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Improved common-path optical heterodyne interferometer for measuring small optical rotation angle of chiral medium

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Abstract

An improved common-path optical heterodyne interferometry for measuring small optical rotation angle of chiral liquid is presented. The optical rotation angle can be determined by the phase difference between the test and reference signals. By means of inserting a phase retarder behind the test sample, the measuring resolution of optical rotation angle is greatly enhanced. The theoretical prediction indicates the resolution is better than 3.5×10^{-5} degree.

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1. Introduction

The chiral or optical activity medium has the ability to rotate the polarization plane of light. The measurement of the rotation angle with an optical polarimeter has existed for several years [1–9]. There are several successful methods such as high accuracy universal polarimetry (HAUP) [4,5] and heterodyne interferometry [6–9] to improve the measurement resolution of the optical rotation angle. The optical configuration of HAUP is the conventional polarimetry setup, and its method is based on a least-squares refinement of the transmitted light intensity as a function of the azimuthal angles of the polarizer and the analyzer. The optical rotation angle is obtained by the fitting parameters provided that the systematic errors have been removed. In heterodyne interferometry, the rotation angle is estimated by means of the measurement of the phase difference between two interference signals. Chou et al. [8] proposed the circular polarized

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optical heterodyne interferometer to effectively measure the optical rotation of a scattered chiral medium. In their works, the measurement resolution is about 10^{-3} degree, and a large amount of denser chiral medium is needed to make the optical rotation angle measurable. On the other hand, Lin et al. [9] presented a heterodyne Mach-Zehnder interferometer to enhance the measurement resolution, and their measurement resolution is about 6×10^{-5} degree. However, Lin's optical configuration and the associated algorithm are more complicated.

In this paper, we present an improved common-path optical heterodyne interferometer [10], which is similar to Chou's configuration [8] to measure the optical rotation angle. In our method, a phase retarder is inserted behind the test sample and the phase difference variation between the test signal and the reference signal is greatly enhanced. Benefiting from the common-path optical configuration and the heterodyne interferometric phase measurement, this method has the advantages of high stability against surrounding vibration, high measurement resolution, and relatively straightforward operation. The theoretical prediction indicates its resolution is better than 3.5×10^{-5} degree, which is almost 30 times better than that obtained with Chou's method. In addition, not only is our measurement resolution higher than that of Lin' s method, but also the optical configuration is more compact. The feasibility is demonstrated.

2. Principle

The schematic diagram of this improved common-path optical heterodyne interferometer is shown in Fig. 1. The light passing through an electro-optic modulator EOM is divided by a beam-splitter BS into two parts-the reflected light and the transmitted light. The reflected light passes through an analyzer AN_r and enters a photodetector D_r. If the amplitude of the light detected by D_r is E_r, then the intensity measured by D_r is $I_r = |E_r|^2$. It is regarded as the reference signal. On the other hand, the transmitted light passes through a half-wave plate H, a test sample S, a phase retarder R, an analyzer AN_t, and finally enters a photodetector D_t. If the amplitudes of the test light is E_t, then the intensity measured by D_t is $I_t = |E_t|^2$. It is regarded as the test signal.

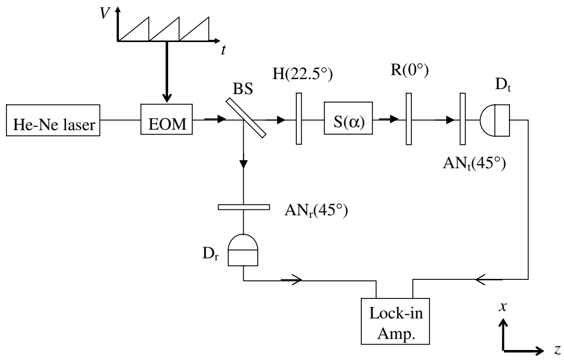

Fig. 1. Schematic diagram for the improved common-path optical heterodyne interferometer. EOM, electro-optic modulator; BS, beam splitter; H, half-wave plate; S, tested sample; R, phase retarder; AN, analyzer; D, photodetector.

For convenience, the z-axis is chosen along the propagation direction and the y-axis is along the vertical direction. Let the light coming from a laser be linearly polarized at 45° with respect to the x-axis, then its Jones vector can be written as

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{1}$$

If the fast axis of EOM is along the x-axis, and an external sawtooth voltage signal with angular frequency ω and amplitude $V_{\lambda/2}$ (the half-wave voltage of EOM) is applied to EOM, then the retardation produced by EOM can be expressed as ωt [9]. If the transmission axis of AN_r is 45° with respect to the x-axis, then we have

$$\begin{aligned} E_r &= AN_r \cdot EOM(\omega t) \cdot E_{in} \\ &= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \cos\left(\frac{\omega t}{2}\right) \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \end{aligned} \tag{2}$$

Hence, the intensity of the reference signal is

$$I_r = |E_r|^2 = \tfrac{1}{2}(1 + \cos \omega t). \tag{3}$$

On the other hand, if the fast axes of H and R, and the transmission axis of AN_t are set to 22.5°, 0°, and 45° with respect to the x-axis, respectively, then the Jones vector of the test beam is

$$E_t = AN_t \cdot R \cdot S(\alpha) \cdot H \cdot EOM(\omega t) \cdot E_{in}$$
$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$
$$\times \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= \frac{1}{2}\left[\left(\cos\alpha\cos\frac{\omega t}{2} + i\sin\alpha\sin\frac{\omega t}{2}\right)e^{\frac{i\delta}{2}} - \left(\sin\alpha\cos\frac{\omega t}{2} - i\cos\alpha\sin\frac{\omega t}{2}\right)e^{-\frac{i\delta}{2}}\right] \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (4)$$

where $S(\alpha)$ is the Jones matrix of the chiral medium [6], and α is its optical rotation angle, and δ is the phase retardation of R. The intensity of the test signal is given by

$$I_t = |E_t|^2 = \tfrac{1}{2}[1 - A\cos(\omega t + \phi)], \quad (5)$$

where

$$A = \sqrt{\sin^2\delta + (\sin 2\alpha\cos\delta)^2} \quad (6)$$

and

$$\phi = \tan^{-1}\left(\frac{\tan\delta}{\sin 2\alpha}\right). \quad (7)$$

If these two sinusoidal signals I_t and I_r are sent to a lock-in amplifier, then their phase difference ϕ can be obtained. The relationships between the phase difference ϕ and the optical rotation angle α for three different phase retardations (i.e., $\delta = 170°$, 175°, and 178°) are shown in Fig. 2. It is clear that the slope of the curve becomes steeper as the phase retardation increases. That is, a phase retarder with large phase retardation will be used to get high measurement resolution. From Fig. 2 or Eq. (7), it can be seen that $\phi = 90°$ as $\alpha = 0°$. Therefore, we define the phase difference variation ψ as

$$\psi = \phi(\alpha) - \phi(0^\circ) = \tan^{-1}\left(\frac{\tan\delta}{\sin 2\alpha}\right) - 90^\circ. \quad (8)$$

Consequently, Eq. (8) can be rewritten as

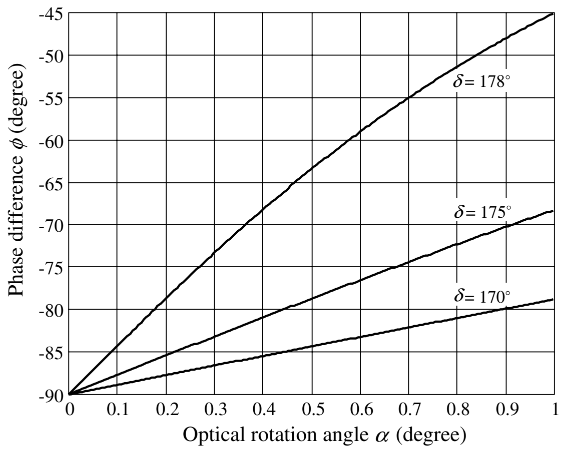

Fig. 2. The relationships between ϕ and α for three different phase retarders.

$$\alpha = -\frac{1}{2}\sin^{-1}(\tan\delta \cdot \tan\psi). \quad (9)$$

It is obvious from Eq. (9) that α can be calculated with the measurement of ψ under experimental conditions in which δ is specified.

3. Experiments and results

To show the validity of this method, a He–Ne laser with wavelength 632.8 nm modulated by an electro-optic modulator (New Focus, Model 4002) was used as a heterodyne light source. The frequency difference between p- and s-polarized components [10] was 1 kHz. The experimental condition $\delta = 178°$ was chosen for high measurement resolution. A half-wave plate is traditionally used as an optical rotator like a Faraday rotator. Therefore, we inserted another half-wave plate to replace the chiral medium S to simulate the optical rotation. The experimental results of ψ versus α are shown in Fig. 3, where the symbol "∘" represents the measured data and the solid curve represents the theoretical curve (curve A) calculated from Eq. (8). It is obvious that they match well. For comparison, the theoretical curves of ψ versus it α calculated by Lin's method (curve B) [9] and Chou's method (curve C) [8] are also shown in Fig. 3. It is clear that the slope of curve A is almost double that of curve B and 30× that of curve C. It

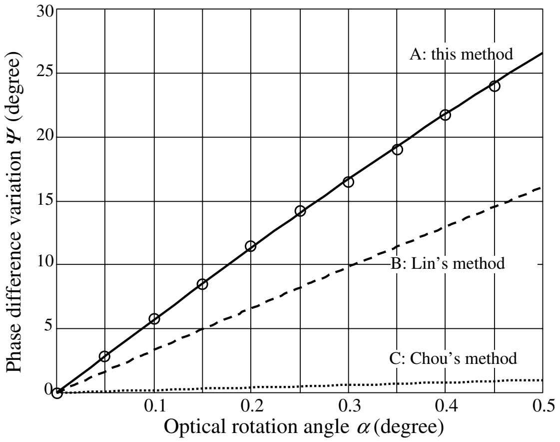

Fig. 3. Measurement results and theoretical curves of ψ versus α.

is thus seen that this method can enhance the measurement resolution.

Next, the optical rotation angles of six d-glucose solutions in different weight percent, namely, 0.1%, 0.3%, 0.5%, 0.7%, 0.9%, and 1% g/cc, were measured. Each solution was held in a quartz cell of 4 cm length. The measured data of ψ and their associated α values are listed in Table 1, where the reference optical rotation angles α_{ref} are also included for comparison. The reference optical rotation can be calculated with the equation [2,10,11]

$$\alpha_{ref} = \alpha_s C_g L, \tag{10}$$

where α_s is the specific rotation in degree/dm/(g/cc), C_g is the concentration in g/cc, and L is the cell length in decimeters, dm. The value of α_s can be

Table 1
Experimental results and the reference data

Concentration (g/cc)	ψ (°)	α (°)	α_{ref} (°)
0.1%	1.081	0.019	0.018
0.3%	3.022	0.053	0.054
0.5%	5.356	0.094	0.090
0.7%	6.823	0.120	0.125
0.9%	9.238	0.163	0.161
1.0%	10.051	0.177	0.179

ψ, phase difference variation; α, optical rotation angle; α_{ref}, reference optical rotation angle calculated from Eq. (10) and [9].

obtained from [9] and it is 44.8°/dm/g/cc at 633 nm wavelength.

4. Discussion

From Eqs. (5) and (6), it can be seen that the amplitude of the test signal depends on α and δ. In our experiments the condition $\delta = 178°$ was chosen, and the minimum contrast of the test signal approximated to 0.035 as $\alpha = 0°$. It is still enough for a lock-in amplifier (Stanford, Model SR850) to measure the phase difference in our experiments. If δ was chosen to approach to 179° or 180°, then the contrast of the test signal would be too low to be measured.

From Eq. (9), we can get

$$\Delta\alpha = -\frac{(\tan\delta \cdot \sec^2\psi)}{2\cos 2\alpha}\Delta\psi, \tag{11}$$

where $\Delta\alpha$ and $\Delta\psi$ are the errors in α and ψ, respectively. As α approaches zero, Eq. (11) can be rewritten as

$$\Delta\alpha \cong -\frac{\tan\delta}{2}\Delta\psi. \tag{12}$$

Considering the angular resolution of the lock-in amplifier, the second harmonic error and the polarization-mixing error, the total errors $\Delta\psi$ decrease to 0.002° in our experiments [9]. Substituting $\delta = 178°$ and $\Delta\psi = 0.002°$ into Eq. (12), we get $\Delta\alpha = 3.5 \times 10^{-5}$ degree.

If a scattered chiral medium is tested, an extra phase difference ϕ_s occurs. According to [8], it can be expressed as

$$\phi_s = \frac{n\omega}{c}\left(\frac{3\mu_s'}{4\mu_a}\right)^{1/2} L, \tag{13}$$

where n is the refractive index, c is the speed of light, μ_s' and μ_a are the reduced scattering and absorption coefficients, respectively, and L is the thickness of the scattering medium. If a polystyrene suspension with 1% vol. concentration, $\mu_s' = 0.3\ \mathrm{cm}^{-1}$, $\mu_a = 0.001\ \mathrm{cm}^{-1}$, and $n = 1.33$ is tested under our experimental conditions, we get $\psi_s \approx 10^{-4}$ degree. This value is far smaller than $\Delta\psi$ (0.002°), so the extra phase difference ϕ_s due to scattering can be ignored. Hence, our experimental conditions may be suitable for measuring a scattered chiral medium.

5. Conclusion

An improved common-path optical heterodyne interferometer for the measuring the small optical rotation angle of a chiral medium is presented. By means of inserting a phase retarder behind the test sample and the heterodyne interferometric phase detection technique, it demonstrates a measurement resolution better than 3.5×10^{-5} degree. In addition, this method has some merits such as simple optical configuration, real-time measurement, and high stability against surrounding vibration and air turbulence.

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