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ROYALTY MODELS FOR BUILD-OPERATE-TRANSFER TRANSPORTATION PROJECTS WITH UNCERTAINTIES

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ROYALTY MODELS FOR BUILD-OPERATE-TRANSFER TRANSPORTATION PROJECTS WITH UNCERTAINTIES

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This paper develops royalty models for build-operate-transfer (BOT) transportation projects with uncertainties about self-liquidating ratios (SLR), equity return rates (IRR_E), annual debt service coverage ratios (DSCR) and patronage. The core logic of the modeling is to maximize public sectors' total discounted royalties while meeting private investors' requirements for IRR_E and DSCR. A real case of BOT project is tested under various royalty schemes with sensitivity analyses for some uncertain factors. The policy implications, based on the case-specific results, suggest that the public sectors should adopt two-part or increasingly multi-part royalty schemes rather than uniform-rate or decreasingly multi-part ones. In addition, if the governments attempt to make low SLR transportation BOT projects financially viable, the public sectors should provide subsidy rather than partial investment.

KEYWORDS: BOT projects, royalty models, royalty schemes

1. INTRODUCTION

Private participation in public works such as transportation infrastructure projects with means like build-operate-transfer (BOT) has long been implemented in many countries. Its main purposes are to lessen the government financial burden, to expedite the construction progress and to enhance the operation efficiency (Sidney, 1996). Numerous renowned transportation BOT projects in the world, such as Eurotunnel, Taiwan High-Speed Railway, Sydney Harbor Tunnel, Hong Kong Eastern and Western Harbor Tunnels and Malaysian Tolled-Freeways, have been implemented. Without in-depth investigations by proper royalty models at the open-tendering stage, the terms and conditions of BOT projects could be either too loose or too tight to private investors during the concession period. In the former case, the private investors would yield substantially excess profit, which is not justified from general public equity perspectives, since the public assets or rights for BOT projects are granted by the government. In the later case, however, the private investors are likely bankrupted, which could cause tremendous troublesome to the society at large. In word, financial viability can be viewed as one of the most crucial factors affecting the success of transportation BOT projects.

The amount of royalties paid by the private institutions and/or the financial supports provided by the public sectors can determine the financial viability of a BOT project. For low self-liquidating ratio (*SLR*) projects, the public sectors normally require providing appropriate financial supports, such as annual subsidy or partial investment, to make the projects financially sustainable during the concession period. For high *SLR* projects, the public sectors do not need to provide financial supports; on the contrary, they can collect initial and/or following annual royalties from the private institutions to which the concession rights are granted. Unfortunately, it is difficult to determine proper royalty schemes at the open-tendering stage due to the uncertainties of future environment. To

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facilitate negotiating and contracting, it is essential to develop various royalty models with consideration of future uncertainties so as to test if the BOT projects are financially viable under different situations.

In the past, substantial research has been found in the analysis of benefit-cost ratio, net present value, internal rate of return and payback period to evaluate the financial viability of BOT projects (see, for example, Lohmann and Baksh, 1996; Xing and Wu, 2000; Chang and Chen, 2001; Ng and Skitmore, 2001; Asensio and Roca, 2001; Daniel, 2002). To deal with the uncertainties or risks that may encounter during the BOT concession period, fuzzy set theory has been largely used (e.g., Buckley, 1987; Chui and Park, 1994; Wang and Liang, 1995; Dompere, 1997). There are various shapes of membership functions, such as trapezoidal, triangular and bell-shaped, to represent the fuzzy numbers (Terano et al., 1992; Tsoukalas and Uhrig, 1997). Of which, triangular is perhaps the one that can best reflect the financing uncertainties (Buckley, 1987; Chui and Park, 1994). Royalty schemes can also be found in previous works related to franchising a system. For instance, Bousquet et al. (1998) proposed three schemes to collect the royalty for licensing: fixed fee, *ad* valorem and per unit. Fixed fee scheme is to collect a fixed amount of royalty; *ad* valorem scheme is to collect royalties proportional to sale revenues; per unit scheme is to collect royalties proportional to sale units. Generally, fixed fee scheme is suitable for relatively certain environment; *ad* valorem scheme (along with a fixed fee) is appropriate under demand uncertainty; per unit scheme (along with a fixed fee) is adopted under cost uncertainty. Windsperger (2001) developed a two-part royalty scheme for franchising a leaguing system. The initial fee paid by the leaguer is to purchase the technology know-how and brand of the leaguing system. The following annual royalty is to encourage the upstream firms to continuously develop and provide innovative technologies. Kaufmann and Dant (2001) also addressed the necessity for collecting royalty with similar scheme with initial leaguing fee and following annual royalty. To the best of authors' knowledge, no study has been devoted to the development of royalty models with various royalty schemes under uncertain contexts.

Depending on the situations of *SLR* and means of government financial supports, this paper attempts to develop three crisp royalty models under predetermined environment that the predicted patronage (revenue) and requirements by private participations and financiers are assumed known with certainty. However, this assumption may not be always true in reality because uncertainties of BOT projects arise mainly from imprecise predicted-patronage (ambiguity) or negotiable equity return rates and annual debt service coverage ratios requested by related parties (fuzziness) or a combination of both. Therefore, this paper further develops nine fuzzy royalty models, based upon the three crisp (baseline) models, with uncertainties about self-liquidating ratios, equity return rates, annual debt service coverage ratios and patronage. Our proposed fuzzy royalty models can be viewed as fuzzy mathematical programming (FMP) problems, which are generally divided into three categories (Inuiguchi and Ramik, 2000): flexible programming (dealt with vague objective or right-hand-side constraints), possibilistic programming (associated with ambiguous parameters) and robust programming (related to vagueness and ambiguity for both flexible and possibilistic programming models).

Numerous related FMP models can be found in the literature and their solving techniques have been well developed. For instance, Hu and Fang (1998) proposed a max-min arithmetical method and induced a new variable to simplify the multi-objective programming problem transformed from FMP. Buckley (1995), Ramik and Rommelfanger (1996), Nakahara (1998), Parra et al. (1999), and Sakawa and Nishizaki

(2002) employed the possibility distribution to transform fuzzy numbers into crisp ones in order to simplify FMP as a linear programming problem. Rommelfanger (1996) proposed a method to transform the fuzzy constraint into a crisp constraint and a crisp objective function and then employed the max-min method to obtain the compromise solutions. Buckley and Feuring (2000) utilized Hamming distance to define fuzzy objective function and employed fuzzy numbers ranking method to represent the satisfaction of fuzzy constraints. To solve for our proposed nine fuzzy royalty models, we will employ these appropriate techniques.

In this paper, four royalty schemes are identified and a real case of car park BOT project will be tested. Section 2 details the three crisp and nine fuzzy royalty models. Section 3 identifies the four royalty schemes, including uniform-rate, two-part, increasingly multi-part and decreasingly multi-part. Section 4 presents the case study with comprehensive sensitivity analysis. Section 5 summarizes the results and directs the future explorations.

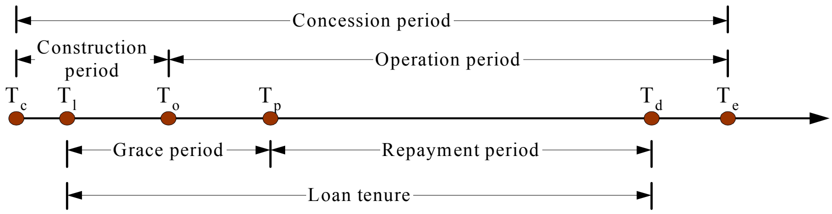
2. THE MODELS

2.1 Certain environment (crisp models)

Let the financial status of the BOT projects be represented by self-liquidation ratio (*SLR*), which is defined as the ratio of total discounted net cash flows to total discounted construction costs. *SLR* greater than 1 represents that the construction investment and annual operation expenses of private institutions can be fully covered by the future revenues. In this case the governments do not have to provide any subsidy or partial investment; instead, the governments can collect a certain amount of royalty from the private institutions. If *SLR* is less than 1, however, the governments often require providing subsidy or partial investment to create enough impetus for private participation and to make the BOT projects financially viable.

Depending on the situations of *SLR* and financial supports from the governments, we develop three crisp royalty (base) models under certain environment: model 1 is for the situation of *SLR* > 1 without government financial supports; model 2 is for the situation of *SLR* < 1 with government subsidy; model 3 is for the situation of *SLR* < 1 with government partial investment. Each model is developed from the governments' perspective but must meet the requirements from private institutions and financiers. We assume that the governments aim to collect highest amount of royalty or providing lowest amount of financial supports (in this paper, we treat it as negative royalties) while franchising the transport infrastructure BOT projects. Thus, the objective functions of the models are to maximize the total discounted royalty. We also assume that the private institutions would demand for a certain level of return rate on their equity (IRR_E) and that the financiers would request a certain level of annual debt service coverage ratio (*DSCR*) to assure the loan being repaid on schedule. Both requirements from the private institutions and the financiers are taken as the constraints.

To facilitate the models development, some key time points of BOT projects are defined in Figure 1. The concession period (T_c to T_e) contains construction period (T_c to T_o) and operation period (T_o to T_e). The loan tenure (T_l to T_d) is divided into grace period (T_l to T_p) and repayment period (T_p to T_d). Three cases of crisp royalty models are considered as the base models as follows.



T_c: The beginning year of construction. T_c=1.
T_l: The year of loan borrowed.
T_o: The beginning year of operation.
T_p: The beginning year of loan repaid.
T_d: The ending year of load repaid.
T_e: The ending year of concession.

FIGURE 1: Definition of key time points for BOT projects

2.1.1 Case (1) SLR > 1 without government financial supports

Model 1, denoted as [LP_1], is developed for the situation of *SLR* > 1 without government financial supports. It is formulated as:

[LP_1]
$$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}, \tag{1}$$

subject to

$$\alpha_t \geq D, t = T_p, \ldots, T_d, \tag{2}$$

$$\beta \geq 0, \tag{3}$$

$$x_t \geq 0, t = 1, 2, \ldots, T_e, \tag{4}$$

where Z is the total discounted royalty, x_t is the royalty of tth concession year, r is the discount rate, α_t is the *DSCR* of tth concession year, D is the level of *DSCR* required by financier, and β is the net present value. Equation (1) represents the objective function of [LP_1]. Equation (2) is the constraint to satisfy the requirement of financiers. Equation (3) is the constraint to meet the requirement of private institutions. Equation (4) is the non-negativity of royalty due to *SLR* > 1 without financial supports. Furthermore, α_t can be calculated by equation (5):

$$\alpha_t = \frac{FB_t + NFB_t - (OC_t + NOC_t + RC_t + x_t)}{PMT}, t = T_p, \ldots, T_d, \tag{5}$$

where FB_t is the fare-box (direct) revenue of tth concession year, NFB_t is the non-fare-box (indirect) revenue of tth concession year, OC_t is the operating cost of tth concession year, which includes the expenses of activities that are directly related to the operation, such as personnel, land rent, public utility, and maintenance costs, etc., NOC_t is the non-operating cost of tth concession year, which includes the expenses of activities that are not directly related to the operation, such as investment loss, inventory loss and ancillary cost, RC_t is the re-installation cost of tth concession year, which depends upon the scale of facilities needed to be replaced, and *PMT* is the annual repayment for both loan and interest, which is calculated as:

$$PMT = \frac{D_b}{\sum_{t=T_p}^{T_d} \frac{1}{(1+I)^t}}, \tag{6}$$

where D_b is the amount of loan, and I is the average loan interest rate. In addition, β can be calculated by equation (7):

$$\beta = \sum_{t=1}^{T_e} \frac{CF_t}{\left(1+WACC\right)^t} \ , \tag{7}$$

where CF_t is the cash flow of tth concession year, $WACC$ is the weighted average cost of capital, $CF_t=C_t \times E/A$ for $t=T_c,\ldots,T_o$-1 (i.e., during the construction period), C_t is the construction cost of tth construction year, E is total equity, A is total asset, $CF_t=FB_t+NFB-(IE_t+PMT+OC_t+NOC_t+RC_t+DP_t+x_t)-tax_t$, for $t=T_o,\ldots,T_e$ (i.e., during the operation period), IE_t is the interest paid during grace period, $IE_t=D_b \times I$, DP_t is the depreciation cost of tth concession year, and tax_t is the tax (with tax rate θ) of tth concession year, which can be calculated as:

$$tax_t=(FB_t+NFB_t-IE_t-PMT-OC_t-NOC_t-RC_t-DP-x_t)\times\theta, \text{ if } t=T_t,\ldots,T_e; \text{ 0, otherwise}, \tag{8}$$

where T_t is the beginning year of levying the tax. For instance, if the project enjoys a tax exemption for h years, then $T_t=T_o+h$; otherwise, $T_t=T_o$. $WACC$ can be calculated by $WACC=I\times(1-E/A)+IRR_E\times E/A$ where IRR_E is the internal rate of returns on equity required by private institutions.

2.1.2 Case (2) SLR < 1 with government subsidy

Model 2, denoted as [LP_2], is developed for the situation of SLR < 1 with government subsidy. It can be formulated as:

[LP_2] $$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t} \ , \tag{9}$$

subject to

$$\alpha_t \geq D, t = T_p,\ldots,T_d \ , \tag{10}$$

$$\beta \geq 0 \ . \tag{11}$$

Note that [LP_2] is essentially the same as [LP_1] except that the royalty x_t for each concession year is allowed to be negative, representing the amount of subsidy (negative royalty) that the government would provide in the tth concession year.

2.1.3 Case (3) SLR < 1 with government partial investment

Model 3, denoted as [LP_3], is developed for the situation of SLR < 1 with government partial investment. It is formulated as:

[LP_3] $$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t} - GI \ , \tag{12}$$

subject to

$$\alpha_t \geq D, t = T_p,\ldots,T_d \ , \tag{13}$$

$$\beta \geq 0 \ , \tag{14}$$

$$GI \geq 0 \ , \tag{15}$$

$$x_t \geq 0 \ , \tag{16}$$

where GI is the amount of the government partial investment of the construction. Note that [LP_3] is essentially the same as [LP_1] except that the governments provide a nonnegative investment at the beginning of the concession period and that both equation (6) and IE_t should be modified as $PMT = (D_b - GI) / \left(\sum_{t=1}^{l-m} 1/(1+I)^t \right)$ and $IE_t = (D_b - GI) \times I$.

2.2 Uncertain environment (fuzzy models)

The above three crisp royalty models are served as the base cases. These crisp models are to determine the optimal annual royalty under certain environment where the predicted patronage (revenue) and the requirements from private institutions and financiers are assumed known as crisp values. In practice, this assumption may not hold due to the uncertainty attributed from informational vagueness (e.g., imprecise prediction of patronage) and intrinsic vagueness (e.g., negotiable IRR_E and *DSCR*). Corresponding to the above three crisp models (base cases) and in association with the three sources of uncertainties (one informational and two intrinsic vagueness), nine fuzzy royalty models can be formulated as follows:

2.2.1 Case (1-1) vagueness of DSCR

If the level of *DSCR* required by the financiers is vague, the fuzzy royalty model, denoted as [FLP_{11}], can be formulated as:

[FLP_{11}] $$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}, \quad (17)$$

subject to

$$\alpha_t \tilde{\geq} D, t = T_p, \ldots, T_d, \quad (18)$$

$$\beta \geq 0, \quad (19)$$

$$x_t \geq 0, \quad (20)$$

where, except for equation (18), all equations of [FLP_{11}] are exactly the same as those of [LP_1]. Equation (18), modified from equation (2), can be interpreted as the *DSCR* of *t*th concession year being “fuzzy no less than” the level required by the financiers. According to Rommelfanger (1996), the “fuzzy no less than” constraint can be transformed into a crisp constraint and a crisp objective function and then solved by the max-min method. We will use Rommelfanger (1996) method to solve for [FLP_{11}].

2.2.2. Case (1-2) vagueness of IRR_E

If the level of IRR_E required by the private institutions is vague, the fuzzy royalty model, denoted as [FLP_{12}], can be formulated as:

[FLP_{12}] $$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}, \quad (21)$$

subject to

$$\alpha_t \geq D, t = T_p, \ldots, T_d, \quad (22)$$

$$\beta \tilde{\geq} 0, \quad (23)$$

$$x_t \geq 0, \quad (24)$$

where, except for equation (23), all equations of [FLP_{12}] are exactly the same as those of [LP_1]. Equation (23), modified from equation (3), represents the *NPV* of *t*th concession year being "fuzzy no less than" zero, which is discounted by the vague level of IRR_E required by the private institutions. Again, Rommelfanger (1996) method is employed in this paper to solve for [FLP_{12}].

2.2.3 Case (1-3) ambiguity of patronage

If the predicted patronage is indefinite (ambiguous), the fuzzy royalty model, denoted as [FLP_{13}], can be formulated as:

[FLP_{13}]
$$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}, \quad (25)$$

subject to

$$\tilde{\alpha}_t \geq D, t = T_p, \ldots, T_d, \quad (26)$$

$$\tilde{\beta} \geq 0, \quad (27)$$

$$x_t \geq 0, \quad (28)$$

where, $\tilde{\alpha}_t$ and $\tilde{\beta}$ are fuzzy numbers. According to Inuiguchi and Ramik (2000), we can use possibility distribution to transform these fuzzy numbers into crisp ones and then solve the linear programming problem.

To avoid repeat and lengthy presentation, the remaining six fuzzy royalty models, including ([FLP_{21}], [FLP_{22}], [FLP_{23}]) associated with [LP_2] and ([FLP_{31}], [FLP_{32}], [FLP_{33}]) associated with [LP_3], are only summarized without explanations as follows.

2.2.4 Case (2-1) vagueness of DSCR

[FLP_{21}]
$$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t},$$

subject to

$$\alpha_t \tilde{\geq} D, t = T_p, \ldots, T_d,$$

$$\beta \geq 0.$$

2.2.5 Case (2-2) vagueness of IRR_E

[FLP_{22}]
$$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}$$

subject to

$$\alpha_t \geq D, t = T_p, \ldots, T_d,$$

$$\beta \tilde{\geq} 0.$$

2.2.6 Case (2-3) ambiguity of patronage

[FLP_{23}]
$$\text{Maximize } Z = \sum_{t=0} \frac{x_t}{(1+r)^t}$$

subject to

$$\tilde{\alpha}_t \geq D, t = T_p, ..., T_d \ ,$$
$$\tilde{\beta} \geq 0 \ .$$

2.2.7 Case (3-1) vagueness of DSCR

[FLP_{31}] Maximize $Z = \sum_{t=0} \frac{x_t}{(1+r)^t} - GI$

subject to

$$\alpha_t \tilde{\geqq} D, t = T_p, ..., T_d \ ,$$
$$\beta \geq 0 \ ,$$
$$GI \geq 0 \ ,$$
$$x_t \geq 0 \ .$$

2.2.8 Case (3-2) vagueness of IRR_E

[FLP_{32}] Maximize $Z = \sum_{t=0} \frac{x_t}{(1+r)^t} - GI$

subject to

$$\alpha_t \geq D, t = T_p, ..., T_d \ ,$$
$$\beta \tilde{\geqq} 0 \ ,$$
$$GI \geq 0 \ ,$$
$$x_t \geq 0 \ .$$

2.2.9 Case (3-3) ambiguity of patronage

[FLP_{33}] Maximize $Z = \sum_{t=0} \frac{x_t}{(1+r)^t} - GI$

subject to

$$\tilde{\alpha}_t \geq D, t = T_p, ..., T_d \ ,$$
$$\tilde{\beta} \geq 0 \ ,$$
$$GI \geq 0 \ ,$$
$$x_t \geq 0 \ .$$

3. ROYALTY SCHEMES

A free-form royalty scheme is adopted in the present paper to solve the optimal annual royalty of [LP_1]~[LP_3] problems under certain environment or of [FLP_{11}]~[FLP_{33}] problems under uncertain context. Although the free-form scheme can determine optimal annual royalty without any preset structures, it is difficult to be set as a basis for negotiating and contracting. In practice, to facilitate the negotiations for documenting onto the contracts, a definite royalty scheme must be given and accepted by different parties, including private investors and financiers. This paper demonstrates with four definite royalty schemes, including uniform-rate, two-part, increasingly multi-part and decreasingly multi-part structures, as follows.

Uniform-rate scheme is to collect the same amount of royalty every year. The uniform-rate royalty for each year is defined as:

$$x_t = \rho \times PB_t, t = T_o, T_o + 1, \ldots, T_e, \tag{29}$$

where PB_t represents the royalty calculation base. If PB_t is the fare-box revenue, total revenue or profit, its value represents a ratio. If PB_t is the patronage, its value represents dollars. The optimal uniform-rate scheme can be solved by substituting the decision variables x_t in [LP_1]-[LP_3] or [FLP_{11}]-[FLP_{33}] with equation (29).

Two-part scheme is to collect an initial royalty (*FX*) upon the concession agreement being reached and then followed by the same amount of annual royalty. This scheme is expressed as:

$$x_1 = FX \tag{30}$$

$$x_t = \rho \times PB_t, t = T_o, T_o + 1, \ldots, T_e. \tag{31}$$

Increasingly multi-part scheme is to collect annual royalty with monotonically progressive rates, expressed as:

$$x_t = \rho_1 \times PB_t, t = T_o, T_o + 1, \ldots, T_o + k, \tag{32}$$

$$x_t = \rho_2 \times PB_t, t = T_o + k + 1, T_o + k + 2, \ldots, T_o + 2k, \tag{33}$$

$$\vdots$$

$$x_t = \rho_n \times PB_t, t = T_o + (n-1)k + 1, T_o + (n-1)k + 2, \ldots, T_e, \tag{34}$$

$$\rho_i < \rho_{i+1}, i = 1, 2, \ldots, n, \tag{35}$$

where *k* is the period that the uniform-rate remains unchanged. *k* can be set arbitrarily (e.g., 5 years). *n* is the number of parts, which is determined by dividing the operation period by *k*. The optimal multi-part uniform rates can be solved by substituting the decision variables x_t in [LP_1]-[LP_3] or [FLP_{11}]-[FLP_{33}] with equations (32)-(34), respectively. Equation (35) is additional constraint to assure the monotonic increase of the royalty scheme.

Decreasingly multi-part scheme is to collect annual royalty with monotonically regressive rates, expressed as:

$$x_t = \rho_1 \times PB_t, t = T_o, T_o + 1, \ldots, T_o + k, \tag{36}$$

$$x_t = \rho_2 \times PB_t, t = T_o + k + 1, T_o + k + 2, \ldots, T_o + 2k, \tag{37}$$

$$\vdots$$

$$x_t = \rho_n \times PB_t, t = T_o + (n-1)k + 1, T_o + (n-1)k + 2, \ldots, T_e, \tag{38}$$

$$\rho_i > \rho_{i+1}, i = 1, 2, \ldots, n, \tag{39}$$

where *k* and *n* are the same as in the increasingly multi-part scheme. Equation (39) is to assure the decreasing trend of the scheme. The optimal multi-part uniform rates can be solved by substituting the decision variables x_t in [LP_1]-[LP_3] or [FLP_{11}]-[FLP_{33}] with equations (36)- (38), respectively.

4. CASE STUDY

4.1 Data

To demonstrate the applicability of our proposed royalty models, we select the Pei-Tou Car Park BOT project in Taipei City as the case study. This transportation BOT project was invested by a single private institution with a sole financier. At the tendering stage, the project was preset by Taipei City Government with two years of construction and 17 years of concession.

After negotiations among different parties, the ratio of self-fund to loan was agreed as 3:7. The IRR_E required by the private institution was 12%. Financial interest rate (I) was 6%. Weighted average capital cost was calculated as 12%×0.3+6%×0.7=7.8%. Discount rate (r) was 8%. Inflation rate was 3.5%. The *DSCR* requested by the financier was 1.2. The total construction cost (CC_t) was 50 millions (hereafter in NT dollars; 33NT$ equivalent to 1US$), which was evenly distributed in the two-year construction period. A loan of 35 millions was borrowed in the second year. Grace period for this loan was two years and repayment period was 10 years. The interest (IE_t) in the grace period was 2.1 millions for t=3 and 4. The *PMT* in the period of repayment period was 5,696,089 dollars for t=5, 6,…,14.

The annual operating cost (OC_t) for personnel, land rent and public utility was 6 millions and for ancillary items was 0.3 million for t=3, 4,…,17. The replacement costs were 500,000, 573,762, 658,405 dollars that took place at t=6, 10 and 14, respectively. The first five years of operation were exempted from tax but the following years were subject to business income tax (tax_t) with tax rate 25% for t=8, 9,…,17. The depreciation cost was 1 million per year, for t=3, 4,…,17.

In the following analysis, patronage is used as the royalty calculation base (PB_t). The average parking fee is estimated with 50 dollars per stay (transaction). Two patronage scenarios, optimistic and pessimistic, are examined: (1) The optimistic patronage scenario of the first-year operation is 0.25 million transactions with an annual growth rate of 3%. The corresponding ancillary revenue (NFB_t) is 0.25 million in the first year with an annual growth rate of 3% for t=3,4,…,17. (2) The pessimistic patronage scenario of the first-year operation is 0.22 million transactions with an annual growth rate of 3%. The corresponding ancillary revenue (NFB_t) was 0.22 million in the first year with an annual growth rate of 3% for t=3,4,…,17.

Based upon the data given above, the *SLR* of this project is higher than 1 for the optimistic scenario (thus, no government financial supports would be provided) and less than 1 for the pessimistic scenario (thus, government annual subsidy or partial investment would be provided). Therefore, the following will conduct comprehensive analysis for three scenarios using crisp royalty models as the bases and corresponding fuzzy models as the contrasts: (1) $SLR > 1$ without government financial supports, (2) $SLR < 1$ with government subsidy, and (3) $SLR < 1$ with government partial investment are analyzed as follows.

4.2 Results

The annual royalties of free-form scheme, determined by the three crisp royalty models ([LP_1], [LP_2], [LP_3]) and six fuzzy royalty models ([FLP_{11}], [FLP_{12}], [FLP_{13}], [FLP_{21}], [FLP_{22}], [FLP_{23}], [FLP_{31}], [FLP_{32}], [FLP_{33}]), are solved by the above-mentioned techniques. Their results are discussed as follows.

4.2.1 Scenario (1) SLR > 1 without government financial supports

[LP_1] model and its corresponding three fuzzy models ([FLP_{11}], [FLP_{12}], [FLP_{13}]) are solved under this optimistic scenario. In case of vagueness of *DSCR*, we assume that the level of *DSCR* required by the financier is 1.2, but it is still acceptable as long as the level is above 1.0. Therefore, the cortex of fuzzy *DSCR* is 1.2 and spread is 1.2-1.0=0.2. A total discounted royalty of 29.27 millions is resulted from crisp model [LP_1]. In contrast, the total discounted royalty resulted from fuzzy royalty model [FLP_{11}] is 30.64 millions, slightly higher than the royalty collected by crisp model.

The optimal annual royalties for crisp and fuzzy models are compared in Figure 2a. According to the annual royalties determined by both models, it is recommended that the city government collect the annual royalties in the 8th -17th concession years. Notice that the annual royalties or subsidies calculated by fuzzy model are almost the same as those obtained by crisp model except for the 10th and 14th-17th concession years with higher royalties. As expected, replacement costs in the concession years of 10 and 14 have caused the royalties slight drop.

In case of vagueness of IRR_E, we assumed that the level of IRR_E required by the private institution is 12%, but it is still acceptable as long as the level is above 10% (namely, the cortex is 12% and the spread is 2%). A total discounted royalty of 35.45 millions is resulted from fuzzy royalty model [FLP_{12}], which is even higher than that by [FLP_{11}]. The optimal annual royalties for crisp and fuzzy models are compared in Figure 2b. According to the annual royalties determined by both models, it is also recommended that the city government collect royalties in the 8th -17th concession years. The annual royalties or subsidies calculated by fuzzy model are almost the same as those by crisp model except for the 14th-17th concession years with much higher royalties.

In case of ambiguity of patronage (imprecise prediction of revenue), we assume that the patronage is an isosceles triangular fuzzy number with the cortex of 0.25 million and the spread of 0.01 million in the first year of operation to represent the 4% of uncertainty about predicted patronage. A maximum discounted royalty of 16.23 millions is resulted from fuzzy royalty model [FLP_{13}], which is far less than both [FLP_{11}] and [FLP_{12}] and even less than the crisp one [LP_1]. Figure 2c compares the optimal annual royalties for both models.

4.2.2 Scenario (2) SLR < 1 with government subsidy

[LP_2] model and its corresponding three fuzzy models ([FLP_{21}], [FLP_{22}], [FLP_{23}]) are solved under this pessimistic scenario. In case of vagueness of *DSCR*, the cortex and spread of fuzzy *DSCR* are assumed the same as in scenario (1). A total discounted royalty of 10.38 millions can be collected by crisp royalty model [LP_2]. In contrast, the maximum royalty collected by fuzzy royalty model [FLP_{21}] is 10.78 millions, slightly higher than the royalty collected by crisp model.

The optimal annual royalties for crisp and fuzzy models are depicted in Figure 3a. According to the annual royalties determined by both models, it is recommended that the government collect royalties in the 3rd and 8th -17th concession years and provide subsidies in the 4th-7th concession years. The annual royalties or subsidies calculated by fuzzy model are almost the same as those by crisp model except for the 3rd, 15th and 17th concession years with higher royalties and 4th and 5th concession years with lower subsidies. As expected, the replacement costs in the years of 6, 10 and 14 have caused sharp rise in subsidy and drop in royalty.

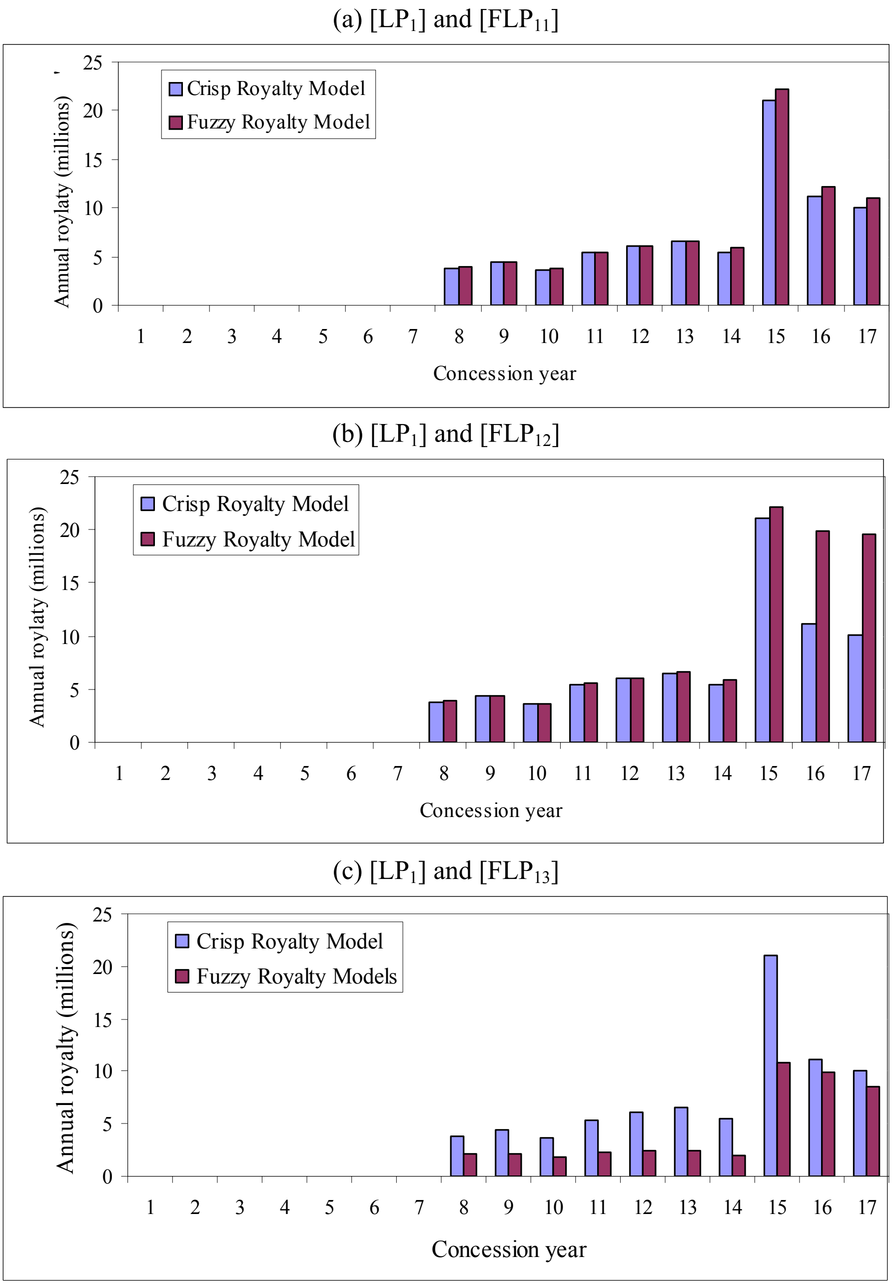
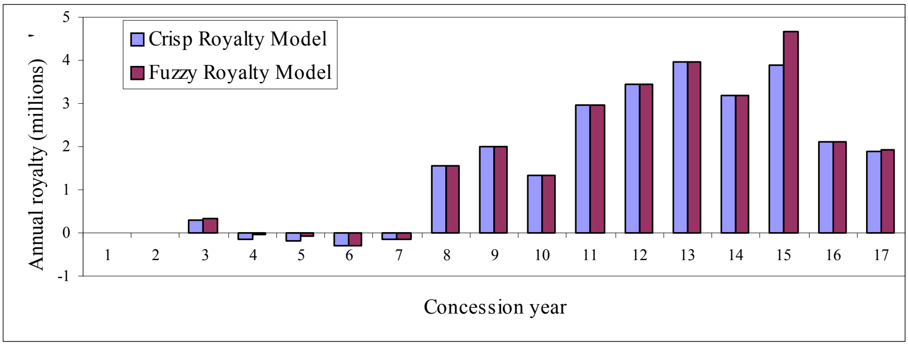
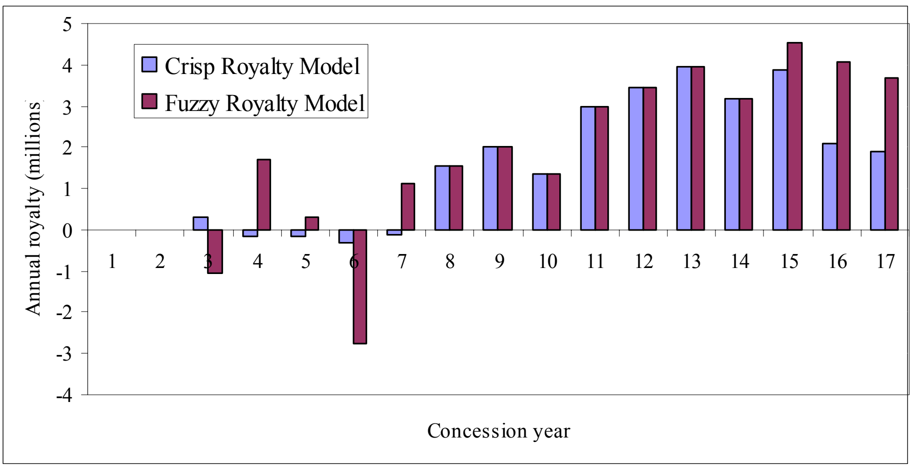


FIGURE 2: Optimal annual royalties for case (1): *SLR* > 1 without government financial supports

(a) [LP_2] and [FLP_{21}]



(b) [LP_2] and [FLP_{22}]



(c) [LP_2] and [FLP_{23}]

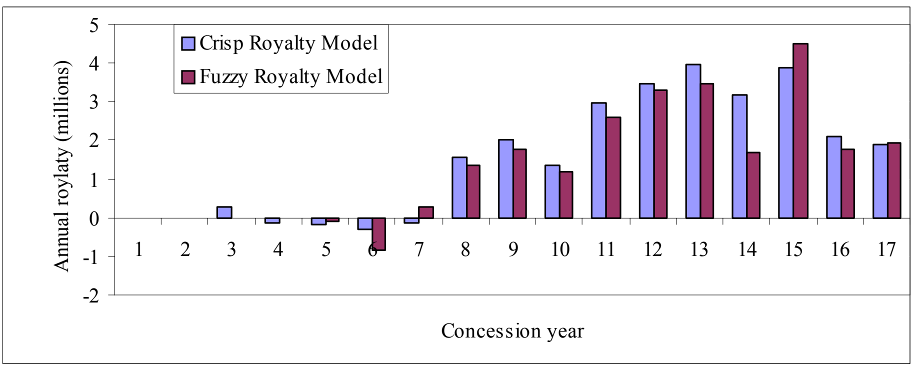


FIGURE 3: Optimal annual royalties for case (2): *SLR* < 1 with government subsidy

In case of vagueness of IRR_E, the cortex and spread of fuzzy IRR_E are assumed the same as in scenario (1). A total discounted royalty of 11.50 millions can be collected by fuzzy royalty model [FLP_{22}], even higher than [FLP_{21}]. The optimal annual royalties for crisp and fuzzy models are depicted in Figure 3b. The patterns of annual royalties determined by fuzzy model are rather different from those by crisp model in the beginning and ending years of operation, but the royalties determined by both models are exactly the same in 8th - 14th concession years.

Similarly, in case of ambiguity of patronage and the fuzzy patronage is assumed the same as in scenario (1). A total discounted royalty of 9.02 millions would be collected by fuzzy royalty model [FLP_{23}], which is far less than both [FLP_{21}] and [FLP_{22}] and even less than the crisp one [LP_2]. Figure 3c presents the optimal annual royalties for both models.

4.2.3 Scenario (3) SLR < 1 with government partial investment

[LP_3] model and its corresponding three fuzzy models ([FLP_{31}], [FLP_{31}], [FLP_{33}]) are solved under this pessimistic scenario. In case of vagueness of *DSCR* and the cortex and spread of fuzzy *DSCR* are the same as in scenario (1). A total discounted royalty of 8.24 millions would be collected by crisp royalty model [LP_3], lower than that by [LP_2]. In contrast, the maximum royalty collected by fuzzy royalty model [FLP_{31}] is 9.13 millions, slightly higher than that by crisp model.

The optimal annual royalties for crisp and fuzzy models are shown in Figure 4a. According to the annual royalties determined by both models, it is recommended that the government provide partial investment of 1.45 millions at the construction stage but collect annual royalties in the 8th - 17th concession years. Notice that the pattern of annual royalties by fuzzy model is similar to the pattern by crisp model, but with higher royalties.

In case of vagueness of IRR_E and the cortex and spread of fuzzy IRR_E are the same as in scenario (1). A total discounted royalty of 10.61 millions would be collected by fuzzy royalty model [FLP_{32}], which is even higher than [FLP_{31}]. The optimal annual royalties for crisp and fuzzy models are presented in Figure 4b. The pattern of annual royalties determined by fuzzy model is rather different from that by crisp model. The fuzzy model results in higher royalties in the 7th-13th concession years, except for the 9th year, but lower royalties in the 14th-17th concession years.

Finally, in case of ambiguity of patronage and the fuzzy patronage is the same as in scenario (1), but with the cortex of 0.22 million and the spread of 0.01 million in the first year of operation. A total discounted royalty of 5.95 millions would be collected by fuzzy royalty model [FLP_{33}], which is far less than both [FLP_{31}] and [FLP_{32}] and even less than the crisp model [LP_3]. The optimal annual royalties for both crisp and fuzzy models are shown in Figure 4c. Note that fuzzy model will yield lower royalties in the 8th -14th concession year, but higher royalties in the 15th -17th concession year.

4.3 Sensitivity analysis

In Section 4.2, the three fuzzy numbers *DSCR*, IRR_E and patronage are given subjectively. We are interested in the sensitivities of fuzzy numbers to the total discounted royalty. The following sensitivity analyses evaluate the total discounted royalties under different royalty schemes and with cortexes and spreads varied by ±10% and ±20%, respectively. However, some variation might not be practically acceptable.

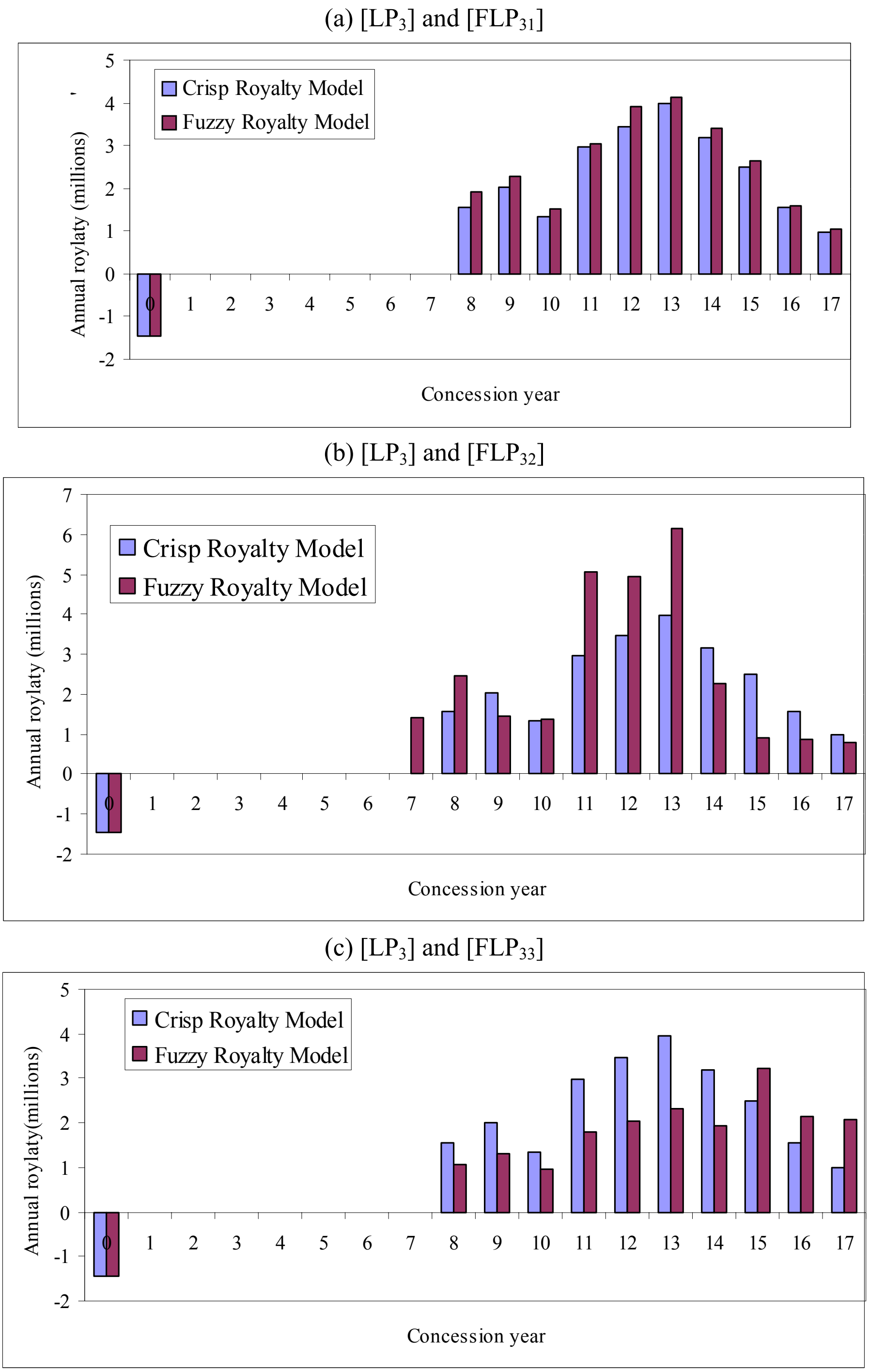


FIGURE 4: Optimal annual royalties for case (3): *SLR* < 1 with government partial investment

For example, the bottom-line requirement of DSCR is set as 1.0, but the –10% or -20% variation on the cortex or spread would make it below 1.0, which might not be accepted by the financers.

Table 1 summarizes the results of sensitivity analysis for scenario (1): *SLR* > 1 without government financial supports. The effects of *DSCR* show that total discounted royalties determined by uniform-rate and decreasingly multi-part schemes are sensitively decreased with the increase of the cortex. However, the total discounted royalties for all schemes are insensitive to the change in spread. The effects of IRR_E show that total discounted royalties for all schemes are not so sensitive to the changes in cortex or spread. The effects of patronage show that total discounted royalty determined by different schemes are all sensitively increased with the increase of the cortex but sensitively decreased with the increase of the spread. The comparisons among different royalty schemes consistently conclude that free-form scheme would obtain the highest total discounted royalty, followed by two-part or increasingly multi-part schemes. Both uniform-rate and decreasingly multi-part schemes would have the lowest total discounted royalty. These results are expected mainly due to the patronage effect.

Table 2 reports the results of sensitivity analysis for scenario (2): *SLR* < 1 with government subsidy. The effects of *DSCR* show that total discounted royalty determined by uniform-rate, increasingly multi-part and decreasingly multi-part schemes are sensitively decreased with the increase of the cortex; while total discounted royalty for uniform-rate and decreasingly multi-part schemes are sensitively increased with the increase of the spread. In additions, the total discounted royalties determined by free-form scheme and two-part scheme are far higher than the other three schemes. The effects of IRR_E show that total discounted royalties for all schemes are not sensitive to the changes in cortex or spread. The effects of patronage show that total discounted royalties determined by different schemes are all sensitively increased as the cortex increases. However, the total discounted royalties for all schemes are almost all insensitive to the change in spread. Again, free-form scheme would have the highest total discounted royalty, followed by two-part or increasingly multi-part schemes. Uniform-rate and decreasingly multi-part schemes have the lowest total discounted royalty.

Table 3 presents the results of sensitivity analysis for scenario (3): *SLR* < 1 with government partial investment. The effects of *DSCR* show that total discounted royalties determined by uniform-rate, increasingly multi-part and decreasingly multi-part schemes are sensitively decreased with the increase of the cortex; while total discounted royalty for uniform-rate and decreasingly multi-part schemes are sensitively increased with the increase of the spread. The total discounted royalties determined by free-form scheme and two-part scheme are significantly higher than the other three schemes. The effects of IRR_E show that total discounted royalties for all schemes are not sensitive to the change in cortex or spread. The effects of patronage show that total discounted royalties determined by different schemes are all sensitively increased as the cortex increases; while total discounted royalty for uniform-rate scheme is sensitively decreased as the spread increases. Again, the free-form scheme can yield the highest total discounted royalty, followed by increasingly multi-part or two-part schemes. Uniform-rate and decreasingly multi-part schemes obtain the lowest total discounted royalty.

TABLE 1: Sensitive analysis for case (1): *SLR* > 1 without government financial supports

Royalty	Cortex					Spread				
scheme	-20%	-10%	0%	10%	20%	-20%	-10%	0%	10%	20%
(a) DSCR										
Uniform-	27.12	24.99	20.39	14.48	8.57	19.40	19.89	20.39	20.88	21.33
rate	**33%**	**23%**	0%	**-29%**	**-58%**	-5%	-2%	0%	2%	5%
Two-part	27.12	26.84	26.23	25.45	24.67	26.10	26.16	26.23	26.30	26.36
	3%	2%	0%	-3%	-6%	0%	0%	0%	0%	0.00
Increasingly	29.84	29.14	27.97	26.55	25.31	27.69	27.84	27.97	28.10	28.23
multi-part	7%	4%	0%	-5%	-10%	-1%	0%	0%	0%	0.01
Decreasingly	26.92	24.25	19.25	13.34	7.44	18.27	18.76	19.25	19.74	20.23
multi-part	**40%**	**26%**	0%	**-31%**	**-61%**	-5%	-3%	0%	3%	0.05
Free-form	30.67	30.66	30.64	30.62	30.60	30.64	30.64	30.64	30.64	30.64
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
(b) IRR_E										
Uniform-	15.06	15.06	15.06	15.06	15.06	15.06	15.06	15.06	15.06	15.06
rate	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Two-part	26.02	26.00	25.97	25.94	25.91	25.96	25.96	25.97	25.97	25.98
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Increasingly	24.73	24.73	24.73	24.73	24.73	24.73	24.73	24.73	24.73	24.73
multi-part	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Decreasingly	13.96	13.96	13.96	13.96	13.96	13.96	13.96	13.96	13.96	13.96
multi-part	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Free-form	29.88	29.84	29.80	29.77	29.73	29.79	29.80	29.80	29.81	29.82
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
(a) Patronage										
Uniform-	0.00	2.42	8.30	14.18	20.05	22.06	14.88	8.30	2.30	0.00
rate	**-100%**	**-71%**	0%	**71%**	**142%**	**166%**	**79%**	0%	**-72%**	**-100%**
Two-part	2.55	8.43	13.61	20.19	26.07	28.67	21.20	13.61	8.01	2.30
	-81%	**-38%**	0%	**48%**	**92%**	**111%**	**56%**	0%	**-41%**	**-83%**
Increasingly	1.76	7.65	13.52	19.40	25.27	27.80	20.37	13.52	7.26	1.59
multi-part	**-87%**	**-43%**	0%	**43%**	**87%**	**106%**	**51%**	0%	**-46%**	**-88%**
Decreasingly	0.00	1.87	7.71	13.53	19.36	21.29	14.21	7.71	1.78	0.00
multi-part	**-100%**	**-76%**	0%	**76%**	**151%**	**176%**	**84%**	0%	**-77%**	**-100%**
Free-form	3.29	8.47	16.23	22.70	29.16	32.08	23.83	16.23	8.05	2.96
	-80%	**-48%**	0%	**40%**	**80%**	**98%**	**47%**	0%	**-50%**	**-82%**

Note:

1. The total discounted royalties and percentage changes are given in the first and second rows in each scheme, respectively.
2. The percentage changes represented in bold face indicate that they are sensitive, depending on whether their percentage changes in total discounted royalty exceed the percentage changes in cortex or spread or not.

TABLE 2: Sensitive analysis for case (2): *SLR* < 1 with government subsidy

Royalty	Cortex					Spread				
scheme	-20%	-10%	0%	10%	20%	-20%	-10%	0%	10%	20%
(a) DSCR										
Uniform-rate	9.52	7.32	2.56	-3.54	-9.66	3.05	3.46	3.87	4.28	4.64
	272%	**186%**	0%	**-239%**	**-478%**	**-21%**	**-11%**	0%	**10%**	**20%**
Two-part	9.52	9.23	8.59	7.79	6.98	7.67	7.85	8.02	8.18	8.22
	11%	7%	0%	-9%	-19%	-4%	-2%	0%	2%	3%
Increasingly multi-part	9.52	7.96	3.57	-2.54	-8.64	3.88	4.30	4.71	5.09	5.29
	167%	**123%**	0%	**-171%**	**-342%**	-18%	-9%	0%	8%	12%
Decreasingly multi-part	9.36	6.65	1.54	-4.56	-10.66	2.20	2.61	3.02	3.44	3.85
	508%	**332%**	0%	**-397%**	**-793%**	**-27%**	**-14%**	0%	**14%**	**27%**
Free-form	10.80	10.79	10.78	10.78	10.76	10.66	10.66	10.67	10.67	10.67
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
(b) IRR_E										
Uniform-rate	-2.39	-2.39	-2.39	-2.39	-2.39	5.40	5.40	5.40	5.40	5.40
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Two-part	8.58	8.55	8.52	8.49	8.46	9.30	9.30	9.31	9.31	9.31
	1%	0%	0%	0%	-1%	0%	0%	0%	0%	0%
Increasingly multi-part	7.28	7.28	7.28	7.28	7.28	8.86	8.86	8.86	8.86	8.86
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Decreasingly multi-part	-3.34	-3.34	-3.34	-3.34	-3.34	5.00	5.00	5.00	5.00	5.00
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Free-form	10.75	10.72	10.68	10.54	10.61	10.68	10.68	10.68	10.68	10.69
	1%	0%	0%	-1%	-1%	0%	0%	0%	0%	0%
(a) Patronage										
Uniform-rate	-18.92	-10.37	-1.83	6.71	15.24	-1.53	-1.68	-1.83	-1.98	-2.22
	-933%	**-466%**	0%	**466%**	**932%**	17%	8%	0%	-8%	-21%
Two-part	-10.16	-1.61	6.94	15.47	24.01	7.77	7.35	6.94	6.54	6.15
	-246%	**-123%**	0%	**123%**	**246%**	12%	6%	0%	-6%	-11%
Increasingly multi-part	-10.28	-1.74	6.80	15.33	23.86	7.62	7.21	6.80	6.40	6.01
	-251%	**-126%**	0%	**126%**	**251%**	12%	6%	0%	-6%	-12%
Decreasingly multi-part	-19.62	-11.15	-2.68	5.78	14.24	-2.30	-2.49	-2.68	-2.87	-3.20
	-633%	**-316%**	0%	**316%**	**632%**	14%	7%	0%	-7%	**-20%**
Free-form	-4.52	3.77	9.02	18.65	28.29	9.93	9.47	9.02	8.58	8.12
	-150%	**-58%**	0%	**107%**	**214%**	10%	5%	0%	-5%	-10%

Note:

1. The total discounted royalties and percentage changes are given in the first and second rows in each scheme, respectively.
2. The percentage changes represented in bold face indicate that they are sensitive, depending on whether their percentage changes in total discounted royalty exceed the percentage changes in cortex or spread or not.

TABLE 3: Sensitive analysis for case (3): *SLR* < 1 with government partial investment

Royalty scheme	Cortex -20%	Cortex -10%	Cortex 0%	Cortex 10%	Cortex 20%	Spread -20%	Spread -10%	Spread 0%	Spread 10%	Spread 20%
(a) DSCR										
Uniform-rate	8.06	6.51	3.31	-0.50	-3.97	2.60	2.96	3.31	3.66	3.97
	143%	**96%**	0%	**-115%**	**-220%**	**-21%**	**-11%**	0%	**10%**	**20%**
Two-part	8.06	7.81	6.86	6.25	5.90	6.56	6.72	6.86	7.00	7.03
	17%	14%	0%	-9%	-14%	-4%	-2%	0%	2%	3%
Increasingly multi-part	7.98	6.98	4.03	0.20	-3.29	3.32	3.68	4.03	4.36	4.53
	98%	**73%**	0%	**-95%**	**-182%**	-18%	-9%	0%	8%	12%
Decreasingly multi-part	7.93	6.00	2.59	-1.19	-4.63	1.88	2.23	2.59	2.94	3.29
	207%	**132%**	0%	**-146%**	**-279%**	**-27%**	**-14%**	0%	**14%**	**27%**
Free-form	9.15	9.14	9.13	9.10	9.11	9.12	9.12	9.13	9.13	9.13
	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
(b) IRR_E										
Uniform-rate	-0.13	-0.13	-0.15	-0.16	-0.16	-0.15	-0.15	-0.15	-0.15	-0.15
	-9%	-9%	0%	9%	9%	0%	0%	0%	0%	0%
Two-part	4.77	4.73	4.69	4.65	4.61	4.68	4.68	4.69	4.69	4.70
	2%	1%	0%	-1%	-2%	0%	0%	0%	0%	0%
Increasingly multi-part	0.93	0.93	0.92	0.90	0.90	0.92	0.92	0.92	0.92	0.92
	1%	1%	0%	-1%	-1%	0%	0%	0%	0%	0%
Decreasingly multi-part	-1.20	-1.20	-1.21	-1.23	-1.23	-1.21	-1.21	-1.21	-1.21	-1.21
	-1%	-1%	0%	1%	1%	0%	0%	0%	0%	0%
Free-form	8.70	8.65	8.61	8.56	8.50	8.58	8.60	8.61	8.61	8.62
	1%	0%	0%	-1%	-1%	0%	0%	0%	0%	0%
(a) Patronage										
Uniform-rate	-10.17	-5.52	-0.14	6.36	13.23	-0.11	-0.13	-0.14	-0.16	-0.18
	-7174%	**-3848%**	0%	**4648%**	**9565%**	**22%**	9%	0%	**-13%**	**-26%**
Two-part	-6.86	-1.24	4.49	10.40	16.39	4.94	4.72	4.49	4.27	4.04
	-253%	**-128%**	0%	**132%**	**265%**	10%	5%	0%	-5%	-10%
Increasingly multi-part	-6.06	-0.94	4.71	11.28	18.36	5.21	4.96	4.71	4.46	4.22
	-229%	**-120%**	0%	**139%**	**290%**	11%	5%	0%	-5%	-10%
Decreasingly multi-part	-10.58	-5.96	-0.61	5.84	12.78	-0.53	-0.57	-0.61	-0.65	-0.72
	-1641%	**-881%**	0%	**1061%**	**2202%**	13%	6%	0%	-7%	-19%
Free-form	-2.91	0.77	5.95	12.53	19.11	6.55	6.25	5.95	5.65	5.35
	-149%	**-87%**	0%	**111%**	**221%**	10%	5%	0%	-5%	-10%

Note:
1. The total discounted royalties and percentage changes are given in the first and second rows in each scheme, respectively.
2. The percentage changes represented in bold face indicate that they are sensitive, depending on whether their percentage changes in total discounted royalty exceed the percentage changes in cortex or spread or not.

4.4 Policy implications

The total discounted royalties obtained by three crisp and nine fuzzy royalty models are summarized in Table 4. Optimistically, the total discounted royalty for this 50-million dollars car park BOT project with 17-year concession period may reach 29.27 millions under certain environments and from 16.23 to 30.64 millions under uncertain environments. Pessimistically, it may only have 10.38 millions under certain environments and from 9.02 to 10.78 millions under uncertain environments if the city government provides subsidy. It may only yield 8.24 millions under certain environments and from 5.95 to 9.13 millions under uncertain environments if the city government provides partial investment. The policy implications suggest that for low self-liquidating ratio BOT projects, subsidy approach is superior to the partial investment approach. Namely, the public sectors should consider adopting subsidy strategy rather than partial investment strategy to make low SLR projects financially viable.

We note that different scenarios have come up with consistent results -- vagueness of IRR_E would obtain the highest royalty, followed by vagueness of *DSCR*, followed by crisp conditions, and the ambiguity of patronage has the lowest royalty. The results indicate that the amount of royalties with vague *DSCR*, IRR_E and patronage would respectively range from 104% to 111%, 102% to 104%, and 55% to 87% compared with the corresponding crisp (base) conditions. It suggests that increasing the flexibility of *DSCR* and IRR_E of investors and financiers and forecasting the patronage more accurately can further enlarge the royalty. In addition, different royalty schemes have consistently concluded that free-form scheme can always obtain the highest total discounted royalty, followed by two-part or increasingly multi-part scheme. Uniform-rate and decreasingly multi-part schemes would yield the lowest total discounted royalty. Due to the indefinite structure of free-form royalty, it is almost impossible to document it onto the contracts at the open-tendering stage; therefore, two-part or increasingly multi-part royalty scheme should be taken for its simple and definite structure.

TABLE 4: Comparison of total discounted royalty by different royalty models (NT$ million)

Models	Optimistic scenario	Pessimistic scenarios	
	Case (1): *SLR*>1 without government financial supports	Case (2): *SLR*<1 with government annual subsidy	Case (3): *SLR*<1 with government partial investment
Crisp (baseline) model	29.27 (100%) [LP_1]	10.38 (100%) [LP_2]	8.24 (100%) [LP_3]
Vagueness of *DSCR*	30.64 (105%) [FLP_{11}]	10.78 (104%) [FLP_{21}]	9.13 (111%) [FLP_{31}]
Vagueness of IRR_E	29.80 (102%) [FLP_{12}]	10.68 (103%) [FLP_{22}]	8.61 (104%) [FLP_{32}]
Ambiguity of patronage	16.23 (55%) [FLP_{13}]	9.02 (87%) [FLP_{23}]	5.95 (72%) [FLP_{33}]

Note: The percentage in parenthesis represents the proportion of total discounted royalties collected by different royalty models relative to the crisp model of each case.

5. CONCLUDING REMARKS

In consideration of different situations of self-liquidating ratios (*SLR*) and environmental uncertainties, we develop three crisp royalty models as the bases and nine fuzzy royalty models, corresponding to these three bases, for transportation BOT projects. Since the public assets or rights for transportation BOT projects are granted by the government, the public sectors usually collect initial and/or following annual royalties from the private institutions to which the concession rights are granted. Besides, the total discounted royalty can be used for general purposes, thus the core logic of the modeling is to maximize the public sectors' total discounted royalty while satisfying the private investors' equity return rates (IRR_E) and annual debt service coverage ratios (*DSCR*) requirements. We test the proposed models with four different royalty schemes for a real car park BOT project. The results have consistently agreed that free-form scheme can always obtain the highest total discounted royalty, followed by two-part or increasingly multi-part scheme; while uniform-rate or decreasingly multi-part scheme will yield the lowest total discounted royalty. It should be noted that the conclusions drawn here are based on the particular BOT case being studied. We need to test more BOT cases to draw general conclusions.

Based on the findings, we recommend the public sectors to adopt two-part or increasingly multi-part royalty scheme rather than a uniform-rate or decreasingly multi-part scheme for similar BOT projects like the case being tested. If the public sectors intend to make any low *SLR* BOT project financially viable, we also recommend the public sectors to provide annual subsidy rather than initially partial investment to maximize the total discounted royalty. Since the total discounted royalties are increased with the degrees of uncertainty in *DSCR* and IRR_E, but largely decreased with the degree of uncertainty in patronage, allowing the investors and financiers with higher flexibility in *DSCR* and IRR_E and predicting the patronage with higher accuracy would help yield the total discounted royalty.

Our analyses of fuzzy royalty models are based on subjective settings of spreads of fuzzy numbers. In real operation, however, the settings of spreads of *DSCR*, IRR_E can be jointly determined by the related parties according to their preferences and settings of patronage can be determined by referring to that of similar transportation projects. Moreover, our proposed fuzzy royalty models treat the uncertainties of *DSCR*, IRR_E and patronage separately. More sophisticated fuzzy royalty models with joint consideration of uncertainties of *DSCR*, IRR_E, patronage and other related parameters deserve further exploration. Finally, different scales of BOT projects should be examined with our proposed royalty models so as to reach generalized or robust conclusions.

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