Determination of optical parameters of a twisted-nematic liquid crystal by phase-sensitive optical heterodyne interferometric ellipsometry

Chien-Chung Tsai, Chien Chou, Chien-Yuan Han, Cheng-Hung Hsieh, Kuo-Yu Liao, and Yu-Faye Chao

What is believed to be a novel phase-sensitive optical heterodyne interferometric ellipsometer is set up to characterize a twisted-nematic liquid crystal (TN-LC) by the elliptical parameters of the output polarization state. This ellipsometer presents the advantages of both polarized optical heterodyne interferometry and optical photometry, which introduce a polarization modulation that is capable of performing with high-sensitivity on phase detection in real time. The twist angle Φ and the untwisted phase retardation Γ of TN-LC are measured precisely. The experimental results verify that a TN-LC can be treated as identical to an elliptical retarder. © 2005 Optical Society of America

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1. Introduction

The optical properties of an anisotropic material can generally be divided into linear birefringence (LB) and circular birefringence (CB).[1,2] LB results from different phase velocities of two orthogonal linear-polarization eigenstates in material.[3] In contrast, CB is caused by different phase velocities of two orthogonal circular polarization eigenstates in the medium.[4] However, an anisotropic material, such as a twisted-nematic liquid crystal (TN-LC), shows its optical properties of LB and CB at the same time.[5,6] Then a pair of orthogonal elliptical polarizations becomes the eigenpolarizations of the TN-LC. In other words, a TN-LC[5,7–13] can be treated as an elliptical phase retarder theoretically. Therefore the optical properties of a TN-LC can be characterized in view of the elliptical birefringence by means of the phase retardation γ of the elliptical eigenpolarizations and the elliptical angle ε produced by two linearly polarized P and S waves of each elliptical polarization eigenstate emerging from the TN-LC. In the meantime, the untwisted phase retardation (which is related to cell thickness), twist angle, and the entrance orientation of molecular direction (the director angle) of a twisted-nematic cell are of great importance for LC display applications because those parameters are closely related to the optical quality of LC display devices.[14] The poor uniformity of cell thickness results in a decrease in picture quality. These parameters are usually determined from measuring the intensity transmittance of the TN-LC device sandwiched between two polarizers and from curve fitting of the theoretical prediction based on a Jones matrix to experimental data. However, a twist angle with 180° ambiguity and a director angle with 90° ambiguity resulted from this approach.[15] Kim and Lee[16] proposed a method of curve fitting by the Jones matrix to measure the TN-LC parameters with no ambiguity by measuring the intensity transmittance with and without a quarter-wave plate. In this paper, we propose what we believe to be a novel method in which only the phase retardation between orthogonal linearly polarized P and S waves emerging from the TN-LC in the polarized optical heterodyne interferometer is detected by a lock-in amplifier. This method is different from conventional methods that rely on intensity detection by a photometric technique to determine TN-LC parameters.[6,9,14] In contrast, phase detection by this optical heterodyne interferometry has the advantage of avoiding the ambiguities of twist angle and entrance director angle of a TN-LC. In addition, the detection sensitivity of

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phase retardation or birefringence of the TN-LC with this method is better than that of conventional photometric methods because of a higher signal-to-noise ratio (SNR) of optical heterodyne detection. However, photometric methods provide real-time measurement ability. Consequently, this proposed method of combining optical heterodyne interferometry with optical photometry by continuously rotating a TN-LC to generate the polarization modulation not only increases the SNR of the heterodyne signal but also provides the phase detection in real time. To characterize a TN-LC in terms of elliptical birefringence quantitatively, a polarized common-path optical heterodyne interferometer is set up, in which the phase lock-in technique is incorporated.[1,4,7] Thus the optical parameters of a TN-LC, such as twist angle Φ and untwisted phase retardation Γ, can be determined by this method when the pretilt angle of the TN-LC is at 0°, where no electric field is applied.[8] Both Φ and Γ are measured by analysis of the state of output elliptical polarization from the TN-LC when a linearly polarized laser beam is incident upon the TN-LC perpendicularly. The elliptical parameters, the amplitude ratio, and the phase retardation of P- and S-polarized optical heterodyne signals are measured with a lock-in amplifier.[4] A TN-LC can also be characterized in view of the elliptical polarization state directly by continuous rotation of the TN-LC. Thus we can treat the TN-LC as identical to an elliptical phase retarder for which the parameters Φ and Γ of the TN-LC can be measured properly in terms of phase retardation of elliptical eigenpolarizations. Finally the identity between the TN-LC and the elliptical phase retarder is verified experimentally, in which the equivalent optical axis of the TN-LC is proved to align along the central line between the directions of rubbing in and rubbing out by this novel polarized interferometric ellipsometer.

2. Working Principles

The optical setup of the polarized optical heterodyne interferometric ellipsometer is shown in Fig. 1, in which a frequency-stabilized linearly polarized laser source is introduced. There are two acousto-optic modulators (AOMs) adopted in this interferometer: AOM_1, driven at frequency ω_1, is in the reference channel, and AOM_2, driven at frequency ω_2, is in the signal channel simultaneously. A P-polarized heterodyne signal ($P_1 + P_2$) in which the P_1 wave from the reference channel and the P_2 wave from the signal channel are heterodyned with a beat frequency $\Delta\omega = \omega_1 - \omega_2$ generated by photodetector D_P. At the same time, the S-polarized heterodyne signal ($S_1 + S_2$) is

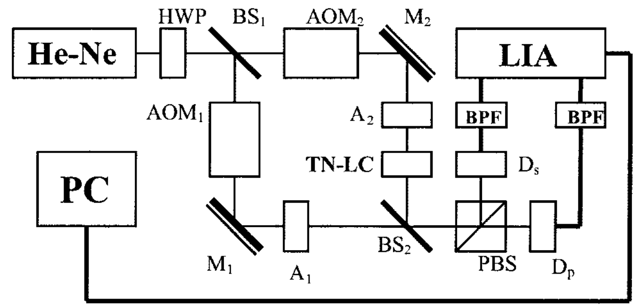

Fig. 1. Experimental setup: HWP, half-wave plate; BS, beam splitter; AOM, acousto-optic modulator; M, mirror; A, analyzer; PBS, polarization beam splitter; D, detector; BPF, bandpass filter; LIA, lock-in amplifier; PC, personal computer.

produced at photodetector D_S independently. They are

$$I_p(\Delta\omega t) = \left|E_{p_1} + E_{p_2}\right|^2 = I_{p_1} + I_{p_2} + 2\sqrt{I_{p_1} I_{p_2}} \cos(\Delta\omega t + \delta_p), \quad (1)$$

$$I_s(\Delta\omega t) = \left|E_{s_1} + E_{s_2}\right|^2 = I_{s_1} + I_{s_2} + 2\sqrt{I_{s_1} I_{s_2}} \cos(\Delta\omega t + \delta_s), \quad (2)$$

where δ_p and δ_s are defined as $\delta_{p_1} - \delta_{p_2}$ and $\delta_{s_1} - \delta_{s_2}$, respectively. I_{p_1}, I_{p_2}, I_{s_1}, and I_{s_2} are the intensities of P_1, P_2, S_1, and S_2, respectively, whereas δ_{p_1}, δ_{p_2}, δ_{s_1}, and δ_{s_2} are the phases of P_1, P_2, S_1, and S_2, respectively. By definition,[4] a Jones vector of a polarization state is defined by $[E_p \exp(i\delta_p), E_s \exp(i\delta_s)]^T$. To characterize the polarization state of a light wave by the elliptical parameters, the ratio of the amplitude $|X| = E_s/E_p \simeq (I_{s_2}/I_{p_2})^{1/2}$ and the phase difference $\delta = \delta_s - \delta_p \simeq \delta_{s_2} - \delta_{p_2}$ of P_2 and S_2 polarization states from X can be given as

$$X = \frac{E_s}{E_p} \exp[i(\delta_s - \delta_p)] = |X| \exp(i\delta) \quad (3)$$

according to the proper definition of ellipsometry.[2] Thus the input $X^{(i)}$ and the output $X^{(o)}$ of the TN-LC are expressed[2] by

$$X^{(o)} = \frac{T_{22} X^{(i)} + T_{21}}{T_{12} X^{(i)} + T_{11}}, \quad (4)$$

where the transfer matrix is

$$T_\varepsilon = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos\frac{\gamma}{2} + i\cos 2\varepsilon \sin\frac{\gamma}{2} \cos 2\left(\theta + \frac{\pi}{2}\right) & \sin\frac{\gamma}{2} \sin 2\varepsilon + i\cos 2\varepsilon \sin\frac{\gamma}{2} \sin 2\left(\theta + \frac{\pi}{2}\right) \\ -\sin\frac{\gamma}{2} \sin 2\varepsilon + i\cos 2\varepsilon \sin\frac{\gamma}{2} \sin 2\left(\theta + \frac{\pi}{2}\right) & \cos\frac{\gamma}{2} - i\cos 2\varepsilon \sin\frac{\gamma}{2} \cos 2\left(\theta + \frac{\pi}{2}\right) \end{bmatrix}, \quad (5)$$

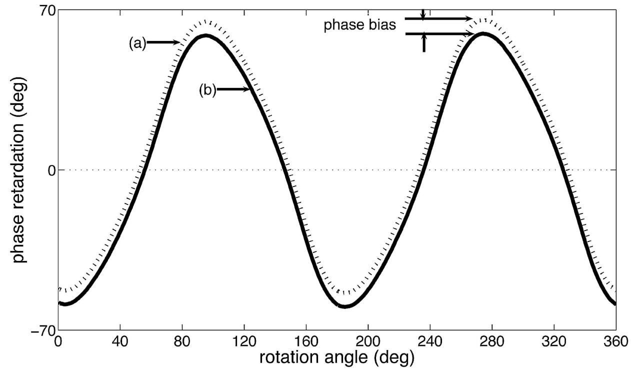

Fig. 2. Experimental data of phase retardation versus rotation angle of TN-LC by this setup: (a) without phase-bias correction, (b) with phase-bias correction.

where ε is the elliptical angle and θ is the azimuth angle of the elliptical phase retarder; $\gamma = 2\pi(n_f - n_s)d/\lambda$ is the phase retardation between the fast and the slow elliptical eigenpolarizations; n_f and n_s are the refractive indices of these eigenpolarizations, respectively; d is the thickness of the TN-LC; and λ is the wavelength of laser beam. The refractive indices for which $n_e > n_o$ is always true for a TN-LC implies that the optical axis of a TN-LC is along the direction of the slow axis.[2,6] To calibrate this ellipsometer, the tested TN-LC is not inserted into the interferometer. Two analyzers, A_1 and A_2, are introduced in the reference and signal channels, respectively. Both azimuth angles of A_1 and A_2 are adjusted near 45° to the x axis to satisfy the condition of $|X^{(i)}| = 1$ at this stage. Therefore a calibration process is necessary for ensuring a zero phase difference $[\delta^{(i)} = 0]$ at the same time for the measurement. A dc phase bias can be obtained by averaging of the measured phase difference versus the rotated angle β of the TN-LC within the range of $0° < \beta < 360°$ as shown in Fig. 2. Then $X^{(i)} = 1$ of the initial condition of this polarized interferometric ellipsometer is satisfied. Thus the polarization state of output $X^{(o)}$ emerging from the TN-LC becomes

$$X^{(o)} = \frac{T_{22} + T_{21}}{T_{12} + T_{11}}. \tag{6}$$

According to Eq. (5) and the relation of

$$X^{(o)} = \left|X^{(o)}\right| \exp\left[i\delta^{(o)}\right], \tag{7}$$

$$\delta_\varepsilon^{(0)} = \tan^{-1}\Bigg(\Bigg\{2 \sin\frac{\gamma}{2} \cos 2\varepsilon\Bigg[\sin\frac{\gamma}{2} \sin 2\varepsilon \times \sin 2\left(\theta + \frac{\pi}{2}\right) - \cos\frac{\gamma}{2} \cos 2\left(\theta + \frac{\pi}{2}\right)\Bigg]\Bigg\} \Bigg/ \Bigg[\cos^2\frac{\gamma}{2} - \sin^2\frac{\gamma}{2} \sin^2 2\varepsilon - \frac{1}{2}\sin^2\frac{\gamma}{2}\cos^2 2\varepsilon \times \cos 4\left(\theta + \frac{\pi}{2}\right)\Bigg]\Bigg). \tag{8}$$

We can then precisely calculate ε and γ in terms of θ and $\delta_\varepsilon^{(o)}$ by constantly rotating the azimuth angle θ of the TN-LC and fitting the parameters of ε and γ with the measured data $[\beta,\ \delta^{(o)}(\beta)]$. Generally the rotation of the TN-LC in this experiment produces polarization modulation in this ellipsometer. It shows the same feature as that of the optical photometric method that makes possible phase detection in real time. According to the theory of TN-LC,[6] the transfer matrix T_{LC} of the TN-LC is expressed by

$$T_{LC} = \begin{bmatrix} T_{11}' & T_{12}' \\ T_{21}' & T_{22}' \end{bmatrix} = \begin{bmatrix} \cos\chi\cos\Phi + \frac{\Phi}{\chi}\sin\chi\sin\Phi - i\frac{\Gamma/2}{\chi}\sin\chi\cos(2\alpha+\Phi) & -\cos\chi\sin\Phi + \frac{\Phi}{\chi}\sin\chi\cos\Phi - i\frac{\Gamma/2}{\chi}\sin\chi\sin(2\alpha+\Phi) \\ \cos\chi\sin\Phi - \frac{\Phi}{\chi}\sin\chi\cos\Phi - i\frac{\Gamma/2}{\chi}\sin\chi\sin(2\alpha+\Phi) & \cos\chi\cos\Phi + \frac{\Phi}{\chi}\sin\chi\sin\Phi + i\frac{\Gamma/2}{\chi}\sin\chi\cos(2\alpha+\Phi) \end{bmatrix}. \tag{9}$$

Then the phase retardation becomes $\delta_{LC}^{(o)}$, as expressed by

$$\delta_{LC}^{(o)} = \tan^{-1}\Bigg(\Bigg\{2\left(\frac{\Gamma/2}{\chi}\right)\sin\chi\Bigg[\cos\chi\cos 2\alpha - \left(\frac{\Phi}{\chi}\right) \times \sin\chi\sin 2\alpha\Bigg]\Bigg\} \Bigg/ \Bigg\{\cos 2\Phi\Bigg[\cos^2\chi - \left(\frac{\Phi}{\chi}\right)^2 \times \sin^2\chi\Bigg] + \left(\frac{\Phi}{\chi}\right)\sin 2\chi\sin 2\Phi - \left(\frac{\Gamma/2}{\chi}\right)^2 \times \sin^2\chi\cos(4\alpha + 2\Phi)\Bigg\}\Bigg), \tag{10}$$

where α is the angle of orientation of the LC molecules of the first layer (direction of rubbing in or director angle) relative to the x axis, Φ is the twist angle, and $\Gamma = 2\pi(n_e - n_o)d/\lambda$ indicates the phase retardation of the TN-LC at $\Phi = 0$. The refractive indices n_e and n_o are the extraordinary and ordinary linear eigenpolarizations of the TN-LC, respectively.

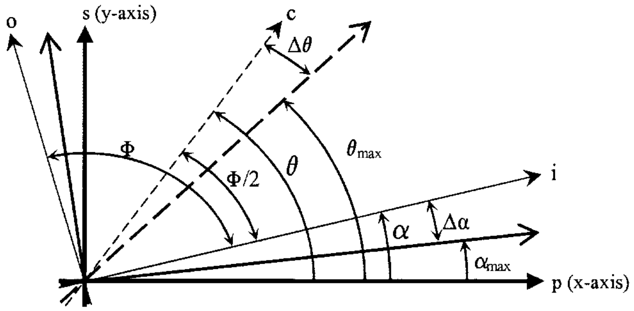

Fig. 3. Schematic diagram of azimuth angle θ and rubbing angle α of the TN-LC: c indicates the optical axis, i is the direction of rubbing in of the TN-LC, and o is the direction of rubbing out of the TN-LC.

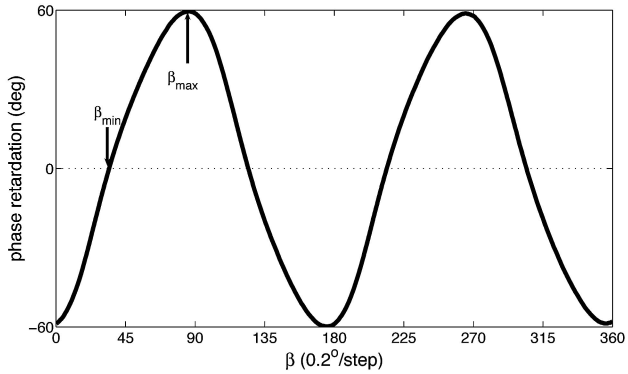

Fig. 4. Measured phase retardation versus mechanical rotation angle β of the TN-LC.

Therefore,

$$\chi = \sqrt{\Phi^2 + \left(\frac{\Gamma}{2}\right)^2}, \tag{11}$$

which is a parameter for characterizing the TN-LC instead of using the original transfer matrix T_{LC}. Lin[5] has shown that the equivalent optical axis (or slow axis) of the TN-LC is aligned in the direction of the central line between the directions of rubbing in and rubbing out as shown in Fig. 3 theoretically. Thus the phase retardation $\delta_\varepsilon^{(o)}$ as the elliptical phase retarder of Eq. (8) related to the phase retardation as the TN-LC of Eq. (10) can be equivalent to

$$\delta_\varepsilon^{(o)}(\gamma,\ \varepsilon,\ \theta) = \delta_{LC}^{(o)}(\Gamma,\ \Phi,\ \alpha) \tag{12}$$

$$\alpha = \theta - \frac{\Phi}{2}. \tag{13}$$

Therefore, if ε and γ of the TN-LC are given, then Φ and Γ can be determined by use of the best-fitting algorithm of two curves of $\delta_\varepsilon^{(o)}$ and $\delta_{LC}^{(o)}$ under the condition of Eq. (13).

3. Experimental Setup and Results

Figure 1 is the optical setup of this phase-sensitive polarized interferometric ellipsometer, in which a 1-mW frequency-stablized linearly polarized He–Ne laser was used. The wavelength was 632.8 nm. The driving frequency of AOM_1 and AOM_2 were $\omega_1 = 80.0000$ MHz and $\omega_2 = 80.0329$, respectively, which resulted in a beat frequency of $\Delta\omega = 32.9$ kHz in this arrangement. During the measurement, the TN-LC was rotated constantly (each rotation angle was $\Delta\beta = 0.2°$) by a digitally controlled rotation stage. The phase retardations $\delta^{(o)}$ of *P*- and *S*-polarized optical heterodyne signals were measured simultaneously. Figure 2 shows the experimental results of phase retardation $\delta^{(o)}(\beta)$ versus rotation angle β to the *x* coordinate, of which a full range of $0° < \beta < 360°$ was scanned. Figure 2(a) shows the measured data for which the phase bias is not corrected. In contrast, Fig. 2(b) calibrates the data of the phase retardation in which phase bias is corrected numerically. In this experiment, the phase bias can be obtained by proper averaging of the measured data from Fig. 2(a), and the result is shown in Fig. 4. To fit $\delta^{(o)}(\beta)$ with the theory of $\delta_\varepsilon^{(o)}(\theta)$, based on a TN-LC treated as an elliptical phase retarder (see Fig. 5), with $\Delta\theta = 0.2°$ per step, $\delta^{(o)}(\beta) \equiv \delta_\varepsilon^{(o)}(\varepsilon, \gamma, \theta)$ is matched numerically by the MATLAB software for this measurement. A bias between β and θ was found numerically according to the following constraints: $\beta_{max} - \beta_{min} = \theta_{max} - \theta_{min} = 51.30°$, $\delta^{(o)}(\beta)_{max} = \delta_\varepsilon^{(o)}(\varepsilon, \gamma, \theta)_{max} = 59.238°$, and $\delta^{(o)}(\beta)_{min} = \delta_\varepsilon^{(o)}(\varepsilon, \gamma, \theta)_{min} = 0°$ in the fitting process. As a result, $\varepsilon = -31.741°$ and $\gamma = 142.810°$ of the elliptical phase retarder were found numerically. Similarly, the measured phase retardation $\delta^{(o)}(\beta)$ matches $\delta_{LC}^{(o)}(\Phi, \Gamma, \alpha)$ in view of the LC and then follows the same procedures as described in the elliptical phase retarder previously. Then $\Phi = 89.804°$, $\Gamma = 235.337°$, and $\chi = 240.533°$ were found. Thus the theoretical response of $\delta_\varepsilon^{(o)}(\varepsilon, \gamma, \theta)$ versus θ based on obtained (ε, γ) is shown in Fig. 5. Likewise the theoretical response of $\delta_{LC}^{(o)}(\Phi,\ \Gamma,\ \alpha)$ versus α is shown in Fig. 6 accordingly when $(\Phi,\ \Gamma)$ is given. Therefore, to match the curves in Figs. 4–6 simultaneously such that $\delta^{(o)} = \delta_\varepsilon^{(o)} = \delta_{LC}^{(o)}$ is satisfied, the angles of the coordinates of

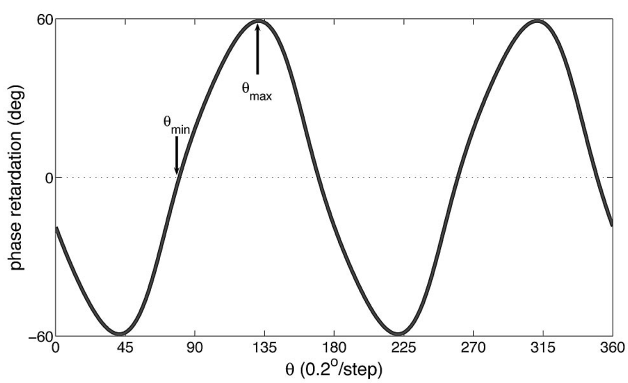

Fig. 5. Theoretical calculation of phase retardation versus azimuth angle θ of the TN-LC.

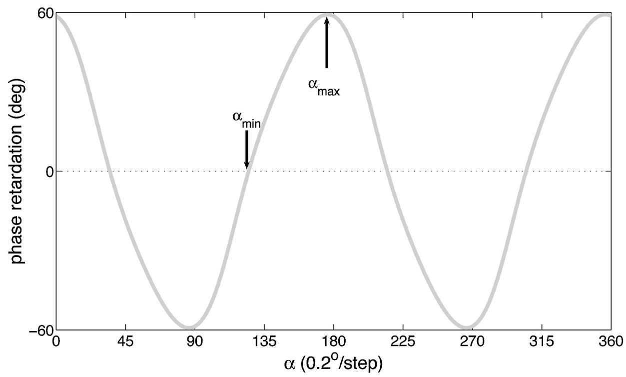

Fig. 6. Theoretical calculation of phase retardation versus rubbing angle α of the TN-LC.

$\theta_s = 44.6°$ in Fig. 5 and $\alpha_s = 89.5°$ in Fig. 6 are shifted necessarily, which makes $\Phi = 2(\theta_s - \alpha_s) = -89.8°$ in Fig. 3. This is a good agreement with the result of $\Phi = -89.804°$ when these three curves are matched simultaneously in Fig. 7.

To correlate the experimental result (Fig. 4) with the theoretical predictions based on an elliptical phase retarder (Fig. 5) and a TN-LC (Fig. 6), the respective correlation coefficients between Figs. 4 and 5 is 0.999918 and between Figs. 4 and 6 is 0.999925. As a result, $\Phi = -89.804°$ and the parameter $\Delta nd = 413.67$ nm are obtained by this experiment. These results agree with the given data (estimated) of the tested TN-LC of $\Phi \cong -90°$ and $\Delta nd \cong 408$ nm in this measurement. From the results, this polarized interferometric ellipsometer enables us to characterize TN-LC very precisely both from the consideration of the TN-LC as an elliptical phase retarder and the LC experimentally. To our knowledge, this method provides the highest accuracy of characterizing a TN-LC device compared with the conventional method.[14–19] In addition, this method also shows the potential capability of monitoring the parameters (Φ, Γ) of a TN-LC in real time. It is essential to the performance of TN-LC in the temporal domain.

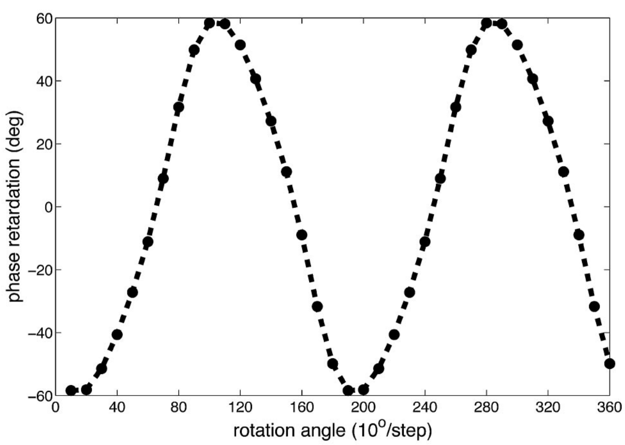

Fig. 7. Phase retardation versus rotation angle of the TN-LC under the conditions of (a) the mechanical rotation of the TN-LC as marker (●), (b) the theoretical calculation based on the elliptical phase retarder as marker (--), (c) the theoretical calculation based on a linear TN-LC coincided with (b).

4. Discussion and Conclusions

This research proposes a phase-sensitive polarized optical heterodyne interferometric ellipsometer in which the tested TN-LC cell is rotated continuously to introduce polarization modulation during the measurement. The ellipsometer combines the features of optical heterodyne interferometry with optical photometry, which makes it possible to present a higher sensitivity of phase detection based on synchronized detection. In the meantime, a capability of real-time measurement is conducted by means of continuous-phase detection. It is similar to the polarization modulation technique in an optical photometric method by use of a phase lock-in technique. Therefore, when the optical heterodyne interferometry and optical photometry are incorporated in this novel method, it is possible to improve not only the detection sensitivity but also the dynamic range of the phase measurement compared with the conventional photometric method. In addition, this polarized interferometric ellipsometer also shows a feature of common-path configuration of *P*- and *S*-polarized optical heterodyne signals: A common-phase rejection mode is able to reduce the background-induced phase noise and the laser frequency noise significantly. Then an enhancement of the SNR of the detected signal is apparently produced. As a result, for identifying a TN-LC as an elliptical phase retarder, a high accuracy of determining the TN-LC parameters is applicable by this method. This method is also independent of the absorption of the TN-LC device because of phase-sensitive detection in this setup.

Therefore the proposed polarized optical heterodyne interferometric ellipsometry enables us to characterize a TN-LC very precisely in terms of the phase retardation of *P*- and *S*-polarized light waves when the TN-LC is treated identically as an elliptical phase retarder and a LC. The advantages of this polarized interferometric ellipsometry can be summarized by (1) a common-path configuration of the polarized optical heterodyne signals of the interferometer, (2) a synchronized detection of the interferometer resulting in a high SNR and high sensitivity, (3) a common-phase noise-rejection mode that reduces the background phase noise and laser frequency noise, (4) the feature of optical photometry with polarization modulation that provides a real-time measurement of phase retardation, and (5) a phase-sensitive detection that is independent of the absorption of the TN-LC device.

In conclusion, the parameters of a TN-LC can be precisely determined by this polarized optical interferometric ellipsometer based on the fact that a TN-LC is identical to an elliptical phase retarder. In addition, the equivalent optical axis is verified experimentally along the direction of the central line between the directions of rubbing in and rubbing out of the TN-LC.

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References

1. C. Chou, Y. C. Huang, and M. Chang, "Polarized common path optical heterodyne interferometer for measuring the elliptical birefringence of a quartz wave plate," Jpn. J. Appl. Phys. **35,** 5526–5530 (1996).
2. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, 1979), p. 99.
3. H. K. Teng, C. Chou, C. N. Chang, C. W. Lyu, and Y. C. Huang, "Linear birefringence measurement with a differential-phase optical heterodyne polarimeter," Jpn. J. Appl. Phys. **41,** 3140–3144 (2002).
4. C. H. Lin, C. Chou, and K. S. Chang, "Real time interferometric ellipsometry with optical heterodyne and phase lock-in techniques," Appl. Opt. **29,** 5159–5162 (1990).
5. B. R. Lin, "TN-LC as an elliptical retarder," Master's thesis (Institute of Electro-Optical Engineering, National Chao-Tung University, 2004), p. 19.
6. P. Yeh and C. Gu, *Optics of Liquid Crystal Display* (Wiley, 1999), pp. 129–130.
7. Y. C. Huang, M. Chang, and C. Chou, "Effect of elliptical birefringence on the measurement of the phase retardation of a quartz wave plate by an optical heterodyne polarimeter," J. Opt. Soc. Am. A **14,** 1367–1372 (1997).
8. T. Nishiok and T. Kurata, "Novel pretilt angle measure method for twisted-nematic liquid-display cells by apparent retardation measurement," Jpn. J. Appl. Phys. **40,** 6017–6023 (2001).
9. J. A. Davis, I. Moreno, and P. Tsai, "Polarization eigenstates for twisted-nematic liquid-crstal display," Appl. Opt. **37,** 937–945 (1998).
10. I. Moreno, J. A. Davis, K. G. D'Nelly, and D. B. Allison, "Transmission and phase measurement for polarization eigenvectors in twisted-nematic liquid crystal spatial light modulators," Opt. Eng. **37,** 3048–3052 (1998).
11. X. Zhu, Q. Hong, Y. Huang, and S. T. Wu, "Eigenmodes of a reflective twisted-nematic liquid-crystal cell," J. Appl. Phys. **94,** 2868–2873 (2003).
12. I. Scierski and F. Ratajczyk, "The Jones matrix of the real dicroic elliptic object," Optik **68,** 121–125 (1984).
13. J. A. Davis, J. Nicolas, and A. Márquez, "Phasor analysis of eigenvectors generated in liquid-crystal displays," Appl. Opt. **22,** 4579–4584 (2002).
14. Y. Zhou, Z. He, and S. Sato, "A novel method determining the cell thickness and twist angle of a twisted nematic cell by Stokes parameter measurement," Jpn. J. Appl. Phys. **36,** 2760–2764 (1997).
15. J. A. Davis, D. B. Allison, K. G. D'Nelly, M. L. Wilson, and I. Moreno, "Ambiguities in measuring the physical parameters for twisted-nematic liquid-crystal spatial light modulators," Opt. Eng. **38,** 705–709 (1999).
16. H. Kim and Y. H. Lee, "Unique measurement of the parameters of a twisted-nematic liquid-crystal display," Appl. Opt. **44,** 1642–1649 (2005).
17. N. Konforti, E. Marom, and S. T. Wu, "Phase-only modulation with twisted nematic liquid-crystal spatial light modulators," Opt. Let. **13,** 251–253 (1998).
18. S. Valyukh, A. Slobodyanyuk, and V. Sorokin, "Simulation of obliquely incident light propagation through a general twisted nematic liquid crystal cell by the Jones matrix technique," Semicond. Phys., Quantum Electron. Optoelectron. **3,** 258–263 (2000).
19. R. Giust and J. P. Goedgebuer, "Determination of the twist angle and the retardation properties of twisted nematic liquid crystal television by spectral measurements," Opt. Eng. **37,** 629–634 (1998).