

# 考慮債務限制條款下的企業信用風險模式

## Predicting the Default Risk of Firms: A Model with Safety Covenants

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**摘要：**有鑑於企業違約預警對於經濟體系的重要性，本文應用障礙選擇權理論，建構一個較符合實際經濟社會違約過程的違約預警模型。實證結果顯示，相較於傳統 BSM (Black and Scholes, 1973 與 Merton, 1974) 利用市場資訊所估算之結構式模型，以障礙選擇權理論建立的 DOC(down-and-out call option) 模型，其違約預警能力相較 BSM 模型而言有提升之效果。另外，透過 censored Tobit 迴歸模式觀察影響兩種結構式模型表現的相關因素，亦可以發現，DOC 模型於建構過程中，較傳統 BSM 模型多考慮企業獲利性層面因素，故更能有效地偵測企業違約之發生。因此，本文認為 DOC 模型亦可作為另一個判斷企業違約風險的預警工具。

**關鍵詞：**信用風險模型；Black-Scholes-Merton (BSM)模型；障礙選擇權模型；Tobit 迴歸

**Abstract :** This study uses barrier option theory to establish a credit risk model with greater relevance to the process of default by firms in the real world. As

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compared to the traditional Black-Scholes-Merton (BSM) structural model, which makes use of market information along with the results of the empirical testing of default predicting performance, we suggest that our down-and-out call option (DOC) model, established on the basis of barrier option theory, provides superior performance. When factoring in profitability, and when using the censored Tobit regression model to observe the characteristics of these two structural models, we find that the DOC model is more effective at predicting default events; we therefore conclude that the DOC model is another appropriate model for the measurement of the credit risk of firms.

**Keywords:** Credit risk model; Black-Scholes-Merton (BSM) model; Barrier option model; Tobit regression.

## 1. Introduction

As a result of the continuing transition and development of the financial environment, there has been significant growth over recent years in the importance of both the measurement and management of credit risk. From examples such as Enron, Worldcom, Infodisc, Summit Computer and Procomp, it is now common knowledge that financial distress can affect major companies across the globe, with the incidents involving these companies having serious impacts not only on investors, creditors, company employees and financial institutions, but on society as a whole. Following the New Basel Capital Accord (Basel II) aimed at ensuring that domestic financial markets meet the international trend, greater emphasis is now being placed on the evaluation of the default risk of firms.

Any corporate operational difficulties or financial distress has significant adverse impacts on society, which ultimately affect the rights of both creditors and investors. A prerequisite to meeting the requirements of the Basel II credit risk management protocol is the establishment of an effective, discriminating and predictive model of firm default capable of detecting real-time default signals. For financial institutions, the accurate, real-time detection of the default risk of borrowers will clearly mitigate their operational risk; whilst for investors, creditors and managers, an early-warning system could prompt them to take the

necessary precautionary measures that will enable them to avoid massive losses resulting from the financial distress of firms.

The quantitative credit risk model studies can be largely divided into accounting-based models (adopting historical financial data) and market-based models (those which use information on the equity and bond markets). The early credit evaluation models, which were invariably accounting-based, featured univariate analysis (Beaver, 1966), multiple discriminant analysis (Altman, 1968), logit analysis (Ohlson, 1980), probit analysis (Zmijewski, 1984) and neural network analysis (Atiya, 2001), all of which are designed to discriminate between defaulting and non-defaulting firms prior to defaults occurring.

Since they use historical financial data (with no consideration of future information), accounting-based methods cannot fully reflect the actual economic condition of a company, as no forward-looking default prediction methods are involved. In contrast, the structural models (which regard the equity and liabilities of a company as the contingent claims on its assets based on information from the equity markets) use the standard European call options of the option-pricing models following the 'BSM' (Black and Scholes, 1973; Merton, 1974) model. The reduced-form models (based on information from the bond markets) are grounded in theories with insufficient economic implications, and since the bond market in Taiwan is less developed than the equity market, the use of bond market information for the prediction of firm defaults clearly has its limitations.

Comparatively speaking, the structural models have many advantages, not least of which is their grounding in robust theory, whilst the use of high-frequency equity market information also ensures that these models are more forward-looking. The structural models are therefore more likely to have good predictive ability on firm default and rating changes; indeed, McQuown (1993) notes that the measurement of default probability by rating agencies based upon an historical average value is incapable of responding to changes in credit risk, whereas the BSM model, built on market trade price information, can rapidly reflect variations in the credit risk of a firm. Thus, from the results of their empirical testing of the BSM hypotheses, Farnen, Westhaard, and van der Wijst (2004) conclude that the BSM model is an appropriate model for credit risk applications.

In their comparison of the relative information content of the default probability measures using the Altman (1968) Z-Score, the Ohlson (1980) O-Score and the BSM model, Hillegeist et al. (2004) find that the default probability measured by the BSM model provides significantly more information than either of the other two accounting-based models. Vassalou and Xing (2004) examine the differences between the predictive performance of market-based structural model and accounting-based models, noting that the latter models do not take into account the volatility of a firm's assets when estimating its default risk; thus, they argue that the accounting-based models imply that firms with similar financial ratios will have a similar likelihood of risk. This is not, however, the case for the BSM model. Although firms may have similar levels of equity and debt, if there are significant differences in the volatility of their assets, such firms can have starkly contrasting risks of default. Therefore, given that market-based structural models can truly reflect the daily volatility of firms through their trading price, such models clearly provide both forward-looking information and the expectations of investors on the future performance of such firms.

Based upon their empirical examination of both default risk and credit spreads, Patel and Pereira (2007) clearly demonstrate that the structural models have more predictive power when firms are close to financial distress. Furthermore, following Altman (1968), in their empirical comparison of the performance of accounting-based and structural models, Benos and Papanastasopoulos (2007) consider various combinations of 22 explanatory variables before selecting the five with the highest predictive power. Their results demonstrate that the structural model has more accurate predictive power of defaults than the accounting-based model.

The prior empirical studies on default probability provide clear support for the replacement of the accounting-based model by the BSM model. Accordingly, not only do the structural models provide superior empirical predictive performance, but they also have better theoretical grounding. However, whilst the structural BSM model does have the advantage of combining its real-time nature with market information, the assumption of debt maturity as the default date oversimplifies the firm default process from both theoretical and practical perspectives, since this

assumption is inconsistent with the process of firm default observed in real life.

As noted by Brockman and Turtle (2003), there are fundamental differences between the valuation of corporate securities in the real world and the standard European call options of the BSM model; this is essentially because the standard call option of the BSM model assumes that corporate securities are path-independent, with the payoff being dependent on the value of the underlying assets only at the maturity date, and not on the particular path followed prior to maturity. This indicates that the standard European call option remains alive, regardless of any rise or fall in asset value during the life of the option; nevertheless, in the real world, if asset values fall below a pre-specified level, often related to debt loading, corporate equity can be wiped out by default.

Clearly, therefore, corporate securities should be path-dependent options, the payoffs of which are dependent on the particular path followed by the underlying asset; and indeed, Brockman and Turtle (2003) use such a path-dependent barrier option framework to replace the traditional path-independent BSM approach for the valuation of corporate securities, treating corporate equity as a down-and-out call option on the corporate assets with a strike price which is the face value of debt, and applying the down-and-out call option framework to the predicting of default by observing such defaults when the value of the asset falls below a predetermined barrier level prior to the maturity of the debt.

The prior empirical research demonstrates that under most scenarios, a default prediction model based upon a down-and-out call option has significantly greater predictive ability than the Altman Z-score, which also infers that the performance of a barrier option framework is superior to that of the BSM model. For example, Reisz and Perlich (2007) also see corporate equity as a down-and-out call option on corporate assets with a strike price which is equal to the face value of the debt; however, they improve on the shortcomings of the Brockman and Turtle (2003) model by carrying out empirical comparisons between the predictive ability of their barrier option model, and that of the BSM and KMV models. Their results reveal that the predictive ability of the barrier option model is superior to that of both the BSM and KMV approaches.

According to Giesecke (2004), the extant literature on the evaluation of firm credit risk through the use of a barrier option model can be divided into two groups, the first of which represents the case where the constant default barrier exceeds the face value of the debt. If the firm's asset value never falls below the default barrier over the term of the bond, the bond holders receive the face value at debt maturity and the equity holders receive the remainder; however, if the firm's asset value falls below the barrier at some point during the term of the bond, then the firm defaults. Under such a scenario, the firm ceases operations, the bond holders take over its assets and the equity holders receive nothing; thus, the bond holders are fully protected, ultimately receiving at least the face value of the debt upon default, and the bond is no longer subject to default risk.

The second case is where both the constant default barrier below the face value of the debt and the bond holders are exposed to some default risk. If the firm's asset value never falls below the default barrier during the term of the bond, and also exceeds the face value of the bonds at debt maturity, then the bond holders receive the face value of the debt, and the equity holders receive the remainder. If the firm's asset value never falls below the default barrier over the term of the bond but is below the face value of the bond at debt maturity, then the firm defaults. Under such a scenario, since the remaining assets are insufficient to pay off the debt in full, only the bond holders get the remaining assets and the equity becomes worthless; however, if the firm's asset value falls below the default barrier over the term of the bond, the firm again defaults, and the bond holders receive the barrier value at default, with the equity once again becoming worthless.<sup>2</sup>

The suggestion from the above discussion is that if the bond holders are not subject to any default risk in the case where the default barrier exceeds the face value of the debt only; however, this no longer reflects economic reality. The purpose of this study is therefore to establish a credit risk model through an approach which differs from the standard European call option in the relevant literature, by defining a totally different barrier level to that adopted in the prior literature along with relaxation of the assumption of the BSM model, where only

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<sup>2</sup> Graphical illustrations of these cases are provided in the Appendix.

defaults occurring at debt maturity are considered. In order to observe the early defaults that occur when the asset value is falling to a certain barrier level, we establish a mechanism with a set of strict safety covenants for the protection of creditors, such that creditors have the right to trigger company bankruptcy or restructuring when corporate performance fails to reach the barrier level.<sup>3</sup>

Under our framework, if the firm's asset value exceeds the barrier level prior to debt maturity and it can repay corporate debts at debt maturity, the shareholders would obtain the right to control the corporate assets. However, if the corporate asset value is above the barrier level over the term of the debt, but falls below the face value of the debt at debt maturity, corporate default occurs; at this point, the corporate will be taken over by creditors. Alternatively, if the asset value falls below the barrier level over the term of the debt maturity, thereby triggering bankruptcy, creditors will again take over the company, and shareholders will lose their right to dispose of corporate assets. This is the early default scenario emphasized in this study; hence, we measure default risk using a barrier option framework under a more realistic scenario by considering the default process as a European down-and-out call (DOC) option.

Under the setup of the model adopted in this study, it is possible to observe the variations in the asset value during the term of the debt, thereby improving on the shortcomings of the BSM model, which ignores the variations in asset value over the term of the debt by assuming that defaults occur only at debt maturity. We undertake an empirical comparison between our DOC model and the BSM model (constructed under a traditional standard call option framework) to observe whether the relaxation of this assumption in the BSM model provides the credit risk model with more accurate discriminant predictive ability of the default of a firm.

We also address a gap in the extant literature, where default risk models are

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<sup>3</sup> The concept of safety covenants is emphasized in the studies of Black and Cox (1976), Geske (1977), Leland and Toft (1996), Briys and de Varenne (1997), Chesney and Gibson-Asner (1999), Brockman and Turtle (2003), Giesecke (2004), Elizalde (2005) and Reisz and Perlich (2007); such covenants are commonplace in real life in the form of restrictions on either net value or liquidity.

constructed with no analysis being undertaken of the reasons for the differences in the predictive ability of the various models. We therefore apply a Tobit regression analysis in an attempt to explain the reasons for the differences in the predictive ability of the two structural models examined in this study.

The remainder of this paper is organized as follows. Section 2 provides a description of the methodology adopted for this study, followed in Section 3 by details of the data sources and the definition of the variables used. Presentation and analysis of the empirical results is provided in Section 4, with the final section presenting the conclusions drawn from this study, along with some suggestions for further extensions of this area of research.

## **2. Methodology**

We adopt a two-stage approach in this study, using the structural models and a Tobit regression. Firstly, we use the BSM and DOC structural models to estimate the default probability of the firms, observing the default prediction performance for each model. Secondly, we employ the Tobit regression model to observe the factors influencing the differences in the default prediction performance for these two structural models.

### **2.1 The Structural Models and the Measurement of Predictive Ability**

#### **2.1.1 The BSM Model**

Merton (1974) makes use of the Black and Scholes (1973) option-pricing model to value corporate liabilities on the basis of such liabilities being contingent claims on the assets of the firm. The capital structure of a firm in the BSM model comprises of equity and a zero-coupon bond with maturity,  $T$ , and face value,  $F$ . The asset value of the firm is simply the sum of the value of its equity and bonds.

Under these assumptions, the equity of a firm is viewed as a European call option on the firm's assets value, with a strike price which is equal to the book value of the firm's debt,  $F$ , and a debt maturity,  $T$ . The debt issue can be regarded as a portfolio comprising of a default-free bond, with face value,  $F$ , and a short European put on the assets of the firm with a strike price,  $F$ . Since the firm's equity can be



treated as a European call option on the firm's asset value, the firm's equity value can be expressed as:

$$E_t = V_t N(d_1) - e^{-r\tau} FN(d_2) \quad (1)$$

where  $E$  is the market value of the equity;  $V$  is the market value of the asset;  $\sigma_V$  is the volatility of the asset;  $r$  is the risk-free rate;  $N(\cdot)$  is the standard cumulative normal distribution function; and  $\tau = T - t$  refers to the maturity of the debt contract.

$$d_1 = \frac{\ln(V_t/F) + (r + \sigma_V^2/2)\tau}{\sigma_V \sqrt{\tau}} = d_2 + \sigma_V \sqrt{\tau}$$

Applying Itô's Lemma to Equation (1), we take the first derivative on both sides of the equation and then compare the coefficients; we can then show that the volatility of both the equity and the asset are related by the following equation:<sup>4</sup>

$$\sigma_E = \frac{V_t}{E_t} \frac{\partial E_t}{\partial V_t} \sigma_V = \frac{V_t}{E_t} N(d_1) \sigma_V \quad (2)$$

At debt maturity,  $T$ , since the firm asset value is less than the face value of the debt, the firm will default;<sup>5</sup> under such a scenario, the bond holders take control of the firm, and the equity holders receive nothing. The BSM option-pricing model assumes that the random component of the firm's asset returns has normal distribution,  $\varepsilon \sim N(0,1)$ ; thus, the risk-neutral default probability of the firm can be written in terms of the cumulative normal distribution, as follows:<sup>6</sup>

$$P_{BSM} = N\left(-\frac{\ln(V_t/F) + (r - \sigma_V^2/2)\tau}{\sigma_V \sqrt{\tau}}\right) = N(-d_2) \quad (3)$$

### 2.1.2 The DOC Model

<sup>4</sup> For examples, Jones, Scott, and Rosenfeld (1984) and Ronn and Verma (1986).

<sup>5</sup> This is the default barrier value in the BSM model; that is, the face value of the debt,  $F$ , is the default point in the BSM model.

<sup>6</sup> Risk-neutral refers to the assumption that, regardless of the preferences of investors or the assets that they hold, they all see risk-free rates on their returns.

Although the BSM model assumes that firm defaults occur only at debt maturity, in the real world, defaults can occur at any time during the term of the debt. Thus, we apply the concept of a barrier option, using a structural DOC model to construct a more realistic default process which relaxes the assumption of the BSM model; in our model, default can occur whenever the asset value passes through the barrier level.

We assume that the corporation is fully financed with a share of equity and a single zero-coupon bond, both of which are traded in a perfect financial market. Since the sum of the bond and the stock value is equal to the firm's asset value, we can consider the firm's asset value as a traded security. Under a risk-neutral probability measure, the firm's asset value process follows a geometric Brownian motion of the form:

$$dV_t = rV_t dt + \sigma_V V_t dW_t \quad (4)$$

where  $dW$  is assumed to follow a Wiener process.

The firm's debt, issued at time  $t$ , has the form of a pure discount bond of promised payment,  $F$ , which matures at time  $T$ ; therefore, prior to the debt maturity date,  $T$ , if the firm's asset value does not cross the barrier level, the equity holders will honor the face value of the debt to the bond holders. Thus, the equity of a firm can be seen as a down-and-out call option on the value of the firm's assets, with a barrier level,  $B$ , strike price,  $F$ , and maturity,  $\tau = T - t$ . In the case of a firm defaulting, and with no consideration of bankruptcy costs, creditors can obtain the remaining value of the firm's assets.<sup>7</sup> By applying the concept of a barrier level, this study introduces a safety covenant mechanism for the protection of creditors, defining the barrier level as the historical recovery ratio of the repayment of the face value of the debt at maturity:

$$B = \alpha F e^{-r\tau} < F \quad (5)$$

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<sup>7</sup> As noted by Black and Cox (1976), bankruptcy costs are unlikely to alter the qualitative results of the structural models.

where  $\alpha$  is the exogenous recovery rate.<sup>8</sup> If the firm's asset value falls below the barrier level at any time prior to the maturity of the debt, or if the asset value remains above the barrier level prior to maturity but is below the face value of the debt at maturity, default occurs. Thus, the firm's equity can be expressed as:

$$E_t = V_t N(d_1) - Fe^{-r\tau} N(d_1 - \sigma_V \sqrt{\tau}) - [V_t (B/V_t)^{\frac{2r}{\sigma_V^2} + 1} N(d_1^B) - Fe^{-r\tau} (B/V_t)^{\frac{2r}{\sigma_V^2}} N(d_1^B - \sigma_V \sqrt{\tau})] \tag{6}$$

where  $d_1^B = \frac{\ln(B^2/V_t F) + (r + \sigma_V^2/2)\tau}{\sigma_V \sqrt{\tau}}$ .

The equity value in Equation (6) can be decomposed into two elements, the value of a standard European call option on the value of the firm's asset, and the loss of shareholder equity value due to early default triggered by creditors. We can determine that the volatility of the equity and the asset are related, according to Itô's Lemma, as follows:

$$\begin{aligned} \sigma_E &= \frac{V_t}{E_t} \frac{\partial E_t}{\partial V_t} \sigma_V \\ &= \frac{V_t}{E_t} \left\{ N(d_1) + (B/V_t)^{\frac{2r}{\sigma_V^2} - 1} \left\{ \frac{Fe^{-r\tau}}{V_t} N(d_1^B - \sigma_V \sqrt{\tau}) \right. \right. \\ &\quad \left. \left. + \left( \frac{2r}{\sigma_V^2} \right) \left[ \frac{B^2}{V_t^2} N(d_1^B) - \frac{Fe^{-r\tau}}{V_t} N(d_1^B - \sigma_V \sqrt{\tau}) \right] \right\} \right\} \sigma_V \end{aligned} \tag{7}$$

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<sup>8</sup> The barrier level in this paper, which is exogenous, is based on Black and Cox (1976) and Longstaff and Schwartz (1995). It differs from the constant barrier level used in many of the prior studies because it is time variable and because we also consider the debt recovery rate; thus, it is reasonable to assume that the barrier level will be lower than the face value of the debt. The level set in the DOC model in this study takes into account the actual default process observed in real life, thereby potentially improving the oversimplification problem of the default process in the BSM model, where default is assumed to occur at the debt maturity date.

Based on a risk-neutral probability measure, early defaults can be defined under the DOC framework as the asset value crossing the barrier level at any time prior to the debt maturity date,  $T$ ; that is:

$$P_{DOC}(t^* \leq T) = N\left(\frac{\ln(B/V_t) - (r - \sigma^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) + (B/V_t)^{2r/\sigma_v^2 - 1} N\left(\frac{\ln(B/V_t) + (r - \sigma^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) \quad (8)$$

where  $t^*$  is the first occurrence of the asset value passing through the default barrier,  $B$ . In those cases where the asset value stays above the barrier level prior to maturity, the probability of default at maturity is:<sup>9</sup>

$$P_{DOC}(B \leq V_T < F, \min_{t \leq s \leq T} V_s \geq B) = N\left(\frac{\ln(V_t/B) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) - N\left(\frac{\ln(V_t/F) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) - (B/V_t)^{2r/\sigma_v^2 - 1} \left\{ N\left(\frac{\ln(B/V_t) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) - N\left(\frac{\ln(B^2/FV_t) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) \right\} \quad (9)$$

Thus, the total risk-neutral default probability is the sum of the probability of default prior to maturity, as described in Equation (8), and the probability of default at the maturity of the debt contract, as described in Equation (9), that is:<sup>10</sup>

$$P_{DOC} = 1 - N\left(\frac{\ln(V_t/F) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right) \quad (10)$$

<sup>9</sup> In those cases where  $B > F$ , default will only occur prior to the maturity of the debt.

<sup>10</sup> See Reisz and Perlich (2007) for details of the derivation process.

$$+ (B/V_t)^{2r/\sigma_v^2 - 1} N\left(\frac{\ln(B^2/FV_t) + (r - \sigma_v^2/2)\tau}{\sigma_v \sqrt{\tau}}\right)$$

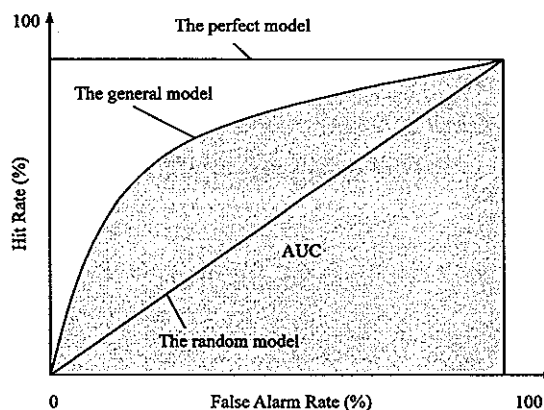
**2.1.3 Analysis of the Predictive Ability of the Models**

We compute the three absolute values of AUC, AR and KS through our respective analyses of the ‘receiver operating characteristics’ (ROC) curve, the ‘cumulative accuracy profiles’ (CAP) curve and the ‘Kolmogorov-Smirnov’ (KS) test. Using these three values, we can then investigate the default prediction performance of the two structural models. The various theories on the default prediction power of the models are described in the following sub-sections.

**2.1.3.1 Receiver Operating Characteristics (ROC)**

The ROC curve depicts the predictive ability levels of different models given the same threshold values, with the models first of all estimating the scores of defaulting firms and normal firms. In order to analyze the irrelevance of the threshold values and the discriminant capabilities of the models, it is necessary to calculate a false alarm rate (the proportion of normal firms mistakenly identified as defaulting firms) and a hit rate (the proportion of defaulting firms accurately identified as defaulting firms) under each threshold value, between the maximum and minimum scores of each model. Finally, all of the dots in a two-dimensional space are connected to generate the ROC curve, as shown in Figure 1.

**Figure 1**  
**Receiver Operating Characteristic Curves and Area Under the Curve**



In a perfect model, the ROC curve would be plotted along the line of the points (0,0), (0,1) and (1,1); in a pure random model, the curve would be plotted along the diagonal from the origin. The area under the curve (AUC) has a well-defined statistical implication, representing the probability that, from all of the firms in the sample, the predicted default probability of a randomly-selected defaulting firm will be greater than the predicted default probability of a randomly-selected non-defaulting firm.

Based upon a probabilistic interpretation of the AUC, two firms are drawn at random, the first from the distribution of defaulting firms, and the second from the distribution of non-defaulting firms. Let  $S_D$  be the score for defaulting firms and  $S_{ND}$  be the score for non-defaulting firms. A rational decision maker may surmise that the defaulting firm would be the one with the higher rating score, but if both firms have the same score, his decision would be random in nature.

Therefore, the probability of a correct decision by this rational decision maker is  $P(S_D > S_{ND}) + 0.5 P(S_D = S_{ND})$ , which is the AUC value. The AUC ratio is between 0 and 1; the closer AUC is to 1, the higher the accuracy, and the better the discriminant capability of the model forecasts. When AUC is 0.5, the discrimination between normal firms and default firms will be a random process; hence, the model has no discriminant capability whatsoever at this point. When AUC is equal to 1, this implies that the model is the best possible, with complete discriminant capability.

### **2.1.3.2 Cumulative Accuracy Profiles (CAP)**

In order to establish the CAP curve, firms are first ordered by default risks rating scores (from riskiest to safest) from each of the structural models. The CAP curve is constructed with the proportion of the riskiest firms ( $X\%$ ) of the total number of firms on the horizontal axis (the alarm rate), and the cumulative proportion of all defaulting firms ( $X\%$ ) on the vertical axis (the hit rate), as shown in Figure 2. The steeper the CAP curve at the beginning, the more accurate the prediction process will be.

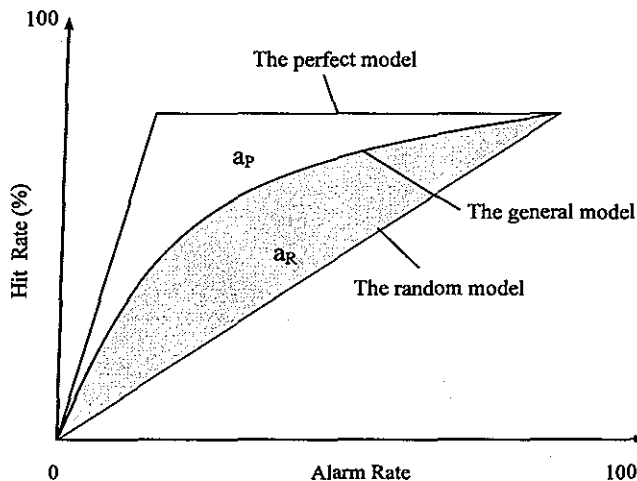
Ideally, the perfect model would show that all non-defaulting firms have the

lowest default risk, in which case the CAP curve would rise linearly at the beginning before leveling off to become horizontal. The other extreme example would be a pure random model, which would have no discriminatory power whatsoever. In this case, the CAP curve would be the diagonal shown in Figure 2.

In reality, the models are neither perfect nor random; hence, the corresponding CAP curve will be somewhere between these two extremes. The 'accuracy ratio' (AR) can be used as a single indicator to measure the predictive ability of the models with a CAP curve. The area between a perfect model and a random model in Figure 2 is indicated by  $a_p$ , and the area between an actual model and a random model is indicated by  $a_R$ . AR is defined as:

$$AR = \frac{a_R}{a_p}; 0 \leq AR \leq 1. \quad (11)$$

**Figure 2**  
**Cumulative Accuracy Profile Curves and the Accuracy Ratio**



### 2.1.3.3 Kolmogorov-Smirnov (KS) Statistics

The KS statistics are the results of observations on the way in which the credit risk models discriminate between normal firms and defaulting firms using

non-parametric tests. The first step involves computation of the cumulative probabilities under different scoring stages of defaulting and non-defaulting firms. This is followed by the calculation of the difference in the cumulative probability in the different stages in order to derive the biggest difference in cumulative probability; that is, the KS value. A good model should be capable of significant differentiation of the variations between non-defaulting and defaulting firms. In other words, there should be considerable variances in the distribution between the cumulative difference of defaulting firms and that of non-defaulting firms. If the variance in cumulative probability is significant at any particular stage, then this indicates that there is a possibility that the two samples came from different populations. Accordingly, when the KS value is great, the null hypothesis that the populations are the same should be rejected. The relationships between different KS values and discriminant capabilities of the models under various scenarios are summarized in Table 1.<sup>11</sup>

**Table 1**  
**KS Quality Values and Model Discriminatory Power**

KS Value (%)	Discriminatory Power of the Model
<20	Probably not worth using
21-40	Fair
41-50	Good
51-60	Very good
61-75	Awesome
>75	Probably too good to be true

## 2.2 Tobit Regression Analysis

In order to observe the factors influencing the predictive ability of the structural models, following assessment of the predictive ability of each model, we conduct a regression analysis on the market and financial variables for the estimation of default probability. The financial variables widely used for default

<sup>11</sup> See Mays (2001) for a detailed description of this relationship.



prediction within the literature are adopted, along with those variables which we believe can influence firm default; a total of 10 explanatory variables are selected for analysis.

Any dependent variable should be between 0 and 1, and since we use default probability as the dependent variable, the regression model in this study is a censored sample model, where independent variables correspond to any observation values, but dependent variables correspond only to certain observation values. Since the estimated parameters generated by the OLS approach can be biased and inconsistent, we use the Tobit regression model to examine the influence of relevant default variables on default probability. The function of this econometric model is as follows:

$$Y = f(CACL, WCTA, CLTA, OPOR, EBITOR, SCAI, TLTA, MVETL, LLTA, VOL, CASHTA, ORAFA, ETA, SIZE, CG, AGE) \quad (12)$$

where  $Y$  is the default probability estimated by the structural models;  $CACL$  is the current ratio;  $WCTA$  is the working capital/total assets;  $CLTA$  is the current liabilities/total assets;  $OPOR$  is the operating profit ratio;  $EBITOR$  is the net profit margin;  $SCAI$  is the inventory turnover ratio;  $TLTA$  is the liability ratio;  $MVETL$  is the market value of equity/total liabilities;  $LLTA$  is the long-term debt ratio;  $VOL$  is the stock price volatility;  $CASHTA$  is the cash and cash equivalents/total assets;  $ORAFA$  is the fixed asset turnover ratio;  $ETA$  is the total equity/fixed assets;  $SIZE$  is the logarithm of the firm's asset scale;  $CG$  is the pledged shares held by board directors; and  $AGE$  is the number of established years of the firm.

We divide the ten independent variables in Equation (12) into two constructs and four categories based upon their respective characteristics. The two constructs are the financial statements construct (which comprises of liquidity, profitability and solvency variables) and the stock price construct (which contains the market information variables).  $CACL$ ,  $WCTA$  and  $CLTA$  are the liquidity variables;  $OPOR$ ,  $EBITOR$  and  $SCAI$  are the profitability variables;  $TLTA$ ,  $MVETL$  and  $LLTA$  are the solvency variables, and  $VOL$  is the market information

variable. We also include a liquidity factor (*CASHTA*), an operating capability factor (*ORAF*), a solvency factor (*ETA*), a firm scale factor (*SIZE*), a corporate governance factor (*CG*) and a non-financial variable (*AGE*) as control variables in the regression; these six control variables are used in this study to observe the robustness of the previous ten financial and market information variables in regression Equation (12).

### **3. Data Source and Variable Definitions**

#### **3.1 Data Source and Sample Selection**

Our study sample comprises of all listed companies in Taiwan, along with variable data taken from the Taiwan Economic Journal (TEJ) database. A series of cases of financial distress have occurred in Taiwan since 1998, including dishonored checks, misappropriation of assets and default in the delivery of securities. We select 118 listed companies in Taiwan which defaulted between 1998 and June 2006.<sup>12</sup> 'Normal' companies are other listed companies with no reported defaults and continuing normal operations as at June 2006; these companies each had positive net values during the prediction periods.

#### **3.2 Variable Definitions**

Our empirical study is divided into two stages: (i) the analysis of the predictive ability of the structural models; and (ii) a Tobit regression analysis. The first stage, involving the assessment of corporate credit risk, is based mainly on 'option-pricing' theory. According to Equation (3), when using a BSM model, five input variables are required for the prediction of default probability; these are: asset value ( $V$ ), asset volatility ( $\sigma_v$ ), the default point ( $F$ ), the risk-free interest rate ( $r$ ) and duration ( $\tau$ ). Given the known default point, the risk-free interest rate and the

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<sup>12</sup> We adopt the TEJ definition of financial distress, where default is defined as the occurrence of any of the following events: bankruptcies and closures, restructuring, dishonored checks, bailouts, takeovers, accountant's questions on the company's prospects as a going concern, negative net value, delisting or suspension of operations due to tight financial situations. Although the finance and insurance industry has significant influence on the finance system, their accounting systems differ from those of other industries due to the uniqueness of their business; since such accounting differences may lead to problems in data matching, we do not include any of the firms in this industry in our sample.

duration, we can derive the value and volatility of the asset using simultaneous Equations (1) and (2).<sup>13</sup>

For our computation of credit risk, we calculate the market value of equity,  $E$ , based upon its closing price multiplied by the number of outstanding shares at the credit risks estimation computation cut-off point. Equity volatility,  $\sigma_E$ , is derived from the annual standard deviation of weekly equity returns of the one-year-ahead credit risk cut-off point.<sup>14</sup> According to the empirical study of KMV, the default points are usually found between the current liabilities and the total liabilities; therefore, we measure the default point in the BSM model as current liabilities plus half of the long-term liabilities.<sup>15</sup>

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<sup>13</sup> The use of simultaneous equations, please refer to Hull (2006). In order to ensure the accuracy of our results, we estimate the two unknown variables, the value and volatility of the assets in the BSM and DOC models simultaneously, using the maximum likelihood method suggested by Duan (1994, 2000), Duan, Gauthier, and Simonato (2004) and Chou and Wang (2007), to derive the default probabilities of the two structural models. We also use Spearman rank correlation coefficients to observe whether the default probabilities derived from the simultaneous equations and maximum likelihood methods are consistent with the risk ranking orders. The results show that the different parameter estimation methods have highly correlated Spearman rank correlation coefficients for the two models, at around 90% (all with statistical significance,  $p < 0.001$ ), indicating consistent ranking of the default risks for the two structural models by the two estimation methods. No significant variance is found in the ranking of defaulting and non-defaulting firms by these two credit risk models; therefore, the use of different parameter estimation methods has no significant impact on the results of this study. We greatly appreciate the recommendation from an anonymous reviewer regarding our analysis of the results.

<sup>14</sup> The default probability estimation in the structural models requires the estimation of the value and volatility of the asset, as the two unknown variables; however, these are estimated based on stock price information. Given the upper and lower limits on stock price fluctuations in Taiwan, the use of daily stock price data cannot fully reflect stock price volatility; therefore, we use weekly stock price data to avoid any distortion, first computing equity returns based upon weekly stock price data, then calculating the weekly standard deviations of equity returns, and finally using  $\sqrt{52}$  to convert them into annual standard deviations.

<sup>15</sup> According to Bohn (1999), hundreds of firms were observed by KMV, from which the asset values at the time of defaults were largely found to be between current liabilities and total liabilities (both expressed in terms of book value). Therefore, the use of the reference that the asset value is lower than the total liability value as the default point, may not correctly measure default probability. KMV therefore first calculated the default distance denoted by the number of standard deviations between the distribution of the asset value and the default points, prior to the computation of the expected default probability. Their empirical study concluded that the default points were approximately equal to current liabilities plus half of the long-term liabilities.

Generally speaking, risk-free interest rates are defined as the rates for treasury bills or time deposits with minimum default risks. As regards the calculation of credit risk in the present study, the risk-free rate,  $r$ , refers to the one-year time deposit rate offered by the Bank of Taiwan at the cut-off point. In order to analyze whether different prediction periods have diverse influences on the discriminant capability of the models, we compute the financial and market information data for the most recent two-year period prior to the default points, and then derive separate default probabilities for the one-year and two-year-ahead periods. In other words, the one-year and two-year periods which are used as the measure terms,  $\tau$ , respectively represent the short- and long-term default prediction periods.

As regards the default probabilities of the DOC model in Equation (10), in addition to the five variables noted above, it is also necessary to estimate the barrier level,  $B$ , which is set up in this study by considering the debt recovery rate and the changing values over time. The recovery rate of the barrier level ( $\alpha$ ) is based upon prior study within the literature, with Tsai and Shen (2003) having dealt with this issue in Taiwan.<sup>16</sup> All of our listed firms are classified into the four categories of industries, comprising of: (i) the electronics industry; (ii) the construction-related industries, including cement, steel and construction; (iii) general manufacturing-related industries, including food, plastics, textiles, machinery, household appliances, chemicals, glass and ceramics, paper and pulp, rubber and automobiles; and (iv) 'other' industries, including marine transportation, tourism, retail and department stores, conglomerates, and others. We define the different recovery rates for each different industry in Taiwan (with the exception of our exclusion of the finance and insurance industry).

Following the calculation of the barrier levels for the different periods, it is possible to derive the asset value ( $V$ ) and asset volatility ( $\sigma_v$ ) using simultaneous Equations (6) and (7), prior to the estimation of the default probability for the

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<sup>16</sup> To the best of our knowledge, the study of Tsai and Shen (2003) represents the only study to have been carried out in Taiwan on the recovery rate of bad debts. We define the recovery rates for the four categories of industries by referring to the rates used in their study, as follows: electronics firms, 64.11 per cent; construction-related firms, 49.38 per cent; general manufacturing-related firms, 35.83 per cent; others 41.45 per cent.

DOC model using Equation (10). We also examine whether the predictive ability of the DOC model constructed in this study is superior to that of the traditional BSM model, observing the level of inflection in the ROC and CAP curves to determine the quality of the related models.

It can be extremely difficult to distinguish between the curves of individual models unless there are significant differences in quality, which can ultimately lead to erroneous judgement; we therefore use the absolute values, AUC and AR, to compare the strength of the predictive ability of the two structural models. In order to validate the overall performance of these models, we use three methods for the estimation of predictive ability in our empirical analysis, calculating the AUC, AR and KS values of the BSM and DOC models for the two forecast years. Finally, the Tobit regression, which is undertaken in the second stage, comprises mainly of a factor analysis on the predictive ability difference of the two structural models.

We refer to the default probabilities derived from the structural models in the first stage as the dependent variables of the regression model, and use the financial and market information variables as the explanatory variables.<sup>17</sup> In order to observe whether the explanatory power of the independent variables remains robust, we also use six control variables to carry out subsequent robustness analysis of the models.<sup>18</sup> In assessing the predictive capabilities of the two structural models, given that this study refers to the financial and market information variables one and two years ahead of the default time for the estimation of the default probability, we also use the annual financial indicators in the Tobit regression analysis.

#### **4. Empirical Analysis and Results**

Our empirical study comprises of two parts, the first of which involves the analysis of the predictive ability of the BSM and DOC structural models. We estimate the probability of firms defaulting using the two models, and then use three validation methods to assess their respective predictive capability. In addition to providing an

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<sup>17</sup> As noted earlier, a total of ten explanatory variables are selected.

<sup>18</sup> A description of the variables is provided in Section 2.

explanation of the results on the assessment of predictive performance, we also compare their performance on data for different forecast periods. In the second part of our analysis, we provide an explanation of the reasons for the variances in the predictive capabilities of the two models through the variables relating to defaults by our sample firms.

#### **4.1 Comparison of the Predictive Ability of the Models**

Since the purpose of credit risk models is to predict the future possibility of firms defaulting, we examine the predictive capabilities of the BSM and DOC models on our sample of firms during the two-year period prior to defaults by referring to their AUC, AR and KS values, so as to determine whether the models can accurately predict these actual defaults. A comparison of the variances in the predictive ability of the two structural models for the different forecast periods is provided in Table 2 (Table 3) for one-year- (two-year-) ahead default predictions. We find that the closer the default time, the better the discriminant capability of the two structural models in detecting default risk, and that the emphasis in the structural models is on real-time detection.

Generally speaking, the AUC values of the BSM and DOC models both exceed 0.7, whilst the AR values are above 0.5 and the KS values exceed 0.4; these values show that the two structural models have a certain degree of default prediction ability.<sup>19</sup> The use of historical data, stock price information or financial statements that are too remote from the debt maturity may undermine the predictive ability of the models because such data lack timeliness. The main advantage of the structural models is their ability to assess real-time stock market price data to detect the changes in the default risk of firms; thus, the above results demonstrate that structural models are more suitable for the measurement of default risk over shorter horizons.

According to the assumption of the traditional BSM model, defaults only

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<sup>19</sup> Hosmer and Lemeshow (2000) note that in the relationship between the AUC value and the discriminant capability of the models, good discriminant capability is demonstrated by the models when the AUC value ranges between 0.7 and 0.8. Furthermore, according to Engelmann, Hayden, and Tasche (2003), AUC and AR have a linear conversion relationship; that is,  $AR = 2AUC - 1$ , with the model being regarded as having good discriminant capability when AR ranges between 0.4 and 0.6.

occur at debt maturity. However, in the real world, many firms will often default long before the maturity of the debt; thus, the barrier option model should be more consistent with the actual process of defaulting. By relaxing the assumption of the BSM model on the setting of the default point, it is possible to observe defaults prior to the maturity of the debt. Using the DOC model, which treats company equity as a down-and-out call option on the firm's assets with a strike price which is equal to the face value of the debt, it is possible to derive a more general outcome.

In order to determine whether this general model is more effective than the BSM model in terms of detecting defaults, using the same full samples, we compare the predictive capabilities of the two models with the predictive capability values listed in Tables 2 and 3. As the tables show, the DOC model has higher AUC, AR and KS values than the BSM model; thus, our study shows that the general DOC model has superior predictive ability to that of the traditional BSM model, a result which may be attributable to the fact that the structural models reflect the short-term situations in firms based upon real-time stock price data.

The BSM model assumes that the default time is at debt maturity, which eliminates any possibility of early defaults; as a result, there can be no early detection of companies with problems. In contrast, the DOC model considers the default situation in the real world, allowing defaults to occur prior to debt maturity. Thus, in cases where there is a discernible excessive fall in the asset value of a firm over a certain period of time, there may be a greater likelihood of the firm defaulting prior to the maturity of the debt; hence, the DOC model exhibits superior predictive ability to that of the BSM model.

To summarize, the structural models use stock price data to predict firm defaults. The closer the default time, the greater the information that is factored into the stock price data; therefore, the structural models are better suited to short-term forecasting. Although the BSM model also exhibits good predictive ability, it remains necessary to strive to improve predictive accuracy, essentially because firm defaults are detrimental to society as a whole, and any improvement in predictive accuracy can reduce such losses to society. The improved predictive ability of the DOC model constructed in this study could serve as the foundation

for ex-ante prevention of firm defaults by reducing the potential for huge losses resulting from such defaults.

**Table 2**  
**Comparative Performance of the Two Structural Models for**  
**One-Year Ahead Default Predictions**

	BSM	DOC
AUC	0.7534	0.7799
AR	0.5069	0.5597
KS	0.4424	0.4591

**Table 3**  
**Comparative Performance of the Two Structural Models for**  
**Two-Year Ahead Default Predictions**

	BSM	DOC
AUC	0.6581	0.6692
AR	0.3162	0.3384
KS	0.3021	0.3118

#### **4.2 Differences in the Predictive Ability of the Models**

According to our previous analyses, the DOC model constructed in this study has superior predictive ability to that of the traditional BSM model. In order to gain a clear understanding of whether the variations in the predictive ability of these two structural models are statistically significant, we go on to apply a paired sample test. As the results show, with regard to the measurement of default risk, the variations between the two models are statistically significant at the 5% significance level.

We also carry out a Tobit regression, exploring the factors relevant to firm defaults and their influence on such defaults, so as to observe the variations in the



predictive ability of the two models.<sup>20</sup> Prior to our analysis of the variations in the performance of the models, we first conduct a multicollinearity test on the explanatory variables used in the Tobit regression model. In accordance with the variance inflation factor (VIF) test, any variable with a VIF value of greater than 10 is gradually deleted until all of the VIF values in the model are below 10. After deleting all of the variables within the model with serious multicollinearity, eight explanatory variables are selected; these are: *CACL*, *WCTA*, *CLTA*, *EBITOR*, *SCAI*, *MVETL*, *LLTA* and *VOL*. We then carry out the Tobit regression analysis on these variables, with the results forming the input for the first regression equation in Tables 4 and 5.

*CLTA* has statistical significance at the 1% level (with a positive coefficient) in both the BSM and DOC models in the first regression equation, which indicates that firms with greater current liabilities have a higher probability of defaulting. *MVETL* has statistical significance at the 1% level (with a negative coefficient) in both the BSM and DOC models, which indicates that firms with greater owned capital have better levels of protection for creditors' rights; thus the probability of defaulting is lower.

*VOL* also has statistical significance at the 1% level (with a positive coefficient) in both the BSM and DOC models, which indicates that greater stock price volatility will lead to greater uncertainty in investors' expectations of the firm's performance in future, thus the probability of defaulting will be higher. *LLTA* has statistical significance at the 5% level (with a positive coefficient) but only in the BSM model. Since *LLTA* in the DOC model does not have statistical significance, this indicates that the BSM model performs better than the DOC model in the measurement of non-current liabilities. If a firm has a higher ratio of long-term liabilities to total assets, this indicates that the capital structure of the firm is unstable, thus the probability of default will be higher.

*EBITOR* has statistical significance at the 1% level (with a negative

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<sup>20</sup> Referring to the previous section, defaults can be determined based upon the predictive ability of the two structural models at one- and two-year-ahead periods, with the performance of the former proving to be superior; thus, we analyze the differences in the predictive ability of the models for one-year-ahead default probability, and the financial and market information variables for one-year prior to the default.

coefficient) but only in the DOC model. Since *EBITOR* in the BSM model does not have statistical significance, this indicates that the DOC model takes into account the profit-making characteristics of the firm when predicting its default risk. In other words, when the net profit margin is higher, the contribution to profit is higher for every dollar of sales; thus, the profit capability of the firm is higher and its default probability will therefore be lower.

To summarize, when constructing the BSM model, only the liquidity variable (*CLTA*), solvency variables (*MVETL* and *LLTA*) and market information variable (*VOL*) are considered; when constructing the DOC model, in addition to considering the *CLTA*, *MVETL* and *VOL* variables, the profitability index (*EBITOR*) is also considered. Based upon the differential analysis of the predictive ability of the BSM and DOC structural models examined in this study, along with the use of the Tobit regression model, we demonstrate that the variable constructs affecting the BSM model include the solvency variables and the liquidity variable in the financial statements construct, as well as the market information variable in the stock price construct. The variable constructs affecting the DOC model include the solvency, liquidity and profitability variables in the financial statements construct, and the market information variable in the stock price construct.

Thus, during the construction of the DOC model, this study considers not only solvency factors, such as the matured debts and liquidity debts, and market information factors, such as stock price volatility, but also the probability of default prior to the maturity of the debts arising from a fall in profitability; hence, the DOC model may well demonstrate improvements on the shortcomings of the traditional BSM model, which considers only the solvency and market information factors.

As compared to the traditional BSM model, the DOC model constructed in this study could help in the timely identification of potential default situations within firms resulting from a reduction in profitability; thus, it might have superior default prediction capabilities to that of the traditional BSM model. Not only does the DOC model offer the possibility of real-time detection over the traditional BSM model, but it also considers a wide range of financial situations.

The selection of the explanatory variables in the abovementioned Tobit

regression equations is based upon the variables used in the literature on default prediction, as well as the analysis of the eight basic variables selected through the VIF process. Following on from our previous analyses, we adopt a robustness analysis to observe whether the explanatory power of the basic variables remains robust, incorporating the liquidity variable *CASHTA*, operating capability variable *ORAF*, solvency variable *ETA*, scale variable *SIZE* and corporate governance variable *CG*, along with the non-financial *AGE* variable as the control variables. In order to observe whether the factors influencing default risk in the two structural models remain stable, these control variables are included in the Tobit regression equation constructed using the abovementioned basic variables. The second to the seventh regression equations in Tables 4 and 5 include the control variables in the regression results of the BSM and DOC models.

According to the results of these regression equations, current liabilities/total assets (*CLTA*), market value of equity/total liabilities (*MVETL*), long-term liability ratio (*LLTA*) and stock price volatility (*VOL*) are the variables respectively representing liquidity, solvency and marketability in the BSM model; these variables are found to have important explanatory power on the default risks estimated by the BSM model. Current liabilities/total assets (*CLTA*), net profit margin (*EBITOR*), market value of equity/total liabilities (*MVETL*) and stock price volatility (*VOL*) are the variables respectively representing liquidity, profitability, solvency and marketability in the DOC model; these variables are also found to have important explanatory power on the default risks estimated by the DOC model.

In other words, following the inclusion of the control variables, the significance levels of the variables of the two structural models are found to be largely consistent with the regression results of the equations containing only the basic variables in the first regression equation in Tables 4 and 5. Therefore, based upon the second-stage analysis undertaken in this paper, since the DOC model takes into account a wider range of factors than the BSM model within the model construction process, we could conclude that the DOC model has superior predictive ability to that of the BSM model.

## 5. Conclusions

To the economic system as a whole, it is important to have advance warnings of firm defaults; however, since the traditional market-based BSM structural model assumes that defaults occur only at the maturity of the debt, thereby oversimplifying the process of defaulting, there is a discrepancy between this model and the actual process of defaulting in the real world. Accordingly, in this study, we use barrier option theory to consider a default warning model that may better reflect the process of defaulting within the economic system. Our results show that as compared to the traditional BSM structural model, which is based upon stock market prices, the DOC model constructed in this study has improved default warning capability. Thus, we propose the application of the DOC model to the measurement of default probability, under the internal rating-based approaches of Basel II Accord, to establish a firm credit risk quantification index.

This study not only evaluates the predictive ability of the two structural models, but also uses the censored Tobit regression model to examine the related factors expressed by the two models. The results reveal that the traditional BSM model considers only firm solvency and market information factors, whereas the DOC model places greater emphasis on profitability factors, thereby providing potentially better forecasting of defaults. In the advance detection of defaulting firms, the BSM model considers only defaults resulting from debts not to be repaid on the maturity date; this method of risk assessment is too conservative and has predictive errors. In contrast, with the DOC model established in this study serving as a market-based BSM model, it represents an effective default warning tool for determining default risk.

As discussed in the preceding sections, this study presents important results that are worthy of further study. In terms of the research methodology, our relaxation of the settings of the BSM model provides the potential for the early detection of defaulting firms under safety covenants by setting up an exogenous barrier level that changes with time. Future studies could further discuss whether there is an endogenous barrier level in the duration period of the debt, and compare the results with those reported in this study. In the construction of the default

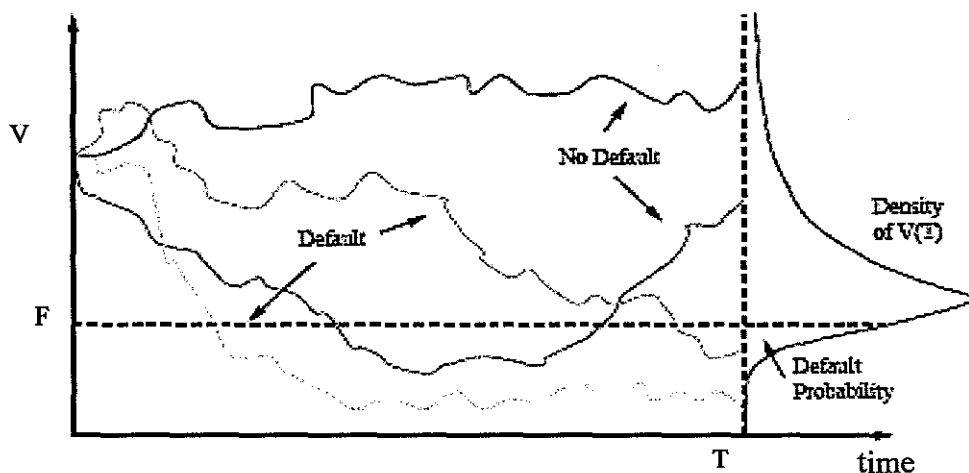
warning model, this study demonstrates that the DOC model has better default predictive ability; thus, future studies should consider the construction of a default warning model based on the significant financial and market information variables of the Tobit regression model, as in the second stage of our study, and compare the results with those of the DOC and BSM models reported here.

## 6. Appendix: Default Figures

Appendix Figures A-1 to A-3 are taken from Giesecke (2004), whilst Appendix Figure A-4 is the DOC model established in this study. Figure A-1 is the case of the BSM model, whilst Figures A-2 and A-3 are the cases of the DOC model with a constant default barrier value, where  $M_t = \min_{s \leq t} V_s$  is the historical low of the asset value of the firm.

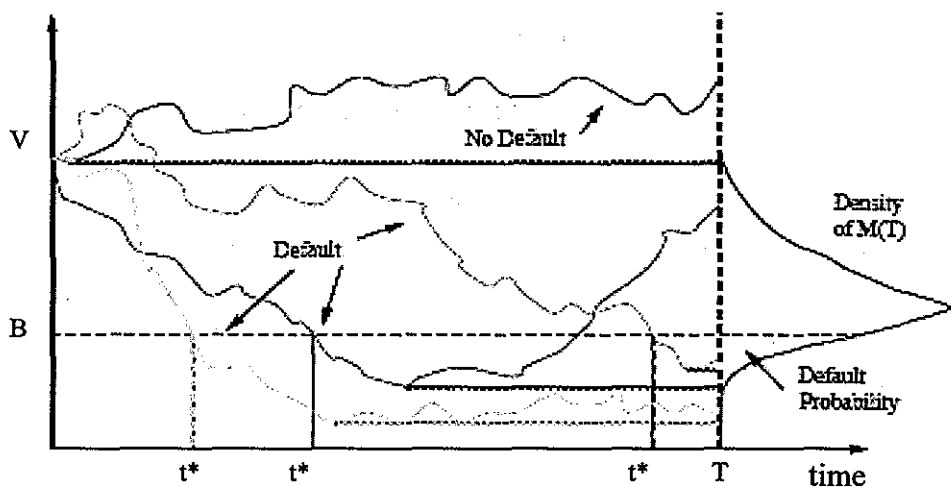
Figure A-1

### The BSM Model : Default Only at Debt Maturity



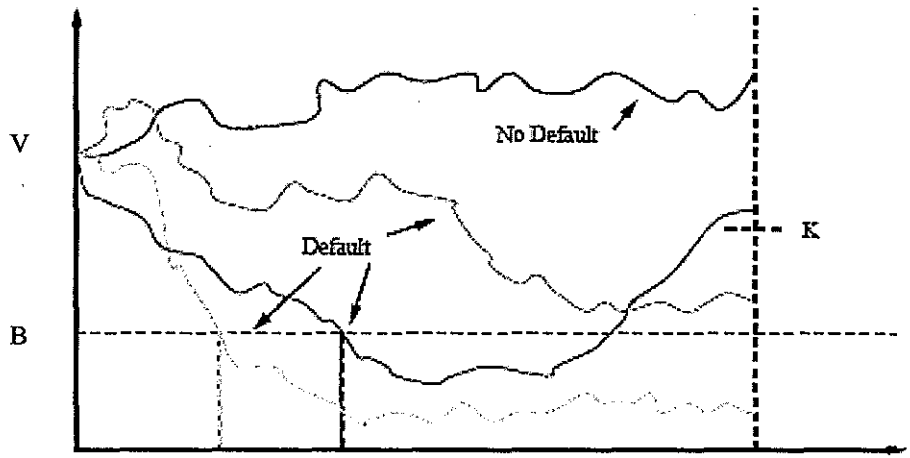
**Figure A-2**

**The DOC Model (Where  $B > F$ ): Default Only Prior to Debt Maturity**



**Figure A-3**

**The DOC Model (Where  $B < F$ ):  
Default During the Term of the Debt Maturity**



**Table 4**  
**Tobit Regression Results on the Robustness of the BSM Model**

Constructs	Categories	Variable Codes	Models								
			1	2	3	4	5	6	7		
Financial Statements	Liquidity	Constant	-0.12075*** (-8.83)	-0.1197*** (-8.79)	-0.1193*** (-8.68)	-0.1218*** (-8.75)	-0.0509 (-1.15)	-0.1201*** (-8.76)	-0.1282*** (-6.99)		
		<i>CACL</i>	0.0020 (0.95)	0.0023 (1.14)	0.0019 (0.88)	0.0022 (1.01)	0.0017 (0.82)	0.0019 (0.92)	0.0018 (0.87)		
		<i>WCTA</i>	0.0113 (0.57)	0.0221 (1.07)	0.0111 (0.57)	0.0110 (0.56)	0.0083 (0.42)	0.0205 (1.01)	0.0156 (0.75)		
		<i>CLTA</i>	0.1545*** (6.47)	0.1582*** (6.62)	0.1487*** (6.08)	0.1571*** (6.35)	0.1545*** (6.47)	0.1516*** (6.3)	0.1589*** (6.37)		
		<i>EBITOR</i>	-0.0001 (-0.2)	-0.0001 (-0.15)	-0.0002 (-0.21)	-0.0001 (-0.20)	-0.0002 (-0.23)	-0.00005 (-0.07)	-0.0001 (-0.17)		
		<i>SCAI</i>	0.000017 (0.57)	0.00002 (0.66)	0.00001 (0.45)	0.00002 (0.65)	0.00002 (0.66)	0.00002 (0.55)	0.00002 (0.61)		
		<i>MVETL</i>	-0.0038*** (-3.1)	-0.0036*** (-2.93)	-0.0038*** (-3.11)	-0.0038*** (-3.09)	-0.0036*** (-2.92)	-0.0040*** (-3.26)	-0.0036*** (-2.88)		
		<i>LLTA</i>	0.0721** (2.03)	0.0759** (2.14)	0.0695** (1.95)	0.0726** (2.04)	0.0862** (2.36)	0.0537 (1.49)	0.0772** (2.12)		
		Stock Price	Market Information	<i>VOL</i>	0.2016*** (10.01)	0.2000*** (9.95)	0.2023*** (10.05)	0.2018*** (10.02)	0.2009*** (9.99)	0.1986*** (9.71)	0.2026*** (10.03)
				Liquidity		<i>CASHTA</i>					
Control Variables	Operating Capability	<i>ORAF A</i>			0.00004 (1.1)						
		Solvency				-0.00006 (-0.41)					
	Size	<i>SIZE</i>					-0.0103 (-1.65)				
		Corporate Governance	<i>CG</i>					0.0121 (1.01)			
	Non-financial Variable	<i>AGE</i>							0.0002 (0.61)		
		Sigma	0.0739*** (34.98)	0.0738*** (34.97)	0.0739*** (34.98)	0.0739*** (34.98)	0.0738*** (34.98)	0.0735*** (34.78)	0.0739*** (34.98)		

Notes: Figures in parentheses refer to t-values. \* indicates significance at the 10% level; \*\* indicates significance at the 5% level; and \*\*\* indicates significance at the 1% level.

**Table 5**  
**Tobit Regression Results on the Robustness of the DOC Model**

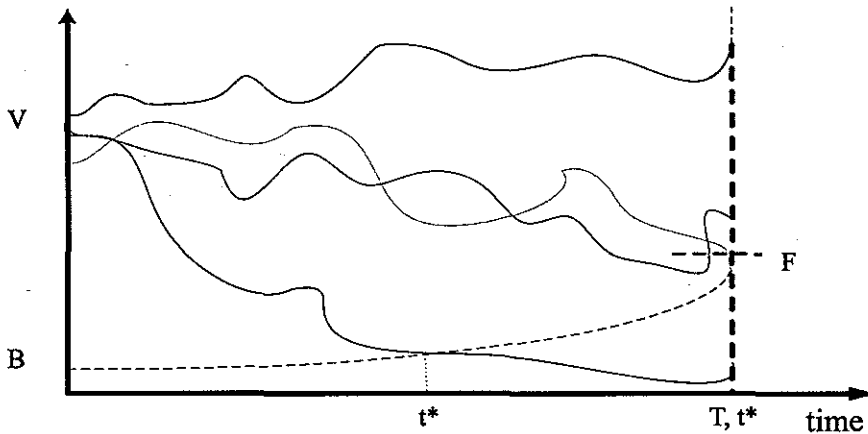
Constructs	Categories	Variable Codes	Models								
			1	2	3	4	5	6	7		
Financial Statements		Constant	-0.0617*** (-14.19)	-0.0616*** (-14.18)	-0.0616*** (-14.09)	-0.0620*** (-14.04)	-0.0332** (-2.37)	-0.0608*** (-13.88)	-0.0671*** (-11.54)		
		<i>CACL</i>	0.0003 (0.48)	0.0004 (0.53)	0.0003 (0.46)	0.0004 (0.54)	0.0002 (0.31)	0.0003 (0.47)	0.0002 (0.30)		
		Liquidity	<i>WCTA</i>	0.0040 (0.64)	0.0050 (0.76)	0.0040 (0.64)	0.0039 (0.63)	0.0027 (0.44)	0.00322 (0.49)	0.0071 (1.07)	
			<i>CLTA</i>	0.0203*** (2.68)	0.0207*** (2.73)	0.0199*** (2.57)	0.0211*** (2.69)	0.0205*** (2.72)	0.0194*** (2.53)	0.0236*** (2.99)	
		Profitability	<i>EBITOR</i>	-0.0006*** (-2.80)	-0.0006*** (-2.78)	-0.0006*** (-2.80)	-0.0006*** (-2.80)	-0.0006*** (-2.85)	-0.0006*** (-2.86)	-0.0006*** (-2.74)	
			<i>SCAI</i>	0.00001 (1.09)	0.00001 (1.12)	0.00001 (1.05)	0.00001 (1.15)	0.00001 (1.21)	0.00001 (1.08)	0.00001 (1.17)	
		Solvency	<i>MVETL</i>	-0.0013*** (-3.11)	-0.0013*** (-3.04)	-0.0013*** (-3.11)	-0.0013*** (-3.09)	-0.0012*** (-2.86)	-0.0013*** (-3.12)	-0.0012*** (-2.73)	
			<i>LLTA</i>	0.0057 (0.51)	0.0061 (0.55)	0.0055 (0.49)	0.0059 (0.53)	0.0117 (1.01)	0.0051 (0.45)	0.0095 (0.82)	
		Stock Price	Market Information	<i>VOL</i>	0.1404*** (22.07)	0.1403*** (22.03)	0.1405*** (22.07)	0.1405*** (22.07)	0.1401*** (22.10)	0.1399*** (21.53)	0.1411*** (22.15)
		Control Variables	Liquidity	<i>CASHTA</i>		-0.0070 (-0.49)					
Operating Capability	<i>ORAF A</i>				0.000003 (0.25)						
Solvency	<i>ETA</i>					-0.00002 (-0.39)					
Size	<i>SIZE</i>						-0.0042** (-2.14)				
Corporate Governance	<i>CG</i>							-0.0017 (-0.46)			
	Non-financial Variable	<i>AGE</i>						0.0001 (1.38)			
		Sigma	0.0230*** (35.15)	0.0230*** (35.14)	0.0230*** (35.15)	0.0230*** (35.15)	0.0229*** (35.14)	0.0230*** (34.95)	0.0230*** (35.14)		

Notes: Figures in parentheses refer to t-values. \* indicates significance at the 10% level; \*\* indicates significance at the 5% level; and \*\*\* indicates significance at the 1% level.



Figure A-4

## The DOC Model Established in this Study



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