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Critical acceptance values and sample sizes of a variables sampling plan for very low fraction of defectives

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Abstract

Acceptance sampling plans are practical tools for quality control applications, which involve quality contracting on product orders between the vendor and the buyer. Those sampling plans provide the vendor and the buyer rules for lot sentencing while meeting their preset requirements on product quality. In this paper, we introduce a variables sampling plan for unilateral processes based on the one-sided process capability indices C_{PU} (or C_{PL}), to deal with lot sentencing problem with very low fraction of defectives. The proposed new sampling plan is developed based on the exact sampling distribution rather than approximation. Practitioners can use the proposed sampling plan to determine accurate number of product items to be inspected and the corresponding critical acceptance value, to make reliable decisions. We also tabulate the required sample size n and the corresponding critical acceptance value C_0 for various α-risks, β-risks, and the levels of lot or process fraction of defectives that correspond to acceptable and rejecting quality levels.

Keywords: Acceptance sampling plan; Critical acceptance value; Fraction of defectives; Process capability indices

1. Introduction

Acceptance sampling plan has been one of the most practical tools in statistical quality control applications, which involves quality contracting on product orders between the factories and customers. It provides the vendor and the buyer a general criterion for lot sentencing while meeting their preset requirements on product quality. A well-designed sampling plan can significantly reduce the difference between the required (expected) and the actual supplied product quality. Acceptance sampling plan, however, cannot avoid the risk of accepting bad product lots or rejecting good product lots even when 100% inspection is implemented, because of human error and fatigue, we are never ensured that the decision will be the right one. Acceptance sampling plan is a statement regarding the required sample size for product inspection and the associated acceptance or rejection criteria for sentencing each individual lot. The criteria used for measuring the performance in an acceptance sampling plan, is usually based on the operating characteristic (OC) curve which quantifies the risk for vendors and buyers.

The OC curve plots the probability of accepting the lot against actual lot fraction defective, which displays the discriminatory power of the sampling plan. For product quality protection and company's profit, both the vendor and the buyer would focus on certain points on the OC curve to reflect their benchmarking risk. The vendor (supplier) usually would focus on a specific level of product quality, traditionally called acceptable quality level (AQL), which

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would yield a high probability for accepting a lot. The AQL also represents the poorest level of quality for the vendor's process that the consumer would consider acceptable as a process average. Therefore, a preferred sampling procedure would be one, which gives a high probability of acceptance at the AQL that is normally specified in the contract. The consumer would also focus on another point at the other end of the OC curve, traditionally called lot tolerance percent defective (LTPD). Alternate names for the LTPD are the rejecting quality level (RQL) and limiting quality level (LQL). The LTPD is the poorest level of quality that the consumer is willing to accept for an individual lot. Note that the LTPD is a level of quality specified by the buyer, representing the specified low probability of accepting a lot with defect level as high as the LTPD.

A well-designed sampling plan must provide a probability of at least $1-\alpha$ of accepting a lot if the lot fraction of defectives is at the contracted AQL. The sampling plan must also provide a probability of acceptance no more than β if the lot fraction of defectives is at the LTPD level, the designated undesired level preset by the buyer. Thus, the acceptance sampling plan must have its OC curve passing through those two designated points (AQL, $1-\alpha$) and (LTPD, β). There are a number of different ways to classify acceptance-sampling plans. One major classification is by attributes and variables. When a quality characteristic is measurable on a continuous scale and is known to have a distribution of a specified type, it may be possible to use as a substitute for an attributes sampling plan based on sample measurements such as the mean and the standard deviation of the sample. These variables sampling plans have the primary advantage that the same OC curve can be obtained with a smaller sample then is required by attributes plan. The precise measurements required by a variables plan would probably cost more than the simple classification of items required by an attributes plan, but the reduction in sample size may more than offset this exact expense. Such saving may be especially marked if inspection is destructive and the item is expensive (see Refs. [1,2]).

Guenther [3] developed a systematic search procedure, which can be used with published tables of binomial, hypergeometric, and Poisson distributions to obtain the desired acceptance sampling plans. Stephens [4] provided a closed form solution for single sample acceptance sampling plans using a normal approximation to the binomial distribution. Hailey [5] presented a computer program to obtain minimum sample size single sampling plans based on either the Poisson or binomial distribution. Hald [6] gave a systematic exposition of the existing statistical theory of lot-by-lot sampling inspection by attributes and provided some tables for the sampling plans. Other researches related to the classical acceptance sampling plans include Jennett and Welch [7], Wallis [8,9], Jacobson [10], Lieberman and Resnikoff [11], Das and Mitra [12], Owen [13], Kao [14], Hamaker [15], Bender [16], Govindaraju and Soundararajan [17], and Suresh and Ramanathan [18]. In addition to the graphical procedure for designing sampling plans with specified OC curves, tabular procedures are also available for the same purpose. Duncan [19] gave a good description of these techniques. In this paper, we consider a variable sampling plan for product lots (or processes) with very low fraction of defectives. The proposed sampling plan is based on analytical exact formulas hence the decisions made are reliable.

2. Process capability indices approach

Process capability indices, including C_p, C_{PU}, C_{PL}, and C_{pk}, have been popularly used in the manufacturing industry to measure whether a process is capable of reproducing product items within the specified manufacturing tolerance. Those indices provide common quantitative measures on process potential and performance [20–23], are defined in the following, where USL and LSL are the upper and lower specification limits, respectively, μ is the process mean, and σ is the process standard deviation.

$$C_p = \frac{\text{USL}-\text{LSL}}{6\sigma}, \quad C_{PU} = \frac{\text{USL}-\mu}{3\sigma},$$
$$C_{PL} = \frac{\mu-\text{LSL}}{3\sigma}, \quad C_{pk} = \min\left\{\frac{\text{USL}-\mu}{3\sigma}, \frac{\mu-\text{LSL}}{3\sigma}\right\}.$$

While C_p and C_{pk} are appropriate measures for processes with two-sided specifications (which require both USL and LSL), C_{PU} and C_{PL} have been designed specifically for processes with one-sided specifications (which require only USL or LSL). The index C_{PU} measures the capability of a smaller-the-better process with an upper specification limit USL, whereas the index C_{PL} measures the capability of a larger-the-better process with a lower specification limit LSL.

For normally distributed processes with one-sided specification limit USL, the process yield is:

$$\begin{aligned} P(X<\text{USL}) &= P\left(\frac{X-\mu}{3\sigma} < \frac{\text{USL}-\mu}{3\sigma}\right) \\ &= P\left(\frac{1}{3}Z < C_{PU}\right) = P(Z<3C_{PU}) \\ &= \Phi(3C_{PU}), \end{aligned}$$

where Z follows the standard normal distribution $N(0,1)$ with the cumulative distribution function $\Phi(x) = (2\pi)^{-1/2}\int_{-\infty}^{x}\exp(-t^2/2)\mathrm{d}t$. Similarly, for normally distributed processes with one-sided specification limit LSL, the process yield can be calculated as $P(X>\text{LSL}) = P(-Z/3<C_{PL}) = 1-\Phi(-3C_{PL}) = \Phi(3C_{PL})$. For convenience of presentation, we let C_I denote either C_{PU} or C_{PL}. Therefore, the corresponding non-conforming units in parts per million (NCPPM) for a well controlled normally distributed process can be expressed as NCPPM $= p = 10^6 \times [1-\Phi(3C_I)]$. Thus, process capability indices C_I provide an exact measure of process yield. Table 1 displays some values of C_I and the corresponding NCPPM.

Table 1
Various C_{I} values and the corresponding NCPPM

C_{PU} or C_{PL}	NCPPM	C_{PU} or C_{PL}	NCPPM
1.00	1349.90	1.50	3.40
1.15	280.29	1.60	0.7933
1.25	88.42	1.67	0.2722
1.30	48.10	1.70	0.1698
1.33	33.04	1.90	0.0060
1.45	6.81	2.00	0.0010

In practice, sample data must be collected in order to calculate those indices since the process mean μ and standard deviation σ are usually unknown. To estimate the indices C_{PU} and C_{PL}, Chou and Owen [24] considered $\hat{C}_{\mathrm{PU}}$ and $\hat{C}_{\mathrm{PL}}$, the natural estimators of C_{PU} and C_{PL}, which are defined below, where $\bar{X} = \sum_{i=1}^{n} X_i/n$ is the sample mean, and $S^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the sample variance, which may be obtained from a process that is demonstrably stable (under statistical control). Under the normality assumption, the estimator $\hat{C}_{\mathrm{PU}}$ is distributed as $(3\sqrt{n})^{-1}t_{n-1}(\delta)$, where $t_{n-1}(\delta)$ is a non-central t distribution with $n-1$ degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n}C_{\mathrm{PU}}$. The estimator $\hat{C}_{\mathrm{PL}}$ has the same sampling distribution as $\hat{C}_{\mathrm{PU}}$ but with $\delta = 3\sqrt{n}C_{\mathrm{PL}}$. However, both estimators $\hat{C}_{\mathrm{PU}}$and $\hat{C}_{\mathrm{PL}}$ are biased. Pearn and Chen [25] showed that by adding the correction factor $b_{n-1}=[2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ to $\hat{C}_{\mathrm{PU}}$ and $\hat{C}_{\mathrm{PL}}$, we could obtain unbiased estimators $b_{n-1}\hat{C}_{\mathrm{PU}}$ and $b_{n-1}\hat{C}_{\mathrm{PL}}$ which have been denoted as $\tilde{C}_{\mathrm{PU}}$ and $\tilde{C}_{\mathrm{PL}}$. That is, $E(\tilde{C}_{\mathrm{PU}}) = C_{\mathrm{PU}}$, and $E(\tilde{C}_{\mathrm{PL}}) = C_{\mathrm{PL}}$. Since $b_{n-1} < 1$, then $\mathrm{Var}(\tilde{C}_{\mathrm{PU}}) < \mathrm{Var}(\hat{C}_{\mathrm{PU}})$ and $\mathrm{Var}(\tilde{C}_{\mathrm{PL}}) < \mathrm{Var}(\hat{C}_{\mathrm{PL}})$. And since the estimators $\tilde{C}_{\mathrm{PU}}$ and $\tilde{C}_{\mathrm{PL}}$are only based on the complete and sufficient statistics $(\overline{X}, S^2)$, it can conclude that $\tilde{C}_{\mathrm{PU}}$ and $\tilde{C}_{\mathrm{PL}}$ are the uniformly minimum variance unbiased estimators (UMVUEs) of C_{PU} and C_{PL}, respectively. Therefore, in practice using the UMVUEs $\tilde{C}_{\mathrm{PU}}$ and $\tilde{C}_{\mathrm{PL}}$ to calculate the capability measures would be desirable.

$$\hat{C}_{\mathrm{PU}} = \frac{\mathrm{USL} - \bar{X}}{3S}, \quad \hat{C}_{\mathrm{PL}} = \frac{\bar{X} - \mathrm{LSL}}{3S}.$$

To test whether the process meets the capability requirement, we consider the following testing hypothesis with H_0: $C_{\mathrm{I}} \leqslant C$ (the process is incapable), versus the alternative H_1: $C_{\mathrm{I}} > C$ (the process is capable). Thus, we may consider the test $\phi^*(x) = 1$ if $C_{\mathrm{I}} > C_0$, and $\phi^*(x) = 0$, otherwise. The test ϕ^* rejects the null hypothesis if $\tilde{C}_{\mathrm{I}} > C_0$, with type I error $\alpha(C_0) = \alpha$, the chance of incorrectly judging an incapable process as a capable one. Thus, the power of the test can be calculated as

$$\pi(C_{\mathrm{I}}) = P(\tilde{C}_{\mathrm{I}} \geqslant C_0 \mid C_{\mathrm{I}}) = P\left(t_{n-1}(\delta) \geqslant \frac{3\sqrt{n}C_0}{b_{n-1}}\right),$$

which under H_0 for $E_{C_{\mathrm{I}}}(\phi^*(x)) = \alpha$, we can obtain that the critical value $C_0 = b_{n-1}(3\sqrt{n})^{-1}t_{n-1,\alpha}(\delta)$, where $\delta = 3\sqrt{n}C_I$ and $t_{n-1,\alpha}(\delta)$ is the upper α quantile of non-central t distribution with $n-1$ degrees of freedom satisfies $P(t_{n-1}(\delta) \geqslant t_{n-1,\alpha}(\delta)) = \alpha$. And from the probability density function of $\tilde{C}_I$, we let

$$\lambda(x) = \frac{f(x, C'_{\mathrm{I}})}{f(x, C_{\mathrm{I}})} = \frac{\int_0^\infty y^{(n-2)/2} \exp\left\{-\frac{1}{2}\left[y + \left(\frac{3x\sqrt{ny}}{b_{n-1}\sqrt{n-1}} - 3\sqrt{n}C'_{\mathrm{I}}\right)^2\right]\right\} \mathrm{d}y}{\int_0^\infty y^{(n-2)/2} \exp\left\{-\frac{1}{2}\left[y + \left(\frac{3x\sqrt{ny}}{b_{n-1}\sqrt{n-1}} - 3\sqrt{n}C_{\mathrm{I}}\right)^2\right]\right\} \mathrm{d}y}.$$

Since for $C'_{\mathrm{I}} > C_{\mathrm{I}} > 0$, $\lambda(x)$ is a non-decreasing function of x, then $\{(f_{\tilde{C}_{\mathrm{I}}}(x, C_{\mathrm{I}}) \mid C_{\mathrm{I}} > 0\}$ has monotone likelihood ratio (MLR) property in $\tilde{C}_{\mathrm{I}}$. Therefore, we can conclude that the test ϕ^* is uniformly most powerful (UMP) test of its size $E_{C_{\mathrm{I}}}(\phi^*(x) \mid \mathrm{H}_0) = \alpha$. In fact, the decision rule of the UMP test can be constructed as

The lot is accepted, $\quad if\ \tilde{C}_{\mathrm{I}} \geqslant \frac{b_{n-1}}{3\sqrt{n}} t_{n-1,\alpha}(\delta = 3\sqrt{n}C_I)$.
The lot is rejected, $\quad$ otherwise.

3. Designing variables sampling plan based on C_{PU} and C_{PL}

Consider a variables sampling plan to control the lot or process fraction defective (or nonconforming). Since the quality characteristic is a variables, there will exist either an USL or a LSL, or both, that define the acceptable values of this parameter. As indicated earlier, selection of a meaningful critical value for a capability test requires specification of an AQL and a LTPD for the C_{I} value. The AQL is simply a standard against which to judge the lots. It is hoped that the vendor's process will operate at a fallout level that is considerably better than the AQL. Both the vendor and the buyer will lay down their requirements in the contract: the former demands that not too many "good" lots shall be rejected by the sampling inspection, while the latter demands that not too many "bad" lots shall be accepted. A sampling plan attempts will be chosen to meet these somewhat opposing requirements. Let $(\mathrm{AQL}, 1-\alpha)$ and (LTPD, β) be the two points on the OC curve of interest. To determine whether a given process is capable, we can first consider the following testing hypothesis:

H_0: $\quad p = \mathrm{AQL}$ (process is capable),

H_1: $\quad p = \mathrm{LTPD}$ (process is not capable).

Process capability index is a function of process parameters and manufacturing specifications. It measures the ability of the process to reproduce product units that meet the specifications. For processes with one-sided specification limits, thus, C_{I} can be used as a quality benchmark for acceptance of a product lot. That is, the null hypothesis with process fraction of defectives, $\mathrm{H}_0 : p = \mathrm{AQL}$, is equivalent to test process capability index with $\mathrm{H}_0 : C_{\mathrm{I}} \geqslant C_{\mathrm{AQL}}$, where

C_{AQL} is the level of acceptable quality for C_I index correspond to the lot or process fraction of nonconformities AQL as $\Phi^{-1}(1 - \text{AQL} \times 10^{-6})/3$. For instance, if the proportion defective $p = \text{AQL}$ of vendor's product is less than 88 NCPPM, then the probability of consumer accept the lots will greater than $100(1-\alpha)\%$. On the other hand, if the proportion defective of vendor's product, $p = \text{LTPD}$, is more than 1350 NCPPM, then the probability of consumer would accept no more than $100\beta\%$. Thus, from the relationship between the index value and fraction of defectives, we could obtain the equivalent $C_{AQL} = 1.25$ and the $C_{LPTD} = 1.00$. Then, the required inspection sample size n and critical acceptance value C_0 for the sampling plans are the solution to the following two nonlinear simultaneous equations.

$$P\{\text{Accepting the lot}|\text{proportion defective } p = \text{AQL}\} \geqslant 1 - \alpha,$$

$$P\{\text{Accepting the lot}|\text{proportion defective } p = \text{LTPD}\} \leqslant \beta.$$

As described before, the sampling distribution of $\tilde{C}_I$ is distributed as $b_{n-1}(3\sqrt{n})^{-1}t_{n-1}(\delta)$, where $t_{n-1}(\delta)$ is a non-central t distribution with $n-1$ degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n}C_I$. Thus, the probability of accepting the lot can be expressed as

$$\pi_A(C_I) = P(\tilde{C}_I \geqslant C_0 \mid C_I = C) = P\left(t_{n-1}(\delta) \geqslant \frac{3\sqrt{n}C_0}{b_{n-1}}\right).$$

Therefore, the required inspection sample size n and critical acceptance value C_0 of $\tilde{C}_I$ for the sampling plan are the solution to the following two nonlinear simultaneous equations (1) and (2).

$$P\left(t_{n-1}(\delta_1) \geqslant \frac{3\sqrt{n}C_0}{b_{n-1}}\right) \geqslant 1 - \alpha, \quad (1)$$

$$P\left(t_{n-1}(\delta_2) \geqslant \frac{3\sqrt{n}C_0}{b_{n-1}}\right) \leqslant \beta, \quad (2)$$

where $\delta_1 = 3\sqrt{n}C_{AQL}$ and $\delta_2 = 3\sqrt{n}C_{LTPD}$, $C_{AQL} > C_{LPTD}$. We note that the required sample size n is the smallest possible value of n satisfying Eqs. (1) and (2), and determining the $\lceil n \rceil$ as sample size, where $\lceil n \rceil$ means the least integer greater than or equal to n. Moreover, to illustrate how we solve the above two nonlinear simultaneous equations (1) and (2), let

$$S_1(n, C_0) = \frac{1}{2^{\frac{n-3}{2}}\Gamma\left[\frac{n-1}{2}\right]}\int_0^\infty x^{n-2}e^{-x^2/2}\frac{1}{\sqrt{2\pi}} \times \int_0^{3\sqrt{n}C_0x/(b_{n-1}\sqrt{n-1})} \times \exp\left[-\frac{1}{2}(u-\delta_1)^2\right]du\,dx - \alpha \quad (3)$$

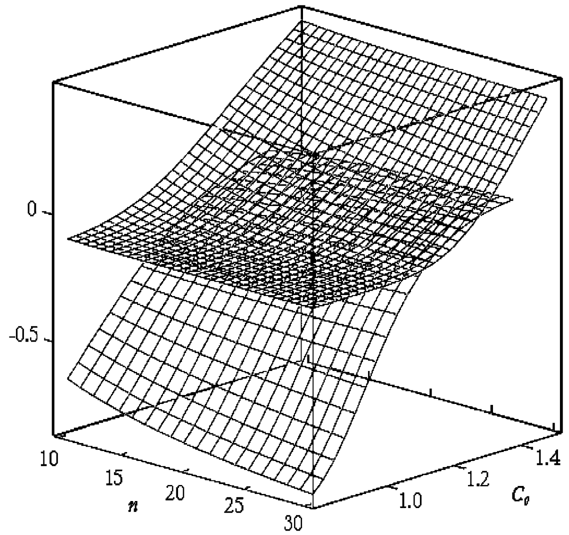

Fig. 1. Surface plot of S_1 and S_2.

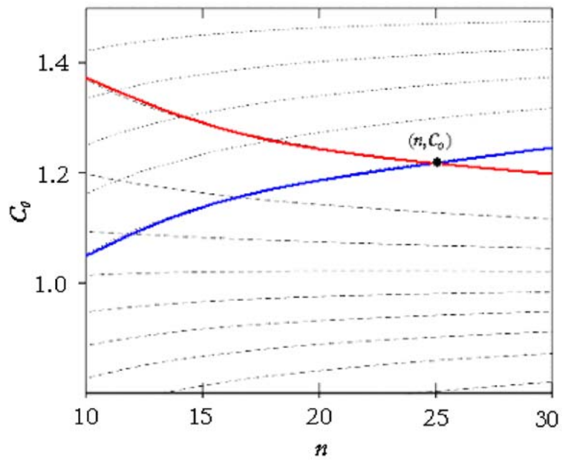

Fig. 2. Contour plot of S_1 and S_2.

$$S_2(n, C_0) = \frac{1}{2^{\frac{n-3}{2}}\Gamma\left[\frac{n-1}{2}\right]}\int_0^\infty x^{n-2}e^{-x^2/2}\frac{1}{\sqrt{2\pi}} \times \int_0^{3\sqrt{n}C_0x/(b_{n-1}\sqrt{n-1})} \times \exp\left[-\frac{1}{2}(u-\delta_2)^2\right]du\,dx - (1-\beta). \quad (4)$$

Figs. 1 and 2 display the surface and contour plots of Eqs. (3) and (4) simultaneously with α-risk $= 0.10$ and β-risk $= 0.10$ for $C_{AQL} = 1.50$ and $C_{LPTD} = 1.00$, respectively. From Fig. 2, we can see that the interaction of $S_1(n, C_0)$ and $S_2(n, C_0)$ contour curves at level 0 is $(n, C_0) = (24.49, 1.2200)$, which is the solution to nonlinear simultaneous equations (1) and (2). That is, in this case, the minimum required sample size is $\lceil n \rceil = 25$, and the critical acceptance value is $C_0 = 1.2200$ for the sampling plan based on the one-sided capability index C_{I}.

To investigate the behaviour of the critical acceptance values and required sample sizes with various parameters, we perform extensive calculations to obtain the solution of (1) and (2). The results indicated that the larger of the risks which producer or customer would suffer, the smaller sample size n is required for inspection. This phenomenon can be interpreted intuitively, as if we would want the chance of wrongly concluding a bad lot (process) as a good one, or a good lot (process) as a bad one, to be smaller, we would need more sample information to make the judgement. Further, for fixed α, β risks and C_{LTPD}, the required sample sizes become smaller when the C_{AQL} becomes larger. This can also be explained by the same reasoning, since the judgement will be more correct with a larger value of the difference between the C_{AQL} and C_{LTPD}. For practical applications purpose, we calculate and tabulate the critical acceptance values and sample sizes required for the sampling plans, with commonly used α, β, C_{AQL} and C_{LTPD}. The values of (n, C_0) for producer's α-risk $= 0.01(0.01)0.10$, buyer's β-risk $= 0.01(0.01)0.10$, with various benchmarking quality levels, $(C_{\mathrm{AQL}}, C_{\mathrm{LPTD}})=$ (1.25, 1.00), (1.45, 1.00), (1.60, 1.00), (1.45, 1.25), (1.60, 1.25) and (1.60, 1.45) are displayed in the Appendix. As an example, if the benchmarking quality level $(C_{\mathrm{AQL}}, C_{\mathrm{LPTD}})$ is set to (1.45, 1.00) with producer's α-risk $= 0.01$ and buyer's β-risk $= 0.05$, then the corresponding sample size and critical acceptance value can be calculated as $(n, C_0) = (66, 1.1749)$. The lot will be accepted if the 66 inspected product items yield measurements with $\tilde{C}_{\mathrm{I}} \geqslant 1.1749$. Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement. In this case, the buyer will reject the lot.

4. Discussions and comparisons

An approximate approach based on measurements similar to process capability indices, was used to designed a variables sampling plan, which was proposed to deal with the lot sentencing problem [1,2,15]. The approximation used the statistic,

$$Z_{\mathrm{LSL}} = \frac{\bar{X} - \mathrm{LSL}}{S}.$$

The relationship between $\tilde{C}_{\mathrm{PL}}$ and Z_{LSL} (or $\tilde{C}_{\mathrm{PU}}$ and Z_{USL}) is that $\tilde{C}_{\mathrm{PL}}$ is a constant multiplied by the Z_{LSL}. Clearly, $\tilde{C}_{\mathrm{I}} > C_0 \;\Leftrightarrow\; Z_{\mathrm{LSL}} > k$, with $k = 3C_0/b_{n-1}$. Taking approximation approach, the values of n and k can be obtained from the following formulas for given p_1, p_2, α, and β. Results should always be rounded up. Formula for n depends upon the given knowledge of the standard deviation.

$$k = \frac{z_\alpha z_{p_2} + z_\beta z_{p_1}}{z_\alpha + z_\beta},$$

$$n = \left(\frac{z_\alpha + z_\beta}{z_{p_1} - z_{p_2}}\right)^2, \quad \text{if } \sigma \text{ is known},$$

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{z_\alpha + z_\beta}{z_{p_1} - z_{p_2}}\right)^2, \quad \text{if } \sigma \text{ is unknown},$$

where the z's are the normal deviates the probability of exceeding which are the p_1, p_2, α, and β. Formulas of n and k are based on the assumption that $\bar{X} \pm ks$ is approximately normally distributed with a mean of $\mu \pm k\sigma$ and a standard deviation equal approximately to

$$\sigma\sqrt{\frac{1}{n} + \frac{k^2}{2n}}.$$

In addition to the formulas derived by Wallis [8,9] Jacobson [10] developed the useful nomograph. Table of n and k for some given p_1's and p_2's is also provided (can be found in Statistical Research Group, Columbia University, *Techniques of Statistical Analysis*, [26, pp. 22–25]).

We compared the sample sizes (n) and the corresponding critical acceptance value (C_0) based on the existing approximation and our proposed (exact) method. We summarized (n, C_0) values for various $C_{\mathrm{AQL}}, C_{\mathrm{LPTD}}$, and the risks of producer and customer (i.e. p_1, p_2, α, and β), as given in the following Tables 2 and 3. Table 2 displays some cases with $\alpha < \beta$. It is noted that for such cases the existing approximation requires larger sample size (hence more cost) and larger critical acceptance values than the proposed exact approach. For such cases, the existing approximation tends to in favor of rejecting the lots, thus provides more protection to the customer (producer will suffer unfairly more risk). Table 3 displays some cases with $\alpha > \beta$. It is noted that for those cases, the existing approximation requires smaller sample size and smaller critical acceptance values than the proposed exact approach. For such cases, the existing approximation tends to in favor of accepting the lots, thus provides more protection to the producer (customer will suffer unfairly more risk in this case). The proposed sampling plan is based on the analytical exact approach, which provides the vendor and the buyer a fair and accurate criterion for lot sentencing, which significantly reduces the difference between the expected and the actual supplied product quality.

5. An application example

Electrically erasable programmable read-only memory (EEPROM) chip is a user-modifiable read-only memory chip

Table 2
(n, C_0) values for $\alpha = 0.01$, $\beta = 0.05$ and 0.10 with various (C_{AQL}, C_{LTPD})

(α, β)		$C_{AQL} = 1.25$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.45$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.60$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.45$ $C_{LTPD} = 1.25$		$C_{AQL} = 1.60$ $C_{LTPD} = 1.25$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
(0.01,0.05)	Exact	185	1.0997	66	1.1749	41	1.2280	398	1.3305	142	1.3880
	Approx.	194	1.1420	71	1.2502	45	1.3288	413	1.3647	151	1.4478
(0.01,0.10)	Exact	150	1.0843	53	1.1465	33	1.1907	325	1.3185	116	1.3673
	Approx.	164	1.1559	61	1.2742	39	1.3600	346	1.3760	128	1.4670

Table 3
(n, C_0) values for $\alpha = 0.05(0.01)0.10$, $\beta = 0.01$ with $C_{AQL} = 1.25$ and $C_{LTPD} = 1.00$

α	β	Exact		Approximation	
		n	C_0	n'	$C_0' = b_{n'-1} \times k/3$
0.05	0.01	193	1.1423	182	1.0990
0.06	0.01	185	1.1456	173	1.0954
0.07	0.01	179	1.1488	165	1.0920
0.08	0.01	173	1.1516	159	1.0890
0.09	0.01	167	1.1541	153	1.0860
0.1	0.01	162	1.1566	147	1.0832

that can be erased and reprogrammed (written onto) repeatedly through the application of higher electrical voltage. It is usually used in portable phones, PHS phones, compact portable terminals, consumer products (such as cordless phones and audio systems); industrial equipment including measuring instruments and PLCs; OA products such as printers and scanners, in-house telephone switches, and other communication equipment. The output leakage current (OLC) is an essential product quality characteristic, which has significant impact to product quality. For the output leakage current of a particular model of EEPROM, the upper specification limit, USL, is set to 5 μA.

In a purchasing contract, a minimum value of the PCI is usually specified. If the prescribed minimum value of the PCI fails to be met, the process is determined to be incapable. Otherwise, the process will be determined to be capable. For processes with one-sided specifications, some minimum capability requirements have been recommended for specific process types must run under some designated quality conditions. In particular, 1.25 for existing processes, and 1.45 for new processes; 1.45 also for existing processes on safety, strength, or critical parameter, and 1.60 for new processes on safety, strength, or critical parameter. The recommended guidelines for minimum quality requirements and the corresponding parts per million (PPM) of non-conformities (NC) for those processes can be found in Montgomery [2]. In recent years, many companies have adopted criteria for evaluating their processes that include process capability objectives that are more stringent than those recommended minimums above. For example, the "Six-Sigma" program pioneered by Motorola essentially requires that when the process mean is in control, it will not be closer than six standard deviations from the nearest specification limit. Thus, in effect, requires that the process capability ration will be at least 2.0.

To illustrate how the sampling plan can be established and applied to the actual data collected from the factories, we consider the following example taken from a company manufacturing and designing standard Flash Memory EEPROM and Mixed-Signal products, such as, PLL, ADC DAC, and many others. The manufacturing specifications for a 128-bit EEPROM chip, has an upper specification limit USL = 5 μA for the output leakage current which are mentioned before. If the OLC is greater than 5 μA, then the EEPROM chip is considered to be nonconforming product, and will not be used to make the EEPROM chip of that particular model.

The capability requirement for this particular model of EEPROM chip was defined as "*Capable*" if $C_{PU} > 1.60$. In the contract, the C_{AQL} and C_{LTPD} are set to 1.60 and 1.25 with the α-risk = 0.01, and β-risk = 0.05 respectively. That is, the sampling plan must provide a probability of at least 99% of accepting the lot if the lot proportion defective is

Table 4
The sample data with 142 observations (unit: μA)

4.1 4	3.914	3.993	3. 39	3. 2	4.201	4.066	4.049	4.210	4.247	4.106	4.650	3. 47
4.420	4.216	3.746	4.5 9	3.945	3. 39	3.342	4.175	4.1	3.644	3.946	4.09	3.696
3.729	4.024	3.975	3.7 2	4.211	3. 44	3.931	4.091	4.057	3.761	3.965	3.976	3.94
4.154	4.156	4.316	3. 7	3.917	3.953	4.145	3.91	4.00	4.0 4	4.170	4.042	3.906
4.26	4.241	4.153	3.620	4.139	3. 2	3. 24	3.752	4.610	4.0 2	3.571	4.015	3. 3
3. 23	4.233	3.905	4.2 9	3.761	4.059	4.333	3.921	3. 30	3. 25	4.040	4.715	4.123
3. 64	4.103	3.957	4.4 0	3.717	3.921	4.515	3.666	3. 74	3.695	4.146	4.025	3.7 4
4.10	4.320	4.127	3. 74	4.191	4.12	4.045	4.2 2	3. 73	4.245	4.279	4.301	3.713
4.046	3.619	4.356	3. 25	3.763	3. 61	4.130	4.075	3. 04	3. 70	3.96	3.943	4.637
3.745	4.199	4.139	3.73	4.3 9	3.442	3.965	4.025	4.166	4.123	3.955	3.773	4.0 6
4.191	3.9 5	3.994	4.005	4.541	4.147	3.767	3.970	3.770	4.324	3. 6	4.140	

at the $C_{\mathrm{AQL}} = 1.60$ (which is equivalent to AQL $= 0.79$ NCPPM), and also provide a probability of no more than 5% of accepting the lot if the lot proportion defective is at the $C_{\mathrm{LTPD}} = 1.25$ (which is equivalent to LTPD $= 88$ NCPPM). Therefore, by checking Table 5 in this Appendix we find the required sample sizes and critical acceptance value $(n, C_0) = (142, 1.3880)$. Hence, 142 inspected, EEPROM chips are taken from the lot randomly and the observed measurements are displayed in Table 4. Based on these inspections, we obtain that

$$\overline{X} = 4.0248, \quad S = 0.2407,$$

$$\tilde{C}_{\mathrm{PU}} = b_{n-1} \times \frac{\mathrm{USL} - \bar{X}}{3S} = 1.3433.$$

Since the sample estimator, 1.3433, is smaller than the critical acceptance value 1.3880 of the sampling plan, the buyer will reject the lot.

6. Concluding remarks

Acceptance sampling plan basically consists of a sample size (n) and an acceptance criterion (C_0). Since the sampling cannot guarantee that every defective item in a lot will be inspected, then the sampling involves risks of not adequately reflecting the quality conditions of the lot. Such risk is even more significant as the rapid advancement of the manufacturing technology and stringent customers demand is enforced. Particularly, when the fraction of defectives is very low, such as in PPM, the required number of inspection items must be enormously large in order to adequately reflecting the actual lot quality. In this paper, we introduce a variables sampling plan for one-sided processes based on the uniformly most powerful test of the capability indices C_{PU} (or C_{PL}), to deal with lot sentencing problem with very low fraction of defectives. The proposed new sampling plan is developed based on the exact sampling distribution rather than approximation. Therefore, practitioners can determine the number of required inspection units and the critical acceptance value, and to make more accurate decision. To illustrate how the sampling plan can be established and applied to the actual data collected from the factories, a real world application to the EEPROM manufacturing process is also provided.

Appendix

The values of (n, C_0) for producers α-risk buyers β-risk with various benchmarking quality levels are shown in Tables 5 and 6.

Table 5
(n, C_0) values for $\alpha = 0.01(0.01)0.050$, $\beta = 0.01(0.01)0.10$ with selected (C_{AQL}, C_{LTPD})

α	β	$C_{AQL} = 1.25$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.45$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.60$ $C_{LTPD} = 1.00$		$C_{AQL} = 1.45$ $C_{LTPD} = 1.25$		$C_{AQL} = 1.60$ $C_{LTPD} = 1.25$		$C_{AQL} = 1.60$ $C_{LTPD} = 1.45$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.01	0.01	259	1.1216	93	1.2145	59	1.2837	554	1.3480	200	1.4192	1229	1.5216
	0.02	228	1.1137	82	1.2006	51	1.2624	489	1.3417	176	1.4081	1096	1.5170
	0.03	209	1.1080	75	1.1904	47	1.2499	449	1.3372	161	1.4000	1015	1.5137
	0.04	195	1.1034	70	1.1821	44	1.2395	421	1.3337	151	1.3940	957	1.5112
	0.05	185	1.0997	66	1.1749	41	1.2280	398	1.3305	142	1.3880	910	1.5089
	0.06	176	1.0962	63	1.1691	39	1.2197	379	1.3277	136	1.3838	872	1.5069
	0.07	168	1.0928	60	1.1629	37	1.2108	363	1.3252	130	1.3792	839	1.5051
	0.08	162	1.0901	57	1.1562	36	1.2060	349	1.3229	125	1.3752	810	1.5034
	0.09	156	1.0873	55	1.1515	34	1.1960	337	1.3207	120	1.3709	784	1.5018
	0.10	150	1.0843	53	1.1465	33	1.1907	325	1.3185	116	1.3673	760	1.5003
0.02	0.01	231	1.1290	84	1.2281	53	1.3001	494	1.3541	179	1.4297	1084	1.5263
	0.02	202	1.1211	73	1.2132	46	1.2800	432	1.3477	156	1.4183	959	1.5217
	0.03	184	1.1152	67	1.2037	42	1.2665	395	1.3432	143	1.4107	884	1.5184
	0.04	172	1.1109	62	1.1947	39	1.2551	369	1.3397	133	1.4041	829	1.5157
	0.05	162	1.1069	58	1.1868	37	1.2467	347	1.3364	125	1.3984	786	1.5134
	0.06	154	1.1034	55	1.1803	35	1.2377	330	1.3336	119	1.3937	750	1.5114
	0.07	146	1.0997	52	1.1733	33	1.2280	315	1.3310	113	1.3886	719	1.5095
	0.08	140	1.0967	50	1.1683	32	1.2228	302	1.3285	108	1.3841	692	1.5077
	0.09	135	1.0941	48	1.1630	30	1.2116	290	1.3262	104	1.3803	668	1.5061
	0.10	130	1.0913	46	1.1574	29	1.2057	280	1.3241	100	1.3762	647	1.5045
0.03	0.01	215	1.1345	78	1.2370	50	1.3133	458	1.3584	167	1.4374	996	1.5296
	0.02	187	1.1265	68	1.2228	43	1.2926	399	1.3521	145	1.4261	877	1.5250
	0.03	170	1.1207	62	1.2128	39	1.2784	363	1.3475	131	1.4175	805	1.5217
	0.04	158	1.1162	57	1.2034	36	1.2662	337	1.3438	122	1.4112	753	1.5190
	0.05	148	1.1119	54	1.1971	34	1.2573	317	1.3406	115	1.4059	712	1.5167
	0.06	140	1.1082	51	1.1903	32	1.2476	300	1.3376	108	1.4000	678	1.5146
	0.07	133	1.1048	48	1.1829	30	1.2370	286	1.3350	103	1.3955	648	1.5127
	0.08	128	1.1021	46	1.1775	29	1.2313	274	1.3326	99	1.3917	623	1.5109
	0.09	122	1.0987	44	1.1719	28	1.2253	263	1.3303	94	1.3865	600	1.5092
	0.10	118	1.0963	42	1.1658	27	1.2190	253	1.3281	91	1.3833	580	1.5076

Table 5 (*continued*)

α	β	$C_{AQL}=1.25$ $C_{LTPD}=1.00$		$C_{AQL}=1.45$ $C_{LTPD}=1.00$		$C_{AQL}=1.60$ $C_{LTPD}=1.00$		$C_{AQL}=1.45$ $C_{LTPD}=1.25$		$C_{AQL}=1.60$ $C_{LTPD}=1.25$		$C_{AQL}=1.60$ $C_{LTPD}=1.45$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.04	0.01	203	1.1387	74	1.2446	47	1.3218	432	1.3619	157	1.4429	933	1.5323
	0.02	175	1.1305	64	1.2300	41	1.3034	374	1.3555	136	1.4318	817	1.5276
	0.03	159	1.1249	58	1.2196	37	1.2887	340	1.3510	123	1.4235	748	1.5243
	0.04	147	1.1201	54	1.2117	34	1.2762	315	1.3473	114	1.4170	698	1.5217
	0.05	138	1.1161	50	1.2029	32	1.2668	295	1.3440	107	1.4114	658	1.5193
	0.06	131	1.1128	47	1.1956	30	1.2566	279	1.3411	101	1.4061	626	1.5173
	0.07	124	1.1091	45	1.1904	28	1.2454	265	1.3384	96	1.4014	598	1.5153
	0.08	118	1.1057	43	1.1848	27	1.2393	253	1.3358	92	1.3974	573	1.5135
	0.09	113	1.1027	41	1.1788	26	1.2329	243	1.3336	88	1.3930	551	1.5118
	0.10	109	1.1002	39	1.1724	25	1.2261	233	1.3312	84	1.3884	531	1.5101
0.05	0.01	193	1.1423	71	1.2514	45	1.3302	411	1.3649	150	1.4482	883	1.5346
	0.02	167	1.1345	61	1.2365	39	1.3112	355	1.3586	129	1.4368	771	1.5300
	0.03	151	1.1288	55	1.2257	35	1.2960	321	1.3540	117	1.4290	703	1.5267
	0.04	139	1.1238	51	1.2176	32	1.2828	297	1.3503	108	1.4223	655	1.5240
	0.05	130	1.1197	48	1.2108	30	1.2730	278	1.3470	101	1.4165	616	1.5216
	0.06	123	1.1162	45	1.2033	28	1.2621	262	1.3440	95	1.4110	585	1.5196
	0.07	117	1.1129	42	1.1951	27	1.2562	249	1.3414	90	1.4060	558	1.5176
	0.08	111	1.1094	40	1.1891	26	1.2500	238	1.3390	86	1.4018	534	1.5158
	0.09	106	1.1063	39	1.1860	25	1.2434	227	1.3364	82	1.3972	513	1.5141
	0.10	102	1.1036	37	1.1793	24	1.2364	218	1.3341	79	1.3936	494	1.5124

Table 6
(n, C_0) values for $\alpha = 0.06(0.01)0.10$, $\beta = 0.01(0.01)0.10$ with selected $(C_{\mathrm{AQL}}, C_{\mathrm{LTPD}})$

α	β	$C_{\mathrm{AQL}} = 1.25$, $C_{\mathrm{LTPD}} = 1.00$		$C_{\mathrm{AQL}} = 1.45$, $C_{\mathrm{LTPD}} = 1.00$		$C_{\mathrm{AQL}} = 1.60$, $C_{\mathrm{LTPD}} = 1.00$		$C_{\mathrm{AQL}} = 1.45$, $C_{\mathrm{LTPD}} = 1.25$		$C_{\mathrm{AQL}} = 1.60$, $C_{\mathrm{LTPD}} = 1.25$		$C_{\mathrm{AQL}} = 1.60$, $C_{\mathrm{LTPD}} = 1.45$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.06	0.01	185	1.1456	68	1.2568	44	1.3396	393	1.3675	144	1.4528	841	1.5367
	0.02	159	1.1376	59	1.2432	37	1.3170	338	1.3611	124	1.4418	732	1.5321
	0.03	144	1.1321	53	1.2323	34	1.3053	306	1.3567	112	1.4338	666	1.5288
	0.04	133	1.1275	49	1.2239	31	1.2920	282	1.3529	103	1.4270	619	1.5261
	0.05	124	1.1232	45	1.2145	29	1.2820	264	1.3498	96	1.4210	582	1.5238
	0.06	117	1.1196	43	1.2093	27	1.2709	249	1.3469	90	1.4154	551	1.5217
	0.07	110	1.1157	40	1.2008	26	1.2649	236	1.3441	86	1.4113	525	1.5197
	0.08	105	1.1126	38	1.1946	25	1.2585	224	1.3414	82	1.4069	502	1.5179
	0.09	100	1.1094	37	1.1914	23	1.2445	214	1.3390	78	1.4022	481	1.5161
	0.10	96	1.1066	35	1.1844	22	1.2368	205	1.3366	74	1.3971	463	1.5145
0.07	0.01	179	1.1488	66	1.2626	42	1.3449	378	1.3699	139	1.4572	806	1.5386
	0.02	153	1.1408	56	1.2471	36	1.3251	325	1.3637	119	1.4459	699	1.5340
	0.03	138	1.1352	51	1.2377	32	1.3090	293	1.3592	107	1.4378	635	1.5308
	0.04	127	1.1304	47	1.2291	30	1.2997	270	1.3555	98	1.4307	589	1.5281
	0.05	118	1.1261	43	1.2194	28	1.2895	252	1.3523	92	1.4255	552	1.5257
	0.06	111	1.1223	41	1.2141	26	1.2781	237	1.3493	86	1.4197	522	1.5236
	0.07	105	1.1188	39	1.2083	25	1.2719	224	1.3465	82	1.4154	497	1.5217
	0.08	100	1.1157	37	1.2021	23	1.2583	213	1.3439	78	1.4109	475	1.5199
	0.09	95	1.1123	35	1.1953	22	1.2508	203	1.3414	74	1.4060	455	1.5181
	0.10	91	1.1094	33	1.1879	21	1.2428	195	1.3393	71	1.4021	437	1.5164
0.08	0.01	173	1.1516	64	1.2676	41	1.3521	365	1.3722	134	1.4609	775	1.5404
	0.02	147	1.1435	54	1.2519	35	1.3322	313	1.3661	115	1.4501	670	1.5358
	0.03	133	1.1381	49	1.2423	31	1.3158	281	1.3615	103	1.4418	607	1.5326
	0.04	122	1.1333	45	1.2335	29	1.3063	259	1.3579	95	1.4354	562	1.5299
	0.05	113	1.1289	42	1.2260	27	1.2959	241	1.3546	88	1.4292	527	1.5276
	0.06	107	1.1256	39	1.2178	25	1.2841	227	1.3517	83	1.4242	497	1.5254
	0.07	101	1.1220	37	1.2118	24	1.2777	214	1.3488	78	1.4188	472	1.5235
	0.08	96	1.1188	35	1.2052	23	1.2709	203	1.3462	74	1.4141	451	1.5217
	0.09	91	1.1154	34	1.2017	22	1.2636	194	1.3439	71	1.4103	431	1.5199
	0.10	87	1.1124	32	1.1942	21	1.2558	185	1.3414	68	1.4063	414	1.5183

Table 6 (*continued*)

α	β	$C_{AQL}=1.25$ $C_{LTPD}=1.00$		$C_{AQL}=1.45$ $C_{LTPD}=1.00$		$C_{AQL}=1.60$ $C_{LTPD}=1.00$		$C_{AQL}=1.45$ $C_{LTPD}=1.25$		$C_{AQL}=1.60$ $C_{LTPD}=1.25$		$C_{AQL}=1.60$ $C_{LTPD}=1.45$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.09	0.01	167	1.1541	62	1.2720	40	1.3586	354	1.3744	130	1.4646	747	1.5420
	0.02	143	1.1465	53	1.2578	34	1.3384	302	1.3682	111	1.4537	644	1.5375
	0.03	128	1.1407	47	1.2461	30	1.3217	271	1.3637	99	1.4452	582	1.5343
	0.04	117	1.1358	43	1.2370	28	1.3121	249	1.3601	91	1.4387	538	1.5316
	0.05	109	1.1318	40	1.2293	26	1.3013	232	1.3569	85	1.4332	504	1.5293
	0.06	102	1.1278	38	1.2237	24	1.2892	217	1.3538	80	1.4282	475	1.5272
	0.07	97	1.1248	36	1.2176	23	1.2825	205	1.3510	75	1.4227	451	1.5253
	0.08	92	1.1215	34	1.2109	22	1.2754	195	1.3486	71	1.4178	430	1.5235
	0.09	87	1.1179	32	1.2037	21	1.2678	185	1.3459	68	1.4139	411	1.5217
	0.10	83	1.1149	31	1.1998	20	1.2596	177	1.3436	65	1.4098	394	1.5200
0.10	0.01	162	1.1566	60	1.2760	33	1.1907	343	1.3764	126	1.4680	722	1.5436
	0.02	138	1.1489	51	1.2615	29	1.2057	292	1.3703	107	1.4569	620	1.5391
	0.03	124	1.1434	46	1.2516	27	1.2190	262	1.3659	96	1.4490	560	1.5360
	0.04	113	1.1384	42	1.2425	25	1.2261	240	1.3622	88	1.4424	517	1.5333
	0.05	105	1.1343	39	1.2347	24	1.2364	223	1.3589	82	1.4368	483	1.5310
	0.06	99	1.1309	37	1.2290	22	1.2368	209	1.3560	77	1.4317	455	1.5289
	0.07	93	1.1272	34	1.2195	21	1.2428	197	1.3532	72	1.4260	431	1.5269
	0.08	88	1.1238	33	1.2161	21	1.2558	187	1.3507	68	1.4211	411	1.5252
	0.09	84	1.1209	31	1.2087	20	1.2596	178	1.3482	65	1.4170	392	1.5234
	0.10	80	1.1177	30	1.2047	19	1.2623	170	1.3459	62	1.4127	376	1.5217

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