

The modified propagation equation for TM polarized subwavelength spatial solitons in a nonlinear planar waveguide

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Abstract

The wave equation of TM polarized subwavelength beam propagations in a nonlinear planar waveguide is derived beyond the paraxial approximation. This modified equation contains more higher-order linear and nonlinear terms than the nonlinear Schrödinger equation. The propagation of fundamental subwavelength spatial solitons is numerically studied. It is shown that the effect of the higher nonlinear terms is significant. That is, for the propagation of narrower beam the modified nonlinear Schrödinger equation is more suitable than the nonlinear Schrödinger equation.

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1. Introduction

Spatial solitons which are the balance of the diffraction and the self-focusing have been studied both theoretically and experimentally in a nonlinear planar waveguide [1,2]. Generally, the nonlinear Schrödinger equation (NLSE) making the paraxial approximation can describe well the propagation of spatial solitons. If the beam width of spatial solitons is as narrow as one wavelength or less, the validity of the paraxial approximation becomes questionable. The full-vector nonlinear Maxwell's equations were used to avert this problem [3], but it is very time consuming. In addition, to enhance the validity of the wave equation, the modified NLSEs are presented [4–7]. The additional terms including a polarization-dependent correction to the soliton propa-

gation constant were considered, the dynamics of a narrow spatial soliton with an arbitrary polarization were affected [4]. The solution of a modified NLSE describing the electric field for TM mode was found, they analyzed the effects of those addition terms on the shapes of bright and dark solitons of TM mode with a fixed polarization [5]. Beyond the paraxial approximation, a modified NLSE describing the electric field for subwavelength TE solitons was derived and an analytical solution for the soliton was found [6]. Very narrow solitons in (1+1)-dimensional and (2+1) dimensional versions of cubic–quintic and full saturable models were analyzed [7]. For the solitons of TE and TM polarizations, it was shown that there is always a finite minimum of the soliton's width, and the solitons cease to exist at a critical value of the propagation constant, at which their width diverges.

In this paper, we will derive the propagation equation in a nonlinear planar waveguide by the iteration method and the order of magnitude method [6,8,9]. The derived equation contains more higher-order linear

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and nonlinear terms than the NLSE. It is different to the one for the propagation of TE polarized subwavelength beams [6]. We also numerically study the propagation of TM polarized subwavelength beams in a nonlinear planar waveguide. It is found that the fundamental spatial soliton is not stable due to these higher-order terms and the effect of the higher nonlinear terms is significant and must be considered.

2. Derivation of the wave equations

We now derive the wave equation which can describe the propagation of TM polarized subwavelength beams in a nonlinear planar waveguide. The electric field \mathbf{E} of the light obeys the vector wave equation

$$\nabla^2 \mathbf{E} - \frac{\omega^2 n_0^2}{c^2} \mathbf{E} + \frac{\omega^2}{c^2 \varepsilon_0} \mathbf{P}_{\text{NL}} + \frac{1}{n_0^2 \varepsilon_0} \nabla(\nabla \cdot \mathbf{P}_{\text{NL}}) = 0 \quad (1)$$

which follows from the Maxwell equations, where ε_0 is the vacuum permittivity, n_0 is linear refractive index, ω is the light frequency, c is the velocity of light in vacuum, and \mathbf{P}_{NL} is the third-order nonlinear polarization and

$$(\mathbf{P}_{\text{NL}})_i = \frac{3\varepsilon_0}{4} \sum_{j,k,l} \chi_{ij,kl}^{(3)}(\omega = \omega_j + \omega_k - \omega_l) E_j E_k E_l^*.$$

Here $\chi^{(3)}(\omega)$ is the third-order susceptibility, i, j, k and l refer to the Cartesian components of the fields. In the following, we will derive the wave equations which describe the propagations of subwavelength beams in a nonlinear planar waveguide.

The electric field of the light can be written as

$$E(x, y, z) = [\hat{x}A_x(x, z) + \hat{z}A_z(x, z)]F(y) \exp(ik_0 z), \quad (2)$$

where $A_x(x, z)$ and $A_z(x, z)$ are slowly varying amplitude envelopes; $F(y)$ is the normalized linear eigenfunction of the mode excited in the nonlinear planar waveguide; $k_0 = n_0 \omega / c$ is the propagation constant, n_0 is linear refractive index, ω is the light frequency, and c is the velocity of light in vacuum. The total refractive index is given by $n = n_0 + n_2 |E|^2$, where n_2 is the Kerr coefficient and $n_2 = 3\chi_{xxxx}^{(3)} / 8n_0$. Substituting Eq. (2) into Eq. (1), we obtain

$$\begin{aligned} i \frac{\partial}{\partial z} A_x + \frac{1}{2k_0} \frac{\partial^2}{\partial x^2} A_x \\ + \gamma |A_x|^2 A_x = -\frac{1}{2k_0} \frac{\partial^2}{\partial z^2} A_x - \frac{\gamma}{k_0^2} \frac{\partial^2}{\partial x^2} (|A_x|^2 A_x) \\ - \frac{2\gamma}{3} |A_z|^2 A_x - \frac{\gamma}{3} (A_z)^2 A_x^*, \end{aligned} \quad (3a)$$

$$\begin{aligned} i \frac{\partial}{\partial z} A_z + \frac{1}{2k_0} \frac{\partial^2}{\partial x^2} A_z \\ + \gamma |A_z|^2 A_z = -\frac{1}{2k_0} \frac{\partial^2}{\partial z^2} A_z \\ - \frac{2\gamma}{3} |A_x|^2 A_z - \frac{\gamma}{3} (A_x)^2 A_z^*, \end{aligned} \quad (3b)$$

where $\gamma = k_0 n_2 / n_0$. For the weakly guided mode, $|A_z| \ll |A_x|$. The relation

$$A_z = \frac{i}{k_0} \frac{\partial A_x}{\partial x} \approx i\sigma A_x$$

is obtained from $\nabla \cdot \mathbf{D} = 0$, where \mathbf{D} is the electric displacement. Therefore, Eq. (3a) can be rewritten as

$$\begin{aligned} i \frac{\partial}{\partial z} A_x + \frac{1}{2k_0} \frac{\partial^2}{\partial x^2} A_x + \gamma |A_x|^2 A_x = -\frac{1}{2k_0} \frac{\partial^2}{\partial z^2} A_x \\ - \frac{\gamma}{k_0^2} \left[\frac{\partial^2}{\partial x^2} (|A_x|^2 A_x) \right. \\ \left. + \frac{2}{3} \left| \frac{\partial}{\partial x} A_x \right|^2 A_x - \frac{1}{3} \left(\frac{\partial}{\partial x} A_x \right)^2 A_x^* \right] \end{aligned} \quad (4)$$

and Eq. (3b) can be neglected. To normalize Eq. (4), we make the following transformations:

$$\begin{aligned} A_x(x, z) = \frac{\sqrt{P_0}}{N} u(\eta, \zeta) = \sigma \sqrt{\frac{n_0}{n_2}} u(\eta, \zeta), \\ x = w_0 \eta, \\ z = L_d \zeta, \end{aligned} \quad (5)$$

where the parameter

$$N = \left[\frac{n_2 P_0}{k_0^2 w^2 n_0} \right]^{1/2}$$

is the order of the spatial soliton, $N = 1$ for the fundamental soliton, $w_0 = w_F / 1.763$ and w_F is the full-width at the half-maximum (FWHM) of the beam, P_0 is peak power of the incident beam, the parameter $\sigma = 1 / (k_0 w_0) = 0.28(\lambda_0 / w_F)$, $\lambda_0 = 2\pi / k_0$ is the wavelength in the waveguide, and $L_d = k_0 w_0^2$ is the diffraction length. Eq. (4) can be normalized to

$$\begin{aligned} \frac{\partial u}{\partial \zeta} = \frac{i}{2} \frac{\partial^2}{\partial \eta^2} u + i|u|^2 u \\ + i\sigma^2 \left[\frac{\partial^2}{\partial \eta^2} (|u|^2 u) + \frac{2}{3} \left| \frac{\partial u}{\partial \eta} \right|^2 u - \frac{1}{3} \left(\frac{\partial u}{\partial \eta} \right)^2 u^* \right] \\ + \frac{i\sigma^2}{2} \frac{\partial^2}{\partial \zeta^2} u. \end{aligned} \quad (6)$$

Then we use the iteration method and the order of magnitude method to derive the propagation equation beyond the paraxial approximation. First, a zero order approximation equation is obtained from Eq. (6) neglected the $\partial^2 u / \partial \zeta^2$ term,

$$\frac{\partial u}{\partial \zeta} = \frac{i}{2} \frac{\partial^2}{\partial \eta^2} u + i|u|^2 u. \quad (7)$$

which is the well-known NLSE for spatial soliton propagations. For the first iteration, we differentiate

Eq. (7) with respect to ξ and obtain

$$\frac{\partial^2}{\partial \xi^2} = -\frac{1}{4} \frac{\partial^4}{\partial \eta^4} u - 2 \left(\frac{\partial^2 u}{\partial \eta^2} \right) |u|^2 - \left(\frac{\partial u}{\partial \eta} \right)^2 u^* - 2 \left| \frac{\partial u}{\partial \eta} \right|^2 u - |u|^4 u. \quad (8)$$

Substituting Eq. (8) into Eq. (6), we obtain the wave equation of the first-order approximation

$$\begin{aligned} \frac{\partial}{\partial \xi} u = & \frac{i}{2} \frac{\partial^2}{\partial \eta^2} u + i |u|^2 u \\ & + i \sigma^2 \left[-\frac{1}{8} \frac{\partial^4}{\partial \eta^4} u + \left(\frac{\partial^2 u}{\partial \eta^2} \right) |u|^2 + \left(\frac{\partial^2 u^*}{\partial \eta^2} \right) u^2 \right. \\ & \left. + \frac{7}{6} \left(\frac{\partial u}{\partial \eta} \right)^2 u^* + \frac{11}{3} \left| \frac{\partial^2 u}{\partial \eta^2} \right|^2 u - \frac{1}{2} |u|^4 u \right] \end{aligned} \quad (9)$$

For the second iteration, we differentiate Eq. (9) with respect to ξ and obtain an expression of $\partial^2 u / \partial \xi^2$ with higher order terms. Substituting this $\partial^2 u / \partial \xi^2$ backs into Eq. (6), we obtain the wave equation of the second-order approximation with σ^4 higher-order terms, which is the same as Eq. (9) up to σ^2 terms.

3. Solution and discussion

When the same σ^2 -order terms in Eq. (9) are ignored, Eq. (9) is approximated to NLSE. Surely, the fundamental spatial soliton is a solution of one. That is, the beam will maintain its shape unchanged after propagating long distance in a nonlinear planar waveguide if the beam width of spatial soliton is much wider than one wavelength. However, when the beam width of spatial soliton is as narrow as one wavelength or less, the higher-order terms cannot be neglected. To show the effects of higher-order terms and the necessity of the modified NLSE, we consider $w_F = 0.75\lambda_0$. And, Eq. (9) is solved by the split-step Fourier method with the initial condition $u(0, \eta) = \text{sech}(\eta)$. Fig. 1 shows the peak power and beam width versus propagation distance with and without higher order terms, respectively. Here z_0 is soliton period, $z_0 = (\pi/2)L_d$. One can see that the changes of the beam width and the peak power are very obviously. At the distance of $7z_0$, the beam width is about $1.86w_F$ and the peak power is down to $0.61P_0$ at the distance of $6z_0$. The changes are due to the effects of higher-order terms. Comparing to without higher-order terms, the results are apparently different. In other words, the propagation of narrower beam must describe by the modified NLSE containing higher-order terms.

4. Conclusion

In conclusion, we have derived an accurate wave equation beyond paraxial approximation by the itera-

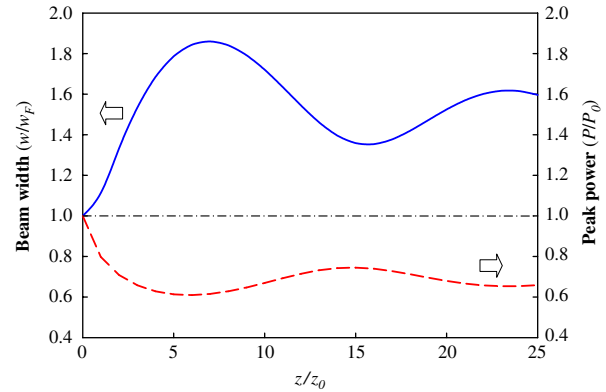


Fig. 1. Peak power (dashed curve) and beam width (solid curve) versus propagation distance simulated by the modified NLSE for $w_F = 0.75\lambda_0$. Dashed-dotted curve is peak power simulated by the NLSE.

tive method and the order of magnitude method for the TM polarized subwavelength optical beam propagation in a nonlinear planar waveguide. The derived equation contains higher-order linear and nonlinear terms than the NLSE. We numerically show that the fundamental subwavelength spatial soliton cannot maintain any more due to these higher-order terms. For $w_F = 0.75\lambda_0$, the changes of the beam width and the peak power are very obviously when the higher-order terms are considered. In other words, for the propagation of narrower beam, the modified NLSE is more suitable than the NLSE.

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