APPENDIX

The analytical model of the large-angle magnetic suspension test facility is

$$\dot{x} = A_m x + B_m u \tag{A1}$$

$$y = C_m x \tag{A2}$$

where $x = \{{x_p \atop \dot{x}_p}\}$, $A_m = \begin{bmatrix} 0_{5\times5} & I_{5\times5} \\ A_{21} & A_{22} \end{bmatrix}$, $B_m = \begin{bmatrix} 0_{5\times5} \\ B_2 \end{bmatrix}$ and $C_m = [C_1 \; 0_{5\times5}]$. The state variable x_p includes pitch and yaw angles and three linear displacements of the cylinder's centroid. The matrices A_{21}, A_{22}, B_2, and C_1 are

The eigenvalues of the system matrix A_m are ±58.78, ±57.81, ±9.78, $\pm j7.97$, and $\pm j0.96$. The matrix C_1 which relates the sensor output voltage to the displacement can be obtained from calibration and is assumed known. To recover the displacement from the sensor output voltage, one can use $x_p = C_1^{-1} y$.

The performance index for the state feedback design is chosen as

$$P.I. = \sum_{k=1}^{\infty} y_k^T Q y_k + u_k^T R u_k \tag{A3}$$

where $Q = (C_1^{-1})^T \operatorname{diag}[1.e3 \; 1.e3 \; 2.e8 \; 2.e8 \; 2.e8] C_1^{-1}$ and $R = I_{5\times5}$.

$$A_{21} = \begin{bmatrix} 3.3415e+03 & 0 & -3.9392e+04 & 4.9534e-12 & 2.0811e-12 \\ 0 & 3.3415e+03 & -4.9534e-12 & 4.8609e-12 & -1.4472e-11 \\ -9.8070e+00 & -2.4664e-15 & 4.9937e+01 & 4.3604e-15 & -2.5089e-02 \\ -3.6031e-15 & 1.9618e-15 & 4.3604e-15 & 9.5577e+01 & -9.0007e-15 \\ -2.3357e-16 & -3.6031e-15 & -2.5089e-02 & -9.0007e-15 & -9.1324e-01 \end{bmatrix},$$

$$A_{22} = 0_{5\times5},$$

$$B_2 = \begin{bmatrix} 3.8370e+01 & 3.8370e+01 & 3.8370e+01 & 3.8370e+01 & 3.8370e+01 \\ 0 & 8.9802e+01 & 5.5514e+01 & -5.5514e+01 & -8.9802e+01 \\ 2.2144e-01 & -1.5274e-01 & 7.8453e-02 & 7.8453e-02 & -1.5274e-01 \\ 0 & 1.2154e-01 & -1.9674e-01 & 1.9674e-01 & -1.2154e-01 \\ -2.7672e-01 & -8.5465e-02 & 2.2388e-01 & 2.2388e-01 & -8.5465e-02 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 8.9024e+01 & 0 & 0 & 0 & 6.0976e+03 \\ 0 & 0 & 7.8740e+03 & 0 & 0 \\ -1.1625e+02 & 0 & 0 & 0 & 6.2500e+03 \\ 0 & 9.5425e+01 & 0 & -6.5359e+03 & 0 \\ 0 & -1.0725e+02 & 0 & -5.1813e+03 & 0 \end{bmatrix}.$$

Feedforward and Feedback Control Strategy for Active Noise Cancellation in Ducts

Jwu-Sheng Hu[1]

This paper presents the theoretical work about active noise cancellation in ducts. The proposed control system is designed based on the assumption of a one-dimensional sound field. The controller consists of a feedforward block which serves as a noise observer. The feedback portion of the control algorithm is to minimize residual disturbances. Closed-loop stability of the MIMO (multiple-input-multiple-output) system is analyzed and the result shows that the dynamic influenced by the space-feedforward and feedback controllers can be decoupled. Both semi-infinite and finite-length ducts are considered in this study and simulation examples are given to illustrate the effectiveness of the proposed controllers.

1 Introduction

Active Noise Control (ANC) utilizes the physical principle of wave superposition to attenuate unwanted noise. During the past decade, much progress has been reported toward applying ANC to attenuate noise in confined spaces. A review by Nelson and Elliott (1992) presents the fundamental principles underlying modern techniques for ANC while Gorden and Vining (1992) provide an overview of the field. The most successful application so far is to cancel noise in ducts, especially cancellation of plane waves. Utilizing the principle of wave propagation, the active controller can be synthesized by introducing delay elements (Eghtesadi and Leventhall, 1982; Guicking and Karcher, 1984). Although the idea is not new, a complete theoretical analysis on its performance and closed loop stability has not been presented, especially in a finite-length duct without prior knowledge of the noise source (Trinder and Nelson, 1983). The concept of controlling wave propagation is also investigated in structural vibration suppression (von Flotow and Shäfer, 1986; von Flotow, 1986; Pines and von Flotow, 1990; Tanaka and Kikushima, 1992). The problem is more complicated because of the distributed-delay dynamic of the plant.

In this paper, theoretical studies about noise cancellation in semi-infinite and finite-length ducts are presented. While many ANC research are conducted in the Fourier Transform domain, the proposed controllers are designed using the Laplace Transform so as to avoid causality problem (Curtis et al., 1987) and to analyze closed loop stability. The plant is derived by assuming a one-dimensional sound field in ducts (Hu, 1993). The controllers' structures are explained by analyzing the interconnection of delay elements in the plant model. For a semi-infinite duct, an upstream microphone is used to observe noise propagating in the downstream direction. This signal is then fed into the controller as a *space-feedforward* command to attenuate noise transmitting downstream. Theoretically, complete cancellation can be achieved through this control configuration. An additional microphone placed at a downstream location to pick up

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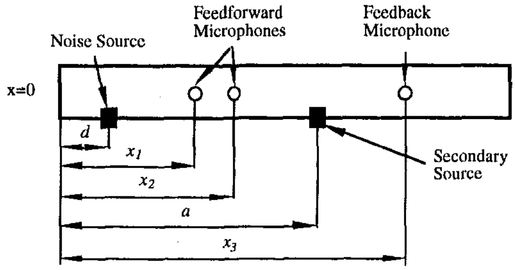

Fig. 1 Schematic diagram of a finite-length duct (all the variable are dimensionless)

any residual noise is used to generate a feedback signal. Closed-loop stability of the MIMO (multiple-input-multiple-output) system is analyzed and the result shows that the dynamic influenced by the space-feedforward and feedback controllers can be decoupled.

For a finite-length duct, the duct's dynamic becomes more complicated due to reflection from the downstream boundary. Consequently, two upstream microphones are needed to observe noise propagating downstream. Unlike the semi-infinite duct, only asymptotic cancellation can be achieved in a finite-length duct using the proposed control strategy. The error dynamic is governed by the characteristic equation of the open loop plant. A complete analysis of the closed loop system including a feedback microphone is also presented.

It should be noted that the objective of the work is to develop a theoretical foundation of noise cancellation based on wave propagation principles and close-form solutions in the Laplace Transform domain. Other important issues such as the robustness of the control system and actuator and sensor dynamics will be considered in the following studies.

2 The Dynamic Model of a Finite-Length Duct

The schematic diagram of a finite-length duct is shown in Fig. 1. It is assumed that the noise (primary source) enters the duct at d, a control speaker (secondary source) is placed at a, and x represents a point within the duct. The length of the duct is L and a, d, and x are dimensionless variables. The specific impedance of both ends is denoted as $z_0(s)$ at $x = 0$ and $z_1(s)$ at $x = 1$ where s is the Laplace variable. It is further assumed that the boundaries are passive. This implies that the impedance functions are positive real (Hu, 1993). Moreover, the pressure reflection coefficients for each end can be defined from the impedance functions as

$$\theta_0(s) = \frac{1 - z_0(s)}{1 + z_0(s)} \quad \text{and} \quad \theta_1(s) = \frac{1 - z_1(s)}{1 + z_1(s)}$$

Denoting the strength of the noise and control source as $N(s)$ and $Q(s)$ respectively, the transfer function of the speaker to the source pressure can be derived as

When $d \le x \le a$,

$$\begin{aligned} p(x, s) &= \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x, s) \\ &\quad + [G_U^+(x, a, s) + G_U^-(x, a, s)]Q(s) \\ &\quad + [G_D^+(x, d, s) + G_D^-(x, d, s)]N(s) \qquad (1a) \end{aligned}$$

and when $d \le a \le x$,

$$\begin{aligned} p(x, s) &= \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x, s) \\ &\quad + [G_D^+(x, a, s) + G_D^-(x, a, s)]Q(s) \\ &\quad + [G_D^+(x, d, s) + G_D^-(x, d, s)]N(s) \qquad (1b) \end{aligned}$$

where

c: speed of sound

$$G_D^+(x, a, s) = e^{-(Ls/c)(x-a)} + \theta_0(s)e^{-(Ls/c)(x+a)} \qquad (1c)$$

$$G_D^-(x, a, s) = -\theta_1(s)e^{-(Ls/c)(2-x-a)} - \theta_0(s)\theta_1(s)e^{-(Ls/c)(2-x+a)} \qquad (1d)$$

$$G_U^-(x, a, s) = -e^{-(Ls/c)(a-x)} - \theta_1(s)e^{-(Ls/c)(2-x-a)} \qquad (1e)$$

$$G_U^+(x, a, s) = \theta_0(s)e^{-(Ls/c)(a+x)} + \theta_0(s)\theta_1(s)e^{-(Ls/c)(2+x-a)} \qquad (1f)$$

The above equations are obtained from Green's function of a one-dimensional wave equation (Hu, 1993). The superscripts/subscripts used in the above equations are based on physical meanings of the transfer function. Subscript D means downstream position, U means upstream position, superscript $+$ means pressure wave propagating in the downstream direction, and $-$ means pressure wave propagating in the upstream direction. For example, $G_D^-(x, a, s)$ represents the influence of the source at downstream position but propagating in the upstream direction. It is easy to see that $G_D^+(x, a, s)$ consists of a direct propagating wave (the first term on the left-hand side) and a wave reflected from the boundary $x = 0$ (the second term). Further, the multiplication of $\theta_0(s)$ in the second term shows that part of the energy is absorbed by the passive boundary. Notice that a delayed superposition appears in Eq. (1a) and (1b). This term represents the wave traveling twice the length of the duct. If no energy is absorbed at both boundaries ($\theta_0(s)$, $\theta_1(s) = \pm 1$), the reflected wave will be reinforced and results in resonance at certain frequencies.

The transfer functions defined in Eq. (1c) to (1f) possess some interesting properties. Let x, y be two arbitrary points and $x, y \ge a$, the following relations can be derived.

$$-G_D^-(x, a, s)G_D^+(x, d, s) + G_D^-(x, d, s)G_D^+(x, a, s) = 0 \qquad (P1)$$

$$-G_D^+(x, a, s)G_D^+(y, d, s) + G_D^+(x, d, s)G_D^+(y, a, s) = 0 \qquad (P2)$$

$$-G_D^-(x, a, s)G_D^+(y, d, s) + G_D^-(x, d, s)G_D^+(y, a, s) = 0 \qquad (P3)$$

These properties will be used later for constructing a noise observer and controller.

3 Noise Cancellation in a Semi-Infinite Duct

The diagram of a semi-infinite duct is shown in Fig. 2. Its dynamic can be derived from Eq. (1) by letting the impedance at $x = 1$ equal 1 (i.e., zero pressure reflection coefficient). The control signal is derived from the microphone placed upstream (at x_2). The purpose of putting another microphone downstream (x_1 in Fig. 2) will be explained later. Notice that in a semi-infinite duct, we can no longer use dimensionless space variables. Assuming that the effect of noise at x_2 is $N(s)$, the pressure response can be written as

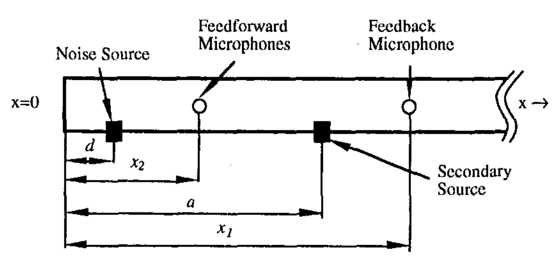

Fig. 2 Schematic diagram of a semi-infinite duct

$$p(x_2, s) = \tfrac{1}{2}(-e^{-(s/c)(a-x_2)} + \theta_0(s)e^{-(s/c)(x_2+a)})Q(s) + N(s) \quad (2a)$$

Moreover, the pressure response at any downstream position $x > a$ is

$$p(x, s) = \tfrac{1}{2}(e^{-(s/c)(x-a)} + \theta_0(s)e^{-(s/c)(x+a)})Q(s) + e^{-(s/c)(x-x_2)}N(s) \quad (2b)$$

The objective is to block the transmission of noise, i.e., $p(x, s) = 0$. Solving $Q(s)$ by letting Eq. (2b) equal zero, we have

$$Q(s) = \frac{-2e^{-(s/c)(x-x_2)}}{e^{-(s/c)(x-a)} + \theta_0(s)e^{-(s/c)(x+a)}} N(s)$$

Substituting Eq. (2a) into the above equation, the control signal can be shown as

$$Q(s) = \frac{-2e^{-(s/c)(a-x_2)}}{1 + e^{-(2s/c)(a-x_2)}} p(x_2, s) \quad (3)$$

The control law can also be derived by combining Eq. (2a) and (2b):

$$p(x, s) - e^{-(s/c)(x-x_2)}p(x_2, s) = \tfrac{1}{2}(e^{-(s/c)(x-a)} + e^{-(s/c)(a+x-2x_2)})Q(s) \quad (4)$$

An interesting observation of Eq. (3) is that the control signal is independent of the impedance at the boundary. In fact, the result is the same as the case of an infinite duct (Eghtesadi and Leventhall, 1982). From Fig. 2, it is obvious that the purpose of placing a microphone upstream is to detect the noise before it arrives at the position of the cancellation speaker. Therefore, it is characterized as a *space-feedforward* control signal. Nevertheless, in time domain, it is still a feedback control system and the closed-loop response at x_2 can be derived as

$$p(x_2, s) = \frac{1 + e^{-(2s/c)(a-x_2)}}{1 + \theta_0(s)e^{-(2s/c)a}} N(s) \quad (5)$$

Equation (5) shows that the cancellation speaker acts like a hard-walled boundary (impedance equals infinity). Since $\theta_0(s)$ is derived from a passive boundary, the closed-loop system is stable (Hu, 1993).

To accommodate the model uncertainty, a "true" feedback signal should be used in the overall control system. Since the target area is the downstream area of the duct, the error signal measured at $x_1(>a)$ is included in the controller as

$$Q(s) = \frac{-2e^{-(s/c)(a-x_2)}}{1 + e^{-(2s/c)(a-x_2)}} p(x_2, s) + \frac{-2C(s)}{1 + e^{-(2s/c)(a-x_2)}} p(x_1, s) \quad (6)$$

where $C(s)$ is the feedback compensator. The block diagram of the overall control system is shown in Fig. 3. Suppose some bounded uncertainty $\delta(s)$ (Fig. 3) is presented at x_1, the response of $p(x_1, s)$ can be shown as

$$p(x_1, s) = \frac{\delta(s)}{1 + C(s)e^{-(s(x_1-a)/c)}} \quad (7)$$

The controller $C(s)$ can be designed to minimize the effect of the disturbance. Since the control system is a multiple-input-multiple-output (MIMO) system, it is essential to analyze the global stability. As will be shown later, the stability is guaranteed if and only if both Eqs. (5) and (7) are stable. Substituting Eq. (6) into Eq. (2), the closed-loop transfer function can be derived as

$$\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} p(x_1, s) \\ p(x_2, s) \end{bmatrix} = \begin{bmatrix} N(s) \\ e^{-(2s/c)(x_1-x_2)}N(s) + \delta(s) \end{bmatrix} \quad (8)$$

where

$$g_{11}(s) = -\frac{e^{-(s/c)(a-x_2)} - \theta_0(s)e^{-(s/c)(a+x_2)}}{(1 + e^{-(2s/c)(a-x_2)})} C(s)$$

$$g_{12}(s) = 1 - \frac{e^{-(2s/c)(a-x_2)} - \theta_0(s)e^{-(2s/c)a}}{(1 + e^{-(2s/c)(a-x_2)})}$$

$$g_{21}(s) = 1 + \frac{e^{-(s/c)(x_1-a)} + \theta_0(s)e^{-(s/c)(x_1+a)}}{1 + e^{-(2s/c)(a-x_2)}} C(s)$$

$$g_{22}(s) = \frac{e^{-(s/c)(x_1-x_2)} + \theta_0(s)e^{-(s/c)(x_1+2a-x_2)}}{(1 + e^{-(2s/c)(a-x_2)})}$$

It is straightforward to verify that the characteristic equation of the above equation is

$$(1 + \theta_0(s)e^{-(2s/c)a})(1 + C(s)e^{-(s(x_1-a)/c)}) = 0 \quad (9)$$

The condition on $C(s)$ to guarantee stability of the system can be checked by using various frequency domain methods (Marshall, 1979). This condition depends on the value of $x_1 - a$. An obvious choice of x_1 to simplify the design is $x_1 = a$ (collocated sensor and actuator).

4 Noise Cancellation in a Finite-Length Duct

The dynamic of sound in a finite-length duct (Section 2) is more complicated than the semi-infinite one. Unlike the semi-infinite duct, the noise travels in the upstream direction due to reflection from the boundary ($x = 1$ in Fig. 1). As a result, the space-feedforward signal can not be constructed by a single microphone only. To better explain the proposed control system, we first examine the following control law:

$$Q(s) = \frac{-G_D^+(x, d, s)}{G_D^+(x, a, s)} N(s) \quad (10)$$

where $x \geq a$ is a downstream location. Substituting Eq. (10) into Eq. (1b) and using (P1), the pressure response at x satisfies

$$p(x, s) - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x, s) = 0 \quad (11)$$

This means that the pressure will go to zero asymptotically. The proposed control law (Eq. (10)) is derived by letting the controlled sound propagating downstream cancel the noise propagating downstream. Once the downstream-propagating noise is canceled, the upstream-propagating noise will vanish automatically as demonstrated by Eq. (11). The same result can also be obtained at other downstream location $y \geq a$. According (P2) and (P3), the response at y can be derived.

$$\begin{aligned} p(y, s) &- \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(y, s) \\ &= [G_D^+(y, a, s) + G_D^-(y, a, s)] \frac{-G_D^+(x, d, s)}{G_D^+(x, a, s)} N(s) \\ &\quad + [G_D^+(y, d, s) + G_D^-(y, d, s)]N(s) = 0 \end{aligned}$$

In order to implement Eq. (10), a noise observer based on microphone measurements has to be constructed. Let two microphones be placed at x_1 and x_2 (Fig. 1), from Eq. (1a), the following relation can be derived.

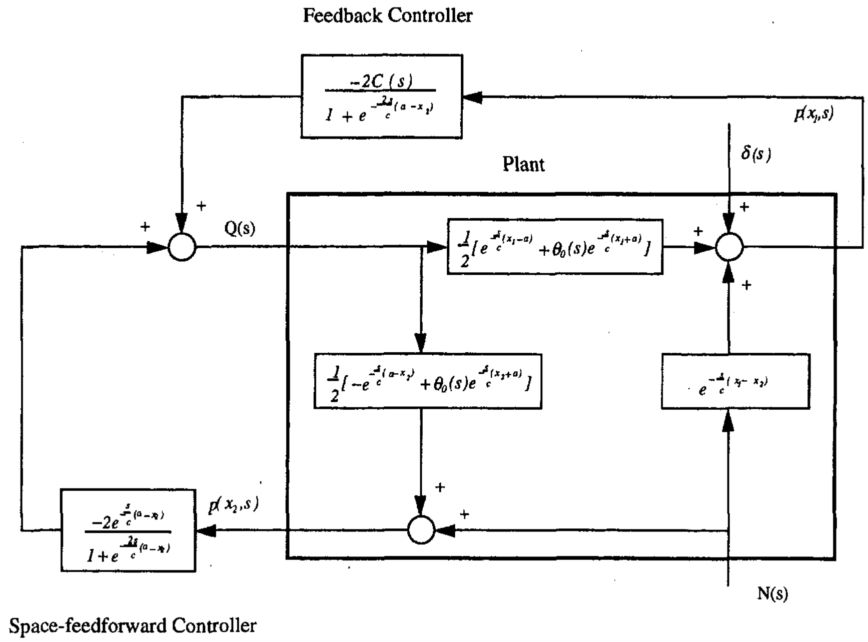

Fig. 3 Block diagram of active noise control system in a semi-infinite duct

$$\hat{p}_1 - \hat{p}_2 e^{-((x_2-x_1)/c)Ls} = (1 - e^{-(2(x_2-x_1)/c)Ls}) \times [G_U^+(x_1, a, s)Q(s) + G_D^+(x_1, d, s)N(s)] \quad (12a)$$

where

$$\hat{p}_1 = p(x_1, s) - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x_1, s) \quad (12b)$$

and

$$\hat{p}_2 = p(x_2, s) - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x_2, s) \quad (12c)$$

Further, the noise propagating downstream satisfies the following equation.

$$G_D^+(x, d, s)N(s) = e^{-((x-x_1)/c)Ls}G_D^+(x_1, d, s)N(s)$$

As a result, the downstream noise observer can be designed as

$$G_D^+(x, d, s)N(s) = \frac{(\hat{p}_1 - \hat{p}_2 e^{-((x_2-x_1)/c)Ls})e^{-((x-x_1)/c)Ls}}{1 - e^{-(2(x_2-x_1)/c)Ls}} - G_U^+(x_1, a, s)e^{-((x-x_1)/c)Ls}Q(s) \quad (13)$$

Substituting Eq. (12*b*), (12*c*), and (13) into Eq. (10), the control law becomes

$$Q(s) = \frac{-(p(x_1, s) - p(x_2, s)e^{-((x_2-x_1)/c)Ls})e^{-((a-x_1)/c)Ls}}{1 - e^{-(2(x_2-x_1)/c)Ls}} \quad (14)$$

It is interesting to see that the control law is independent of the impedance functions at both boundaries. By applying Eq. (14), the pressure response at upstream location is altered. As shown in the Appendix, the pressure response at y, $d < y < a$, is

$$p(y, s) = \frac{(1 + e^{-(2Ls/c)(y-a)})(1 + \theta_0(s)e^{-(2Ls/c)d})e^{(Ls/c)(2a-y-d)}}{1 + \theta_0(s)e^{-(2Ls/c)a}} N(s) \quad (15)$$

As clearly shown in the denominator of Eq. (15), the secondary source acts like a hard-walled boundary. The space-feedforward controller in Eq. (14) is combined with a feedback signal as

$$Q(s) = \frac{-(p(x_1, s) - p(x_2, s)e^{-((x_2-x_1)/c)Ls})e^{-((a-x_1)/c)Ls}}{1 - e^{-(2(x_2-x_1)/c)Ls}} - \frac{C(s)p(x_3, s)}{1 - \theta_1(s)e^{(2Ls/c)(1-x_3)}} \quad (16)$$

where x_3 is a downstream position as indicated in Fig. 1. The complete block diagram of the control system is depicted in Fig. 4 where $\delta(s)$ is a bounded uncertainty presented at x_3. As shown in the Appendix, the pressure response at x_3 is

$$p(x_3, s) = \frac{\delta(s)}{[1 + C(s)e^{-(Ls/c)(x_3-a)}][1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]} \quad (17)$$

Similar to the case of semi-infinite duct, an obvious choice of x_3 is a.

Finally, we examine the closed-loop characteristic equation of the overall system. As shown in the Appendix, the characteristic equation can be derived as

$$[1 + \theta_0(s)e^{-(2Ls/c)a}][1 + C(s)e^{-(Ls/c)(x_3-a)}] \times [1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}] = 0 \quad (18)$$

which consists of the dynamics appeared in Eqs. (15) and (17).

5 Simulation Examples

The following parameters are used to simulate the control system for a semi-infinite duct (Fig. 2).

$$a = 0.7m,\ d = 0.1m,\ x_1 = 0.7m,\ x_2 = 0.4m, \text{ and } q_0 = 0.5.$$

The speed of sound is assumed to be 350 m/s. It is noted that the feedback microphone is placed at the speaker's position (collocated). Figure 5 shows the pressure response of a pure space-feedforward control (assuming $\delta(s) = 0$). The response is measured at $x = 2.3m$. It can be seen that once the controller is turned on, the pressure at downstream position becomes zero as explained in Section 3. Figure 6 shows the pressure response at x_1 when $d(s)$ is not zero and the feedback control law is included. The feedback controller is selected as

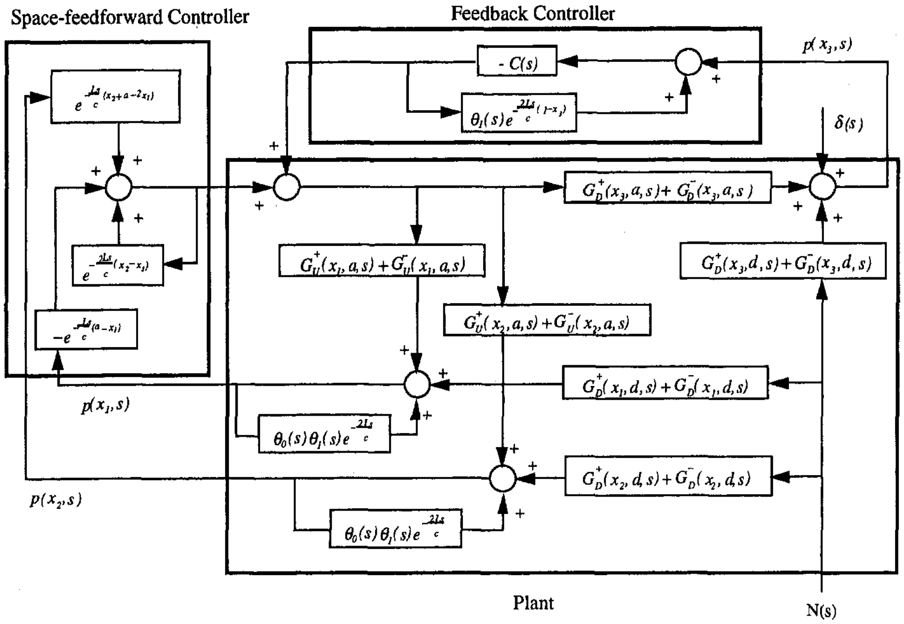

Fig. 4 Block diagram of active noise control system in a finite-length duct

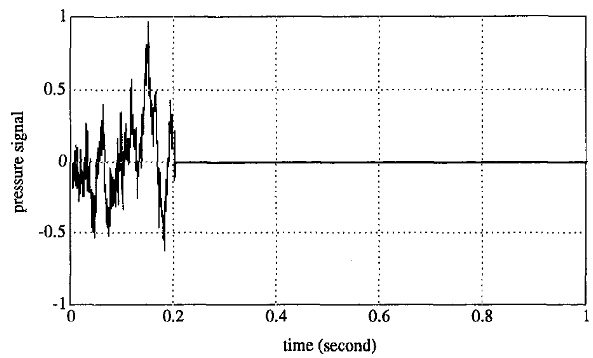

Fig. 5 Pressure response of a pure space-feedforward control in a semi-infinite duct (controller is turned on at $t = 0.2$ s)

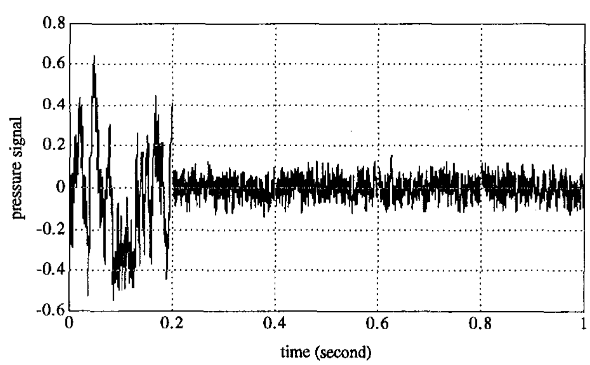

Fig. 6 Pressure response of space-feedforward and feedback control in a semi-infinite duct (non-zero disturbance $\delta(s)$ as shown in Fig. 3)

$$C(s) = \frac{500}{s + 100} \tag{19}$$

The case of sound cancellation in a finite-length duct is also simulated by using the following parameters,

$$L = 3.13m, \quad a = 0.379, \quad d = 0.076, \quad x_1 = 0.228,$$
$$x_2 = 0.176, \quad x_3 = 0.379, \quad \text{and} \quad q_0 = q_1 = 0.5.$$

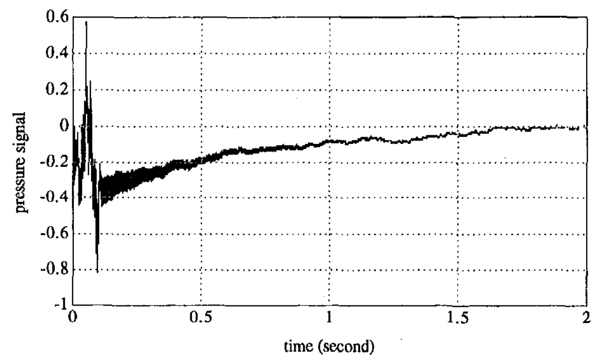

Fig. 7 Pressure response of a pure space-feedforward control in a finite-length duct

Figure 7 shows the result of pure space-feedforward control when $\delta(s) = 0$. The pressure goes to zero asymptotically as indicated by Eq. (11). The same controller $C(s)$ (Eq. (19)) is used to simulate the feedback control. Figure 8 shows the pressure response at x_3.

6 Conclusion

Active noise cancellation in ducts based on delay relations of wave propagation is discussed. The proposed controllers use

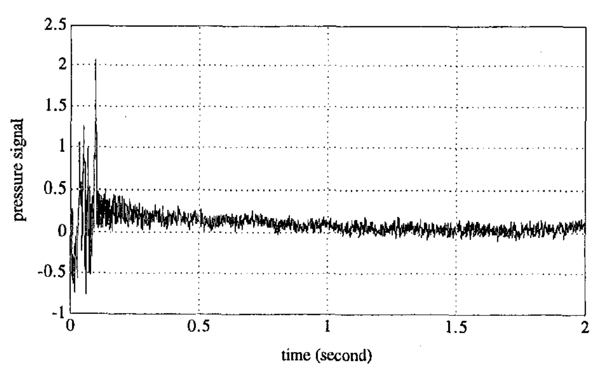

Fig. 8 Pressure response of space-feedforward and feedback control in a finite-length duct (non-zero disturbance $\delta(s)$ as shown in Fig. 4)

both space-feedforward and feedback sound measurements. It is shown that by carefully selecting the delay elements in the controller, prior information of noise can be observed. Most importantly, the design results in simple closed-loop transfer functions which are easy to analyze. More theoretical development as well as experimental verification will be conducted in future research work.

Acknowledgment

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APPENDIX

Proof of Eq. (15):

Defining $\hat{p}_y$ as the following

$$\hat{p}_y = p(y, s) - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(y, s) \tag{A-1}$$

From Eq. (1*a*), we have:

$$\hat{p}_y = [G_U^+(y, a, s) + G_U^-(y, a, s)]Q(s) + [G_D^+(y, d, s) + G_D^-(y, d, s)]N(s)$$

Using control law of Eq. (10), the following equation can be derived.

$$\begin{aligned}\hat{p}_y &= -[G_U^+(y, a, s) + G_U^-(y, a, s)]\frac{G_D^+(x, d, s)}{G_D^+(x, a, s)}N(s) + [G_D^+(y, d, s) + G_D^-(y, d, s)]N(s)\\ &= -[G_U^-(y, a, s) + G_U^-(y, a, s)]\frac{G_D^+(y, d, s)e^{-(x-y)(L/c)s}}{G_D^+(y, a, s)e^{-(x-y)(L/c)s}}N(s) + [G_D^+(y, d, s) + G_D^-(y, d, s)]N(s)\\ &= \frac{[G_D^+(y,d,s)G_D^+(y,a,s) + G_D^-(y,d,s)G_D^+(y,a,s)]N(s)}{G_D^+(y,a,s)} - \frac{[G_U^+(y,a,s)G_D^+(y,d,s) + G_U^-(y,a,s)G_D^+(y,d,s)]N(s)}{G_D^+(y,a,s)}\end{aligned}$$

Expanding the numerators in the above equation and rearranging the all terms, we obtain:

$$\hat{p}_y = \frac{[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}][e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(y-a)}]}{G_D^+(y, a, s)}N(s)$$

Equation (15) can be derived by substituting Eq. (A1) into the above equation.

Proof of Eq. (17):

Substituting Eq. (12*a*) into Eq. (14), we have:

$$\begin{aligned}Q(s) &= \frac{-[G_U^+(y, a, s)Q(s) + G_D^+(y, d, s)N(s)]e^{-(Ls/c)(a-y)}}{1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}} - \frac{C(s)p(x, s)}{1 - \theta_1(s)e^{-(2Ls/c)(1-x)}}\\ &= \frac{-[\theta_0(s)e^{-(2Lsa/c)} + \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]Q(s) - G_D^+(y, d, s)N(s)e^{-(Ls/c)(a-y)}}{1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}} - \frac{C(s)p(x, s)}{1 - \theta_1(s)e^{-(2Ls/c)(1-x)}}\end{aligned}$$

The above equation can be rearranged as follows.

$$\begin{aligned}Q(s) &= -\frac{G_D^+(y, d, s)}{G_D^+(y, a, s)}N(s) - \frac{C(s)[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]e^{-(Ls/c)(y-a)}p(x, s)}{G_D^+(y, a, s)[1 - \theta_1(s)e^{-(2Ls/c)(1-x)}]}\\ &= -\frac{G_D^+(x, d, s)}{G_D^+(x, a, s)}N(s) - \frac{C(s)[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]e^{-(Ls/c)(x-a)}p(x, s)}{G_D^+(x, a, s)[1 - \theta_1(s)e^{-(2Ls/c)(1-x)}]}\end{aligned} \tag{A-2}$$

Substituting the above controller into Eq. (1*b*), and considering that there exists a disturbance $\delta(s)$ at downstream position x, the pressure response can be shown as

$$\begin{aligned}\hat{p}_x &= p(x, s) - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}p(x, s) = [G_D^+(x, a, s) + G_D^-(x, a, s)]Q(s) + [G_D^+(x, d, s) + G_D^-(x, d, s)]N(s) + \delta(s)\\ &= [G_D^+(x, a, s) + G_D^-(x, a, s)]\frac{-G_D^+(x, d, s)}{G_D^+(x, a, s)}N(s) - [G_D^+(x, a, s)\end{aligned}$$

$$+ G_D^-(x, a, s)] \frac{C(s)e^{-(Ls/c)(x-a)}[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]}{G_D^+(x, a, s)[1 - \theta_1(s)e^{-(2Ls/c)(1-x)}]} p(x, s) + [G_D^+(x, d, s) + G_D^-(x, d, s)]N(s) + \delta(s)$$

$$= -[1 - \theta_1(s)e^{-(2Ls/c)(1-x)}] \frac{C(s)[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]e^{-(Ls/c)(x-a)}p(x, s)}{1 - \theta_1(s)e^{-(2Ls/c)(1-x)}} + \delta(s)$$

$$= -C(s)[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]e^{-(Ls/c)(x-a)}p(x, s) + \delta(s)$$

Therefore,

$$p(x, s) = \frac{\delta(s)}{[1 - C(s)e^{-(Ls/c)(x-a)}][1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]} \tag{A-3}$$

Proof of Eq. (18):

Substitute Eq. (A-2) into Eq. (1*b*) for $x = x_1$, we have:

$$\hat{p}_1 = [G_U^+(x_1, a, s) + G_U^-(x_1, a, s)] \frac{-G_D^+(x_3, d, s)}{G_D^+(x_3, a, s)} N(s) - [G_U^+(x_1, a, s)$$

$$+ G_U^-(x_1, a, s)] \frac{C(s)e^{-(Ls/c)(x_1-a)}[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]}{G_D^+(x_1, a, s)[1 - \theta_1(s)e^{-(2Ls/c)(1-x_3)}]} p(x_3, s) + [G_D^+(x_1, d, s) + G_D^-(x_1, d, s)]N(s)$$

$$= \frac{[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}][e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(x_1-a)}]}{G_D^+(x_1, a, s)} N(s) - [G_U^+(x_1, a, s)$$

$$+ G_U^-(x_1, a, s)] \frac{C(s)e^{-(Ls/c)(x_1-a)}[1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}]}{G_D^+(x_1, a, s)[1 - \theta_1(s)e^{-(2Ls/c)(1-x_3)}]} p(x_3, s)$$

The above equation can be rearranged as follows

$$\Delta_{11}p(x_1, s) + \Delta_{13}p(x_3, s) = N(s)$$

where,

$$\Delta_{11} = \frac{G_D^+(x_1, a, s)}{[e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(x_1-a)}]}$$

and

$$\Delta_{13} = \frac{[G_U^+(x_1, a, s) + G_U^-(x_1, a, s)]C(s)e^{-(Ls/c)(x_1-a)}}{[e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(x_1-a)}]}$$

Similarly, for $x = x_2$,

$$\Delta_{22}p(x_2, s) + \Delta_{23}p(x_3, s) = N(s)$$

Where

$$\Delta_{22} = \frac{G_D^+(x_2, a, s)}{[e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(x_2-a)}]}$$

and

$$\Delta_{23} = \frac{[G_U^+(x_2, a, s) + G_U^-(x_2, a, s)]C(s)e^{-(Ls/c)(x_2-a)}}{[e^{-(Ls/c)(a-d)} + \theta_0(s)e^{-(Ls/c)(a+d)}][1 + e^{-(2Ls/c)(x_2-a)}]}$$

From Eq. (A-3), we have:

$$\Delta_{33}p(x_3, s) = \delta(s)$$

where

$$\Delta_{33} = [1 - \theta_0(s)\theta_1(s)e^{-(2Ls/c)}][1 + C(s)e^{-(Ls/c)(x_3-a)}]$$

So, the whole closed-loop system can be expressed by:

$$\begin{bmatrix} \Delta_{11} & 0 & \Delta_{13} \\ 0 & \Delta_{22} & \Delta_{23} \\ 0 & 0 & \Delta_{33} \end{bmatrix} \begin{bmatrix} p(x_1, s) \\ p(x_2, s) \\ p(x_3, s) \end{bmatrix} = \begin{bmatrix} N(s) \\ N(s) \\ \delta(s) \end{bmatrix}$$

The characteristic equation of above equation is

$$\Delta_{11}\Delta_{22}\Delta_{33} = 0$$

which is equivalent to Eq. (18).