Chiao Da Management Review
Vol. 30 No. 1, 2010
pp . 165-206

不完全情報下同質產品最適訂價與生產策略之研究－以雙占廠商為例

The Optimal Pricing and Production Strategy for Homogeneous Product under Incomplete Information and Duopoly

黃允成 Yun-Cheng Huang
國立屏東科技大學 工業管理系
Department of Industrial Management, National Ping-Tung University of Science and Technology
詹家豪[1] Chia-Hao Chan
國立屏東科技大學 工業管理研究所
Institute of Industrial Management, National Ping-Tung University of Science and Technology
鄭文睿 Wen-Ruei Jeng
國立屏東科技大學 工業管理研究所
Institute of Industrial Management, National Ping-Tung University of Science and Technology

摘要：以現今的競爭市場來說，廠商間生產成本皆具有某種程度的差異，而此差異將影響產品間之售價，亦會影響管理者如何在此種環境中，作出最適的訂價與生產策略。然而，廠商在訂定產品的售價時必須同時考量競爭者的生產成本及產品價格後再決定最適價格與生產策略，而此價格則是最直接影響消費者選擇的重要因素。另外，早期之研究僅針對產品最適價格與數量進行探討，並未更貼進市場實際情況，實際的競爭市場中，競爭者成本結構資訊是不透明的，廠商需藉由猜測對手成本結構及評估自身總期望利潤後決定最適價格與生產策略。因此，本研究提出雙占市場，考量競爭者成本結構資

[1] Corresponding author: Institute of Industrial Management, National Ping-Tung University of Science and Technology, Pingtung City, Taiwan. E-mail: a729063@yahoo.com.tw

訊不完全下，評估投入資訊成本對總期望利潤之影響，探討雙占廠商生產同質產品最適訂價與生產策略，並提出數值範例及針對參數進行敏感度分析。最後，根據模式推導與數值結果，提出七點具體結論，包括：(1)經由最佳化與統計期望理論，建構出使總期望利潤最大化之最適策略(2)在資訊未知情況下，為降低市場風險，進而投入經費去蒐集競爭對手的相關資訊，建構出使自身總期望利潤最大化之最適價格策略(3)雙方廠商進行 Cournot 數量競爭時，利用兩廠商之數量反應函數，得到雙方均衡數量與價格(4)當廠商自發性採取價格競爭策略，雙方則會進行價格競爭，最終由成本優勢之廠商獨占市場(5)當自身為中成本時，若第一期採行降價策略且對手跟進之條件下往後各期則以恢復均衡價格為最適決策(6)前一期以維持均衡為前提下，往後各期皆以維持均衡為最適選擇(7)廠商價格係數與單位生產成本增加時，會導致廠商利潤下降。

關鍵詞：Cournot 模式；雙占市場；同質產品；不完全資訊

Abstract：In the competitive markets, for varied firms, there are some differences in production cost under different production structure. The differences not only affect price but also production strategy. Firms can set the optimal price and propose suitable production strategy after considering both production cost and price from opponents. In addition, early studies only focus on optimum price and quantity, but not concern real situation of market. In practice, cost structure of competitor is not clear. Firm required to conjecture cost structure of competitor and evaluated their own expected profit, then determine the most appropriate price and production strategies. Therefore, in this study we evaluate the effect about of information cost affecting the expected total profit under incomplete information and duopoly. Production strategies are proposed for homogeneous product. Numerical example and sensitivity analysis of parameters are also taken. Finally, according to the analysis and results, seven concrete conclusions are presented.

Keywords: Cournot model；Duopoly；Homogeneous product；Incomplete information

1. Introduction

In the market of modern society, some products are dominated by a few firms in oligopoly market-oriented ideology. In oligopoly market, there are a few of firms, but each firm has ability to monopolize the market. However, based on common interests, if each of them in the market cooperates to avoid price competition, then they can gain maximized profits, so it seems to be a good solution for them. Despite of cooperating with each others, but each firm is still production independently, and pursue their maximum profit. So cooperative behavior in oligopoly market whether they can reach a binding agreement or not is a problem. Therefore, in most of time, the main market type is the non-cooperative oligopoly. In this study, we explored the non-cooperative oligopoly behavior and find out the optimal production quantity and price for their firms to maximize their total expected profits.

In actual situation of competitive market, firms must assess the competitor's strategies before making decision, but facing the unknown cost of competitor, firms in order to reduce market risk, and then they invest money to collect the competitor's information and consider their own total expected profit to determine the optimal pricing and production strategy. Finally, the firms need to assess the cost of collecting information whether it is worth or not.

Synthesize the description as above, this study will set the price as a decision variable and explore the optimal pricing and production strategies under homogeneous products and incomplete information of competitor's cost. This paper is organized as follows. Section 2 explores the relevant literatures; Section 3 constructs the mathematical models; Section 4 takes an example to verify the availability and the sensitivity analysis are also taken. Finally, section 5 gives seven brief conclusions.

2. Literature Review

This section is divided into three parts: First, explore the relevant literature of homogeneous duopoly; Second, summary the related papers about price competition and incomplete information; third, give the section a conclusion.

2.1 Homogeneous Duopoly

Quantitative analysis of oligopoly market by Cournot in 1838. The main underlying assumptions are: (1) market demand is a linear function (2) only two firms in the market (3)the product is homogeneous (4) the cost of production is zero (5) the firms determine their own production volume simultaneously, but assume naively that when we adjust the production quantity, the opponent do not change its quantity.

The oligopoly market means that the number of firms is few, but there are a lot of buyers and the firms are in the competition situation. Oligopoly market has the following characteristics: (1) several firms produce all or most of products in the market (2) firms produced homogeneous or heterogeneous products and the buyers are the price receivers (3) based on technologies and patents, the entrance barrier is pretty high (4) firm has strategic behavior, its profit is influenced by competitor (Chang and Wu, 2000). Key feature of oligopoly firms is their mutual interdependence, in order to highlight the dependency between the firms; many scholars assumed that only two firms in the market. Cabral (2000) studied the duopoly firms produce the homogeneous products, the profit come from the R&D cooperation and non-cooperative production, and the success of R & D is uncertainty. Lin *et al.* (2005) explored the duopoly market and used the fuzzy theory in airlines companies, without considering the risk attitude of decision makers. They assessed their own fixed costs and unit variable costs, and finally find out the optimal strategy under the consideration of competitor's production quantities.

2.2 Incomplete Information

In dynamic game under incomplete information, the second-move firms can observe the first-move firm's behavior, and first-move firm knows all about it. Lofaro (2002) compare Cournot and Bertrand models under incomplete information, in the Bertrand model, although the firm's prior expected profit higher than the actual profit, but in the social welfare, Bertrand model is still better than the Cournot model. Wen and David (2001) used Cournot game theory to research oligopoly electricity market, under incomplete information of market, in different ways for providers to assess their profit functions to search for the market equilibrium price.

2.3 Price Competition

Each firm must pursuit optimal output and prices so that maximize their profits. It seems intuitively to set price as high as possible, but in fact may not be able to achieve, because firms choose their price strategy, also need to consider the market state. Under market information is not completely transparent, the firms do not understand the cost structure each others, but price-reducing launched by one firm, its purpose is to force the competitors leave the market, and then maximize its own profit. In pricing behavior, the price-cutting will stimulate customers to buy and price-raising will lose most of the market share, facing the threats by other competitors, the advantageous firm will reduce the price, to force other firms out of the market. In price competition, Singh (1984) compared the Bertrand and the Cournot model about the substitute products and complementary products, the results showed that if product is a substitute, the Cournot profit will be higher than the Bertrand; if the product is a complementary, the Bertrand profit will be higher than Cournot model. Anderson and Leruth [5] propose a portfolio pricing of goods, when the firms carry out a game in one period, two firms adopt a mixed portfolio pricing, when the firms carry out a game in two periods, and the equilibrium is to set individual price each other.

2.4 Summarize

Synthesize the description as above, most scholars assumed that the market information is complete, and they studied the competitive strategies under homogeneous, heterogeneous or partial homogeneous products. But in practical, the cost information of competitor is unknown. In order to reduce the market risk, firm invests money to collect the cost information of competitor, and assess its expected profit to determine the optimal price and production strategy. In this study, we study the optimal pricing and production strategy under duopoly, homogeneous product and incomplete information.

3. Model Construction

In this study, we assumed the duopoly firms produce the homogeneous products, they faced with the incomplete information about the competitor's cost structure. We used the Cournot model to derive the equilibrium price and production quantity under various cost structure combinations. And then based on whether the price competition was taken or not to search for the optimal price to maximize the total expected profit. In duopoly market, one firm's profit is influenced by competitor's pricing strategy. In this study, we assumed that duopoly firms produce homogeneous products, and the demand function is known. The total expected profit model is constructed to determine the optimal price. When one firm take price-reducing strategy, the equilibrium condition was broken, another firm also needs to adopt an optimal pricing strategy to maximize its total expected profit.

3.1 The Basic Assumptions

In this model, the basic assumptions are as follows:

1. The firms produce the homogeneous product.
2. The market demand function is known and linear.
3. The product's life cycle is three periods; each length of period depends on each

industry.

4. Under duopoly market, any firm can launch price-reducing strategy (equivalent to launch a price war), but need to consider the reaction of opponent and its impact on oneself.
5. When one firm starts a price-reducing strategy, the other firms need to consider whether to follow or withdraw from the existing market or remain unchanged. But under the assumption of homogeneity, price remaining unchanged will lose most of the customers. Therefore, remaining unchanged strategy is not taken into considerations. In general, a firm withdrew from the market will be not able to return again in a short term, the cost of withdrawn firm is assumed to be zero in future periods.
6. The choice of consumer only considers the price of product and do not consider the brand loyalty.
7. The two firms only have three possible cost structures, according to different types of production, the cost structures can be divided into high-capital-intensive, middle-capital-intensive and low-capital-intensive.
8. There are two pricing strategies: maintaining equilibrium strategy, and price-reducing strategy.
9. The firm's cost structure will remain unchanged in a short term.

3.2 Notations Definition

Q : Overall market demand quantity for the homogeneous product. Demand function : $Q=(q_i+q_j)=a-\alpha P$, q_i, q_j is the production quantity of firm 1 and firm 2, respectively.

a : Intercept of the demand function, it means the potential market size, that is, when the price is 0, the total market demand is "a"

α : The price coefficient of homogeneous product, That is, when the price changes one unit, the change amounts of market demand is "α"

$Q_{lr_i r_j fk}$: l Firms themselves, firm 1's cost is r_i, firm 2's cost is r_j, f guest the cost structure of competitor, $k=e,b,s$, e means monopolization, b means equilibrium, s means price-reducing.

$p_{lr_ir_j \bullet k}$: In k condition, firm 1 with r_i cost, firm 2 with r_j cost, firms l's market price. $l=1,2$, $k=e,b$, $r_i=h,m,l$, $r_j=h,m,l$, e means monopoly, b means equilibrium, h means high cost, m means middle cost, l means low cost.

$q_{lr_ir_j \bullet}$: Firm 1 with r_i cost, firm 2 with r_j cost, firms l's production quantity $l=1,2$, $r_i=h,m,l$, $r_j=h,m,l$

$q_{1r_i \bullet f}$: Firm 1 with r_i cost speculate the production quantity of firm 2 with f cost $r_i=h,m,l$, $f=h,m,l$

$q_{2 \bullet r_j f}$: Firm 2 with r_j cost speculate the production quantity of firm 1 with f cost $r_j=h,m,l$, $f=h,m,l$

$\Delta \mathrm{I} \pi$: Total expected profit of information

D : Cost of collecting partial information

δ^n : Discount factor in n Period。$0<\delta<1$，$\delta=1/(1+\theta)$，θ for compare-based interest rate

χ_n : The possible cost structure in period n

$s_{1 \bullet \bullet f}$: The price after the price-cutting under firm 1speculate the cost of firm 2 with f $f=h,m,l$

$s_{2 \bullet \bullet f}$: The price after the price-cutting under firm 2 speculate the cost of firm 1 with f $f=h,m,l$

$c_{lr_i \bullet \bullet}$: The actual variable cost of Firm 1 is r_i. $r_i=h,m,l$

$c_{2 \bullet r_j \bullet}$: The actual variable cost of Firm 2 is r_j. $r_j=h,m,l$

$c_{l \bullet \bullet f}$: Firm 1 speculate firm 2's variable cost is f. $f=h,m,l$

$c_{2 \bullet \bullet f}$: Firms 2 speculate firm 1's variable cost is f. $f=h,m,l$

F_1 : Firm 1's fixed cost

F_2 : Firm 2's fixed cost

$\pi_{\bullet lr_ir_j \bullet \bullet}$: The profit of firm l under firm 1 with r_i cost and firm 2 with r_j cost. $l=1,2$，$r_i=h,m,l$，$r_i=h,m,l$

$N\pi_{n1r_i\bullet fk}$: The profit of firm 1 under firm 1 with r_i cost and speculate firm 2 with f cost in k condition and no information. $n=1,2,3$, $r_i=h,m,l$, $k=e,b,s$, e means monopoly , b means equilibrium, s means price-reducing

$P\pi_{nlr_ir_j\bullet k}$: The profit of firm l under firm 1 with r_i cost, firm 2 with r_j cost in k condition and partial information. $n=1,2,3$, $l=1,2$, $r_i=h,m,l$, $r_j=h,m,l$, $k=e,b,s$, e means monopoly, b means equilibrium, s means price-reducing

$P\pi_{n1r_i\bullet fk}$: The profit of firm 1 under firm 1 with r_i cost and speculate firm 2 with r_j cost in k condition and partial information.
$n=1,2,3$, $r_i=h,m,l$, $k=e,b,s$, e means monopoly, b means equilibrium, s means price-reducing

$P\pi_{n2\bullet r_jfk}$: The profit of firm 2 under firm 2 with r_i cost and speculate firm 2 with r_j cost in k condition and partial information. $n=1,2,3$, $r_i=h,m,l$, $k=e,b,s$, e means monopoly, b means equilibrium, s means price-reducing

3.3 Model Reduction

At first, we assumed that the duopoly firms in the market and firm invests money to collect the cost information of competitor, this study stands on the position of firm 1 and we applied Cournot model and the known demand function to derive the equilibrium quantity and price under homogeneous products and possible cost combinations. First, we decide whether invest the money to collect information and then modify the probability of competitor's cost structure. Finally, compute the expected profit of information, expected profit of no information and the difference between these two expected profits:

$$\Delta I\pi = P\pi_{11r_i\bullet fb}(q_{1r_i\bullet f}\big|c_{1r_i\bullet\bullet},c_{1\bullet\bullet f}) - N\pi_{11r_i\bullet fb}(q_{1r_i\bullet f}\big|c_{1r_i\bullet\bullet},c_{1\bullet\bullet f})$$
$$=\sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11r_i\bullet fb}(q_{1r_i\bullet f}\big|c_{1r_i\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \qquad (1)$$
$$-\sum_{n=1}^{3}\left[\frac{1}{9}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11r_i\bullet fb}(q_{1r_i\bullet f}\big|c_{1r_i\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})\right]\delta^n$$

If the incremental profit, $\Delta I\pi$, greater than the cost of collecting information, some of the profits under the cost of information, then the investment on collecting information is worth; otherwise, , the investment is not worth.

3.3.1 Game Model in The First Period

First, obtain the cost structure of duopoly firms of all possible combinations and find an optimal production quantity of firm 1 as follows: When firm 1 and firm 2 are high-cost structures, the profit function is

$$\pi_{\bullet 1hh\bullet\bullet} = (p_{1hh\bullet\bullet} - c_{1h\bullet\bullet})q_{1hh\bullet} - F_1 = (\frac{a - q_{1hh\bullet} - q_{2hh\bullet}}{\alpha} - c_{1h\bullet\bullet})q_{1hh\bullet} - F_1 \quad (2)$$

$$\pi_{\bullet 2hh\bullet\bullet} = (p_{2hh\bullet\bullet} - c_{2\bullet h\bullet})q_{2hh\bullet} - F_2 = (\frac{a - q_{1hh\bullet} - q_{2hh\bullet}}{\alpha} - c_{2\bullet h\bullet})q_{2hh\bullet} - F_2 \quad (3)$$

Let the derivative of profit function of firm 1 by $q_{1hh\bullet\bullet}$, we obtain

$$q_{1hh\bullet} = \frac{a - q_{2hh\bullet} - \alpha c_{1h\bullet\bullet}}{2} \quad (4)$$

By the same way, we can obtain the quantity reaction function as

$$q_{2hh\bullet} = \frac{a - q_{1hh\bullet} - \alpha c_{2\bullet h\bullet}}{2} \quad (5)$$

Solve both equations (4) and (5) simultaneously, we have the equilibrium quantities

$$q_{1hh\bullet} = \frac{a + \alpha c_{2\bullet h\bullet} - 2\alpha c_{1h\bullet\bullet}}{3} \quad (6)$$

$$q_{2hh\bullet} = \frac{a + \alpha c_{1h\bullet\bullet} - 2\alpha c_{2\bullet h\bullet}}{3} \quad (7)$$

The demand function is

$Q = (q_i + q_j) = a - \alpha P$

So,

$$Q_{\bullet hh\bullet\bullet} = (q_{1hh\bullet} + q_{2hh\bullet}) = a - \alpha P_{\bullet hh\bullet\bullet}$$

$$= \frac{a + \alpha c_{2\bullet h\bullet} - 2\alpha c_{1h\bullet\bullet}}{3} + \frac{a + \alpha c_{1h\bullet\bullet} - 2\alpha c_{2\bullet h\bullet}}{3} = a - \alpha P_{\bullet hh\bullet\bullet}$$

The optimal prices of both firms is

$$P_{\bullet hh\bullet\bullet} = \frac{a + \alpha c_{1h\bullet\bullet} + \alpha c_{2\bullet h\bullet}}{3\alpha} \tag{8}$$

Table 1

The Optimal Production Quantity and Equilibrium Price of All Possible Combinations of The Cost Structure of Duopoly Firms

	A equilibrium quantity of firm1	A equilibrium quantity of firm2	Equilibrium price of two firms
high-cost/high-cost	$\frac{a + \alpha c_{2\bullet h\bullet} - 2\alpha c_{1h\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} - 2\alpha c_{2\bullet h\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} + \alpha c_{2\bullet h\bullet}}{3\alpha}$
high-cost/ middle-cost	$\frac{a + \alpha c_{2\bullet m\bullet} - 2\alpha c_{1h\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} - 2\alpha c_{2\bullet m\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} + \alpha c_{2\bullet m\bullet}}{3\alpha}$
high-cost/low-cost	$\frac{a + \alpha c_{2\bullet l\bullet} - 2\alpha c_{1h\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} - 2\alpha c_{2\bullet l\bullet}}{3}$	$\frac{a + \alpha c_{1h\bullet\bullet} + \alpha c_{2\bullet l\bullet}}{3\alpha}$
middle-cost/high-cost	$\frac{a + \alpha c_{2\bullet h\bullet} - 2\alpha c_{1m\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} - 2\alpha c_{2\bullet h\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} + \alpha c_{2\bullet h\bullet}}{3\alpha}$
middle-cost/middle-cost	$\frac{a + \alpha c_{2\bullet m\bullet} - 2\alpha c_{1m\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} - 2\alpha c_{2\bullet m\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} + \alpha c_{2\bullet m\bullet}}{3\alpha}$
middle-cost/low-cost	$\frac{a + \alpha c_{2\bullet l\bullet} - 2\alpha c_{1m\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} - 2\alpha c_{2\bullet l\bullet}}{3}$	$\frac{a + \alpha c_{1m\bullet\bullet} + \alpha c_{2\bullet l\bullet}}{3\alpha}$
low-cost/high-cost	$\frac{a + \alpha c_{2\bullet h\bullet} - 2\alpha c_{1l\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} - 2\alpha c_{2\bullet h\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} + \alpha c_{2\bullet h\bullet}}{3\alpha}$
low-cost/middle-cost	$\frac{a + \alpha c_{2\bullet m\bullet} - 2\alpha c_{1l\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} - 2\alpha c_{2\bullet m\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} + \alpha c_{2\bullet m\bullet}}{3\alpha}$
low-cost/low-cost	$\frac{a + \alpha c_{2\bullet l\bullet} - 2\alpha c_{1l\bullet\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} - 2\alpha c_{2\bullet l\bullet}}{3}$	$\frac{a + \alpha c_{1l\bullet\bullet} + \alpha c_{2\bullet l\bullet}}{3\alpha}$

Similarly, by the same way, we can calculate the equilibrium quantities and price for firm 1 and firm 2 under other cost combination, the results list in Table 1.

Firm 1 hopes using the price-cutting strategy to force firm 2 out of the market, and then occupy the whole market share to achieve the purpose of monopoly, however, the price war may hurt itself. Therefore, in beginning of period 1, firm 1 must assess its advantages and disadvantages of cost structure, and then determining whether launch a price war or not. Firm 1 needs to assess which strategy is better than the other. The strategies are including equilibrium pricing and price-cutting. The dynamic pricing strategies in various cost structures are in figure 1, figure 2 and figure 3.

Figure 1

Dynamic Decision Model under Three Periods and Firms with High-Cost

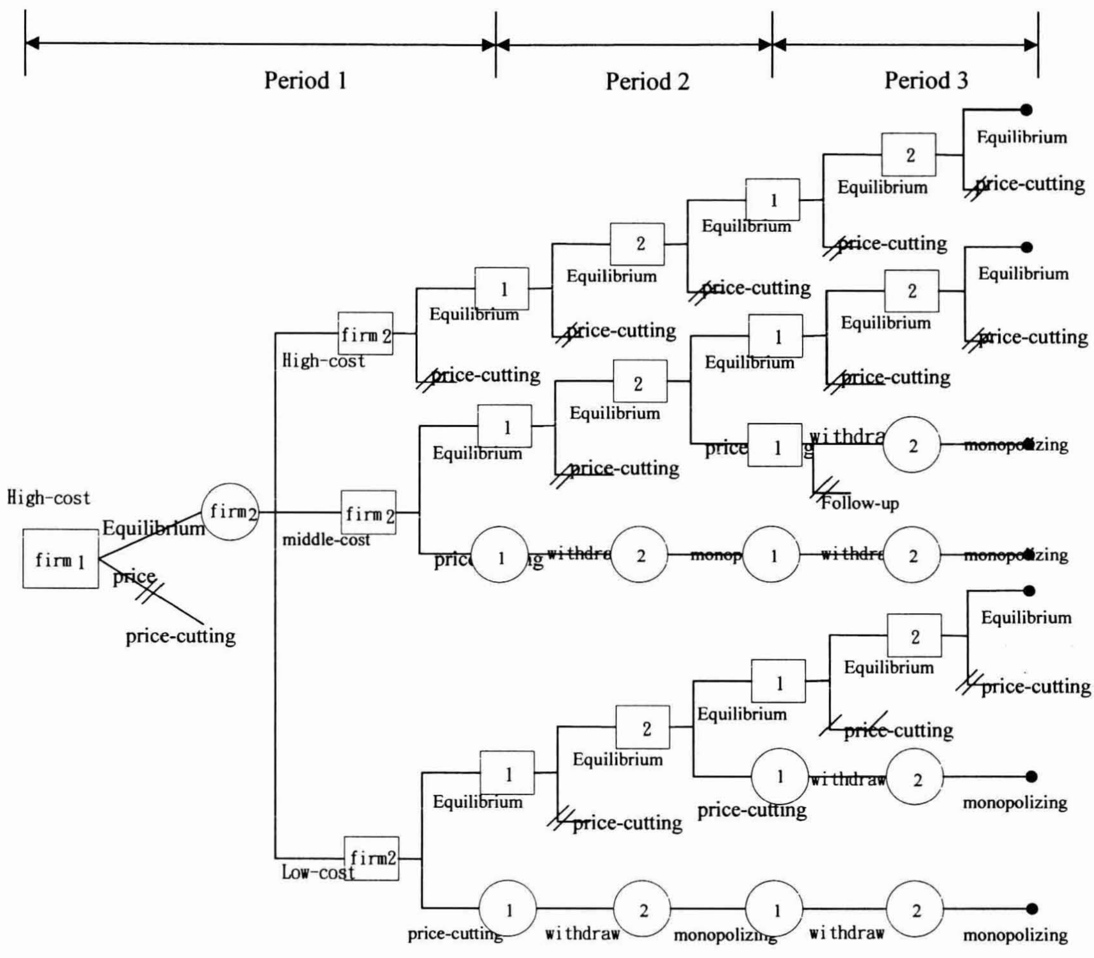


Figure 2

Dynamic Decision Model under Three Periods and Firms with Middle-Cost

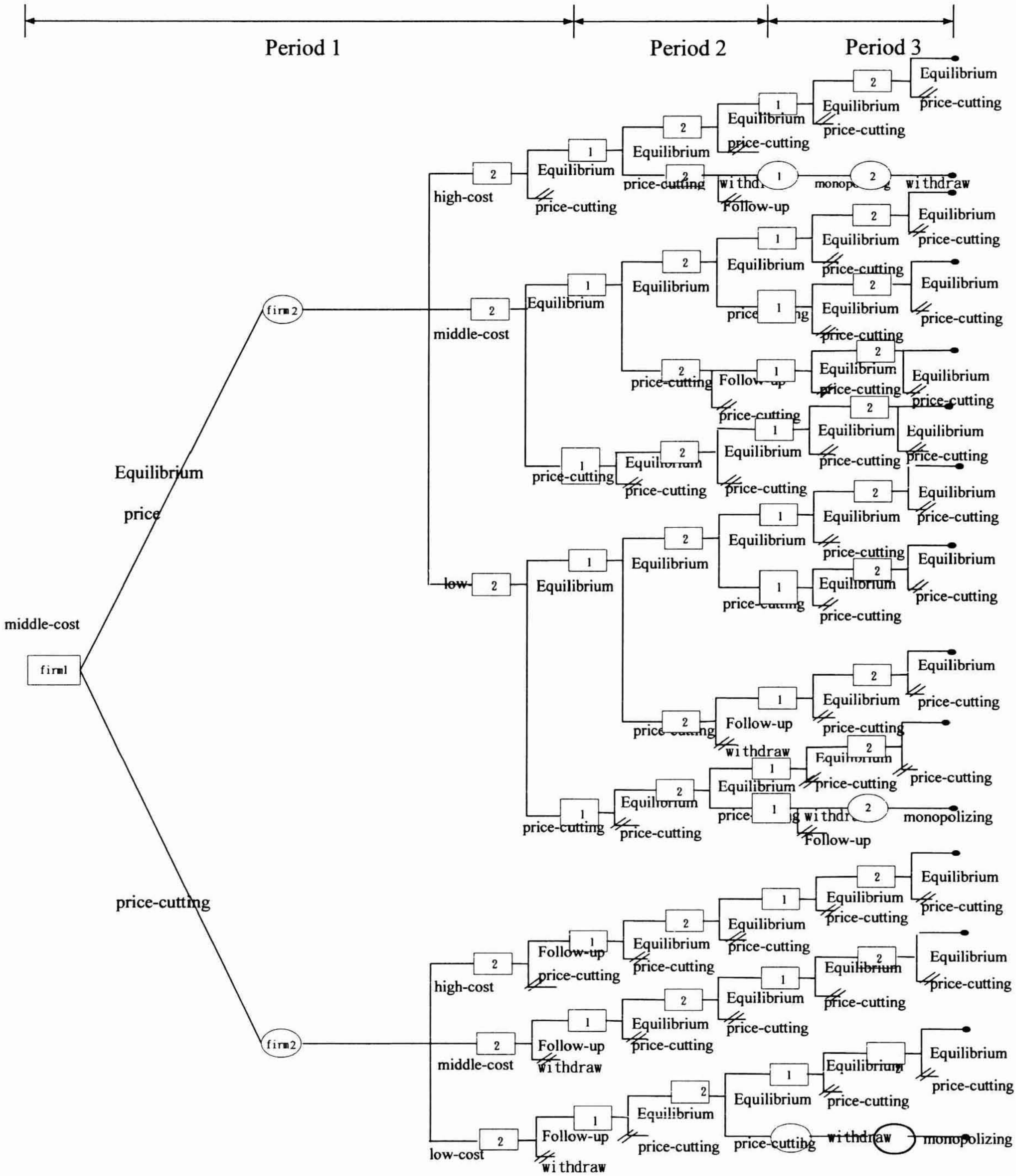
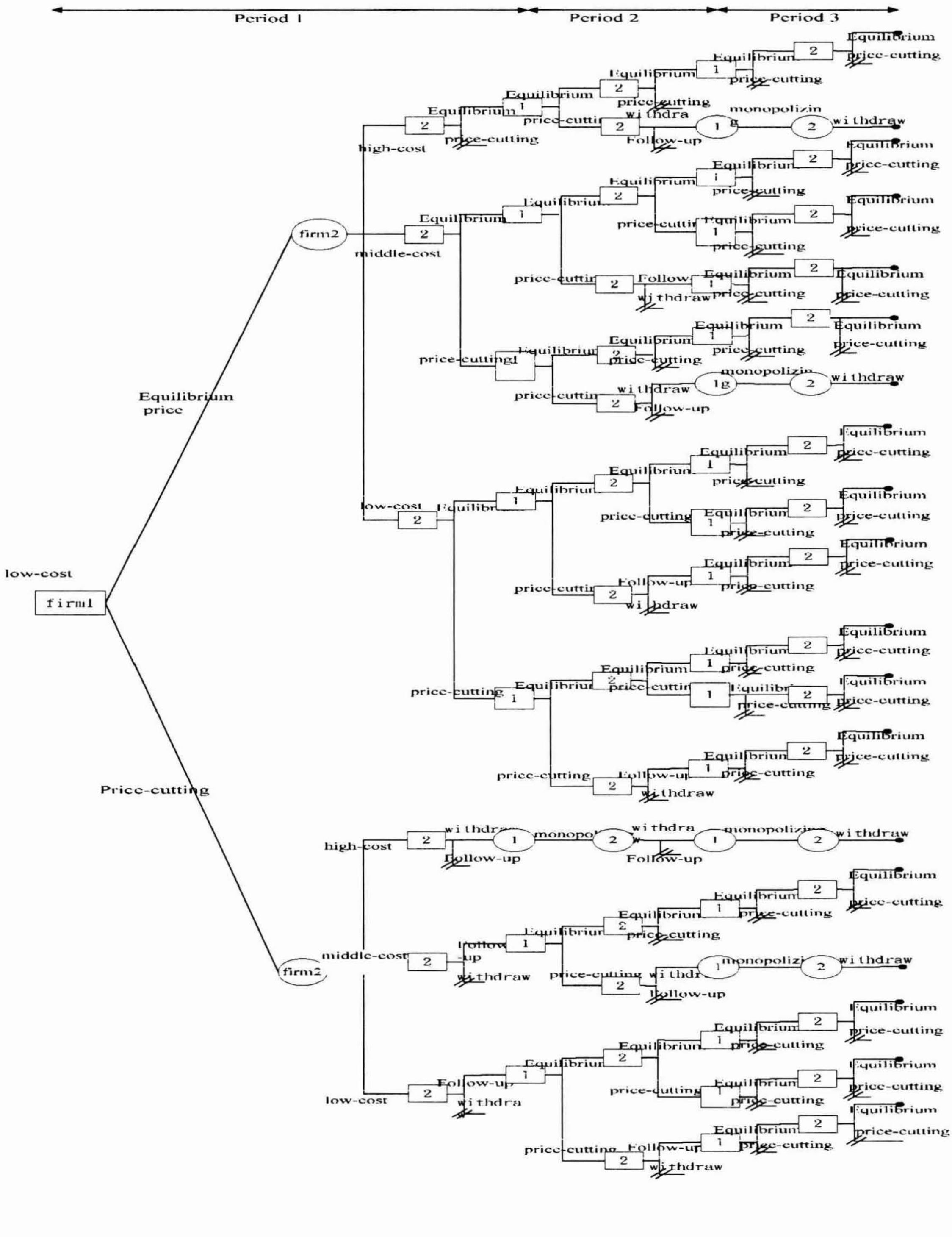


Figure 3
Dynamic Decision Model under Three Periods and Firms with Low-Cost



3.3.1.1 Option 1: An Equilibrium Decision Model in The First Period

When the cost structure information of firm 2 is incomplete, firm 1 invests the money to get the cost information of firm 2 and derive the probabilities of cost structures of firm 2. Furthermore, firm 1 seeks its own total expected profit under equilibrium strategy in three periods, the total expected profit under equilibrium strategy in the first period of firm 1 is

$$P\pi_{11r_i \bullet fk}(q_{1r_i \bullet f} \mid c_{1r_i \bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{r_j \in \chi_n} \sum_{f \in \chi_n} \pi_{1 1r_i \bullet fk}(q_{1r_i \bullet f} \mid c_{1r_i \bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}) p(c_{2\bullet r_j \bullet}) p(c_{2\bullet\bullet f}) \quad (9)$$

Formula (9) means that given firm 1's cost is $c_{1r_i \bullet\bullet}, c_{1\bullet\bullet f}$, and guess firm 2's cost is $c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}$, and then calculated the total expected profit of firm 1.

Decision Model for Firm 1 with High-Cost Structure

When firm 1 is the high-cost structure, no matter what cost structures of firm 2, firm 1's optimal strategy is maintaining equilibrium. Therefore, when firm 1 is the high-cost structure, the equilibrium price is the optimal decision. The total expected profit is

$$P\pi_{11h\bullet fb}(q_{1h\bullet f} \mid c_{1h\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{3} \left[\frac{1}{3} \sum_{r_j \in \chi_1} \sum_{f \in \chi_1} \pi_{11h\bullet fb}(q_{1h\bullet f} \mid c_{1h\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}) p(c_{2\bullet r_j \bullet}) \right] \delta^n \; (10)$$

Decision Model for Firm 1 with Middle-Cost Structure

When firm 1 is the middle-cost structure and under the hypothesis that firm 1 maintains equilibrium, the total expected profit in three periods of firm 1 is

$$P\pi_{11m\bullet fb}(q_{1m\bullet f} \mid c_{1m\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{3} \left[\frac{1}{3} \sum_{r_j \in \chi_1} \sum_{f \in \chi_1} \pi_{11m\bullet fb}(q_{1m\bullet f} \mid c_{1m\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}) p(c_{2\bullet r_j \bullet}) \right] \delta^n \; (11)$$

Decision Model for Firm 1 with Low-Cost Structure

When firm 1 is the low-cost structure and under the hypothesis that firm 1 maintains equilibrium, the total expected profit in three periods of firm 1 is

$$P\pi_{11l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j\bullet}, c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \quad (12)$$

3.3.1.2 Option 2 : Firm 1 Launches Price War in The First Period

When firm 1 launches price war in the first period, the new price is $s_{1\bullet\bullet f}$, and the new price must be less than or equal to unit variable cost of firm 2, $c_{2\bullet r_j\bullet}$. The market price reduce to $s_{l\bullet\bullet f} \le c_{2\bullet r_j\bullet}$ and force firm 2 to withdraw from the market, the price will maintain one period until firm 2 no longer have ability to re-enter the market, firm 1 will monopolize the market in next period.

Decision Model for Firm 1 with High-Cost Structure

When firm 1 is the high-cost structure, launches a price war in the first period, firm 1 will withdraw from the market under negative profit.

Decision Model for Firm 1 with Middle-Cost Structure

When firm 2 is the high-cost structure, launches a price war is the optimal decision for the firm 1. When the firm 2 is the low-cost, launch price war by the firm 1 is the worst decision.

When the firm 2 is the high-cost structure, firm 1 launches the price war in the first period, the new price is $s_{1\bullet\bullet h}$,and the new price must be less than or equal to unit variable cost, $c_{2\bullet h\bullet}$, of firm 2. In the second and the third periods, firm 1 adopt monopoly strategy, production quantity and price are as follows:

Production quantity under monopoly of firm 1 : $Q = \dfrac{a - \alpha c_{1m\bullet\bullet}}{2}$ (13)

Price under monopoly of firm 1 : $p_{1m\bullet\bullet e} = \dfrac{a + \alpha c_{1m\bullet\bullet}}{2\alpha}$ (14)

Put the new price, $s_{1\bullet\bullet h}$, in the first period and monopoly price in the second and the third periods into the total expected profit is

$$
\begin{aligned}
P\pi_{11m\bullet hs} &= ((s_{1\bullet\bullet h} - c_{1m\bullet\bullet})Q - F_1)\delta^1 + ((p_{1m\bullet\bullet e} - c_{1m\bullet\bullet})Q - F_1)(\delta^2 + \delta^3) \\
&= \frac{2\delta^1 a\alpha s_{1\bullet\bullet h} - 2\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1m\bullet\bullet} - 2\delta^1 a\alpha c_{1m\bullet} + 2\delta^1\alpha^2 c_{1m\bullet\bullet}{}^2 + \delta^2 a^2 - 2\delta^2 a\alpha c_{1m\bullet\bullet}}{2\alpha} \\
&\quad \frac{+\delta^2\alpha^2 c_{1m\bullet\bullet}{}^2 + \delta^2\alpha^2 c_{1m\bullet\bullet}{}^2 - \delta^3 a^2 - 2\delta^3 a\alpha c_{1m\bullet\bullet} + \delta^3\alpha^2 c_{1m\bullet\bullet}{}^2}{} - (\delta^1 + \delta^2 + \delta^3)F_1
\end{aligned} \tag{15}
$$

When the firm 2 is the middle-cost structure, firm 1 launches the price war in the first period, the new price is $s_{1\bullet\bullet h}$,and the new price must be less than or equal to unit variable cost, $c_{2\bullet h\bullet}$, of firm 2. In the second and the third periods, firm 1 maintains equilibrium price. The total expected profit of firm 1 is

$$
\begin{aligned}
P\pi_{11m\bullet ms} &= ((s_{1\bullet\bullet h} - c_{1m\bullet\bullet})q_{1m\bullet m} - F_1)\delta^1 + ((p_{1m\bullet\bullet b} - c_{1m\bullet\bullet})q_{1m\bullet m} - F_1)(\delta^2 + \delta^3) \\
&= \frac{3\delta^1 a\alpha s_{1\bullet\bullet h} + 3\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1\bullet\bullet m} - 6\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1m\bullet\bullet} - 3\delta^1 a\alpha c_{1m\bullet\bullet} - 3\delta^1\alpha^2 c_{1m\bullet\bullet}c_{1\bullet\bullet m} + 6\delta^1\alpha^2 c_{1m\bullet\bullet}{}^2 + \delta^2 a^2}{9\alpha} \\
&\quad \frac{+2\delta^2 a\alpha c_{1\bullet\bullet m} - 4\delta^2 a\alpha c_{1m\bullet\bullet} + \delta^2\alpha^2 c_{1\bullet\bullet m} - 4\delta^2\alpha^2 c_{1m\bullet\bullet}c_{1\bullet\bullet m} + 4\delta^2\alpha^2 c_{1\bullet\bullet m} + \delta^3 a^2 + 2\delta^3 a\alpha c_{1\bullet\bullet m}}{9\alpha} \\
&\quad \frac{-4\delta^3 a\alpha c_{1m\bullet\bullet} + \delta^3\alpha^2 c_{1\bullet\bullet m}{}^2 - 4\delta^3\alpha^2 c_{1m\bullet\bullet}c_{1\bullet\bullet m} + 4\delta^3\alpha^2 c_{1m\bullet\bullet}{}^2}{9\alpha} - (\delta^1 + \delta^2 + \delta^3)F_1
\end{aligned} \tag{16}
$$

When firm 2 is the low-cost structure, firm 1 launches the price war in the first period, the new price is $s_{1\bullet\bullet h}$, and the new price must be less than or equal to unit variable cost, $c_{2\bullet h\bullet}$, of firm 2 . After the first period, firm 2 monopolies the market, in the second and the third periods, firm 1 will withdrew from the market. The total expected profit in the first period of firm 1 is

$$
\begin{aligned}
P\pi_{11m\bullet ls} &= ((s_{1\bullet\bullet h} - c_{1m\bullet\bullet})q_{1m\bullet l} - F_1)\delta^1 \\
&= \left(\frac{as_{1\bullet\bullet h} + \alpha s_{1\bullet\bullet h}c_{1\bullet\bullet m} - 2\alpha s_{1\bullet\bullet h}c_{1m\bullet\bullet} - ac_{1m\bullet\bullet} - \alpha c_{1m\bullet\bullet}c_{1\bullet\bullet m} + 2\alpha c_{1m\bullet\bullet}{}^2}{3} - F_1\right)\delta^1
\end{aligned} \tag{17}
$$

Firm 1 launches a price war with middle-cost structure at the first period, the total expected profit is $\rho_1 P\pi_{11m\bullet hs} + \rho_2 P\pi_{11m\bullet ms} + \rho_3 P\pi_{11m\bullet ls} = P\pi_{11m\bullet\bullet s}$. Comparing with the total expected profit of price war $P\pi_{11m\bullet\bullet s}$ and the equilibrium expected profit $P\pi_{11m\bullet fb}$, if $P\pi_{11m\bullet\bullet s} - P\pi_{11m\bullet fb} > 0$, then price war is the optimal decision; otherwise, equilibrium strategy is an optimal decision. Finally, if the difference between total expected profits of partial and no information greater than the information collection cost, D, then the investment in collecting information is worth, otherwise, the investment is failed.

Decision Model for Firm 1 with Low-Cost Structure

When firm 1 is the low-cost structure, and firm 2 is the high-cost or medium-cost structure, price war is launched by firm 1 is the best decision. When the both firms are at a low-cost structure, the equilibrium pricing is the optimal decision. When firm 1 launches a price war in the first period, comparing the total expected profit between $s_{1\bullet\bullet h} \le c_{2\bullet h\bullet}$ and $s_{1\bullet\bullet m} \le c_{2\bullet m\bullet}$ to select an optimal pricing strategy.

When firm 2 is the high-cost structure, firm 1 launches the price war in the first period. In the second and the third periods, firm 1 adopts a monopoly strategy; its production quantity and price are as follows

$$\text{Production quantity under monopoly of firm 1}: Q = \frac{a - \alpha c_{1l\bullet\bullet}}{2} \tag{18}$$

$$\text{Price under monopoly of firm 1}: p_{1l\bullet\bullet e} = \frac{a + \alpha c_{1l\bullet\bullet}}{2\alpha} \tag{19}$$

The new price of market is $s_{1\bullet\bullet h}$ and $s_{1\bullet\bullet h} \le c_{2\bullet h\bullet}$. In the second and the third periods, the total expected profit of firm 1 is as follows

$$\begin{aligned}
P\pi_{11l\bullet hs} &= ((s_{1\bullet\bullet h} - c_{1l\bullet\bullet})Q - F_1)\delta^1 + ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)(\delta^2 + \delta^3) \\
&= \frac{2\delta^1 a\alpha s_{1\bullet\bullet h} - 2\delta^1\alpha^2 s_{1\bullet\bullet h} c_{1l\bullet\bullet} - 2\delta^1 a\alpha c_{1l\bullet\bullet} + 2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^2 a^2 - 2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\
&\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^3 a^2 - 2\delta^3 a\alpha c_{1l\bullet\bullet} + \delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha} - (\delta^1 + \delta^2 + \delta^3)F_1
\end{aligned} \tag{20}$$

When firm 2 is the middle-cost structure, firm 1 launches a price war in the first period with $s_{1\bullet\bullet h} \le c_{2\bullet h\bullet}$, and re-launch price war in the second period with $s_{1\bullet\bullet m} \le c_{2\bullet m\bullet}$, in the third period, firm 1 monopolies the market. The total expected profit of firm 1 is as follows

$$
\begin{aligned}
P\pi_{11l\bullet ms} &= ((s_{1\bullet\bullet h}-c_{1l\bullet\bullet})q_{1l\bullet m}-F_1)\delta^1+((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^2+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)\delta^3 \\
&= \frac{4\delta^1 a\alpha s_{1\bullet\bullet h}+4\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1\bullet\bullet m}-8\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1l\bullet\bullet}-4\delta^1 a\alpha c_{1l\bullet\bullet}-4\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet m}+8\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2}{12\alpha} \\
&\quad \frac{+6\delta^2 a\alpha s_{1\bullet\bullet m}-6\delta^2\alpha^2 c_{1l\bullet\bullet}s_{1\bullet\bullet m}-6\delta^2 a\alpha c_{1l\bullet\bullet}+6\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+3\delta^3 a^2-6\delta^3 a\alpha c_{1l\bullet\bullet}}{12\alpha} \\
&\quad \frac{+3\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{12\alpha}-(\delta^1+\delta^2+\delta^3)F_1
\end{aligned}
\tag{21}
$$

When firm 2 is the low-cost structure, firm 1 launches a price war in the first period with $s_{1\bullet\bullet h} \le c_{2\bullet h\bullet}$, and re-launches a price war in the second period with $s_{1\bullet\bullet m} \le c_{2\bullet m\bullet}$, in the third period, both maintain equilibrium strategy. The total expected profit of firm 1 is as follows

$$
\begin{aligned}
P\pi_{11l\bullet ls} &= ((s_{1\bullet\bullet h}-c_{1l\bullet\bullet})q_{1l\bullet l}-F_1)\delta^1+((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})q_{1l\bullet l}-F_1)\delta^2+((p_{1l\bullet lb}-c_{1l\bullet\bullet})q_{1l\bullet l}-F_1)\delta^3 \\
&= \frac{3\delta^1 a\alpha s_{1\bullet\bullet h}+3\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1\bullet\bullet l}-6\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1l\bullet\bullet}-3\delta^1 a\alpha c_{1l\bullet\bullet}-3\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}+6\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} \\
&\quad \frac{+3\delta^2 a\alpha s_{1\bullet\bullet m}+3\delta^2\alpha^2 s_{1\bullet\bullet m}c_{1\bullet\bullet l}-6\delta^2\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-3\delta^2 a\alpha c_{1l\bullet\bullet}-3\delta^2\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}+6\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} \\
&\quad \frac{+\delta^3 a^2+2\delta^3 a\alpha c_{1\bullet\bullet l}+\alpha^2 c_{1\bullet\bullet l}{}^2-4\delta^3 a\alpha c_{1l\bullet\bullet}+\delta^3\alpha^2 c_{1\bullet\bullet l}{}^2-4\delta^3\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}+4\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} \\
&\quad -(\delta^1+\delta^2+\delta^3)F_1
\end{aligned}
\tag{22}
$$

Firm 1 launches a price war with the low-cost structure, the expected profit is $\rho_1 P\pi_{11l\bullet hs}+\rho_2 P\pi_{11l\bullet ms}+\rho_3 P\pi_{11l\bullet ls}=P\pi_{11l\bullet\bullet s}$. When firm 2 is the high-cost structure, firm 1 launches a price war in the first period with $s_{1\bullet\bullet m} \le c_{2\bullet m\bullet}$, and in the second and the third periods, firm 1 adopts monopoly strategy, its production quantity and price are as follows:

Production quantity under monopoly of firm 1 : $Q=\dfrac{a-\alpha c_{1l\bullet\bullet}}{2}$ (23)

Price under monopoly of firm 1 : $p_{1l\bullet\bullet e}=\dfrac{a+\alpha c_{1l\bullet\bullet}}{2\alpha}$ (24)

The market's new price in is $s_{1\bullet\bullet m} \le c_{2\bullet m\bullet}$, and use monopoly price in the second and the third periods, $p_{1l\bullet\bullet e}$. The total expected profit is

$$
\begin{aligned}
P\pi_{1l\bullet hs} &= ((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^1+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)(\delta^2+\delta^3) \\
&= \frac{2\delta^1 a\alpha s_{1\bullet\bullet m}-2\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-2\delta^1 a\alpha c_{1l\bullet\bullet}+2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^2 a^2-2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\
&\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^3 a^2-2\delta^3 a\alpha c_{1l\bullet\bullet}+\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha}-(\delta^1+\delta^2+\delta^3)F_1
\end{aligned}
\tag{25}
$$

When firm 2 is the middle-cost structure, firm 1 launches a price war in the first period with $s_{1\bullet\bullet m}\le c_{2\bullet m\bullet}$, and the second and the third periods, firm 1 monopolizes the market. The total expected profit is as follows

$$
\begin{aligned}
P\pi_{11l\bullet ms} &= ((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^1+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)(\delta^2+\delta^3) \\
&= \frac{2\delta^1 a\alpha s_{1\bullet\bullet m}-2\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-2\delta^1 a\alpha c_{1l\bullet\bullet}+2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^2 a^2-2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\
&\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^3 a^2-2\delta^3 a\alpha c_{1l\bullet\bullet}+\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha}-(\delta^1+\delta^2+\delta^3)F_1
\end{aligned}
\tag{26}
$$

When firm 2 is the low-cost structure, firm 1 launches a price war in the first period with $s_{1\bullet\bullet m}\le c_{2\bullet m\bullet}$, and in the second and the third periods, both maintain equilibrium strategies. The total expected profit of firm 1 is

$$
\begin{aligned}
P\pi_{11l\bullet ls} &= ((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})q_{1l\bullet l}-F_1)\delta^1+((p_{1l\bullet lb}-c_{1l\bullet\bullet})q_{1l\bullet l}-F_1)(\delta^2+\delta^3) \\
&= \frac{3\delta^1 a\alpha s_{1\bullet\bullet m}+3\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1\bullet\bullet l}-6\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-7\delta^1 a\alpha c_{1l\bullet\bullet}-7\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha} \\
&\quad \frac{+10\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^1 a^2+2\delta^1 a\alpha c_{1\bullet\bullet l}+2\delta^1\alpha^2 c_{1\bullet\bullet l}{}^2+2\delta^2 a\alpha c_{1\bullet\bullet l}-4\delta^2 a\alpha c_{1l\bullet\bullet}}{9\alpha} \\
&\quad \frac{+4\alpha^2 c_{1\bullet\bullet l}{}^2+\delta^2\alpha^2 c_{1\bullet\bullet l}{}^2-4\delta^2\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha}-(\delta^1+\delta^2+\delta^3)F_1
\end{aligned}
\tag{27}
$$

Firm 1 launches a price war with the low-cost structure, the total expected profit is $\rho_1 P\pi_{11l\bullet hs}+\rho_2 P\pi_{11l\bullet ms}+\rho_3 P\pi_{11l\bullet ls}=P\pi_{11l\bullet\bullet s}$, and then compare the total expected profit between sequential and one-shot price-cutting strategy to select optimal pricing strategy. Comparing $P\pi_{11l\bullet\bullet s}$ with $P\pi_{11l\bullet fb}$, if $P\pi_{11l\bullet\bullet s}>P\pi_{11l\bullet fb}$, then the optimal strategy is to launch a price war; otherwise, maintaining an equilibrium strategy. Finally, if the different between total expected profit of partial and no information greater than the information collection cost, D , then the investment in collecting information is worth, otherwise, the investment is failed.

3.3.2 Game Model in The Second Period

Game model in the second period, assumption that in the first period, the optimal strategy is launching a price war or maintaining equilibrium price. Firm 1 in the beginning of the second period will re-assess which strategy is a dominant strategy.

3.3.2.1 Case1: The Decision Model in Second Given an Equilibrium Strategy in The First Period

When firm 1 in the first period maintains equilibrium, it means that equilibrium pricing is better than a price-cutting. However, price war in the first period is better than the second period, so, maintaining equilibrium strategy in the first period is better than the second period.

Decision Model for Firm 1 with High-Cost Structure

If firm 1 with high-cost structure maintains equilibrium in the first period, then the optimal strategy is also maintaining equilibrium in next period. When firm 2 launches a price war then firm 1 is forced to withdraw from the market. In the second and the third periods, the total expected profit under equilibrium strategy is

$$P\pi_{21h\bullet fb}(q_{1h\bullet f}|c_{1h\bullet\bullet},c_{1\bullet\bullet f})=\sum_{n=1}^{2}\left[\frac{1}{3}\sum_{r_j\in\chi_2}\sum_{f\in\chi_2}\pi_{21h\bullet fb}(q_{1h\bullet f}|c_{1h\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \quad (28)$$

Decision Model for Firm 1 with Middle-Cost Structure

If firm 1 with midline-cost structure maintains equilibrium in the first period, then the total expected profit in the second and the third periods under equilibrium strategy is

$$P\pi_{21m\bullet fb}(q_{1m\bullet f}\left|c_{1m\bullet\bullet},c_{1\bullet\bullet f}\right.)=\sum_{n=1}^{2}\left[\frac{1}{3}\sum_{r_j\in\chi_2}\sum_{f\in\chi_2}\pi_{21m\bullet fb}(q_{1m\bullet f}\left|c_{1m\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f}\right.)p(c_{2\bullet r_j\bullet})\right]\delta^n \quad (29)$$

Decision Model for Firm 1 with Low-Cost Structure

If firm 1 with low-cost structure maintains equilibrium in the first period. The total expected profit in the second and the third periods under equilibrium strategy is

$$P\pi_{21l\bullet fb}(q_{1l\bullet f}\left|c_{1l\bullet\bullet},c_{1\bullet\bullet f}\right.)=\sum_{n=1}^{2}\left[\frac{1}{3}\sum_{r_j\in\chi_2}\sum_{f\in\chi_2}\pi_{21l\bullet fb}(q_{1l\bullet f}\left|c_{1l\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f}\right.)p(c_{2\bullet r_j\bullet})\right]\delta^n \quad (30)$$

3.3.2.2 Case 2: The Decision Model in Second Given a Price War in The First Period

When firm 1 launched a price war in the first period, it need to re-assesses which strategy is better between price-cutting and maintaining equilibrium in the second period.

Decision Model for Firm 1 with High-Cost Structure

When firm 1 with high-cost structure launched a price war in the first period, then in the second and the third periods, firm 1 withdrew from the market under negative profit.

Decision Model for Firm 1 with Middle-Cost Structure

When firm 1 with middle-cost structure launched a price war in the first period, and firm 2 is also the middle-cost structure, the optimal decision for firm 1 is maintaining the equilibrium strategy. When firm 2 is the low-cost structure, price war launched by firm 1 is the worst decision.

When firm 2 is the high-cost structure, then in the second and the third periods, firm 1 adopts monopoly strategy; its production quantity and price are as follows:

Production quantity under monopoly of firm 1 : $Q = \frac{a - \alpha c_{1m\bullet\bullet}}{2}$ (31)

Price under monopoly of firm 1 : $p_{1m\bullet\bullet e} = \frac{a + \alpha c_{1m\bullet\bullet}}{2\alpha}$ (32)

Taking the monopoly price into the total expected profit is:

$$\begin{aligned} P\pi_{21mh\bullet e} &= ((p_{1m\bullet\bullet e} - c_{1m\bullet\bullet})Q - F_1)(\delta^1 + \delta^2) \\ &= (\frac{a^2 - 2a\alpha c_{1m\bullet\bullet} + \alpha^2 c_{1m\bullet\bullet}{}^2}{4\alpha} - F_1)(\delta^1 + \delta^2) \end{aligned} \tag{33}$$

When firm 2 is the middle-cost structure, the total expected profit in the second and the third periods under equilibrium strategy of firm 1 is

$$P\pi_{21m\bullet fb}(q_{1m\bullet f} | c_{1m\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{2} \left[\frac{1}{2} \sum_{r_j \in \chi_2} \sum_{f \in \chi_2} \pi_{21m\bullet fb}(q_{1m\bullet f} | c_{1m\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}) p(c_{2\bullet r_j \bullet}) \right] \delta^n \tag{34}$$

When firm 2 is the low-cost structure, firm 2 launched a price war in the second period and monopolize in market in the third period, firm 1 withdraw from the market under negative profit in the second and the third periods.

Decision Model for Firm 1 with Low-Cost Structure

When firm 2 is the middle-cost structure, launched a price war by firm 1 with low-cost structure in the first period is the optimal decision. When firm 2 is the low-cost structure, the optimal decision is maintaining equilibrium.

In the second and the third periods, the total expected profit under equilibrium strategy of firm 1 is

$$P\pi_{21l\bullet fb}(q_{1l\bullet f} | c_{1l\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{2} \left[\frac{1}{2} \sum_{r_j \in \chi_2} \sum_{f \in \chi_2} \pi_{21l\bullet fb}(q_{1l\bullet f} | c_{1l\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}) p(c_{2\bullet r_j \bullet}) \right] \delta^n \tag{35}$$

When firm 2 is the high-cost structure, in the second and the third periods, firm 1 adopts monopoly strategy; its production quantity and price are as follows:

Production quantity under monopoly of firm 1 : $Q = \frac{a - \alpha c_{1l\bullet\bullet}}{2}$ (36)

Price under monopoly of firm 1 : $p_{1l\bullet\bullet e} = \frac{a + \alpha c_{1l\bullet\bullet}}{2\alpha}$ (37)

Taking the monopoly price into the total expected profit is

$$
\begin{aligned}
P\pi_{21lh\bullet e} &= ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)(\delta^1 + \delta^2) \\
&= (\frac{a^2 - 2a\alpha c_{1l\bullet\bullet} + \alpha^2 c_{1l\bullet\bullet}{}^2}{4\alpha} - F_1)(\delta^1 + \delta^2)
\end{aligned} \tag{38}
$$

When firm 2 is the middle-cost structure, firm 1 launched a price war in the second period and monopolize in the third period. The total expected profit $P\pi_{21l\bullet ms}$ is

$$
\begin{aligned}
P\pi_{21l\bullet ms} &= ((s_{1\bullet\bullet m} - c_{1l\bullet\bullet})Q - F_1)\delta^1 + ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)\delta^2 \\
&= \frac{2\delta^1 a\alpha s_{1l\bullet m} - 2\delta^1\alpha^2 s_{1l\bullet m}c_{1l\bullet\bullet} - 2\delta^1 a\alpha c_{1l\bullet\bullet} + 2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^2 a^2 - 2\delta^2 a\alpha c_{1l\bullet\bullet}}{4\alpha} \\
&\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2}{4\alpha} - (\delta^1 + \delta^2)F_1
\end{aligned} \tag{39}
$$

When firm 2 is the low-cost structure, firm 1 launched a price war in the second period and maintains equilibrium in the third period. The total expected profit $P\pi_{21l\bullet ls}$ is

$$
\begin{aligned}
P\pi_{21l\bullet ls} &= ((s_{1\bullet\bullet m} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^1 + ((p_{1l\bullet lb} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^2 \\
&= \frac{3\delta^1 a\alpha s_{1\bullet\bullet m} + 3\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1\bullet\bullet l} - 6\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet} - 3\delta^1 a\alpha c_{1l\bullet\bullet} - 3\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha} \\
&\quad \frac{+6\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^2 a^2 + 2\delta^2 a\alpha c_{1\bullet\bullet l} - 4\delta^2 a\alpha c_{1l\bullet\bullet} + \delta^2\alpha^2 c_{1\bullet\bullet l}{}^2 - 4\delta^2\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha} \\
&\quad \frac{+4\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} - (\delta^1 + \delta^2)F_1
\end{aligned} \tag{40}
$$

Firm 1 launched a price war with low-cost structure in the first period, the total expected profit is $\rho_4 P\pi_{21l\bullet ms} + \rho_5 P\pi_{21l\bullet ls} = P\pi_{21l\bullet\bullet s}$. Comparing the total expected profit $P\pi_{21l\bullet\bullet s}$ with $P\pi_{21l\bullet fb}$, if $P\pi_{21l\bullet\bullet s} - P\pi_{21l\bullet fb} > 0$, then launching a price war is the optimal decision; otherwise, equilibrium strategy is an optimal one.

3.3.3 Game Model in the Third Period

Assessment game model for one period, there are only two strategies: monopoly or maintaining equilibrium.

3.3.3.1 Case1:The Decision Model in The Third Period Given The First and The Second Periods were Maintaining Equilibrium

Because the third period is the last period, therefore, maintaining equilibrium will be the best.

Decision Model for Firm 1 with High-Cost Structure

If firm 1 with high-cost structure maintained equilibrium in the first and the second periods, no matter what cost structures of firm 2, firm 1's optimal strategy is maintaining equilibrium, the total expected profit in the third period is

$$P\pi_{31h\bullet fb}(q_{1h\bullet f}\left|c_{1h\bullet\bullet}, c_{1\bullet\bullet f}\right.) = \frac{1}{3}\sum_{r_j \in \chi_3}\sum_{f \in \chi_3} \pi_{31h\bullet fb}(q_{1h\bullet f}\left|c_{1h\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}\right.)p(c_{2\bullet r_j \bullet})\delta^1 \quad (41)$$

Decision Model for Firm 1 with Middle-Cost Structure

When maintaining equilibrium in the first and the second periods, only one choice of decisions in the third period is maintaining equilibrium. The total expected profit $P\pi_{31m\bullet fb}(q_{1m\bullet f}\left|c_{1m\bullet\bullet}, c_{1\bullet\bullet f}\right.)$ is

$$P\pi_{31m\bullet fb}(q_{1m\bullet f}\left|c_{1m\bullet\bullet}, c_{1\bullet\bullet f}\right.) = \frac{1}{3}\sum_{r_j \in \chi_3}\sum_{f \in \chi_3} \pi_{31m\bullet fb}(q_{1m\bullet f}\left|c_{1m\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f}\right.)p(c_{2\bullet r_j \bullet})\delta^1 \quad (42)$$

Decision Model for Firm 1 with Low-Cost Structure

When maintaining equilibrium in the first and the second periods, only one choice of decisions in the third period is maintaining equilibrium. The total expected profit $P\pi_{31l\bullet fb}(q_{1l\bullet f}\,|c_{1l\bullet\bullet}, c_{1\bullet\bullet f})$ is

$$P\pi_{31l\bullet fb}(q_{1l\bullet f}\,|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}) = \frac{1}{3}\sum_{r_j\in\chi_3}\sum_{f\in\chi_3}\pi_{31l\bullet fb}(q_{1l\bullet f}\,|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j\bullet}, c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\delta^1 \quad (43)$$

3.3.3.2 Case2：The Decision Model in The Third Period given Launching a Price War in The First Period and Maintaining Equilibrium in The Second Period

When firm 1 launched a price war in the first period, maintaining equilibrium in the second period, the analysis of best decision of firm 1 in the third period is as follows:

Decision Model for Firm 1 with High-Cost Structure

When firm 1 is the high-cost structure, firm 1 launched a price war in the first period and withdrew from the market under negative profit.

Decision Model for Firm 1 with Middle-Cost Structure

When firm 1 launched a price war in the first period and maintaining equilibrium in the second period, then the optimal decision is maintaining equilibrium in last period. The total expected profit $P\pi_{31m\bullet fb}(q_{1m\bullet f}\,|c_{1m\bullet\bullet}, c_{1\bullet\bullet f})$ is

$$P\pi_{31m\bullet fb}(q_{1m\bullet f}\,|c_{1m\bullet\bullet}, c_{1\bullet\bullet f}) = \frac{1}{2}\sum_{r_j\in\chi_3}\sum_{f\in\chi_3}\pi_{31m\bullet fb}(q_{1m\bullet f}\,|c_{1m\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j\bullet}, c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\delta^1 \quad (44)$$

Decision Model for Firm 1 with Low-Cost Structure

When firm 1 launched a price war in the first period, maintaining equilibrium price in the second period, then the optimal decision is maintaining equilibrium in last period. The total expected profit $P\pi_{31l\bullet fb}(q_{1l\bullet f}\,|c_{1l\bullet\bullet}, c_{1\bullet\bullet f})$ is

$$P\pi_{31l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet},c_{1\bullet\bullet f})=\frac{1}{2}\sum_{r_j\in\chi_3}\sum_{f\in\chi_3}\pi_{31l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet},c_{1\bullet\bullet f},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\delta^1 \quad (45)$$

3.3.3.3 Case3 : The Decision Model in The Third Period given The First and The Second Periods Launched a Price War

When firm 1 launched a price war in the first and the second periods, the analysis of best decision of firm 1 in the third period is as follows:

Decision Model for Firm 1 with High-Cost Structure

When firm 1 is the high-cost structure, firm 1 launched a price war in the first period and withdrew from the market under negative profit.

Decision Model for Firm 1 with Middle-Cost Structure

When firm 2 is the high-cost structure, firm 1 uses monopoly price in the third period. The total expected profit $P\pi_{31mh\bullet e}$ is

$$\begin{aligned} P\pi_{31mh\bullet e} &= ((p_{1m\bullet\bullet e}-c_{1m\bullet\bullet})Q-F_1)\delta^1 \\ &= (\frac{a^2-2a\alpha c_{1m\bullet\bullet}+\alpha^2 c_{1m\bullet\bullet}{}^2}{4\alpha}-F_1)\delta^1 \end{aligned} \quad (46)$$

When firm 2 is the middle-cost structure, the optimal decision in the second period is maintaining equilibrium; therefore, this situation will be impossible to happen.

When firm 2 is the low-cost structure, firm 2 launched a price war in the second period; firm 2 monopolize the market in the third period. Firm 1 withdrew from the market in the second and the third periods with negative profit.

Decision Model for Firm 1 with Low-Cost Structure

When firm 2 is the high-cost structure, firm 1 adopts monopoly price in the third period. The total profit $P\pi_{31lh\bullet e}$ is

$$P\pi_{31lh\bullet e} = ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)\delta^1 \\ = (\frac{a^2 - 2a\alpha c_{1l\bullet\bullet} + \alpha^2 c_{1l\bullet\bullet}{}^2}{4\alpha} - F_1)\delta^1 \tag{47}$$

When firm 2 is the middle-cost structure, firm 1 adopts monopoly price in the third period. The total profit $P\pi_{31lm\bullet e}$ is

$$P\pi_{31lm\bullet e} = ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)\delta^1 \\ = (\frac{a^2 - 2a\alpha c_{1l\bullet\bullet} + \alpha^2 c_{1l\bullet\bullet}{}^2}{4\alpha} - F_1)\delta^1 \tag{48}$$

When firm 2 is the low-cost structure, firm 1 adopts equilibrium price in the third period. The total profit $P\pi_{31ll\bullet b}$ is

$$P\pi_{31ll\bullet b} = ((p_{1ll\bullet b} - c_{1l\bullet\bullet})q_{1ll\bullet} - F_1)\delta^1 \\ = (\frac{a^2 + 2a\alpha c_{2\bullet l\bullet} - 4a\alpha c_{1l\bullet\bullet} + \alpha^2 c_{2\bullet l\bullet}{}^2 - 4\alpha^2 c_{2\bullet l\bullet} c_{1l\bullet\bullet} + 4\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} - F_1)\delta^1 \tag{49}$$

In this study, we can find the production quantity and profit depend on cost structure. Firm with high-cost structure will produce lower quantity and earn less profit; on the contrary, firm with low-cost structure will produce more quantity and earn more profit. In equilibrium condition, when both firms produce the same products, their setting pricing will be the same. The result is consistent with Cournot model. Then we proposed two propositions to describe the result:

【Proposition 1】 When the firms produce homogeneous product, the price of both firms is the same, and the firm's production quantity inverse proportion to the unit cost of oneself, and direct proportion to the opponent's unit cost.

【Proof】

$$\pi_{\bullet 1r_ir_j\bullet\bullet} = (p_{1r_ir_j\bullet\bullet} - c_{1r_i\bullet\bullet})q_{1r_ir_j\bullet} - F_1 = (\frac{a - q_{1r_ir_j\bullet} - q_{2r_ir_j\bullet}}{\alpha} - c_{1r_i\bullet\bullet})q_{1r_ir_j\bullet} - F_1 \tag{50}$$

$$\pi_{\bullet 2r_ir_j\bullet\bullet} = (p_{2r_ir_j\bullet\bullet} - c_{2\bullet r_j\bullet})q_{2r_ir_i\bullet} - F_2 = (\frac{a - q_{1r_ir_j\bullet} - q_{2r_ir_j\bullet}}{\alpha} - c_{2\bullet r_j\bullet})q_{2r_ir_j\bullet} - F_2 \tag{51}$$

Let the derivative of profit function of firm 1 by $p_{1r_ir_j\bullet\bullet}$ equal to zero, we obtain

$$q_{1r_ir_j\bullet}=\frac{a-q_{2r_ir_j\bullet}-\alpha c_{1r_i\bullet\bullet}}{2} \tag{52}$$

By the same way, we can obtain the quantity reaction function of firm 2 as

$$q_{2r_ir_j\bullet}=\frac{a-q_{1r_ir_j\bullet}-\alpha c_{2\bullet r_j\bullet}}{2} \tag{53}$$

Solve both equations (52) and (53) simultaneously, we have the equilibrium quantities

$$q_{1r_ir_j\bullet}=\frac{a+\alpha c_{2\bullet r_j\bullet}-2\alpha c_{1r_i\bullet\bullet}}{3} \tag{54}$$

$$q_{2r_ir_j\bullet}=\frac{a+\alpha c_{1r_i\bullet\bullet}-2\alpha c_{2\bullet r_j\bullet}}{3} \tag{55}$$

The optimal price of the two firms is

$$P_{\bullet r_ir_j\bullet\bullet}=\frac{a+\alpha c_{1r_i\bullet\bullet}+\alpha c_{2\bullet r_j\bullet}}{3\alpha} \tag{56}$$

Based on (54)、(55)、(56), we can find when both firm's product is homogeneous, both prices will be consistent, and firm's production quantity is influenced by the unit cost of oneself. When the unit cost is higher, the production quantity then is lower, if the opponent unit cost is higher, then own production quantity is also higher.

【Proposition 2】 When firm's price coefficient (α)and production unit cost increase, then the firm's profit will be decreased. Only when the potential scale of market (a) increase, both firm's profits will be increased.

【Proof】 Firm 1 profit is

$$P\pi_{\bullet 1r_ir_jfb}=\frac{1}{3}(\rho_1\pi_{\bullet 1r_ir_jfb}+\rho_2\pi_{\bullet 1r_ir_jfb}+\rho_3\pi_{\bullet 1r_ir_jfb}+\rho_1\pi_{\bullet 1r_ir_jfb} \\ +\rho_2\pi_{\bullet 1r_ir_jfb}+\rho_3\pi_{\bullet 1r_ir_jfb}+\rho_1\pi_{\bullet 1r_ir_jfb}+\rho_2\pi_{\bullet 1r_ir_jfb}+\rho_3\pi_{\bullet 1r_ir_jfb}) \tag{57}$$

Because the analysis of profit under different cost structures is similar, this study only discusses the profit in one cost structure.

$\pi_{\bullet 1r_ir_jfb}=(\dfrac{a-q_{1r_i\bullet f}-q_{2r_i\bullet f}}{\alpha}-c_{1r_i\bullet\bullet})q_{1r_i\bullet f}-F_{1\bullet})$, used first derivative, we have

$$\frac{\partial\pi_{\bullet 1r_ir_jfb}}{\partial a}=\frac{1}{3}(\frac{q_{1r_i\bullet f}}{\alpha})\geq 0 \tag{58}$$

$$\frac{\partial\pi_{\bullet 1r_ir_jfb}}{\partial\alpha}=\frac{1}{3}(\frac{-(a-q_{1r_i\bullet f}-q_{2r_i\bullet f})q_{1r_i\bullet f}}{\alpha^2})\leq 0 \tag{59}$$

$$\frac{\partial\pi_{\bullet 1r_ir_jfb}}{\partial c_{1r_ir_j\bullet}}=-\frac{1}{9}q_{1r_i\bullet f}\leq 0 \tag{60}$$

By the same way, we can obtain

$$\frac{\partial\pi_{\bullet 2r_ir_jfb}}{\partial a}=\frac{1}{9}\frac{q_{2r_i\bullet f}}{\alpha}\geq 0\ ,\ \frac{\partial\pi_{\bullet 2r_ir_jfb}}{\partial\alpha}=\frac{1}{9}\frac{-(a-q_{1r_i\bullet f}-q_{2r_i\bullet f})q_{2r_i\bullet f}}{\alpha^2}\leq 0\ ,\ \frac{\partial\pi_{\bullet 2r_ir_jfb}}{\partial c_{2r_ir_j\bullet}}=-\frac{1}{9}q_{2r_i\bullet f}\leq 0.$$

4. The Example Analysis

There are only two firms, firm 1 and firm 2, to produce 1 G memory card for cellular phone in the market, the memory card almost makes no difference in customer's mind. The customers only take selling price into consideration; one's product can be substituted completely by the other, so there is no loyalty problem. Suppose the life time of the product has three periods, and the demand function is $Q=100000-62P$ .The next we will discuss which strategy is the optimal strategy in duopoly market, the feasible strategies include: price-cutting and maintaining equilibrium. Under the condition of incomplete information of competitors, one firm considers to collect the cost structure information of opponent, and assess which strategy is a dominant strategy. Summary the related parameters and values in table 2 as shown:

Table 2
Related Parameters and Values in Example

Parameter	Value	Parameter	Value
a	100000	F_l	3000000
α	62	F_m	2000000
δ	1	F_h	1000000
c_h	900	ρ_1	0.4
c_m	800	ρ_2	0.4
c_l	700	ρ_3	0.2
D	500000		

4.1 Maintaining Equilibrium by Firm 1

This section analyzes the situation when firm 1 faces the competitor and has no information available. In order to reduce market risk, firm 1 invests money $D = 500000$ to collect cost information of the competitor through by a marketing consultant company with the aim to force the disadvantageous competitor out of the market. After collecting information, firm 1 predicted the cost structure of firm 2 being high, middle, and low cost structure with probabilities are $\rho_1 = 0.4$, $\rho_2 = 0.4$, and $\rho_1 = 0.2$ respectively. When substituted the values of parameters in Table 2 into the total expected profit model that maintaining equilibrium, we acquire

$$P\pi_{11l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}) = \sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j \in \chi_1}\sum_{f \in \chi_1}\pi_{11l\bullet fb}(q_{1l\bullet f}|c_{1l\bullet\bullet}, c_{1\bullet\bullet f}, c_{2\bullet r_j \bullet}, c_{2\bullet\bullet f})p(c_{2\bullet r_j \bullet})\right]\delta^n \quad (61)$$

If firm 1 predicts firm 2 is the high-cost structure, the total expected profit of firm 1 is

$$
\begin{aligned}
P\pi_{11l\bullet hb}(q_{1l\bullet h}|c_{1l\bullet\bullet},c_{1\bullet\bullet h}) &= \sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11l\bullet hb}(q_{1l\bullet h}|c_{1l\bullet\bullet},c_{1\bullet\bullet h},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \\
&= (0.4\times\pi_{11lhhb}+0.4\times\pi_{11lmhb}+0.2\times\pi_{11llhb})1 \\
&= 0.4\times 14297144+0.4\times 9674516+0.2\times 5096774 \\
&= 10608018.8
\end{aligned}
$$

If firm 1 predicts firm 2 is the middle-cost structure, the total expected profit of firm 1 is

$$
\begin{aligned}
P\pi_{11l\bullet mb}(q_{1l\bullet m}|c_{1l\bullet\bullet},c_{1\bullet\bullet m}) &= \sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11l\bullet mb}(q_{1l\bullet m}|c_{1l\bullet\bullet},c_{1\bullet\bullet m},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \\
&= (0.4\times\pi_{11lhmb}+0.4\times\pi_{11lmmb}+0.2\times\pi_{11llmb})\times 1 \\
&= 0.4\times 14297078+0.4\times 10089883+0.2\times 5923541 \\
&= 10939492.6
\end{aligned}
$$

If firm 1 predicts firm 2 is the low-cost structure, its total expected profit of firm 1 is

$$
\begin{aligned}
P\pi_{11l\bullet lb}(q_{1l\bullet l}|c_{1l\bullet\bullet},c_{1\bullet\bullet l}) &= \sum_{n=1}^{3}\left[\frac{1}{3}\sum_{r_j\in\chi_1}\sum_{f\in\chi_1}\pi_{11l\bullet lb}(q_{1l\bullet l}|c_{1l\bullet\bullet},c_{1\bullet\bullet l},c_{2\bullet r_j\bullet},c_{2\bullet\bullet f})p(c_{2\bullet r_j\bullet})\right]\delta^n \\
&= (0.4\times\pi_{11lhlb}+0.4\times\pi_{11lmlb}+0.2\times\pi_{11lllb})\times 1 \\
&= 0.4\times 13883845+0.4\times 10091883+0.2\times 6336741 \\
&= 10857639.4
\end{aligned}
$$

We are able to acquire the low-cost structure of firm 1 who adopts equilibrium strategy and its total expected profit in the first period is shown in Table 3:

Table3
The Production Quantity and Total Expected Profit of Firm 1 with Low-Cost Structure and Maintaining Equilibrium in The First Period

firm 1 Actual Cost	Estimated Cost of firm 2 by firm 1	production quantity of firm 1	Total expected profit of firm 1
	High-cost	23000	10608018.8
Low-cost	Middle-cost	20900	10939492.6
	Low-cost	18867	10857639.4

Table 3 shows that the total expected profit of firm 1 with low-cost structure in the first period is $P\pi_{11l\bullet\bullet b} = 10939492.6$. In the first period, firm 1 compares with launching a price war sequentially and directly based on partial information of firm 2 and then firm 1 selects the optimal price-cutting strategy. Initially, firm 1 adopts a sequential price war.

4.2 Price War Strategy Adopted by Firm 1

When firm 2 is the high-cost structure, firm 1 launched a price war in the first period. In the second and the third periods, firm 1 monopolizes the market, its total expected profit $P\pi_{11l\bullet hs}$ is

$$\begin{aligned} P\pi_{11l\bullet hs} &= ((s_{1\bullet\bullet h} - c_{1l\bullet\bullet})Q - F_1)\delta^1 + ((p_{1l\bullet\bullet e} - c_{1l\bullet\bullet})Q - F_1)(\delta^2 + \delta^3) \\ &= \frac{2\delta^1 a\alpha s_{1\bullet\bullet h} - 2\delta^1\alpha^2 s_{1\bullet\bullet h} c_{1l\bullet\bullet} - 2\delta^1 a\alpha c_{1l\bullet\bullet} + 2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^2 a^2 - 2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\ &\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^3 a^2 - 2\delta^3 a\alpha c_{1l\bullet\bullet} + \delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha} - (\delta^1 + \delta^2 + \delta^3)F_1 \end{aligned} \tag{62}$$

The monopoly production quantity and price of firm 1 are as follows:
Production quantity under monopoly of firm1:

$$Q = \frac{a - \alpha c_{1l\bullet\bullet}}{2} = \frac{100000 - 62 \times 700}{2} = 28300$$

Price under monopoly of firm 1 :

$$p_{1l\bullet\bullet e}=\frac{a+\alpha c_{1l\bullet\bullet}}{2\alpha}=\frac{100000+62\times700}{2\times62}=1156.4$$

Substituting the values into Equation (62). If firm 1 predicts firm 2 is the high-cost structure, the total expected profit of firm 1 is

$$\begin{aligned}P\pi_{11l\bullet hs}&=((900-700)28300-3000000)1^1+((1156.4-700)28300-3000000)(1^2+1^3)\\&=\frac{2\times1^1\times100000\times62\times900-2\times1^1\times62^2\times900\times700-2\times1^1\times100000\times62\times700}{2\alpha}\\&\quad\frac{+2\times1^1\times62^2\times700^2+1^2\times100000^2-2\times1^2\times100000\times62\times700+1^2\times62^2\times700^2}{2\alpha}\\&\quad-\frac{+1^3\times100000^2-2\times1^3\times100000\times62\times700+1^3\times62^2\times700^2}{2\times62}-(1^1+1^2+1^3)3000000\\&=22495161.29\end{aligned}$$

When firm 2 is the middle-cost structure, firm 1 launched a price war in the first and the second periods. In the third period, firm 1 monopolizes the market, its total expected profit $P\pi_{11l\bullet ms}$ is

$$\begin{aligned}P\pi_{11l\bullet ms}&=((s_{1\bullet\bullet h}-c_{1l\bullet\bullet})q_{1l\bullet m}-F_1)\delta^1+((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^2+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)\delta^3\\&=\frac{4\delta^1 a\alpha s_{1\bullet\bullet h}+4\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1\bullet\bullet m}-8\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1l\bullet\bullet}-4\delta^1 a\alpha c_{1l\bullet\bullet}-4\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet m}+8\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2}{12\alpha}\\&\quad\frac{+6\delta^2 a\alpha s_{1\bullet\bullet m}-6\delta^2\alpha^2 c_{1l\bullet\bullet}s_{1\bullet\bullet m}-6\delta^2 a\alpha c_{1l\bullet\bullet}+6\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+3\delta^3 a^2-6\delta^3 a\alpha c_{1l\bullet\bullet}}{12\alpha}\\&\quad\frac{+3\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{12\alpha}-(\delta^1+\delta^2+\delta^3)F_1\end{aligned}\tag{63}$$

Substituting the values into Equation (63). If firm 1 predicts firm 2 is the middle-cost structure, the total expected profit of firm 1 is

$$\begin{aligned}P\pi_{11l\bullet ms}&=((900-700)20933-3000000)1^1+((800-700)28300-3000000)1^2\\&\quad+((1156.4-700)28300-3000000)1^3\\&=10934247\end{aligned}$$

When firm 2 is the low-cost structure, firm 1 launched a price war in the first and the second periods. In the third period, both firms maintain equilibrium strategy. The total expected profit of firm 1 is

$$P\pi_{11l\bullet ls} = ((s_{1\bullet\bullet h} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^1 + ((s_{1\bullet\bullet m} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^2 + ((p_{1l\bullet lb} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^3$$

$$= \frac{3\delta^1 a\alpha s_{1\bullet\bullet h} + 3\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1\bullet\bullet l} - 6\delta^1\alpha^2 s_{1\bullet\bullet h}c_{1l\bullet\bullet} - 3\delta^1 a\alpha c_{1l\bullet\bullet} - 3\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l} + 6\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha}$$

$$\frac{+3\delta^2 a\alpha s_{1\bullet\bullet m} + 3\delta^2\alpha^2 s_{1\bullet\bullet m}c_{1\bullet\bullet l} - 6\delta^2\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet} - 3\delta^2 a\alpha c_{1l\bullet\bullet} - 3\delta^2\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l} + 6\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha} \quad (64)$$

$$\frac{+\delta^3 a^2 + 2\delta^3 a\alpha c_{1\bullet\bullet l} + \alpha^2 c_{1\bullet\bullet l}{}^2 - 4\delta^3 a\alpha c_{1l\bullet\bullet} + \delta^3\alpha^2 c_{1\bullet\bullet l}{}^2 - 4\delta^3\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l} + 4\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{9\alpha}$$

$$-(\delta^1 + \delta^2 + \delta^3)F_1$$

When both firms are low-cost structures, the equilibrium quantity and price of firm 1 are as follows

Equilibrium quantity of firm 1 : $q_{1l\bullet l} = \dfrac{100000 + 62\times 700 - 2\times 62\times 700}{3} = 18866.6$

Equilibrium price of firm 1 : $P_{1l\bullet lb} = \dfrac{100000 + 62\times 700 + 62\times 700}{3\times 62} = 1004.301$

Substituting the values into Equation (64). If firm 1 predicts firm 2 is the low-cost structure, the total expected profit of firm 1 is

$$\begin{aligned} P\pi_{11l\bullet ls} &= ((900 - 700)18866.6 - 3000000)1^1 + ((800 - 700)18866.6 - 3000000)1^2 \\ &+ ((1004.301 - 700)18866.6 - 3000000)1^3 \\ &= 2401146.953 \end{aligned}$$

Table4

Expected Profit of Firm 1 with Sequential Price War in The First Period

Firm 1 Actual Cost	Estimated Cost of firm 2 by firm 1	Profit of firm 1	Total expected profit of firm 1
	High cost	8998064	
Low cost	Middle cost	4373698	13851991
	Low cost	480229	

The probabilities of high, middle, low-cost structures of firm 2 are 0.4, 0.4 and 0.2 respectively. Accordingly, the total expected profit of firm 1with sequential price war in the first period is shown in Table 4.

The value of lowest price is substituted into the constructed model. When firm 2 is the high-cost structure, firm 1 launched a lowest price war in the first period, then in the second and the third periods, firm 1 monopolizes the market, its total expected profit of firm 1 is

$$\begin{aligned} P\pi_{1l\bullet hs} &= ((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^1+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)(\delta^2+\delta^3) \\ &= \frac{2\delta^1 a\alpha s_{1\bullet\bullet m}-2\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-2\delta^1 a\alpha c_{1l\bullet\bullet}+2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^2 a^2-2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\ &\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^3 a^2-2\delta^3 a\alpha c_{1l\bullet\bullet}+\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha}-(\delta^1+\delta^2+\delta^3)F_1 \end{aligned} \tag{65}$$

Substituting the values into Equation (65). If firm 1 predicts firm 2 is the high-cost structure, the total expected profit of firm 1 is

$$\begin{aligned} P\pi_{1l\bullet hs} &= ((800-700)28300-3000000)1^1+((1156.4-700)28300-3000000)(1^2+1^3) \\ &= 19665161.29 \end{aligned}$$

When firm 2 is the middle-cost structure, firm 1 launched a lowest price war in the first period. Therefore, in the second and the third periods, firm 1 monopolizes the market, its total expected profit of firm 1 is

$$\begin{aligned} P\pi_{11l\bullet ms} &= ((s_{1\bullet\bullet m}-c_{1l\bullet\bullet})Q-F_1)\delta^1+((p_{1l\bullet\bullet e}-c_{1l\bullet\bullet})Q-F_1)(\delta^2+\delta^3) \\ &= \frac{2\delta^1 a\alpha s_{1\bullet\bullet m}-2\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet}-2\delta^1 a\alpha c_{1l\bullet\bullet}+2\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^2 a^2-2\delta^2 a\alpha c_{1l\bullet\bullet}}{2\alpha} \\ &\quad \frac{+\delta^2\alpha^2 c_{1l\bullet\bullet}{}^2+\delta^3 a^2-2\delta^3 a\alpha c_{1l\bullet\bullet}+\delta^3\alpha^2 c_{1l\bullet\bullet}{}^2}{2\alpha}-(\delta^1+\delta^2+\delta^3)F_1 \end{aligned} \tag{66}$$

Substituting the values into Equation (66). If firm 1 predicts firm 2 is the middle-cost structure, the total expected profit of firm 1 is

$$P\pi_{11l\bullet ms} = ((800-700)28300-3000000)1^1 + ((1156.4-700)28300-3000000)(1^2+1^3)$$
$$= 19665161.29$$

When firm 2 is the low-cost structure, firm 1 launched a lowest price war in the first period and in the second and the third periods, both firms maintain equilibrium strategy. The total expected profit is as follows

$$\begin{aligned} P\pi_{11l\bullet lS} &= ((s_{1\bullet\bullet m} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)\delta^1 + ((p_{1l\bullet lb} - c_{1l\bullet\bullet})q_{1l\bullet l} - F_1)(\delta^2+\delta^3) \\ &= \frac{3\delta^1 a\alpha s_{1\bullet\bullet m} + 3\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1\bullet\bullet l} - 6\delta^1\alpha^2 s_{1\bullet\bullet m}c_{1l\bullet\bullet} - 7\delta^1 a\alpha c_{1l\bullet\bullet} - 7\delta^1\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha} \\ &\quad \frac{+10\delta^1\alpha^2 c_{1l\bullet\bullet}{}^2 + \delta^1 a^2 + 2\delta^1 a\alpha c_{1\bullet\bullet l} + 2\delta^1\alpha^2 c_{1\bullet\bullet l}{}^2 + 2\delta^2 a\alpha c_{1\bullet\bullet l} - 4\delta^2 a\alpha c_{1l\bullet\bullet}}{9\alpha} \\ &\quad \frac{+4\alpha^2 c_{1\bullet\bullet l}{}^2 + \delta^2\alpha^2 c_{1\bullet\bullet l}{}^2 - 4\delta^2\alpha^2 c_{1l\bullet\bullet}c_{1\bullet\bullet l}}{9\alpha} - (\delta^1+\delta^2+\delta^3)F_1 \end{aligned} \tag{67}$$

Substituting the values into Equation (67). If firm 1 predicts firm 2 is the low-cost structure, the total expected profit of firm 1 is

$$P\pi_{11l\bullet lS} = ((800-700)18866.6-3000000)1^1 + ((1004.3-700)18866.6-3000000)(1^2+1^3)$$
$$= 4368960.5$$

The probabilities of high, middle, low-cost structures of firm 2 are 0.4, 0.4 and 0.2 respectively. Accordingly, the total expected profit of firm 1with lowest price war in the first period is shown in Table 5:

Table5

Total Expected Profit of Firm 1 with the Lowest Price War in The First Period

firm 1 Actual Cost	Estimated Cost of firm 2 by firm 1	Profit of firm 1	Total expected profit of firm 1
	High cost	7866064	
Low cost	Middle cost	7866064	16605920
	Low cost	873792.1	

As shown in Table 4 and 5, we conclude that the total expected profits under sequential price war and the lowest price war are 13851991 and 16605920 respectively. Therefore, the lowest price war is the optimal strategy. Firm 1 in the first period maintains equilibrium and its total expected profit is $P\pi_{11l \bullet \bullet b} = 10939492.6$. When firm 1 launches a lowest price war, its total expected profit is $P\pi_{11l \bullet \bullet s} = 16605920$, if firm 2 is the middle-cost structure, then it is forced out of the market. Firm 1 monopolizes the market in the second and the third periods, its total expected profit is $P\pi_{11l \bullet \bullet e} = 22835161.29$.

Evaluating whether the investment in collecting information of cost structure of firm 2 is worthy or not. The information cost $D = 500000$ and the difference of total expected profit between launching a lowest price war and without available information is $\Delta \mathrm{I}\pi \geq 500000$, so the investment is a corrected decision.

4.1 Sensitivity Analysis

In this part, this study brings the known information into the model and conducts sensitivity analysis about potential market scale (a), price coefficient (α), and unit price $(c_{1l \bullet \bullet})$ to understand the influence of specific parameters to the price and profit of firm 1.

4.1.1 The Profit Influenced by The Changes of Potential Market Scale (a)

When other parameters hold constant, the increase of potential market scale has influenced on profit as shown in figure 4.

As shown in Fig. 4, when other parameters hold constant, the increase of potential market scale will dramatically increases profit. This result is consistent with Equation (58). This is because the increase of potential market scale will push the increase of demand as well as price, and then the profit of firm is also increasing.

4.1.2 The Profit Influenced by The Changes of Price Coefficient (α)

When other parameters hold consistent, the increase of price coefficient has influenced on profit as shown in figure 5.

Figure 4
Sensitivity Analysis of Potential Market Scale

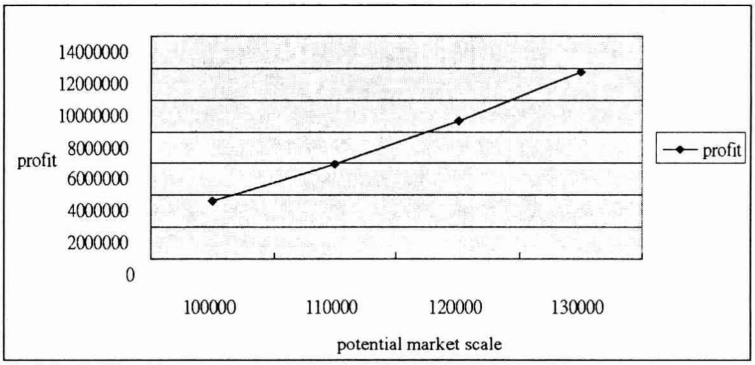
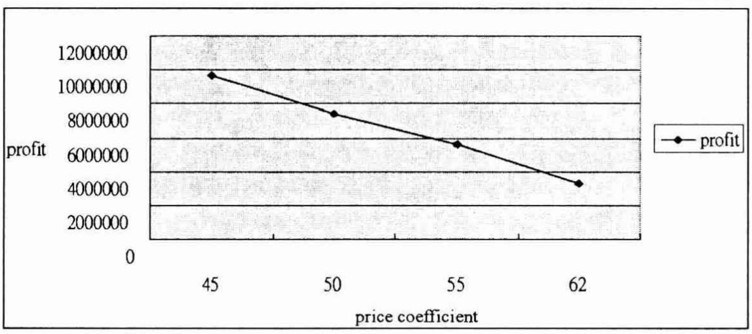


Figure 5
Sensitivity Analysis of Price Coefficient (α)



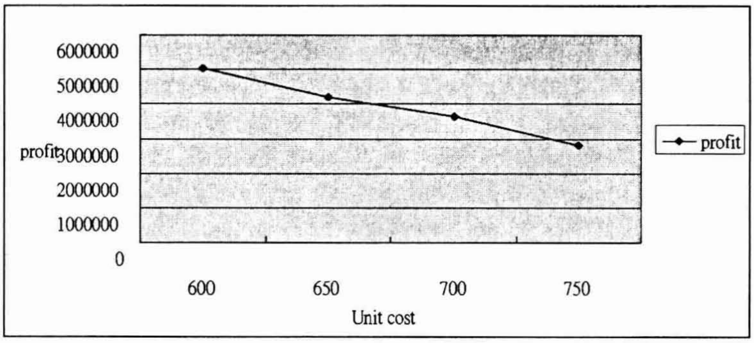
As shown in Fig. 5, when other parameters hold constant, the increase of price coefficient will result in price and profit drop. This is because the increase of price coefficient will reduce the demand and accordingly, revenue and profit of firms will drop. This result is consistent with Equation (59). Thus, firm has to possibly reduce price sensitivity of market to avoid the influence on profit.

4.1.3 The Profit Influenced by The Changes of Unit Cost $(c_{1l\bullet\bullet})$

When other parameters hold constant, the increase of unit cost has influenced on profit as shown in figure 6:

As shown in Fig. 6, when other parameters hold constant, the increase of unit cost will result in price and profit drop. This is because the increase of unit price will raise the price and reduce the demand and accordingly, profit of firm will drop. This result is consistent with Equation (60). Thus, managers should reduce internal operational cost to increase profit.

Figure 6
Sensitivity Analysis of Unit Price $(c_{1l\bullet\bullet})$



5. Conclusions

5.1 Conclusions

This study adopts Cournot model to construct an optimal decision model on production quantity and price based on given demand function, incomplete information of opponent's cost structure and duopoly. Seven concludes are drawn as follows:

1. This study used the given demand function to construct an optimal strategy through by optimization techniques and expectation theory under incomplete information of opponent's cost structure and homogenous product.
2. When cost information of competitor is unknown, in order to reduce the market risk, firm will invest the money to collect relevant information and construct the optimal price strategy to maximize its total expected profit.

Furthermore, firm need to evaluate the difference of expected profit with and without the investment of collecting information. If the difference of expected profit is greater than the information cost, then the investment is worthy; otherwise, the investment is not worthy.

3. Based on Cournot model and applied the quantity reaction function, two firms acquire the equilibrium quantity and price. Firms at the beginning of each period evaluate the equilibrium strategy and price war strategy and estimate the cost structure of competitor based on the result of previous period to search for the optimal strategy to maximize the total expected profit.
4. When a firm launches a price war, the equilibrium price of market will be destroyed and both firms will begin a price competition. Finally, firm with cost advantage will monopolize the market. When a firm is the high-cost structure, it is not suitable to launch a price war and the maintaining equilibrium is the optimal pricing strategy.
5. If a firm with middle-cost structure launched a price war in the first period and competitor followed, then the equilibrium price is the optimal decision.
6. If a firm adopted equilibrium strategy in previous period, then maintaining equilibrium is the optimal strategy in following periods.
7. The increase of price coefficient and unit price will result in the drop of expected profit. However, the increase of potential market scale will also increase the expected profit.

6. References

Lin, L.Y., Liang, G. S. and Liu, G. F. (2005), "Explored the Duopoly Market and Used the Fuzzy Theory in Airlines Companies," *Chung Hua Journal of Management*, 6(2), 15-30.

Chang, Y. C. and Wu, L. E. (2000), *Microeconomics*, Taipei, Taiwan: Xue Fu publishing company.

Cabral, L. M. B. (2000) "R&D Cooperation And Product Market Competition," *International Journal of Industrial Organization,* 18(1), 1033-1047.

Lofaro, A. (2002) "On the Efficiency of Bertrand and Cournot Competition under Incomplete Information," *European Journal of Political Economy*, 18(3), 561-578.

Singh, N. and Vives, X. (1984) "Price and Quantity Competition in a Differentiated Duopoly,"*Rand Journal of Economics*, 15(4), 546-54.

Wen, F. S. and David, A. K. (2001), "Oligopoly Electricity Market Production under Incomplete Information," *IEEE Power Engineering Review,* 21(4), 58-61.