit.

In addition, Fig. 10 shows the fitness score convergence behavior for the circuit optimization with different mutation rate, where the population size = 50. The results suggest that the mutation = 0.5 keeps the population diversity and finally has better evolutionary results.

Finally, the computational efficiency of the proposed simulation-based evolutionary approach is investigated. Fig. 11 shows the score convergence behavior comparison of the standard GA and the simulation-based evolutionary approach. The setting is with the population size = 50 and mutation = 0.5. As shown in this figure, the proposed methodology is superior to the pure GA after 60 generations. The proposed method shows no significant advantage at the beginning because the LM method has not been triggered yet. Once the LM method is activated, based on the result of GA to perform local optimization, the GA follows the local optima obtained by the LM method to keep evolving. Under this mechanism, our simulation-based evolutionary approach shows better trend of convergence and the robustness of our proposed methodology hence is held. We note that the LM method encounters divergent results when the optimization problem is solved.

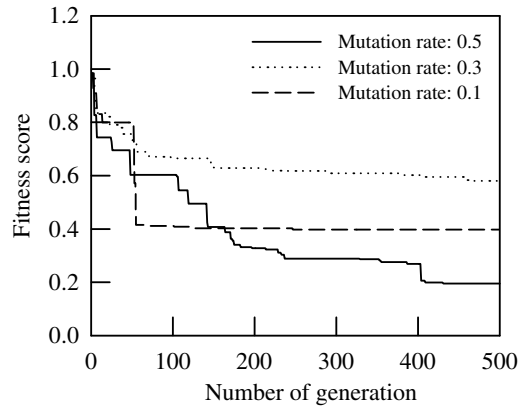


Fig. 10. The fitness score versus the number of generations with respect to different mutation rates.

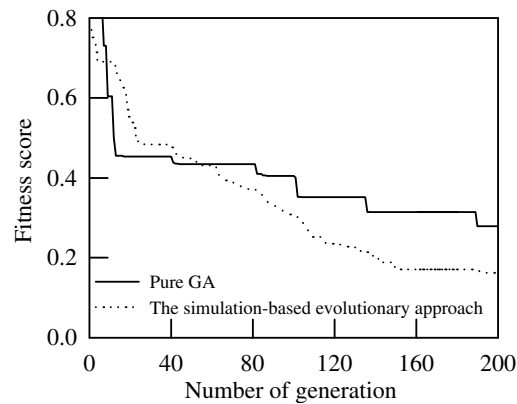


Fig. 11. The fitness score versus the number of generations for the pure GA and the proposed simulation-based evolutionary approach.

5. Conclusions

In this paper, a simulation-based evolutionary approach for optimal design of LNA circuit has been reported. Based on the GA, the LM, and a well-known circuit simulator, the engineering optimization problem has successfully been solved according to the simulation-based evolutionary approach. Electrical characteristics of the explored LNA circuit considered in the optimization process are S_{11} , S_{12} , S_{21} , S_{22} , K factor, the noise figure, and the input third-order intercept point. Testing examples of LNA circuits with the $0.18 \mu\text{m}$ MOSFETs have been examined to show the validity, efficiency, and robustness of the method. To explore the realistic feasibility of optimized designs of the LNA circuit, we are currently fabricating and testing the functionality of the corresponding IC chips. We note that this simulation-based evolutionary approach can also be applied to optimal design of other circuit modules with more advanced technology nodes. Developed algorithms that based on this simulation-based evolutionary approach can be directly incorporated into any existed electronic computer-aided design software which benefits the communities of design and fabrication. Currently, we apply the method to optimal design of operation amplifier and phase-locked loop circuits, and pattern optimization of antenna problems. Furthermore, to enhance the efficiency, distributed computing technique will be considered [20].

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