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經理人人力資本在強制性揭露與自願性揭露中所扮演的角色

The Role of the Manager's Human Capital in Mandatory and Voluntary Disclosures

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摘要：本研究將強制性揭露對自願性揭露之影響納入 Nagar (1999)的模型中，以分析自願性揭露均衡。本研究同時考量強制性揭露對於投資人資訊搜尋的效果，以及強制性揭露提供經理人自願性揭露誘因的作用。本研究發現：相較於 Nagar (1999)的模型，經理人有更強的誘因將其私有資訊揭露給資本市場，故揭露代理問題在雙揭露制交互作用下較只考量自願性揭露制為不嚴重。此外，本研究的模型產生另一異於 Nagar 模型的揭露均衡：當投資人沒有私有資訊時，自願性揭露之自發效果與強制性揭露對於自願性揭露之誘發效果的共同運作下，並不存在充分自願性揭露均衡。最後，自願性揭露之可能性與強制性揭露的資訊品質、自願性揭露的資訊品質之間，均呈現正相關。因此，本研究對於盈餘揭露管制，以及建立良好會計系統以提供經理人公司資產資訊兩方面，具有政策意涵。

關鍵字：自願性揭露；強制性揭露；資訊搜尋；績效評估；經理人才能

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Abstract: This paper analyzes voluntary disclosure equilibria by modifying Nagar's (1999) model and including the effect of mandatory disclosures on his model. We consider mandatory disclosures' effect in triggering investors' information search and in motivating managers to make voluntary disclosures. Our extended model demonstrates that managers have more incentives to disclose private information to the capital market than Nagar's model indicates, which means the managerial disclosure agency problem is mitigated by the interaction between mandatory and voluntary disclosures. Moreover, our model yields additional disclosure equilibria that differ significantly from the equilibria in Nagar (1999): incorporating both self-effect and induced effect on the voluntary disclosure strategy rules out a full-disclosure equilibrium when the investors have no private information. Finally, the likelihood that managers will provide voluntary disclosures is positively related to the quality of both the mandatory and voluntary disclosures. Therefore, this study has policy implications in view of recent calls for regulating the disclosures in press releases related to earnings and setting up good accounting systems that provide managers with more appropriate measures of firm-specific assets.

Keywords: Voluntary disclosure; Mandatory disclosure; Information search; Performance evaluation; Manager's talent

1. Introduction

In this paper we study how managers' concerns about evaluations of their performances and financial-reporting regulations affect their voluntary disclosures. Specifically, we analyze how a mandatory disclosure affects a firm's voluntary disclosures. First, we show that incorporating both self-effect and induced-effect on the voluntary disclosure strategy rules out the full-disclosure equilibrium. We then show what incents a manager to disclose his or her private information to the public. Our model provides an eligible setting in which to examine managers' considerations of how to maximize their welfare under the two disclosure regimes

in light of investors' uncertainty and diversity of opinions and the managers' concerns about future compensation.

The managerial-disclosure agency problem is an important concern for investors, especially given disclosure's key role in capital mark*et al*locations and corporate governance decisions (see, *e.g.*, Bushman and Smith 2001; Verrecchia 2001). Managers with expertise have more information about their firms than the outside investors do, which condition is called information asymmetry[2]. The result of full disclosure of this kind of private information to the public has been shown in the extant literature (Grossman and Hart 1980; Grossman 1981; Milgrom 1981; Wagenhofer 1990). However, firms are regulated to disclose some financial information in specific forms, including financial statements, footnotes, and other regulatory filings. Therefore, some later disclosure models (Verrecchia 1983; Wagenhofer 1990; Sankar 1995; Suijs 1999) that have introduced the costs of disclosure and the threat of market entry and shareholder litigation (Skinner 1994; Kasznik and Lev 1995), and find that the partial disclosure equilibrium occurs.

Mandatory disclosure exists in part to alleviate the agency problem, although voluntary disclosure may well create a path to a more broadly transparent and reliable performance valuation framework. Recent empirical studies have reasoned that the increasing usefulness of earnings announcements over time have increased the absolute or squared abnormal stock returns or abnormal trading volume at earnings announcement dates (Francis *et al.* 2002; Landsman and Maydew 2002). Linsmeier *et al.* (2002) show that, after firms disclose the information mandated by Financial Reporting Release No. 48 related to their exposures to interest, foreign currency exchange rates, and energy prices, trading volume sensitivity to changes in these underlying market rates and prices declines,

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[2] Corporate disclosures have the potential to change firm value. Many studies in corporate disclosures have suggested that voluntary disclosures can release managers' private information about the corporation and so reduce information asymmetry between the corporation and the investors. Therefore, the amount that managers appropriate for themselves (*e.g.*, Kanoida *et al.* 2000; Sapra 2002; Shleifer and Wolfenzon 2002), firms' cost of capital (*e.g.*, Lombardo and Pagano 2002; Lambert, Leuz, and Verrecchia 2007; Hughes, Liu, and Liu 2007), and the investors' uncertainty may all decline.

even after controlling for other factors associated with trading volume. This finding indicates that mandatory disclosures provide useful information to investors.

However, there is little theoretical literature on how changing the properties of the mandatory disclosures may change other sources of information. Modeling the interdependence between the mandatory and voluntary disclosures is an important issue since empirical research that investigates the economic impact on changes in financial reporting typically omits the indirect effect of such changes on other sources of information, such as voluntary disclosures. The perspective that mandatory financial reporting is the primary source of information to the capital market contributes to the lack of prior literature on the relationship between mandatory and voluntary disclosures. From this viewpoint, a market's reaction to changes in the financial reporting itself is a considerably important issue, and the indirect effects of such changes on other sources of information are mostly second-order effects. On the other hand, Ball and Brown (1968) suggest that a mandatory financial-reporting regime based on completed transactions may be better characterized as a source of confirmatory information than as a primary source of timely (forward-looking) information. In this alternative view, the indirect information effects are, by definition, first-order effects (Gigler and Hemmer 1998). Gigler and Hemmer (2001) investigate the link between mandatory and voluntary reporting[3] using the principal-agent model to examine how both liberal and conservative biases affect the usefulness of the mandatory financial reports in disciplining managers' voluntary disclosures. They show that incurring the costs of voluntary preemptive disclosures is optimal only when the firm's accounting system is not too conservative. These important issues motivate this study to investigate the relationship between mandatory and discretionary disclosures.

This study is also motivated by Nagar (1999), who deals with the problem faced by prior disclosure research, which exogenously assumed the firm to be a

[3] Most of the literature omits the managerial disclosure agency problem with respect to the assumption of the preference-consistency between manager and investor (Healy and Palepu 2001).

black box with no agency problems. Nagar's approach concerning the role[4] of the manager's human capital in determining voluntary disclosures is reasonable as long as managers differ in terms of talent. Another assumption, that a disclosure triggers information production by the market, is also reasonable since the manager's and investors' information sets tend to be significantly different in terms of their structure and content (see, *e.g.* Kim 1999)[5]. Therefore, we take these assumptions as the basis of our paper. Moreover, we assume that investors know that the manager has private information and treat nondisclosure as a conscious choice rather than as an absence of private information, even though they could not know whether the absence of disclosure is due to a manager with talent or not.

However, Nagar (1999) assumes that there are no background uncertainties, which implies that the mandatory release of earnings has no effect on voluntary disclosure. We argue that this assumption ignores[6] the fact that mandatory disclosure can increase the manager's uncertainty about the investors' response, while this disclosure triggers acquisition of additional information by the investor. In effect, the risk-averse manager faces a trade-off between a mandatory disclosure's inducing an "inevitable" uncertainty and a voluntary disclosure's triggering an "avoidable" risk. The mandatory disclosure induces the uncertainty. Thus, the mandatory disclosure provides a potential explanation for why some managers are more likely to have incentives to disclose voluntarily, especially when the "inevitable" uncertainty is bigger than the "avoidable" risk. We extend

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[4] Sankar and Subramanyam (2001) also focused on the role of the manager in discussing that the manager uses reporting discretion to communicate private information, thereby increasing the information content of reported earnings. Gigler and Hemmer (2001) took the manager's position to depict the manager's argument. In addition, empirical study has inferred that stock-based incentives can reduce managerial reluctance to disclose private information. Nagar *et al.* (2003) posited a solution to the disclosure agency problem using stock price-based incentives and found that firms' disclosures are positively related to the proportion of CEO compensation that is affected by stock price and the value of the shares held by the CEO.

[5] For example, Dye and Sridhar (2002) presented a model to show that capital market prices can perform simultaneously their conventional role of assessing the future cash flow implications of managers' anticipated actions and the role of directing the firm's managers' actions toward the highest cash--generating activities; that is, information flows from firms to the capital markets as well as from capital markets to firms.

[6] We appreciate the referee's suggestion.

Nagar's model by relaxing this assumption. We consider the role of mandatory disclosures in triggering information production by investors and in incenting managers to make voluntary disclosures. Failure to consider the effects of mandatory disclosures in this way will lead to incorrect inferences about what motivates voluntary disclosures.

Our primary objective is to identify how mandatory disclosures induce managers to make additional voluntary disclosures and, based on our setting, refine Nagar's (1999) voluntary disclosure equilibria. We find that, in contrast to Nagar (1999), voluntary disclosure will always exist since the manager has an incentive to reduce the uncertainty from his or her mandatory disclosure. However, the manager will never make a full disclosure on both the mandatory and voluntary disclosures. Further, we find that the voluntary disclosure increases with (1) a decrease in the noise of the mandatory disclosure and (2) a decrease in the noise of the voluntary disclosure under the comparative statics. Although it uses a different type of analysis, our model complements the findings of Gigler and Hemmer (1998), which suggested that one role of mandatory disclosures may be that of a vehicle that helps create an environment in which managers can credibly communicate their more value-relevant voluntary disclosures. Gigler and Hemmer refer to this role as the "confirmatory role" of the mandatory disclosures, similar to our "induced-effect."

The remainder of the study proceeds as follows: Section 2 describes our model and the major findings, Section 3 provides the comparative statistics, Section 4 presents the implications and discussion, and Section 5 concludes.

2. The Model

Our model is based on that of Nagar (1999), which includes a risk-averse decision-maker (a manager) and a risk-neutral capital market. The manager's objective is to maximize his or her human capital[7], rather than the firm value, so

[7] Human capital is defined as the managers' talent. While investors can "figure out" managers' talent in terms of their functional or educational background, neither the manager nor the

an agency problem exists in our model. The true managerial talent and the true asset types are unknown to the manager and the investors. The beliefs of the manager about the physical asset types and managerial talent are denoted as a and t, respectively, which are random variables drawn from independent normal distributions. Earnings, the manager's gross salary, are produced by the manager's talent for management and the firm-specific assets, $e = a + t + w$, where w is a zero-mean, normally distributed noise term representing exogenous shocks.

The scenario is shown at two stages in one period. In the first stage, earnings are reported in the mandatory financial report. Based on this information, the manager computes the mean of the posterior belief about his talent as $T_0 = E[t|a+t+w]$. Moreover, a public disclosure is assumed to contain truthful information only[8] and to be observed by the capital market.

Next, the investors observe the manager's mandatory disclosure, e and may interpret the disclosure differently from the manager. We model this difference in interpretation as the public disclosure's triggering the market's acquisition of additional information, $d = t + p$, where p is a zero-mean, normally distributed stochastic term.

Based on the mandatory disclosure, the investors update the mean of the manager's talent to $T'_0 = E[t|a+t+w, t+p]$. Under the normality, the T'_0 can be represented as a linear combination of two signals, $T'_0 = T_0 + B'(d - T_0)$, where:

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investors have anything to do with the background information, so they view it as an unavoidable event. Instead, the manager who maximizes the value of his human capital has considerable flexibility in terms of whether and how the supplementary information (via disclosure) is presented in order to affect the market's assessment of his human capital.

[8] The same assumption applies to the models of Grossman (1981), Milgrom (1981), Nagar (1999) and Suijs (1999). One way to make disclosures credible to the public is by contracting with an auditor. Another mechanism is that which Evans III and Sridhar (2002) demonstrated, in which a firm's tradeoffs between reporting good news to reduce the cost of capital and bad news to minimize proprietary costs can induce the firm's managers to provide truthful disclosures when the opposing effects balance each other. On the other hand, Stocken's (2000) model presented the manager as almost always truthfully revealing his or her private information, provided that the manager is sufficiently patient, the accounting report is sufficiently useful for assessing the truthfulness of the manager's voluntary disclosure, and the manager's disclosure performance is evaluated over a sufficiently long period.

$$B' = \frac{Cov(t, t+p|a+t+w)}{Var(t+p, a+t+w)} = \frac{Var(t|a+t+w)}{Var(t|a+t+w)+Var(p)}.$$

The risk-averse manager is uncertain about the capital market' resulting assessment of his or her human capital, T'_0, because of the unknown of d in advance. The uncertainty about T'_0 faced by the manager is represented by the random variable t'_0:

$$t'_0 = T_0 + k',$$

where k' is a normally distributed random variable with zero mean. Because the mean of d is the manager's current assessment of the mean of t or T_0, k' has a zero mean.

Lemma 1: The variance of k' from the manager's perspective is:

$$Var(k') = \frac{[Var(t|a+t+w)]^2}{Var(t|a+t+w)+Var(p)}$$

Proof. See Appendix.

In addition to information about current earnings, the manager privately possesses relevant information about the firm's asset types, written as the signal $Z = a + m$, where m is a zero-mean, normally distributed noise term. Upon getting this signal privately, the manager computes the posterior mean of his ability as $T_1 = E[t|a+t+w, a+m]$.

In the second stage, the manager has the option to disclose Z to the market. This private information may represent, for example, product quality or the numbers of new products and patents; in fact, it can be given any meaning as long as that meaning can be represented by a one-dimensional compact interval. Changes in the capital market's beliefs about a will lead to changes in investors' beliefs about t. In the same manner, for a simplified version, we assume that, if a manager makes a voluntary disclosure, he or she discloses this information truthfully.

The investors observe the manager's disclosure decision and update their

beliefs regarding the manager's talent. Because of the different information sets between the manager and the capital market, the manager's voluntary disclosure will trigger another signal to the market participants about the manager's talent, after which the investors will analyze the disclosed information in the context of their information about the firm and its environment. For mathematical and comparative convenience[9], we represent this difference as $d = t + p$, as in the mandatory case.

Upon the manager's voluntary disclosure, the capital market will update its beliefs to $T_2 = E[t|a+t+w, a+m, t+p]$. In this case, the market's post-voluntary-disclosure beliefs are similar to those after the mandatory disclosure; T_2 can be written as $T_2 = T_1 + B(d - T_1)$, and the uncertainty about T_2 faced by the manager can be captured by t_2:

$$t_2 = T_1 + B(d - T_1),$$
$$t_2 = T_1 + k,$$

where k is a normally distributed random variable with zero mean, and the uncertainty that the manager faces upon the voluntary disclosure.

Lemma 2: The variance of k from the manager's perspective can be shown by:

$$Var(k') = \frac{[Var(t|a+t+w, a+m)]^2}{Var(t|a+t+w, a+m) + Var(p)},$$

where $B = \dfrac{Var(t|a+t+w, a+m)}{Var(t|a+t+w, a+m) + Var(p)}$.

Proof. See Appendix.

Let the variance of a, t, w, and m be A, T, W, and M, respectively. We assume that these random variables are independently distributed. Thus, the

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[9] Of course, one can argue that there are differences between the two disclosing regimes, but there is no exact answer. For the purposes of comparability, we assume that this difference is the same in both disclosure cases in the following context.

conditional variance of the manager's talent to Lemmas 1 and 2 appears in Lemma 3.

Lemma 3: The conditional variance of the manager's talent is:

$$Var(t|a+t+w)=\frac{[AT+TW]}{(A+T+W)}=G \text{ , and}$$

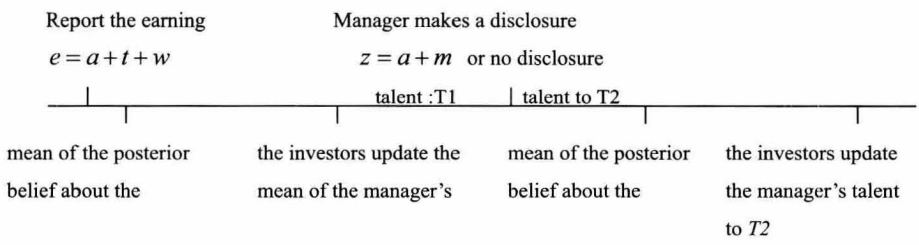
$$Var(t|a+t+w,a+m)=\frac{[AMT+WMT+AWT]}{(AM+AT+TM+AW+MW)}=g \text{ .}$$

Therefore, we get $Var(k')=\frac{G^2}{G+Var(p)}$; $Var(k)=\frac{g^2}{g+Var(p)}$.

Proof. See Appendix.

The two-stage scenario described thus far is shown in the following time line:

Figure1
Time Line



Finally, in order to rule out the condition of collusion between the investors, we assume that firms operate in competitive managerial labor markets. The expected marginal productivity of a manger is his or her talent. Following Nagar (1999), we assume that the manager's future wage is determined by the investor's updated evaluation of the manager's talent, $w(H)$, and that it is strictly concave and satisfies weakly decreasing absolute risk aversion, *i.e.*, that $w''(H)\geq 0$. The fact that risk-averse managers view the uncertainty about future wages as costly makes this assumption hold. Further, because the information sets of the manager and investors are different, the manager has subjective uncertainty about the

investors' assessment of his or her talent in the event of disclosure. Therefore, in our model, when the firm-specific assets are productive but the manager is untalented, the manager chooses nondisclosure to avoid adverse performance evaluation. The manager who is talented but is afraid of disclosure-related uncertainty also chooses nondisclosure.

2.1. Solution of the Model

A voluntary disclosure strategy is in equilibrium if the manager has an incentive to disclose or to withhold while taking into account the beliefs of the capital market. This section first characterizes and then discusses the disclosure equilibrium.

Proposition 1. *The induced effect of mandatory disclosure on voluntary disclosure:* A voluntary disclosure strategy always exists because the risk-averse manager views information as triggering an avoidable risk.

Proof. See Appendix.

The intuition behind this logic is as follows. When the mandatory disclosure is released, the manager's uncertainty about the capital market's response to disclosure arises as a result of the manager's incomplete knowledge of the investors' information. The mandatory disclosure may lead to an equilibrium that entails the manager's uncertainty, to the detriment of the manager. Accordingly, after making rational conjectures regarding the related detriment, the manager selects the optimal disclosure that will maximize his or her utility of future human capital. When the manager is incented to make further voluntary disclosures to eliminate the uncertainty that he or she faces, we call this impact the induced-effect; specifically, the precision of the market's prior beliefs about talent or asset type, that is, A^{-1} and T^{-1}, is low.

This result is contrary to Nagar's result. Nagar assumed that the prospect that a disclosure will trigger the market's acquisition of private information works as a disincentive for the manager to disclose because the risk-averse manager cannot predict with confidence the content of the information the market will acquire. However, in our opinion, the manager will try to reduce all uncertainties that he or

she faces, taking both the mandatory and voluntary disclosures into consideration. Therefore, the mandatory disclosure will also trigger an inevitable uncertainty, and we interpret this uncertainty as the opportunity cost of the voluntary nondisclosure, $Var(k')$. Generally Accepted Accounting Principles (GAAP) allow managerial discretion, subject to certain restrictions, in determining financial reporting policies and procedures. This discretion makes earnings susceptible to manipulation; that is, there is a higher imposition on the manager, $G > g$, even though all discretionary disclosure is truthful. Therefore, the uncertainty from the mandatory disclosure may be greater than the cost of the voluntary disclosure, $Var(k)$. In other words, the mandatory disclosure introduces an "inevitable" uncertainty that is due to the requirements of financial regulations, while the voluntary disclosure triggers the investors to acquire further information and to use that information to reassess the manager's talent. The manager views the information provided in voluntary disclosures as an "avoidable" risk because he or she discloses private information only if it is more favorable than a certain threshold. Therefore, to facilitate the investor's evaluation, the manager appears to voluntarily disclose private information when the manager concerns that the bottom line earning number is less adequate as a performance evaluation metric.

In realistic settings, there exists that shareholders demand more disclosures, and the manager will respond to this demand in order to reduce the opportunity cost of nondisclosure. Therefore, the agency problem is mitigated by the interaction between the two types of disclosure.

Proposition 2. *The effect of the quality of the investor's private information:*

(1) The disclosure threshold decreases (more disclosure is likely) as the noise of the investor's private information, $Var(p)$, increases.

(2) A partial voluntary disclosure occurs if $Var(p) > 2[Gg + Var(p)(G+g)]$.

(3) A full voluntary disclosure strategy is never an equilibrium.

Proof. See Appendix.

The investors may acquire some information from, for instance, other firms, the industry and analysts that is not available to the manager, who is constrained

by his managerial responsibilities and limited human ability. Alternatively, educated investors may use the clues from prior disclosures to seek further information and reduce the noise in conjectures about the manager's talent. The information of uninformed investors is chaotic, so the signal triggered by a disclosure is noisy and will not substantially affect their assessment of the manager.

Therefore, the shareholder composition and analysts' forecast may affect the noise in the capital markets' private information. However, if managing a firm requires specialized knowledge, the information asymmetry is likely to be one-sided, with the capital market's relying on the management's interpretation in the disclosure. Ceteris paribus, the manager will consider $Var(p)$ high in such a condition, and his or her uncertainty resulting from the disclosure will be reduced.

After a mandatory disclosure, it is inevitable for the manager to face the uncertainty triggered by the disclosure. And, the more noise triggered by the capital market, the less uncertainty the manager faces, so the disclosure threshold will decrease both in mandatory and voluntary disclosures. However, it is the decrease of the mandatory disclosure's threshold that reduces the incentive for the voluntary disclosure. Therefore, there is a trade-off relationship between the self-effect of the voluntary disclosure and the induced effect of the mandatory disclosure, both of which are caused by the noise in the investors' private information.

Our result shows that it is impossible to exist the equilibrium of the full voluntary disclosure, and this finding is also contrary to Nagar's. In Nagar's view, in the absence of exogenously imposed proprietary costs, a full-disclosure strategy will not result in reassessment from the capital markets. To account for the induced effect from the mandatory disclosure, we think it is possible to exclude the full voluntary disclosure equilibrium because of the induced effect of the mandatory disclosure, even though there are no proprietary costs in disclosure.

A distinguishing feature of the aforementioned analysis is that the interaction between mandatory and voluntary disclosure removes the possibility of a full voluntary disclosure. Prompted by Pae's (2002) concept, this paper considers a hypothetical benchmark: In our scenario the manager's private information is

publicly known, there is no mandatory disclosure but only voluntary disclosure, and the capital markets have no private knowledge, $Var(p) = \infty$. In this circumstance, the capital market will value the manager's compensation via his or her intrinsic talent, so the manager will face no uncertainty about the disclosure. There is no efficiency loss because the allocation of the resources fully incorporates the information content of the signal. Pareto improvement occurs in welfare since the resource allocation is more accommodating to the manager's talent.

Such an efficient outcome is not achievable when voluntary and mandatory disclosures interact, so full voluntary disclosure is driven out. The mandatory disclosure discourages the voluntary disclosure in this scenario, while it encourages the further disclosure in Proposition 1. This divergence can be explained by the enhanced induced effect of the mandatory disclosure on the voluntary disclosure's only occurring in intervals when the capital market has no private knowledge. From this standpoint, although regulators generally allow the existence of managerial discretions, our model shows that it will deter the full voluntary disclosure[10].

3. Comparative Statics

This section characterizes a comparative statics analysis of the equilibrium disclosure policy with respect to certain information-related parameters. The implications of policy are then outlined.

Proposition 3. *The effect of the quality of the voluntary disclosure:*

The disclosure threshold decreases (more disclosure is likely) as the precision of the manager's private information, M, increases.

Proof. See Appendix.

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[10] As Sankar and Subramanyam (2001) mentioned, that earnings management results in the limited informational value of accounting numbers is almost unanimously accepted in the finance and economic disciplines. While the belief that earnings management impairs earnings' informativeness is widespread among accounting researchers, it is not unanimous. However, this debate is out of our scope.

A result that can be interpreted as similar to Proposition 3 is found in Nagar (1999), except we add the induced effect of the mandatory disclosure to the voluntary disclosure. Intuitively, the more precisely the asset type is known a priori, the less pressure will be exerted on the manager to reveal what he or she knows privately. When the signal becomes more informative, investors, who are aware that the manager faces little uncertainty upon disclosing, increase the penalties for nondisclosure. (Recall that the signal is withheld only when it is unfavorable.) Alternatively, when the manager has an exact idea of his type since he has more precision about assets, he or she will be likely to present his or her views to the investors. Hence, the likelihood of disclosure increases as the quality of the manager's private information improves.

Proposition 4. *The effect of the quality of the mandatory disclosure:*

The disclosure threshold decreases (more disclosure is likely) as the precision of the earnings information, W, increases.

Proof. See Appendix.

Intuitively, if the noise of the earning information decreases, the manager has a more precise idea of his type and the asset's type. In this case, the manager faces less uncertainty upon making these two disclosures, so the threshold of the disclosure decreases. Further, the manager is more likely to disclose in order to reduce the uncertainty from the mandatory disclosure when the precision of the market's prior belief about his or her talent or asset the type, A^{-1} and T^{-1}, is low. Therefore, as the quality of the earnings information improves, a constant disclosure cost tends to reduce the nondisclosure set.

The comparative statics show that voluntary disclosure increases with (1) a decrease in the noise of the mandatory disclosure and (2) a decrease in the noise of the voluntary disclosure. Managers often announce their sources of information along with the type of information, and this contemporaneous disclosure may be useful in identifying the sources of uncertainty. In this paper, we assume that the indirect effects between voluntary and mandatory disclosures will strengthen the direct effects in one of exogenous disclosures.

4. Implications and Discussion

This study has policy implications in view of recent calls for regulating disclosures in press releases about earnings. When the goal that the regulator wants to achieve is to boost transparency in the information environment, the results of our Proposition 1 suggest that financial regulations should focus on imposing mandatory reporting in light of its total effect on a firm's disclosures (both mandatory and voluntary). Regulatory attempts to enhance mandatory disclosures in annual reports spur additional voluntary disclosures that facilitate the investors' ability to interpret the mandatory disclosure because the risk-averse manager cannot predict how the minimum information provided in mandatory disclosures will be interpreted to assess the firm's future prospects and the manager's talent.

In addition, according to our Proposition 2, although lower-quality information held by investors tends to decrease the threshold level of the voluntary disclosure, the induced effect of the mandatory disclosure on the voluntary disclosure may have the opposite effect. As a result of these two countervailing effects, the manager's optimal voluntary disclosure policy is an interval form. In particular, a full voluntary disclosure strategy can be completely ruled out when the investors have no private information. Prior literature[11] has shown that mandatory and voluntary disclosures are substitutes by assuming that an increase in mandatory disclosure is interpreted as either a decrease in the market's prior variance of the firm's liquidation value or as the release of an additional signal correlated with the firm's liquidation value. Rather than focusing on whether the manager knows a signal (*i.e.*, information in voluntary disclosures) correlated with firm value, we provide another reason why mandatory disclosures deter full voluntary disclosures. In the circumstance in which the manager face a large majority of small investors who lack access to proprietary firm- and industry- specific information, the lower subjective uncertainty he or she faces after a mandatory disclosure leads to a decrease in the probability of voluntary

[11] Verrecchia (1983, 1990, 2001), Dye (1985, 1998, 2001), and Fischer and Stocken (2001).

disclosure (*i.e.*, the negative induced effect of the mandatory disclosure on the voluntary disclosure).

Finally, the analyses of our Propositions 3 and 4 show that the likelihood of a voluntary disclosure's being provided by the manager is positively related to the quality of mandatory disclosures and the quality of voluntary disclosures. Therefore, policymakers should set up a good accounting system that provides the manager with the best measures of firm-specific assets. In this scenario, the manager who has an exact idea of his/her type is able to improve the quality of both mandatory and voluntary disclosures. Because of the positive induced effect of the mandatory disclosure on the voluntary disclosure, the positive relationship between the likelihood of the voluntary disclosure and the quality of mandatory/ voluntary disclosures will be stronger when there are both mandatory and voluntary disclosures than when there are only voluntary disclosures.

5. Conclusion

In this paper, we are interested in the disclosure effect on the voluntary disclosure which is succeeding a mandatory disclosure. We modify Nagar's (1999) model to give an insight of the relation between the mandatory disclosure and the voluntary disclosure. We argue that examinations of voluntary disclosure incentives must consider the role that the mandatory disclosures play in shaping firms' voluntary disclosure. Our extended model demonstrates that managers have more incentives to disclose private information to the capital market than Nagar's model indicates, which means the managerial disclosure agency problem is mitigated by the interaction between mandatory and voluntary disclosures. Moreover, our model yields additional disclosure equilibria that differ significantly from the equilibria in Nagar (1999): incorporating both self-effect and induced effect on the voluntary disclosure strategy rules out a full-disclosure equilibrium when the investors have no private information. Finally, the likelihood that managers will provide voluntary disclosures is positively related to the quality of both the mandatory and voluntary disclosures. Therefore, this study has policy implications in view of recent calls for regulating the disclosures in

press releases related to earnings and setting up good accounting systems that provide managers with more appropriate measures of firm-specific assets.

When the goal that the regulator wants to achieve is to boost transparency in the information environment, financial regulations can design a mechanism that will increase the likelihood of voluntary disclosure and the quality of information. For example, because of mandatory disclosures' total effect on a firm's disclosure strategy (mandatory and voluntary), financial regulations should focus on imposing mandatory reporting. In addition, policymakers should set up a good accounting system that provides the manager with appropriate measures of firm-specific assets, leading to a high quality in both mandatory and voluntary disclosures. Because of the positive induced effect of the mandatory disclosure on the voluntary disclosure, the positive relationship between the likelihood of the voluntary disclosure and the quality of mandatory/ voluntary disclosures will be stronger when there are both mandatory and voluntary disclosures, rather than only voluntary disclosures. Accordingly, a firm' strategy for providing voluntary disclosures cannot be studied without taking into account the impact of its mandatory disclosures.

6. References

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APPENDIX

Proof of Lemma 1:

$$Var(k') = B^2[Var(d|a+t+w)]$$
$$= [\frac{Var(t|a+t+w)}{Var(t|a+t+w)+Var(p)}]^2[Var(t+p|a+t+w)]$$
$$= \frac{[Var(t|a+t+w)]^2}{Var(t|a+t+w)+Var(p)}.$$

Proof of Lemma 2:

$$Var(k) = B^2[Var(d|a+t+w,a+m)]$$
$$= [\frac{Var(t|a+t+w,a+m)}{Var(t|a+t+w,a+m)+Var(p)}]^2[Var(t+p|a+t+w,a+m)]$$
$$= \frac{[Var(t|a+t+w,a+m)]^2}{Var(t|a+t+w,a+m)+Var(p)}.$$

Proof of Lemma 3:

The deduction of this proof is the same for both disclosure cases. We first prove the mandatory case. For notational convenience, we make some variable transformation.

Let $X = t$, and $Y = t + b$, because t and b are independent normal distribution, the joint density function of (t,b) is $f(t,b)$, where

$$f(t,b) = \frac{1}{2\pi\sigma_t\sigma_b} e^{\frac{t^2}{2\sigma_t^2}} e^{\frac{b^2}{2\sigma_b^2}}.$$

The Joint density function of (x,y) is $g(X,Y)$

$$h(X,Y) = f(X, Y-X)\|J\|,$$

Where $\|J\|$ represents the Jacobian matrix.

$\because \|J\| = 1$

$\therefore h(X,Y)$

$$= \frac{1}{2\pi\sigma_t\sigma_b} e^{-\frac{X^2}{2\sigma_t^2}} e^{-\frac{(Y-X)^2}{2\sigma_b^2}}$$

$$= \frac{1}{2\pi\sigma_t\sigma_b} \exp\{-\frac{[(\sigma_b^2+\sigma_t^2)X^2 - 2XY\sigma_t^2 + Y^2\sigma_t^2]}{2\sigma_t^2\sigma_b^2}\}$$

$$= \frac{1}{2\pi\sigma_t\sigma_b} \exp\{-\frac{[X - Y(\frac{\sigma_t^2}{\sigma_b^2+\sigma_t^2})]^2}{2\sigma^2}\} \exp\{\frac{-Y^2}{2(\sigma_b^2+\sigma_t^2)}\},$$

where $\sigma^2 = \frac{\sigma_t^2\sigma_b^2}{\sigma_t^2+\sigma_b^2}$.

Therefore, $f(X|Y)$ is a normal distribution with variance equal to $\frac{\sigma_t^2\sigma_b^2}{\sigma_t^2+\sigma_b^2}$;

$$\therefore \quad Var(t|t+b) = \frac{\sigma_t^2\sigma_b^2}{\sigma_t^2+\sigma_b^2}.$$

Let $b = a + w$,if a , t and w are independently normally distributed,

$$Var[t|a+t+w] = \frac{Var(t)[Var(a)+Var(w)]}{Var(t)+[Var(a)+Var(w)]} = \frac{(\text{AT+TW})}{(A+T+W)} = G.$$

Similarly, in voluntary case, one can show that:

$$Var[t|a+t+w, a+m] = \frac{(AMT+WMT+AWT)}{(AM+AT+TM+AW+MW)} = g \text{ [12]}.$$

Proof of Proposition 1

This proof is divided into two parts. The first part follows the proof technique in Nagar (1999) in order to show the unique threshold point, and the second part shows the induced effect of the mandatory disclosure on the voluntary disclosure in order to determine whether a voluntary disclosure strategy always exists.

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[12] Actually, this solution could be found in Nagar (1999).

The manager has to disclose e but has the choice between disclosing and not disclosing Z. The realized earnings, R, are publicly announced as $R = a + t + w$, and the manager gets the private information about the asset type, $Z = a + m$. Then $R - Z = t + w - m$. There is one unique threshold H such that the manager discloses Z if and only if $R - Z > H$. The utility of the voluntary disclosure is $w(E[t|t + w - m > H])$. At the threshold H, the manager is indifferent between disclosing and not disclosing. If the manager chooses the nondisclosure strategy, the investors rationally anticipate the manager's type to be $E[t|t + w - m < H]$, and the utility of nondisclosure to the manager is $w(E[t|t + w - m < H])$. Moreover, a disclosure triggers an additional signal to the investors, namely, $d = t + p$.

Since the manager does not know the content of d, his or her expected utility of the voluntary disclosure will be $E_d w(E[t|t + w - m = H, t + p])$.

Let $T_1 = E[t|t + w - m = H]$. The properties of the normal distribution imply that:

$$E[t|t + w - m = H, t + p]] = T_1 + B(d - T_1) = T_1 + k,$$

where k has a zero mean because the mean of d is the current mean of t from the manager's perspective, namely, T_1.

In the voluntary disclosure case, the manager makes $Ew(T_1 + k) = w(T_1 - RP(T_1))$, where w is a concave function and $RP(\bullet)$ is the risk premium that results from the lottery k. Because w is assumed to satisfy weakly decreasing absolute risk aversion, the higher the T_1, the less costly the lottery k is to the manager. In short, the risk premium of the lottery k is decreasing in T_1:

$$\frac{dRP(T_1)}{dT_1} \leq 0 \quad \longrightarrow \quad \frac{dRP(H)}{dH} \leq 0.$$

Since $w(\bullet)$ is strictly concave and $RP(\bullet)$ is strictly positive, they imply that $RP(t + w - m = H)$ is weakly monotonically decreasing in H. Verrecchia (1983) proved that $E[t|t + w - m = H] - E[t|t + w - m < H]$ is monotonically increasing. Since $E[t|t + w - m = H] - E[t|t + w - m < H]$ goes from zero to

infinity as H goes from negative infinity to positive infinity, we can show that there is one and only one threshold point H where $E[t|t+w-m=H]-E[t|t+w-m<H]=RP(t+w-m=H)$.

In the second part of the proof, we present the induced effect of the mandatory disclosure on the voluntary disclosure. We first determine whether the manager has a choice in terms of whether to disclose. His or her expected utility that is due to the mandatory disclosure would be $E_d w(E[t|a+t+w,t+p])=T_0+B'(d-T_0)=T_0+k'$, where k' has a zero mean. In other words, if his or her option is to disclose in the event of the mandatory disclosure, the manager makes $Ew(T_0+k')=w(T_0-RP(T_0))$.

However, the manager must abide by the results of the mandatory disclosure, so it is expensive for a risk-averse manager to endure the uncertainty, k'. When the manager decides whether to make the voluntary disclosure, the decision implicitly covers the changes in uncertainty that will result from the further disclosure. A self-interested manager may try to abate the uncertainty; one of the best tools is to use the discretionary disclosure to achieve this objective if the variance of k is smaller than that of k'.

From Lemma 3, we can get the uncertainty faced by the manager from the mandatory and voluntary disclosures.

$$\because Var(k')-Var(k)$$

$$=\frac{G^2}{G+Var(p)}-\frac{g^2}{g+Var(p)}$$

$$=\frac{(G-g)[Gg+Var(p)(G+g)]}{[G+Var(p)][g+Var(p)]},$$

$$\because G-g$$

$$=\frac{(\mathrm{AT})^2}{(A+T+W)(AM+AT+TM+AW+MW)}>0,$$

$$\therefore Var(k')-Var(k)>0.$$

Q.E.D.

This is how we show the induced effect of the mandatory disclosure on the voluntary disclosure. Then, there is the unique threshold and a voluntary

disclosure equilibrium given above.

Proof of Proposition 2

(1)

$$Var(k') = \frac{G^2}{G + Var(p)} \longrightarrow \frac{\partial Var(k')}{\partial Var(p)} = \frac{-G^2}{[G + Var(p)]^2} < 0$$

$$Var(k) = \frac{g^2}{g + Var(p)} \longrightarrow \frac{\partial Var(k)}{\partial Var(p)} = \frac{-g^2}{[g + Var(p)]^2} < 0$$

Q.E.D.

(2) and (3)

In our view, a full voluntary disclosure-equilibrium with respect to $Var(p)$ is the condition in which the direct effect of the voluntary disclosure matches the induced effect.

First, we show the direct effect in the case when the investors have no private knowledge: $Var(p) = \infty$.

$Var(p) = \infty$ implies $Var(k) = 0$ and $Var(k') = 0$, and voluntary disclosure yields:

$$E_k w(E[t|t + w - m = H] + k) = w(E[t|t + w - m = H]).$$

Because of the rational expectations, nondisclosure generates $w(E[t|t + w - m < H])$. Similarly, there is a full-disclosure equilibrium in the voluntary disclosure because $w(E[t|t + w - m = H]) > w(E[t|t + w - m < H])$ for all finite H. However, the induced effect of the mandatory disclosure will interfere with the direct effect.

$$\because Var(k') - Var(k) = \frac{(G - g)[Gg + Var(p)(G + g)]}{[G + Var(p)][g + Var(p)]},$$

$$\therefore \frac{\partial[Var(k') - Var(k)]}{\partial Var(p)}$$

$$= \frac{(G - g)(G + g)[G + Var(p)][g + Var(p)]}{\{[G + Var(p)][g + Var(p)]\}^2}$$

$$-\frac{(G-g)[Gg+Var(p)(G+g)][G+Var(p)+g+Var(p)]}{\{[G+Var(p)][g+Var(p)]\}^2}$$

$$=\frac{(G-g)(G+g)Var(p)\{Var(p)-2[Gg+Var(p)(G+g)]\}}{\{[G+Var(p)][g+Var(p)]\}^2}\begin{matrix}>\\=\\<\end{matrix}0$$

$$\text{iff } Var(p)\begin{matrix}>\\=\\<\end{matrix}2[Gg+Var(p)(G+g)]$$

Case 1. If $Var(p)>2[Gg+Var(p)(G+g)]$, then $\frac{\partial[Var(k')-Var(k)]}{\partial Var(p)}>0$.

That means that there is a positive induced effect on voluntary disclosure. However, even though the capital market has no private knowledge—that is, $Var(p)=\infty$—it is impossible for there to be a full-disclosure equilibrium since $Var(p)=\infty$ implies $\frac{\partial[Var(k')-Var(k)]}{\partial Var(p)}=1$.

In this event, the total effect of the quality of the investor's private information drives a partial equilibrium of the voluntary disclosure.

Case 2. If $Var(p)\le 2[Gg+Var(p)(G+g)]$, then $\frac{\partial[Var(k')-Var(k)]}{\partial Var(p)}\le 0$.

In this case, there is a negative induced effect on the voluntary disclosure and the total effect of voluntary disclosure caused by the quality of the investors' information is ambiguous. However, we can only exclude the full voluntary disclosure equilibrium.

Proof of Proposition 3

(1)

$$\because Var(t|a+t+w,a+m)=\frac{(\text{AMT+WMT+AWT})}{(AM+AT+TM+AW+MT)}=g,$$

$$Var(k)=\frac{g^2}{g+Var(p)}.$$

$$\therefore \frac{\partial Var(k)}{\partial M}=\frac{\partial Var(k)}{\partial g}\frac{\partial g}{\partial M}.$$

$$\because \frac{\partial Var(k)}{\partial g} = \frac{[2g(g+Var(p))-g^2]}{[g+Var(p)]^2} = \frac{g^2+2gVar(p)}{(g+Var(p))^2} > 0,$$

$$\frac{\partial g}{\partial M}$$

$$= \frac{(\mathrm{AT+WT})(AM+AT+TM+AW+MW)\text{-}(\mathrm{A+T+W})(\mathrm{AMT+WMT+AWT})}{(AM+AT+TM+AW+MW)^2}$$

$$= \frac{\mathrm{A}^2T^2}{(AM+AT+TM+AW+MT)^2} > 0,$$

$$\therefore \frac{\partial Var(k)}{\partial M} > 0.$$

Q.E.D.

(2)

$$\because Var(k') - Var(k) = \frac{(G-g)[Gg+Var(p)(G+g)]}{[G+Var(p)][g+Var(p)]},$$

$$\therefore \frac{\partial[Var(k')-Var(k)]}{\partial M} = \frac{(-\frac{\partial g}{\partial M})[Var(p)(\frac{\partial g}{\partial M})][G+Var(p)][g+Var(p)]}{[G+Var(p)]^2[g+Var(p)]^2}$$

$$= \frac{\text{-}(\mathrm{AT})^4 Var(p)}{[G+Var(p)][g+Var(p)](AM+AT+TM+AW+MT)^4} < 0.$$

Proof of Proposition 4

(1)

$$Var(k) = \frac{g^2}{g+Var(p)},$$

$$\frac{\partial Var(k)}{\partial W} = \frac{\partial Var(k)}{\partial g}\frac{\partial g}{\partial W},$$

$$\because \frac{\partial Var(k)}{\partial g} = \frac{g^2+2gVar(p)}{(g+Var(p))^2} = \frac{g[g+2Var(p)]}{(g+Var(p))^2} > 0,$$

$$\frac{\partial g}{\partial W} = \frac{(\mathrm{MT+AT})(AM + AT + TM + AW + MW)\text{-}(\mathrm{A+M})(\mathrm{AMT+WMT+AWT})}{(AM + AT + VM + AW + MW)^2}$$

$$= \frac{\mathrm{MT(TA+MT)+A}T(AT + TM)}{(AM + AT + VM + AW + MW)^2} = \frac{(AT + TM)^2}{(AM + AT + VM + AW + MW)^2} > 0,$$

$$\therefore \frac{\partial Var(k)}{\partial W} > 0.$$

$$\because Var(k') = \frac{G^2}{G + Var(p)},$$

$$\therefore \frac{\partial Var(k')}{\partial W} = \frac{\partial Var(k')}{\partial G}\frac{\partial G}{\partial W},$$

$$\because \frac{\partial Var(k')}{\partial G} = \frac{2G[G + Var(p)] - G^2}{[G + Var(p)]^2} = \frac{G[G + 2Var(p)]}{[G + Var(p)]^2} > 0,$$

$$\frac{\partial G}{\partial W} = \frac{T(A + T + W) - (\mathrm{AT+TW})}{(A + T + W)^2} = \frac{T^2}{(A + T + W)^2} > 0,$$

$$\therefore \frac{\partial Var(k')}{\partial W} > 0.$$

Q.E.D.

(2)

$$\frac{\partial[Var(k') - Var(k)]}{\partial W}$$

$$= \frac{(\frac{\partial G}{\partial W} - \frac{\partial g}{\partial W})[\frac{\partial G}{\partial W}\frac{\partial g}{\partial W} + Var(p)(\frac{\partial G}{\partial W} + \frac{\partial g}{\partial W})][G + Var(p)][g + Var(p)]}{[G + Var(p)]^2[g + Var(p)]^2}$$

$$- \frac{(\frac{\partial G}{\partial W}\frac{\partial g}{\partial W})(G - g)[Gg + Var(p)(G + g)]}{[G + Var(p)]^2[g + Var(p)]^2}$$

$$\frac{\partial G}{\partial W}-\frac{\partial g}{\partial W}=\frac{T^2}{(A+T+W)^2}-\frac{(AT+TM)^2}{(AM+AT+VM+AW+MW)^2}$$

$$=\frac{T^2(AM+AT+VM+AW+MW)^2-(AT+TM)^2(A+T+W)^2}{(A+T+W)^2(AM+AT+VM+AW+MW)^2}<0,$$

$$\because T(AM+AT+VM+AW+MW)<(AT+TM)(A+T+W),$$

$$\longrightarrow \quad 0<AT^2,$$

$$\therefore \frac{\partial[Var(k')-Var(k)]}{\partial W}<0.$$

Q.E.D.