

Monotone iterative method for numerical solution of nonlinear ODEs in MOSFET RF circuit simulation

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ABSTRACT

In this paper, we model the metal–oxide–semiconductor field effect transistor (MOSFET) radio frequency (RF) circuit as a system of nonlinear ordinary differential equations. Then we solve them with the waveform relaxation method, the monotone iterative method, and Runge–Kutta method directly in time domain. With the monotone iterative method, we prove that each decoupled and transformed circuit equation converges monotonically. In comparison with the HSPICE outputs, results calculated with our method are stable and robust in both the time and frequency domains. Convergence properties for the monotone iterative and outer iterative loops are also presented and discussed. This method provides an alternative in the time domain numerical solution of MOSFET RF circuit equations.

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1. Introduction

Metal–oxide–semiconductor field effect transistor (MOSFET) devices used for designing radio frequency (RF) integrated circuit becomes a tendency because of the successful experience in digital circuit design [1,2]. Numerical methods for the RF circuit provide an efficient alternative in the development of integrated RF components, such as filter, low-noise amplifier, mixer, and power amplifier. Due to the unusually high linearity of MOSFETs at high frequencies, these active devices have been of great interests for RF and wireless applications in the recent years. General approach to analyze the intermodulation distortion and two-tone characteristics for the MOSFETs is to solve a set of equivalent circuit ordinary differential equations (ODEs) in the frequency domain. The harmonic balanced method is a standard approach for solving such RF problems [3,4]. This frequency domain approach has its merits and limitations in studying the physical properties of MOSFET with time variations. Another approach to the analysis of electrical characteristics for a MOSFET RF circuit is to solve a set of equivalent circuit ODEs in time domain. The time domain results are then further calculated with fast Fourier transformation (FFT) for obtaining its spectrum. However, the decoupled and discretized ODEs in circuit simulation are often solved with conventional Newton's iterative (NI) method [5]. The well-known HSPICE circuit simulator is right adopted the NI method and its variants in its numerical kernel. It is known that the NI method is a local method; in general, it has a quadratic convergence property in a sufficiently small neighborhood of the exact solution, and hence it encounters convergence problem for practical engineering applications [6,7].

In this work, we apply the monotone iterative (MI) method [8–12] to simulate MOSFET RF characteristics with exploiting the basic nonlinear property in the equivalent circuit model. The MI method was successfully applied to the semiconductor device simulation in our recent work [8,9]. By considering the Kirchhoff current law for each node, the circuit governing

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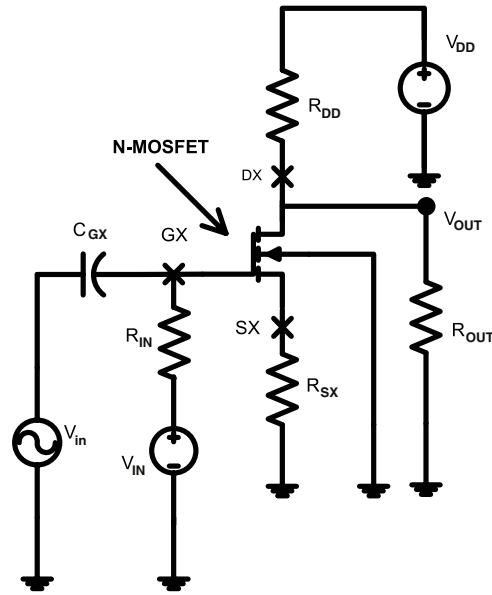


Fig. 1. A common-source amplifier of N-MOSFET used for the RF characteristic simulation.

equations are formulated in terms of the nodal voltages ($V_G, V_D, V_S, V_{DX}, V_{SX}, V_{GX}$). The circuit model is decoupled into several independent ODEs with the waveform relaxation decoupling scheme. The basic idea of the decoupled method for circuit simulation is similar to the well-known Gummel decoupling method for device simulation [13–15,8,9]; it is that the circuit equations are solved sequentially. In the circuit model, the first equation is solved for V_G^{j+1} given the previous states $V_X^j, X = D, S, GX, SX,$ and $DX,$ respectively. For the second equation is solved for V_D^{j+1} given $V_X^j, X = G, S, GX, SX,$ and $DX,$ respectively. We have the same procedure for other ODEs. Each decoupled ODE is transformed and solved with Runge–Kutta (RK) method and the monotone iterative method. We prove the MI method converges monotonically for all decoupled circuit equations. It means that we can solve the circuit ODEs with arbitrary initial guesses in the time domain. Numerical results including convergence properties for a deep-submicron MOSFET circuit operated with a two-tone input signal have been reported to demonstrate the robustness and accuracy of the method.

This paper is organized as follows. In Section 2, we state the MOSFET RF circuit to formulate the ODE system. Section 3 shows the numerical algorithms for the system of ODEs and proves the convergence properties of the monotone iterative method for each decoupled ODEs. A computational procedure is also introduced in this section. Section 4 shows numerical results for a n-type MOSFET circuit operated under two-tone RF range. Different convergence behaviors are also discussed in this section. Finally, Section 5 draws the conclusions and suggests future works.

2. A MOSFET RF circuit model

As shown in Fig. 1, based on the nodal current flow conservation and utilize the EPFL-EKV (EKV) large signal equivalent circuit model, shown in Fig. 2, for the MOSFET device [16,17], the mathematical model is formulated. At nodes B, D, S, and GX we have the following equations.

$$(C_{gs} + C_{gs0}) \left(\frac{dV_S}{dt} - \frac{dV_G}{dt} \right) + (C_{gd} + C_{gd0}) \left(\frac{dV_D}{dt} - \frac{dV_G}{dt} \right) + (C_{gb} + C_{gb0}) \left(\frac{dV_B}{dt} - \frac{dV_G}{dt} \right) + \frac{V_{GX} - V_G}{R_G} = 0, \quad (1)$$

$$-I_{DS} - I_{DB} + (C_{gd} + C_{gd0}) \left(\frac{dV_G}{dt} - \frac{dV_D}{dt} \right) + C_{bd} \left(\frac{dV_B}{dt} - \frac{dV_D}{dt} \right) + \frac{V_{DX} - V_D}{R_D} = 0, \quad (2)$$

$$(C_{gs} + C_{gs0}) \left(\frac{dV_G}{dt} - \frac{dV_S}{dt} \right) + C_{bs} \left(\frac{dV_B}{dt} - \frac{dV_S}{dt} \right) + \frac{V_{SX} - V_S}{R_S} + I_{DS} = 0, \quad (3)$$

$$C_{GX} \left(\frac{dV_{in}}{dt} - \frac{dV_{GX}}{dt} \right) + \frac{V_G - V_{GX}}{R_G} + \frac{V_{IN} - V_{GX}}{R_{IN}} = 0. \quad (4)$$

Similarly, at nodes SX and DX, we formulate the equations as follows:

$$\frac{V_S - V_{SX}}{R_S} + \frac{-V_{SX}}{R_{SX}} = 0 \quad (5)$$

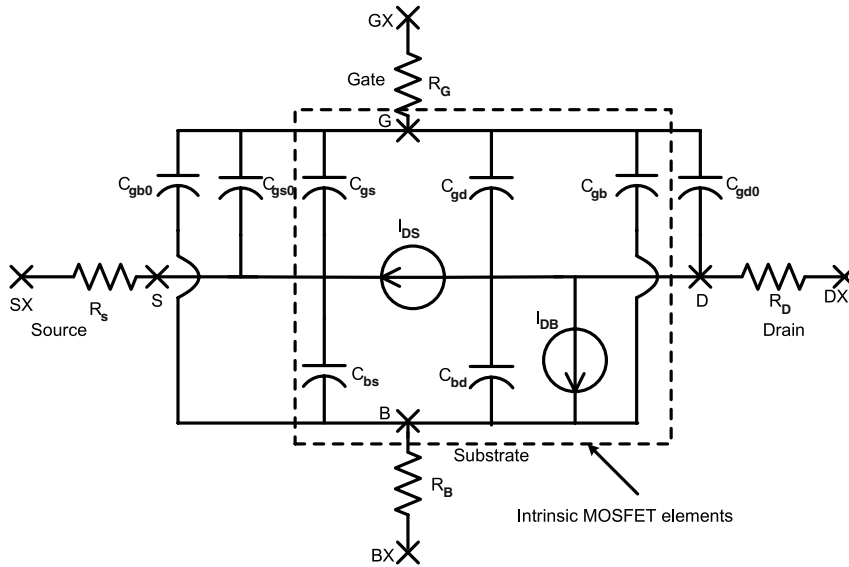


Fig. 2. The MOSFET EKV equivalent circuit.

$$\frac{V_D - V_{DX}}{R_D} + \frac{V_{DD} - V_{DX}}{R_{DD}} + \frac{-V_{DX}}{R_{OUT}} = 0. \tag{6}$$

Eqs. (1)–(4) are the ODEs, and Eqs. (5) and (6) are the algebraic equations. These equations are subject to proper initial values at time $t = 0$ for all unknowns to be solved. All currents I and capacitances C above are nonlinear functions of unknown variables. These nonlinear terms are modelled by the EKV. For finding the characteristic of the RF circuit behavior shown in Fig. 1, there are 4 coupled ODEs with the nonlinear current and capacitance models have to be solved and the unknowns to be calculated in the system of ODEs are V_G , V_D , V_S , and V_{CX} , respectively. Due to the exponential dependence of current and capacitance models, we note that the system consists of strongly coupled nonlinear ODEs.

3. Numerical algorithms for the system of ODEs

We propose here a decoupled and globally convergent simulation technique to solve the system ODEs in the large scale time domain directly. Firstly, under the steady state condition, we find the DC solution as the starting point to compute other time dependent solutions. For a specified time period T , to solve these nonlinear ODEs in the time domain, the computational scheme consists of following steps:

- (1) Let an initial time t and time step Δt be given.
- (2) Use the decoupling method to decouple all Eqs. (1)–(6).
- (3) Each decoupled ODE is solved sequentially with the MI and RK methods.
- (4) Convergence test for each MI loop.
- (5) Convergence test for overall outer loop.
- (6) If the specified stopping criterion is reached for the outer loop, then go to step 7, else update the newer results and back to step 3.
- (7) If $t < T$, $t = t + \Delta t$ and repeat the steps 3–6 until the time step meets the specified time period T .

All coupled ODEs are decoupled by the well-known waveform relaxation (WR) method [13,14]. We state here the WR method in details. Consider a general nonlinear system of \mathcal{N} ODEs with associated initial conditions,

$$\frac{dV}{dt} = f(V, t), \quad V(0) = V_0, \tag{7}$$

where $t \in [0, T]$, $T > 0$, $f : \mathbb{R}^{\mathcal{N}} \times [0, T] \rightarrow \mathbb{R}^{\mathcal{N}}$. $V_0 = [V_{1,0}, V_{2,0}, \dots, V_{\mathcal{N},0}] \in \mathbb{R}^{\mathcal{N}}$ is the initial vector of V , and $V(t) = [V_1(t), V_2(t), \dots, V_{\mathcal{N}}(t)] \in \mathbb{R}^{\mathcal{N}}$ is the solution vector at time t . The system can be written as follows,

$$\begin{cases} \frac{d}{dt} V_1 = f_1(V_1, V_2, \dots, V_{\mathcal{N}}, t), & V_1(0) = V_{1,0} \\ \frac{d}{dt} V_2 = f_2(V_1, V_2, \dots, V_{\mathcal{N}}, t), & V_2(0) = V_{2,0} \\ \vdots \\ \frac{d}{dt} V_{\mathcal{N}} = f_{\mathcal{N}}(V_1, V_2, \dots, V_{\mathcal{N}}, t), & V_{\mathcal{N}}(0) = V_{\mathcal{N},0}. \end{cases} \tag{8}$$

The WR method for solving (7) is a continuous-time iterative method. Therefore, given a function which approximates the solution, it calculates a new approximation along the whole time interval of interest. Clearly, it differs from most standard iterative techniques in that its iterates are functions in time instead of scalar value. The iteration formula is chosen in such a way that one avoids having to solve a large system ODEs. A particularly simple, but often very effective iteration scheme is written below. It maps the old iterate V^{j-1} .

$$\begin{cases} \frac{d}{dt} V_1^j = f_1(V_1^j, V_2^{j-1}, \dots, V_N^{j-1}, t), & V_1^j(0) = V_{1,0} \\ \frac{d}{dt} V_2^j = f_2(\cdot, V_2^j, V_3^{j-1}, \dots, V_N^{j-1}, t), & V_2^j(0) = V_{2,0} \\ \vdots \\ \frac{d}{dt} V_N^j = f_N(\cdot, V_2^j, \dots, V_{N-1}^j, V_N^j, t), & V_N^j(0) = V_{N,0}. \end{cases} \tag{9}$$

It is similar to the Gauss–Seidel method for iteratively solving linear and nonlinear systems of algebraic equations, the so-called Gauss–Seidel waveform relaxation scheme. It converts the task of solving a differential equation in \mathcal{N} variables into the task of solving a sequence of differential equations in a single variable. A closely related iteration is the Jacobi waveform relation scheme, the iteration formula is given by,

$$\begin{cases} \frac{d}{dt} V_1^j = f_1(V_1^j, V_2^{j-1}, \dots, V_N^{j-1}, t), & V_1^j(0) = V_{1,0} \\ \frac{d}{dt} V_2^j = f_2(\cdot, V_2^j, V_3^{j-1}, \dots, V_N^{j-1}, t), & V_2^j(0) = V_{2,0} \\ \vdots \\ \frac{d}{dt} V_N^j = f_N(V_1^{j-1}, \dots, V_{N-1}^{j-1}, V_N^j, t), & V_N^j(0) = V_{N,0}. \end{cases} \tag{10}$$

To solve the described Jacobi waveform relation scheme shown above, the algorithms is used for solving such system ODEs. For a given specified time step t and the previous calculated results, the decoupling algorithm solves the circuit equations sequentially, for instance, the V_G in Eq. (1) is solved for given the previous results ($V_D, V_S, V_{GX}, V_{DX}, V_{SX}$). The V_D in Eq. (2) is solved for newer given V_G and ($V_S, V_{GX}, V_{DX}, V_{SX}$). The V_S in Eq. (3) is solved for newer given (V_G, V_D) and (V_{GX}, V_{DX}, V_{SX}). We have similar procedure for other unknowns. A computational procedure is summarized in the following algorithm.

Algorithm:

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Given  $j = 0$ ;
Choose  $V_x^0(t)$  for  $t \in [0, T], x = 1, 2, \dots, \mathcal{N}$ 
do
   $j = j + 1$ 
  for  $x = 1, 2, \dots, \mathcal{N}$ 
    solve  $\frac{d}{dt} V_x^j = f_x(V_1^j, V_2^j, \dots, V_{x-1}^j, V_x^j, V_{x+1}^{j-1}, \dots, V_N^{j-1}, t)$ 
    with  $V_x^j(0) = V_{x,0}$ 
  end for
while  $|V_x^j - V_x^{j-1}| < \text{TOL}$ , for  $x = 1, 2, \dots, \mathcal{N}$ .
    
```

Each decoupled ODE is solved with the MI method [8–12]. In the following subsection, we state the MI method and show the convergence property.

3.1. Monotone Iterative method for each decoupled ODE

To clarify the MI method for the numerical solution of the decoupled nonlinear ODEs, we write the above decoupled ODEs as the following form

$$\begin{aligned} \frac{dV_x^j}{dt} &= f(V_x^j, t), \\ V_x^j(0) &= V_{x,0} \end{aligned} \tag{11}$$

where V_x^j is the unknowns to be solved, j is the decoupling index $j = 0, 1, 2, \dots$. We note that the f is the collection of the nonlinear functions and $f \in C[\mathbf{R} \times I, \mathbf{R}]$ and $I = [0, T]$. We may assume the upper and lower solutions are \bar{V}_x^j and \underline{V}_x^j exist

in the circuit for a fixed index j and X , and $\bar{V}_X^j \geq \underline{V}_X^j$, we can prove the solution existence in the set Ω .

$$\Omega = \{(V_X^j, t) \mid \bar{V}_X^j \geq V_X^j \geq \underline{V}_X^j, \forall t \in I\} \tag{12}$$

for each decoupled circuit ODE.

We define the MI parameter $\lambda = \frac{\partial f}{\partial V_X}$ and insert the λ into Eq. (11), then we have the MI equation

$$\frac{dV_X^j}{dt} = f(\eta, t) - \lambda(V_X^j - \eta) \tag{13}$$

where $\bar{V}_X^j \leq \eta \leq \underline{V}_X^j$ is a value in $[0, T]$.

In the following, we use Theorem 1 to provide the existence of the solution of the system ODEs. We also describe a monotone constructive method for the simulation methodology of the circuit ODEs. The constructed sequences will converge to the solution of Eq. (11) for all decoupled ODEs in the circuit simulation. In this condition, instead of original nonlinear ODE to be solved, a transformed ODE is solved with such as the RK method with the MI scheme formulated as Eq. (13). Next, we state the main result for the solution of each decoupled MOSFET circuit ODEs by proving Theorems 2 and 3.

Theorem 1. Let \bar{V}_X^j and \underline{V}_X^j are the upper and lower solutions of Eq. (11) in $C^1[\mathbf{R} \times I, \mathbf{R}]$ such that $\bar{V}_X^j \geq \underline{V}_X^j$ in the time interval I and $f \in C[\mathbf{R} \times I, \mathbf{R}]$. Then there exists a solution V_X^j of Eq. (11) such that $\bar{V}_X^j \geq \underline{V}_X^j$ in the time interval I .

Proof. It is a direct result with the continuous property of f , here the comparison theorem is applied [10–12,18].

We note that the nonlinear function f is nonincreasing function of the unknown V_X^j and the upper and lower solutions $\bar{V}_X^j(0)$ and $\underline{V}_X^j(0)$ of Eq. (11) in I can be found for each decoupled ODE. Further more, we can prove there exists a unique solution V_X^j of Eq. (11) in I and $\bar{V}_X^j(0) \geq V_X^j \geq \underline{V}_X^j(0)$.

Theorem 2. Let the $f \in C[\mathbf{R} \times I, \mathbf{R}]$, $\bar{V}_X^j(0)$ and $\underline{V}_X^j(0)$ are the upper and lower solutions of Eq. (11) in I . Since $f(V_X^j, t) - f(\tilde{V}_X^j, t) \geq -\lambda(V_X^j - \tilde{V}_X^j)$, $\bar{V}_X^j(0) \geq V_X^j \geq \tilde{V}_X^j \geq \underline{V}_X^j(0)$ and $\lambda \geq 0$. There exist sequences $\{\bar{V}_{X_n}^j\}_{n=1}^\infty$ converge to \bar{V}_X^j uniformly and $\{\underline{V}_{X_n}^j\}_{n=1}^\infty$ converge to \underline{V}_X^j uniformly as $n \rightarrow \infty$ monotonically in I .

Proof. For $\mathcal{V} \in C[\mathbf{R} \times I, \mathbf{R}]$, such that $\bar{V}_X^j \geq \mathcal{V} \geq \underline{V}_X^j$ we consider the following transformed ODE equation for the fixed j and X

$$\begin{aligned} \frac{dV_X^j}{dt} &= f(\mathcal{V}, t) - \lambda(V_X^j - \mathcal{V}) \\ V_X^j(0) &= V_{X_0}^j, \end{aligned} \tag{14}$$

then $\forall \mathcal{V}, \exists! V_X^j$ of Eq. (11) in I .

Define $\Psi V = V_X^j$ then we can verify:

- (i) $\Psi \bar{V}_X^j(0) \geq \bar{V}_X^j(0)$ and $\Psi \underline{V}_X^j(0) \leq \underline{V}_X^j(0)$
- (ii) Ψ is a monotone operator in

$$\left[\bar{V}_X^j(0), \underline{V}_X^j(0) \right] \equiv \left[V_X^j \in C[I, \mathbf{R}] \mid \bar{V}_X^j(0) \geq V_X^j \geq \underline{V}_X^j(0) \right]. \tag{15}$$

Now we construct two sequences by using the mapping $\Psi: \Psi \bar{V}_{X_n}^j = \bar{V}_{X_{n+1}}^j$ and $\Psi \underline{V}_{X_n}^j = \underline{V}_{X_{n+1}}^j$ and by the above observations, the following relation holds

$$\bar{V}_{X_0}^j \geq \dots \geq \bar{V}_{X_n}^j \geq \underline{V}_{X_n}^j \geq \dots \geq \underline{V}_{X_0}^j \tag{16}$$

in I . Hence $\bar{V}_{X_n}^j \xrightarrow{unif.} \bar{V}_X^j$ and $\underline{V}_{X_n}^j \xrightarrow{unif.} \underline{V}_X^j$ as $n \rightarrow \infty$ monotonically in I . Furthermore the $\bar{V}_{X_n}^j$ and $\underline{V}_{X_n}^j$ satisfy

$$\begin{aligned} \frac{dV_{X_{n+1}}^j}{dt} &= f(\underline{V}_{X_n}^j, t) - \lambda(V_{X_{n+1}}^j - \underline{V}_{X_n}^j) \\ \underline{V}_{X_n}^j(0) &= V_{X_0}^j, \end{aligned} \tag{17}$$

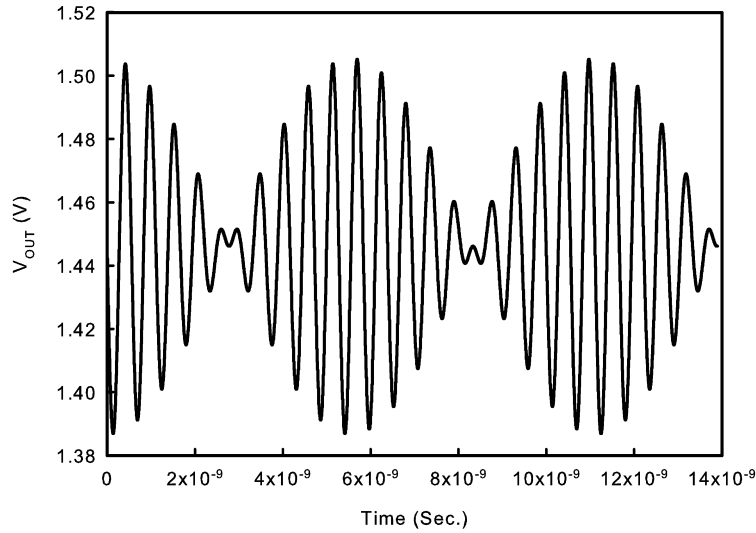


Fig. 3. Our time domain outputs.

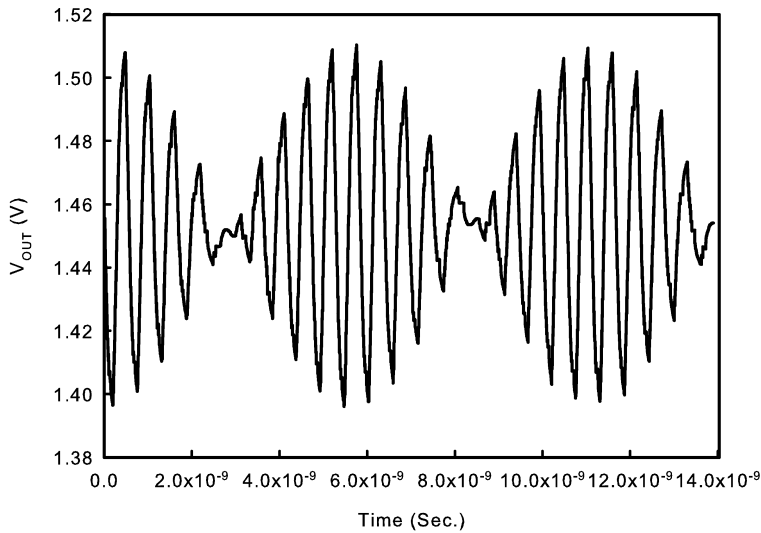


Fig. 4. The HSPICE time domain outputs.

and

$$\frac{d\bar{V}_{X_{n+1}}^j}{dt} = f(\bar{V}_{X_n}^j, t) - \lambda(\bar{V}_{X_{n+1}}^j - \bar{V}_{X_n}^j)$$

$$\bar{V}_{X_n}^j(0) = V_{X_0}^j, \tag{18}$$

respectively. Thus \bar{V}_X^j and \underline{V}_X^j are the solutions of Eq. (11).

Theorem 3. For decoupled ODEs, the nonlinear function f is nonincreasing in V_X^j and $f(V_{X_1}^j, t) - f(V_{X_2}^j, t) \geq -\lambda(V_{X_1}^j - V_{X_2}^j)$, $\forall V_{X_1}^j \geq V_{X_2}^j$. Thus $\{\bar{V}_{X_n}^j\}_{n=1}^\infty$ and $\{\underline{V}_{X_n}^j\}_{n=1}^\infty$ converge uniformly and monotonically to the unique solution V_X^j of Eq. (11).

Proof. By Theorem 2 and note that the nonincreasing property of f in the EKV models of N-MOSFET [8,16,17], the result is followed directly.

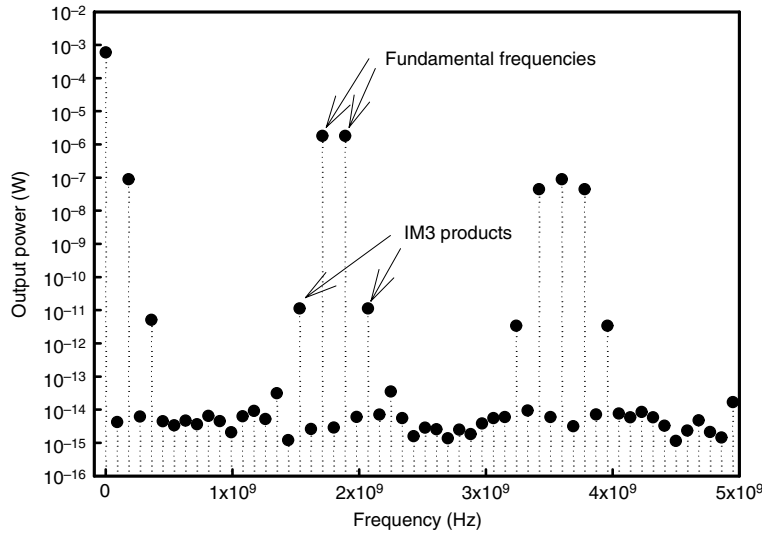


Fig. 5. Our output power in frequency domain.

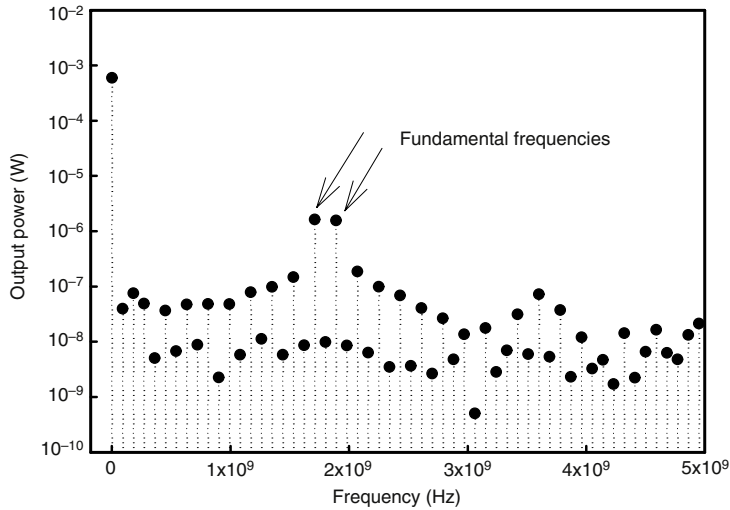


Fig. 6. HSPICE output power in frequency domain.

4. Results and discussion

As shown in Fig. 1, the input signal V_{IN} is the DC bias and the expression of the two-tone input signal V_{in} is as follows:

$$V_{in} = V_m \sin(2\pi f_1 t) + V_m \sin(2\pi f_2 t). \tag{19}$$

The input two-tone signal has an amplitude $V_m = 0.005$ V, and fundamental frequencies f_1 and f_2 are 1.71 and 1.89 GHz, respectively. Figs. 3 and 4 shows our simulation results and the HSPICE results in time domain, respectively. We also can find the difference in the Figs. 3 and 4 at the time between 2.5 ns and 3.5 ns, the HSPICE results show some non-smooth calculation, but our calculation does not have this phenomenon [8]. With the time domain results, we calculate the spectra of the output power by the FFT. Fig. 5 shows the corresponding spectra with our time domain results. We can find the 3rd-order intermodulation (IM3) products at $2f_1 - f_2$ and $2f_2 - f_1$ clearly. Fig. 6 shows the corresponding spectra with HSPICE time domain result. However, it is difficult to identify the two IM3 products.

The stopping criteria for the monotone iterative loop is the maximum norm error $< 10^{-10}$ and for the outer iterative loop is the maximum norm error $< 10^{-9}$, respectively. Figs. 7–9 show the convergence property of our simulation kernel. Fig. 7a shows the inner loop iteration versus the maximum norm error at the beginning ($t = 0$ s) of the simulation. Fig. 8 also shows the inner loop iteration versus the maximum norm error in time point at $t = 13.8$ fs. This two figures show the inner loop iteration has strictly convergent behavior and have $n \log(n)$ convergence where n is the number of iterations. Fig. 9 shows

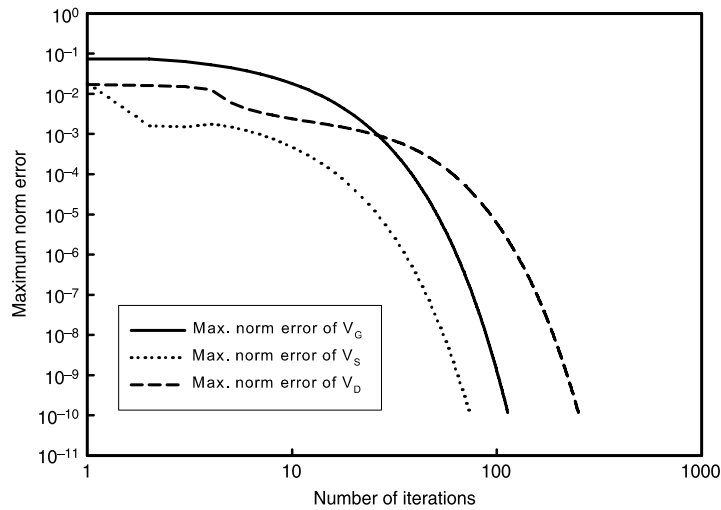


Fig. 7. The maximum norm error versus the number of inner iterations at the $t = 0$ s.

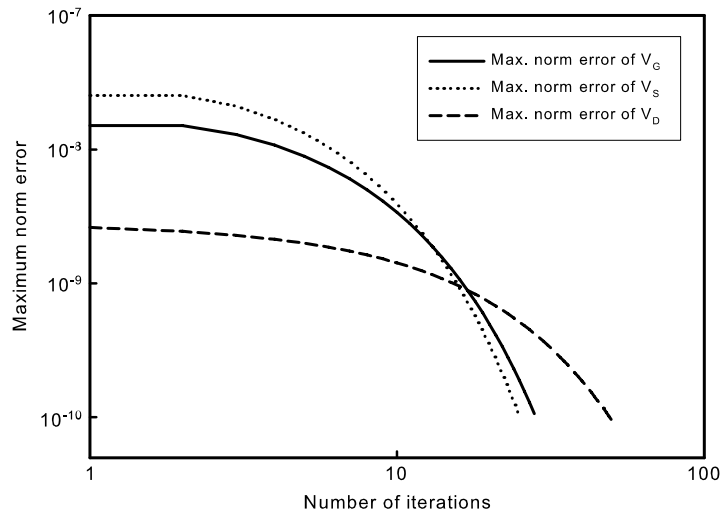


Fig. 8. The maximum norm error versus the number of inner iterations at the $t = 13.8$ fs.

the outer loop iteration convergence property in different time periods. As we can see the speed of convergence is almost around n^2 .

5. Conclusions

A novel RF circuit simulation method based on the waveform relaxation and monotone iterative methods has been proposed. With the monotone iterative technique, we have proved each decoupled circuit ODE converges monotonically. The proposed method here is an alternative in the numerical solution of electric circuit equations. Computational results have been reported in this work to demonstrate the robustness of the method. This method not only provides a computational technique for the time domain solution of circuit ODEs but also can be generalized for circuit simulation including more transistors. We are currently parallelizing this method for large-scale circuit simulation.

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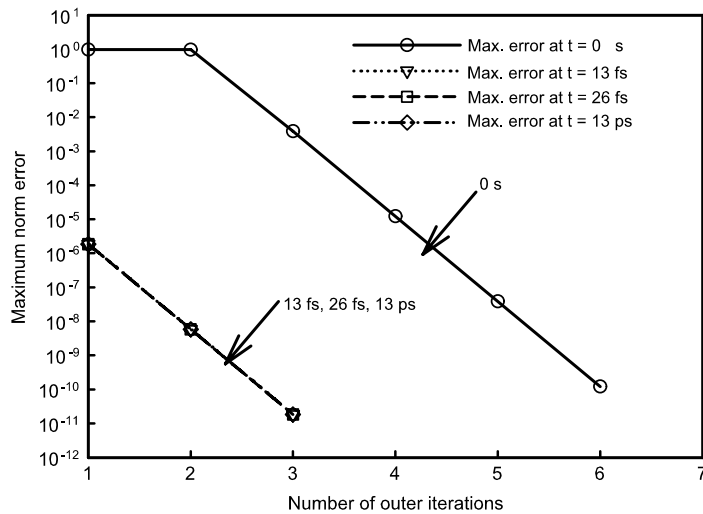


Fig. 9. The maximum norm error versus the number of outer iterations.

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