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# Fragmentation fractions of two-body $b$-baryon decays 

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#### Abstract

We study the fragmentation fractions $\left(f_{\mathbf{B}_{b}}\right)$ of the $b$-quark to b-baryons ( $\mathbf{B}_{b}$ ). By the assumption of $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)=0.25 \pm 0.15$ in accordance with the measurements by LEP, CDF and LHCb Collaborations, we estimate that $f_{\Lambda_{b}}=0.175 \pm 0.106$ and $f_{\Xi_{b}^{-, 0}}=0.019 \pm 0.013$. From these fragmentation fractions, we derive $\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)=(3.3 \pm 2.1) \times 10^{-4}, \mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)=(5.3 \pm 3.9) \times 10^{-4}$ and $\mathcal{B}\left(\Omega_{b}^{-} \rightarrow\right.$ $\left.J / \psi \Omega^{-}\right)>1.9 \times 10^{-5}$. The predictions of $\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)$ and $\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)$clearly enable us to test the theoretical models, such as the QCD factorization approach in the $b$-baryon decays. © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP ${ }^{3}$.


## 1. Introduction

The LHCb Collaboration has recently published the measurements of the $b$-baryon $\left(\mathbf{B}_{b}\right)$ decays $[1-3]$, such as the charmful $\Lambda_{b}$ decays of $\Lambda_{b} \rightarrow \Lambda_{c}^{+}\left(K^{-}, \pi^{-}\right), \Lambda_{b} \rightarrow \Lambda_{c}^{+}\left(D^{-}, D_{s}^{-}\right), \Lambda_{b} \rightarrow$ $D^{0} p\left(K^{-}, \pi^{-}\right)$, and $\Lambda_{b} \rightarrow J / \psi p\left(K^{-}, \pi^{-}\right)$, which are important and interesting results. For example, while the $p \pi$ mass distribution in $\Lambda_{b} \rightarrow J / \psi p \pi^{-}$[2] suggests the existence of the higherwave baryon, such as $N(1520)$ or $N(1535)$, a peaking data point in the $D p$ mass distribution in $\Lambda_{b} \rightarrow D^{0} p\left(K^{-}, \pi^{-}\right)$[3] hints at the resonant $\Sigma_{c}(2880)$ state. On the other hand, it is typical to have the partial observations for the decay branching ratios, given by [4]

$$
\begin{align*}
\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right) f_{\Lambda_{b}} & =(5.8 \pm 0.8) \times 10^{-5} \\
\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right) f_{\Xi_{b}^{-}} & =\left(1.02_{-0.21}^{+0.26}\right) \times 10^{-5} \\
\mathcal{B}\left(\Omega_{b}^{-} \rightarrow J / \psi \Omega^{-}\right) f_{\Omega_{b}^{-}} & =\left(2.9_{-0.8}^{+1.1}\right) \times 10^{-6} \tag{1}
\end{align*}
$$

where $f_{\mathbf{B}_{b}}$ are the fragmentation fractions of the $b$ quark to $b$-baryons $\mathbf{B}_{b}=\Lambda_{b}, \Xi_{b}^{-}$and $\Omega_{b}^{-}$. The partial observations in Eq. (1) along with the measurements of the $\Xi_{b}^{0}$ decays [3-5] are due to the fact that $f_{\Lambda_{b}, \Xi_{b}^{-, 0}, \Omega_{b}^{-}}$are not well determined. In the assumption of $f_{\Lambda_{b}} \simeq f_{\text {baryon }}$ with $f_{\text {baryon }} \equiv \mathbf{B}(b \rightarrow$ all $b$-baryons $)$, it is often

[^0]adopted that $f_{\Lambda_{b}}=0.1[6,7] .{ }^{1}$ However, according to the recent observations of the relatively less decays associated with $\Xi_{b}^{-, 0}$ and $\Omega_{b}^{-}[8], f_{\Lambda_{b}} \simeq f_{\text {baryon }}$ is no longer true. As a result, it is urgent to improve the value of $f_{\Lambda_{b}}$ and obtain the less known ones of $f_{\Xi_{b}^{-, 0}}$.

Although it is possible to estimate $f_{\Lambda_{b}}$ by the ratio of $f_{\Lambda_{b}} /\left(f_{u}+\right.$ $f_{d}$ ) with $f_{u, d, s} \equiv \mathcal{B}\left(b \rightarrow B^{-}, \bar{B}^{0}, \bar{B}_{s}^{0}\right)$, different measurements on $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ are not in good agreement, given by
$f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)=0.281 \pm 0.012(\text { stat })_{-0.056}^{+0.058}(\mathrm{sys})_{-0.087}^{+0.128}(\mathrm{Br})[9]$,
$f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)=0.125 \pm 0.020[4]$,
with the uncertainty related to Br due to the uncertainties on the measured branching ratios, where the first relation given by the CDF Collaboration [9] is obviously twice larger than the world averaged value of the second one [4], dominated by the LEP measurements on Z decays. Moreover, since the recent measurements by the LHCb Collaboration also indicate this inconsistency [10-12], it is clear that the values of $f_{\Lambda_{b}}$ and $f_{\Xi_{b}^{0,-}}$ cannot be experimentally determined yet. In this paper, we will demonstrate the possible range for $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ in accordance with the measurements by LEP, CDF and LHCb Collaborations and give the theoretical estimations of $f_{\Lambda_{b}}$ and $f_{\Xi_{b}^{0,-}}$, which allow us to extract $\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)$, $\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)$, and $\mathcal{B}\left(\Omega_{b}^{-} \rightarrow J / \psi \Omega^{-}\right)$from the data in Eq. (1).

[^1]Consequently, we are able to test the theoretical approach based on the factorization ansatz, which have been used to calculate the two-body $\mathbf{B}_{b}$ decays [7,13-19].

## 2. Estimations of $f_{\Lambda_{b}}$ and $f_{\Xi_{b}^{-}, 0}$

Experimentally, in terms of the specific cases of the charmful $\Lambda_{b} \rightarrow \Lambda_{c}^{+} \pi^{-}$and $\bar{B}^{0} \rightarrow D^{+} \pi^{-}$decays or the semileptonic $\Lambda_{b} \rightarrow \Lambda_{c}^{+} \mu^{-} \bar{\nu} X$ and $\bar{B} \rightarrow D \mu^{-} \bar{\nu} X$ decays detected with the bins of $p_{T}$ and $\eta$, where $p_{T}$ is the transverse momentum and $\eta=$ $-\ln (\tan \theta / 2)$ is the pseudorapidity defined by the polar angle $\theta$ with respect to the beam direction [9-11], the ratio of $f_{\Lambda_{b}} /\left(f_{u}+\right.$ $f_{d}$ ) can be related to $p_{T}$ and $\eta$. This explains the inconsistency between the results from CDF and LEP with $p_{T}=15$ and 45 GeV , respectively. While $f_{s} / f_{u}$ is measured with slightly dependences on $p_{T}$ and $\eta$ [20], $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ is fitted as the linear form in Ref. [10] with $p_{T}=0-14 \mathrm{GeV}$ and the exponential form in Refs. [11,12] with $p_{T}=0-50 \mathrm{GeV}$, respectively, for the certain range of $\eta$.

### 2.1. The present status of $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$

With the semileptonic $\Lambda_{b} \rightarrow \Lambda_{c}^{+} \mu^{-} \bar{\nu} X$ and $\bar{B} \rightarrow D \mu^{-} \bar{\nu} X$ decays, the LHCb Collaboration has shown the dependence of $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ on $p_{T}$ in the range of $p_{T}=0-14 \mathrm{GeV}$ to be the linear form, given by [11]

$$
\begin{align*}
& f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right) \\
& \quad=(0.404 \pm 0.017(\text { stat }) \pm 0.027(\text { syst }) \pm 0.105(\mathrm{Br})) \\
& \quad\left(1-[0.031 \pm 0.004(\text { stat }) \pm 0.003(\text { syst })] p_{T}\right) \tag{3}
\end{align*}
$$

where Br arises from the absolute scale uncertainty due to the poorly known branching ratio of $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)$. By averaging $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ with $p_{T}=0-14 \mathrm{GeV}$, we obtain
$\bar{f}_{\Lambda_{b}}=(0.316 \pm 0.087)\left(f_{u}+f_{d}\right)$,
which agrees with the first relation in Eq. (2) given by the CDF Collaboration with $p_{T} \simeq 15 \mathrm{GeV}$. On the other hand, with the charmful $\Lambda_{b} \rightarrow \Lambda_{c}^{+} \pi^{-}$and $\bar{B}^{0} \rightarrow D^{+} \pi^{-}$decays, another analysis by the LHCb Collaboration presents the exponential dependence of $f_{\Lambda_{b}} / f_{d}$ on $p_{T}[11,12]:$

$$
\begin{align*}
f_{\Lambda_{b}} / f_{d}= & (0.151 \pm 0.030) \\
& +\exp \left\{-(0.57 \pm 0.11)-(0.095 \pm 0.016) p_{T}\right\} \tag{5}
\end{align*}
$$

with the wider range of $p_{T}=0-50 \mathrm{GeV}$. By averaging the value in Eq. (5) with $p_{T}=0-50 \mathrm{GeV}$, we find
$\bar{f}_{\Lambda_{b}}=(0.269 \pm 0.040) f_{d}=(0.135 \pm 0.020)\left(f_{u}+f_{d}\right)$,
with $f_{u}=f_{d}$ due to the isospin symmetry, where the error has combined the uncertainties in Eq. (5). It is interesting to note that, as the relation in Eq. (5) with $p_{T}=0-50 \mathrm{GeV}$ overlaps $p_{T} \simeq 45 \mathrm{GeV}$ for the second relation from LEP in Eq. (2), its value of $\bar{f}_{\Lambda_{b}}=(0.135 \pm 0.020)\left(f_{u}+f_{d}\right)$ is close to the LEP result of $f_{\Lambda_{b}}=(0.125 \pm 0.020)\left(f_{u}+f_{d}\right)$. Apart form the values in Eqs. (4) and (6), the reanalyzed results by CDF and LHCb Collaborations give $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ to be $0.212 \pm 0.058$ and $0.223 \pm 0.022$ with the averaged $p_{T} \simeq 13$ and 7 GeV , respectively [12]. We hence make the assumption of
$R_{\Lambda_{b}} \equiv f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)=0.25 \pm 0.15$,
to cover the possible range in accordance with the measurements from the three Collaborations of LEP, CDF and LHCb, which will be used to estimate the values of $f_{\Lambda_{b}}$ and $f_{\Xi_{b}^{0,-}}$ in the following.


Fig. 1. The $\mathbf{B}_{b} \rightarrow \mathbf{B}_{n} J / \psi$ decays via the internal $W$-boson emission diagram.

### 2.2. Theoretical determination of $f_{\Xi_{b}^{-}} / f_{\Lambda_{b}}$

In principle, when the ratios of $f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)$ and $f_{\Xi_{b}^{0,-}} / f_{\Lambda_{b}}$ are both known, by adding the relations of $[4,20]$
$f_{u}+f_{d}+f_{s}+f_{\text {baryon }}=1$,
$f_{\text {baryon }} \simeq f_{\Lambda_{b}}+f_{\Xi_{b}^{-}}+f_{\Xi_{b}^{0}}$,
$f_{s}=(0.256 \pm 0.020) f_{d}$,
and $f_{u}=f_{d}$ as well as $f_{\Xi_{b}^{-}}=f_{\Xi_{b}^{0}}$ due to the isospin symmetry, we can derive the values of $f_{u}, f_{d}, f_{s}, f_{\Lambda_{b}}, f_{\Xi_{b}^{-}}$and $f_{\Xi_{b}^{0}}$. For $f_{\Xi_{b}^{-}} / f_{\Lambda_{b}}$, it was once given that
$f_{\Xi_{b}^{-}} / f_{\Lambda_{b}} \simeq f_{s} / f_{u}[8,21]$,
$f_{\Xi_{b}^{0}} / f_{\Lambda_{b}} \simeq 0.2$ [22],
where the first relation from Refs. $[8,21]$ requires the assumption of $R_{1} \equiv \mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right) / \mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right) \simeq 1$ [11], while the second one from Ref. [22] uses $R_{2} \equiv \mathcal{B}\left(\Xi_{b}^{0} \rightarrow \Xi_{c}^{+} \pi^{-}\right) / \mathcal{B}\left(\Lambda_{b} \rightarrow\right.$ $\left.\Lambda_{c}^{+} \pi^{-}\right) \simeq 1$ along with $R_{3} \equiv \mathcal{B}\left(\Xi_{c}^{+} \rightarrow p K^{-} \pi^{+}\right) / \mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)$ $\simeq 0.1$ from the naive Cabibbo factors. However, we note that the theoretical calculations provide us with more understanding of $b$-baryon decays, such as the difference between the $\Lambda_{b} \rightarrow \Lambda$ and $\Xi_{b}^{-} \rightarrow \Xi^{-}$transitions, based on the $S U(3)$ flavor and $S U(2)$ spin symmetries. As a result, the assumption of $R_{1}=R_{2} \simeq 1$ might be too naive. Since the theoretical approach with the factorization ansatz well explains $\mathcal{B}\left(\Lambda_{b} \rightarrow p \pi^{-}\right)$and $\mathcal{B}\left(\Lambda_{b} \rightarrow p K^{-}\right)$, and particularly the ratio of $\mathcal{B}\left(\Lambda_{b} \rightarrow p \pi^{-}\right) / \mathcal{B}\left(\Lambda_{b} \rightarrow p K^{-}\right) \sim 0.84$ [23], it can be reliable to determine $f_{\Xi_{b}^{-}} / f_{\Lambda_{b}}$.

Theoretically, we use the factorization approach to calculate the two-body $b$-baryon decay, such that the amplitude corresponds to the decaying process of the $\mathbf{B}_{b} \rightarrow \mathbf{B}_{n}$ transition with the recoiled meson. Explicitly, as shown in Fig. 1, where the $W$-boson emission is internal, the amplitude via the quark-level $b \rightarrow c \bar{c} s$ transition can be factorized as

$$
\begin{align*}
& \mathcal{A}\left(\mathbf{B}_{b} \rightarrow \mathbf{B}_{n} J / \psi\right) \\
& \quad=\frac{G_{F}}{\sqrt{2}} V_{c b} V_{c s}^{*} a_{2}\langle J / \psi| \bar{c} \gamma^{\mu}\left(1-\gamma_{5}\right) c|0\rangle\left\langle\mathbf{B}_{n}\right| \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b\left|\mathbf{B}_{b}\right\rangle, \tag{10}
\end{align*}
$$

for $\Lambda_{b} \rightarrow \Lambda J / \psi$ or $\Xi_{b}^{-} \rightarrow \Xi^{-} J / \psi$, where the parameter $a_{2}$ is given by $[24,25]$
$a_{2}=c_{2}^{e f f}+\frac{c_{1}^{e f f}}{N_{c}}$,
with the effective Wilson coefficients $\left(c_{1}^{\text {eff }}, c_{2}^{\text {eff }}\right)=(1.168,-0.365)$. Note that the color number $N_{c}$ originally being equal to 3 in the naive factorization, which gives $a_{2}=0.024$ in Eq. (11), should be taken as a floating number from $2 \rightarrow \infty$ to account for the non-factorizable effects in the generalized factorization. The matrix element for the $J / \psi$ production is given by $\langle J / \psi| \bar{c} \gamma_{\mu} c|0\rangle=$
$m_{J / \psi} f_{J / \psi} \varepsilon_{\mu}^{*}$ with $m_{J / \psi}, f_{J / \psi}$, and $\varepsilon_{\mu}^{*}$ as the mass, decay constant, and polarization vector, respectively. The matrix elements of the $\mathbf{B}_{b} \rightarrow \mathbf{B}_{n}$ baryon transition in Eq. (10) have the general forms:

$$
\begin{align*}
& \left\langle\mathbf{B}_{n}\right| \bar{q} \gamma_{\mu} b\left|\mathbf{B}_{b}\right\rangle=\bar{u}_{\mathbf{B}_{n}}\left[f_{1} \gamma_{\mu}+\frac{f_{2}}{m_{\mathbf{B}_{b}}} i \sigma_{\mu \nu} q^{\nu}+\frac{f_{3}}{m_{\mathbf{B}_{b}}} q_{\mu}\right] u_{\mathbf{B}_{b}}, \\
& \left\langle\mathbf{B}_{n}\right| \bar{q} \gamma_{\mu} \gamma_{5} b\left|\mathbf{B}_{b}\right\rangle=\bar{u}_{\mathbf{B}_{n}}\left[g_{1} \gamma_{\mu}+\frac{g_{2}}{m_{\mathbf{B}_{b}}} i \sigma_{\mu \nu} q^{\nu}+\frac{g_{3}}{m_{\mathbf{B}_{b}}} q_{\mu}\right] \gamma_{5} u_{\mathbf{B}_{b}}, \tag{12}
\end{align*}
$$

where $f_{j}\left(g_{j}\right)(j=1,2,3)$ are the form factors, with $f_{2,3}=0$ and $g_{2,3}=0$ due to the helicity conservation, as derived in Refs. [7, $14,26]$. It is interesting to note that, as the helicity-flip terms, the theoretical calculations from the loop contributions to $f_{2,3}\left(g_{2,3}\right)$ indeed result in the values to be one order of magnitude smaller than that to $f_{1}\left(g_{1}\right)$, which can be safely neglected. In the doublepole momentum dependences, one has that [23]
$F\left(q^{2}\right)=\frac{F(0)}{\left(1-q^{2} / m_{\mathbf{B}_{b}}^{2}\right)^{2}} \quad\left(F=f_{1}, g_{1}\right)$.
We are able to relate different $\mathbf{B}_{b} \rightarrow \mathbf{B}_{n}$ transition form factors based on $S U(3)$ flavor and $S U(2)$ spin symmetries, which have been used to connect the space-like $\mathbf{B}_{n} \rightarrow \mathbf{B}_{n}^{\prime}$ transition form factors in the neutron decays [27], and the time-like $0 \rightarrow \mathbf{B}_{n} \overline{\mathbf{B}}_{n}^{\prime}$ baryonic as well as $B \rightarrow \mathbf{B}_{n} \overline{\mathbf{B}}_{n}^{\prime}$ transition form factors in the baryonic $B$ decays [28-32]. As a result, we obtain $\left(f_{1}(0), g_{1}(0)\right)=$ $(C, C),(-\sqrt{2 / 3} C,-\sqrt{2 / 3} C)$, and $(0,0)$ with $C$ a constant for $\langle p| \bar{u} \gamma_{\mu}\left(\gamma_{5}\right) b\left|\Lambda_{b}\right\rangle,\langle\Lambda| \bar{s} \gamma_{\mu}\left(\gamma_{5}\right) b\left|\Lambda_{b}\right\rangle$, and $\left\langle\Sigma^{0}\right| \bar{s} \gamma_{\mu}\left(\gamma_{5}\right) b\left|\Lambda_{b}\right\rangle$, which are the same as those in Ref. [26] based on the heavy-quark and large-energy symmetries for the $\Lambda_{b} \rightarrow\left(p, \Lambda, \Sigma^{0}\right)$ transitions, respectively. In addition, we have $f_{1}(0)=g_{1}(0)=C$ for $\left\langle\Xi^{-}\right| \bar{s} \gamma_{\mu}\left(\gamma_{5}\right) b\left|\Xi_{b}^{-}\right\rangle$. To obtain the branching ratio for the two-body decays, the equation is given by [4]
$\mathcal{B}\left(\mathbf{B}_{b} \rightarrow J / \psi \mathbf{B}_{n}\right)=\frac{\Gamma\left(\mathbf{B}_{b} \rightarrow J / \psi \mathbf{B}_{n}\right) \tau_{\mathbf{B}_{b}}}{6.582 \times 10^{-25}}$,
with $\tau_{\mathbf{B}_{b}}$ the life time, where
$\Gamma\left(\mathbf{B}_{b} \rightarrow J / \psi \mathbf{B}_{n}\right)=\frac{\left|\vec{P}_{J / \psi}\right|}{8 \pi m_{\mathbf{B}_{b}}^{2}}\left|\mathcal{A}\left(\mathbf{B}_{b} \rightarrow J / \psi \mathbf{B}_{n}\right)\right|^{2}$,
with $\left|\vec{P}_{J / \psi}\right|=\left|\vec{P}_{\mathbf{B}_{n}}\right|=\left\{\left[m_{\mathbf{B}_{b}}^{2}-\left(m_{J / \psi}+m_{\mathbf{B}_{n}}\right)^{2}\right]\left[m_{\mathbf{B}_{b}}^{2}-\left(m_{J / \psi}-\right.\right.\right.$ $\left.\left.\left.m_{\mathbf{B}_{n}}\right)^{2}\right]\right\}^{1 / 2} /\left(2 m_{\mathbf{B}_{b}}\right)$. As a result, we obtain
$\frac{\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)}{\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)}=\frac{\tau_{\Xi_{b}^{-}}}{\tau_{\Lambda_{b}}} \frac{C^{2}}{(-\sqrt{2 / 3 C})^{2}}=1.63 \pm 0.04$,
with $\tau_{\Xi_{b}^{-}} / \tau_{\Lambda_{b}}=1.089 \pm 0.026 \pm 0.011$ [33]. We note that, theoretically, $R_{1}=1.63$ apparently deviates by $63 \%$ from $R_{1}=1$ in the simple assumption. To determine $f_{\Xi_{b}^{-}} / f_{\Lambda_{b}}$, we relate Eq. (16) to (1) to give
$f_{\Xi_{b}^{-}}=(0.108 \pm 0.034) f_{\Lambda_{b}}$,
which is different from the numbers in Eq. (9).

### 2.3. Determinations of $f_{\Xi_{b}^{-, 0}}$ and $f_{\Lambda_{b}}$

According to Eqs. (4), (7), (8) and (17), we derive the values of $f_{u}, f_{d}, f_{s}, f_{\Lambda_{b}}, f_{\Xi_{b}^{-}}$and $f_{\Xi_{b}^{0}}$ in Table 1, which agree with the data in the PDG [4]. Note that $f_{\Omega_{b}^{-}}<0.108$ is from the error in $f_{\text {baryon }}$. In addition, $f_{\text {baryon }}=0.213 \pm 0.108$, which overlaps $0.089 \pm$ 0.015 from Z-decays [4] and $0.237 \pm 0.067$ from Tevatron [4], is

Table 1
Results of $f_{i}\left(i=u, d, s\right.$, baryon, $\Lambda_{b}, \Xi_{b}^{-, 0}$, and $\left.\Omega_{b}^{-}\right)$, compared with those from Z-decays and Tevatron in PDG [4].

|  | Our result | Z-decays [4] | Tevatron [4] |
| :--- | :--- | :--- | :--- |
| $f_{u}=f_{d}$ | $0.349 \pm 0.037$ | $0.404 \pm 0.009$ | $0.330 \pm 0.030$ |
| $f_{s}$ | $0.089 \pm 0.018$ | $0.103 \pm 0.009$ | $0.103 \pm 0.012$ |
| $f_{\text {baryon }}$ | $0.213 \pm 0.108$ | $0.089 \pm 0.015$ | $0.237 \pm 0.067$ |
| $f_{\Lambda_{b}}$ | $0.175 \pm 0.106$ | - | - |
| $f_{\Xi_{b}^{-}}=f_{\Xi_{b}^{0}}$ | $0.019 \pm 0.013$ | - | - |
| $f_{\Omega_{b}^{-}}$ | $<0.108$ | - | - |

due to the assumption of $R_{\Lambda_{b}}=0.25 \pm 0.15$ in Eq. (7) to cover the possible range from the data. Similarly, $f_{\Lambda_{b}}=0.175 \pm 0.106$ overlaps $f_{\Lambda_{b}}=0.07$ from the LEP measurements [34], while $f_{\Xi_{b}^{-}}=$ $f_{\Xi_{b}^{0}}=0.019 \pm 0.013$ is consistent with $f_{\Xi_{b}^{-}}=0.011 \pm 0.005$ from the measurement [35]. We hence convert the data in Eq. (1) to be

$$
\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)=(3.3 \pm 2.1) \times 10^{-4}
$$

$\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)=(5.3 \pm 3.9) \times 10^{-4}$,
$\mathcal{B}\left(\Omega_{b}^{-} \rightarrow J / \psi \Omega^{-}\right)>1.9 \times 10^{-5}$,
with $\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right) \simeq 1.6 \mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)$ to be in accordance with Eq. (16). With the use of $f_{\Xi_{b}^{0,-}}$, we can also estimate the $\Xi_{b}^{0,-}$ decays [4,5], given by

$$
\begin{align*}
\mathcal{B}\left(\Xi_{b}^{-} \rightarrow \Xi^{-} \ell^{-} \bar{v}_{\ell} X\right) & =(2.1 \pm 1.5) \times 10^{-2}, \\
\mathcal{B}\left(\Xi_{b}^{0} \rightarrow \bar{K}^{0} p \pi^{-}\right) & =(1.1 \pm 1.5) \times 10^{-5}, \\
\mathcal{B}\left(\Xi_{b}^{0} \rightarrow \bar{K}^{0} p K^{-}\right) & =(1.1 \pm 1.1) \times 10^{-5}, \\
\mathcal{B}\left(\Xi_{b}^{0} \rightarrow D^{0} p K^{-}\right) & =(9.5 \pm 9.4) \times 10^{-5}, \\
\mathcal{B}\left(\Xi_{b}^{0} \rightarrow \Lambda_{c}^{+} K^{-}\right) & =(4.2 \pm 4.7) \times 10^{-5} . \tag{19}
\end{align*}
$$

### 2.4. Test of the non-factorizable effects

To numerically test the non-factorizable effects, the CKM matrix elements in the Wolfenstein parameterization are taken as $\left(V_{c b}, V_{c s}\right)=\left(A \lambda^{2}, 1-\lambda^{2} / 2\right)$ with $(\lambda, A)=(0.225,0.814)[4]$, while $f_{J / \psi}=418 \pm 9 \mathrm{MeV}$ [36]. The constant value of $C$ in Ref. [23] is fitted to be $C=0.136 \pm 0.009$ to explain the branching ratios and predict the CP violating asymmetries of $\Lambda_{b} \rightarrow p\left(K^{-}, \pi^{-}\right)$, which is also consistent with the value of $0.14 \pm 0.03$ in the light-cone sum rules [26] and those in Refs. [7,14].

To explain the branching ratios of $\Lambda_{b} \rightarrow J / \psi \Lambda$ and $\Xi_{b}^{-} \rightarrow$ $J / \psi \Xi^{-}$in Eq. (18), the floating color number $N_{c}$ is evaluated to be
$N_{c}=2.15 \pm 0.17$,
which corresponds to $a_{2}=0.18 \pm 0.04$, in comparison with $a_{2}=$ 0.024 for $N_{c}=3$. Note that since $N_{c}=2.15$ in Eq. (20) is not far from 3, we conclude that the non-factorizable effects are controllable. As a result, the theoretical approach based on the factorization ansatz is demonstrated to be reliable to explain the two-body $\mathbf{B}_{b}$ decays.

## 3. Conclusions

In sum, we made the assumption of $R_{\Lambda_{b}}=f_{\Lambda_{b}} /\left(f_{u}+f_{d}\right)=$ $0.25 \pm 0.15$, which is in accordance with the measurements by LEP, CDF and LHCb Collaborations. We have estimated that $f_{\Lambda_{b}}=$ $0.175 \pm 0.106$ and $f_{\Xi_{b}^{-, 0}}=0.019 \pm 0.013$, which can be used to extract the branching ratios of the anti-triplet $b$-baryon decays.

Explicitly, we have found $\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)=(3.3 \pm 2.1) \times 10^{-4}$, $\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)=(5.3 \pm 3.9) \times 10^{-4}$ and $\mathcal{B}\left(\Omega_{b}^{-} \rightarrow J / \psi \Omega^{-}\right)>$ $1.9 \times 10^{-5}$. We have also demonstrated that the predictions of $\mathcal{B}\left(\Lambda_{b} \rightarrow J / \psi \Lambda\right)$ and $\mathcal{B}\left(\Xi_{b}^{-} \rightarrow J / \psi \Xi^{-}\right)$would help us to test the theoretical models, such as the factorization approach.

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## References

[1] R. Aaij, et al., LHCb Collaboration, Phys. Rev. Lett. 112 (2014) 202001.
[2] R. Aaij, et al., LHCb Collaboration, J. High Energy Phys. 1407 (2014) 103.
[3] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 89 (2014) 032001.
[4] K.A. Olive, et al., Particle Data Group Collaboration, Chin. Phys. C 38 (2014) 090001.
[5] R. Aaij, et al., LHCb Collaboration, J. High Energy Phys. 1404 (2014) 087.
[6] J. Abdallah, et al., DELPHI Collaboration, Phys. Lett. B 585 (2004) 63.
[7] T. Gutsche, et al., Phys. Rev. D 88 (2013) 114018.
[8] I. Heredia-De La Cruz, D0 Collaboration, arXiv:1109.6083 [hep-ex].
[9] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. D 77 (2008) 072003.
[10] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 85 (2012) 032008.
[11] R. Aaij, et al., LHCb Collaboration, J. High Energy Phys. 1408 (2014) 143.
[12] Y. Amhis, et al., HFAG Collaboration, arXiv:1412.7515 [hep-ex].
[13] Y. Liu, X.H. Guo, C. Wang, Phys. Rev. D 91 (2015) 016006.
[14] Z.T. Wei, H.W. Ke, X.Q. Li, Phys. Rev. D 80 (2009) 094016.
[15] Fayyazuddin Riazuddin, Phys. Rev. D 58 (1998) 014016.
[16] C.H. Chou, H.H. Shih, S.C. Lee, H.n. Li, Phys. Rev. D 65 (2002) 074030.
[17] M.A. Ivanov, et al., Phys. Rev. D 57 (1998) 5632.
[18] H.Y. Cheng, Phys. Rev. D 56 (1997) 2799.
[19] A. Ahmed, arXiv:1106.0740 [hep-ph].
[20] R. Aaij, et al., LHCb Collaboration, J. High Energy Phys. 1304 (2013) 001.
[21] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. Lett. 99 (2007) 052001.
[22] R. Aaij, et al., LHCb Collaboration, Phys. Rev. Lett. 113 (2014) 032001.
[23] Y.K. Hsiao, C.Q. Geng, Phys. Rev. D 91 (2015) 116007.
[24] Y.H. Chen, et al., Phys. Rev. D 60 (1999) 094014; H.Y. Cheng, K.C. Yang, Phys. Rev. D D62 (2000) 054029.
[25] A. Ali, G. Kramer, C.D. Lu, Phys. Rev. D 58 (1998) 094009.
[26] A. Khodjamirian, et al., J. High Energy Phys. 1109 (2011) 106; T. Mannel, Y.M. Wang, J. High Energy Phys. 1112 (2011) 067.
[27] G.P. Lepage, S.J. Brodsky, Phys. Rev. Lett. 43 (1979) 545; G.P. Lepage, S.J. Brodsky, Phys. Rev. Lett. 43 (1979) 1625 (Erratum).
[28] C.K. Chua, W.S. Hou, S.Y. Tsai, Phys. Rev. D 66 (2002) 054004.
[29] C.K. Chua, W.S. Hou, Eur. Phys. J. C 29 (2003) 27.
[30] C.H. Chen, H.Y. Cheng, C.Q. Geng, Y.K. Hsiao, Phys. Rev. D 78 (2008) 054016.
[31] C.Q. Geng, Y.K. Hsiao, Phys. Rev. D 85 (2012) 017501.
[32] Y.K. Hsiao, C.Q. Geng, Phys. Rev. D 91 (2015) 077501.
[33] R. Aaij, et al., LHCb Collaboration, Phys. Rev. Lett. 113 (2014) 242002.
[34] M. Galanti, et al., arXiv:1505.02771 [hep-ph].
[35] J. Abdallah, et al., DELPHI Collaboration, Phys. Lett. B 576 (2003) 29.
[36] D. Becirevic, et al., Nucl. Phys. B 883 (2014) 306.


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[^1]:    ${ }^{1} f_{\text {baryon }} \sim 0.1$ was also taken in the previous versions of the PDG.

